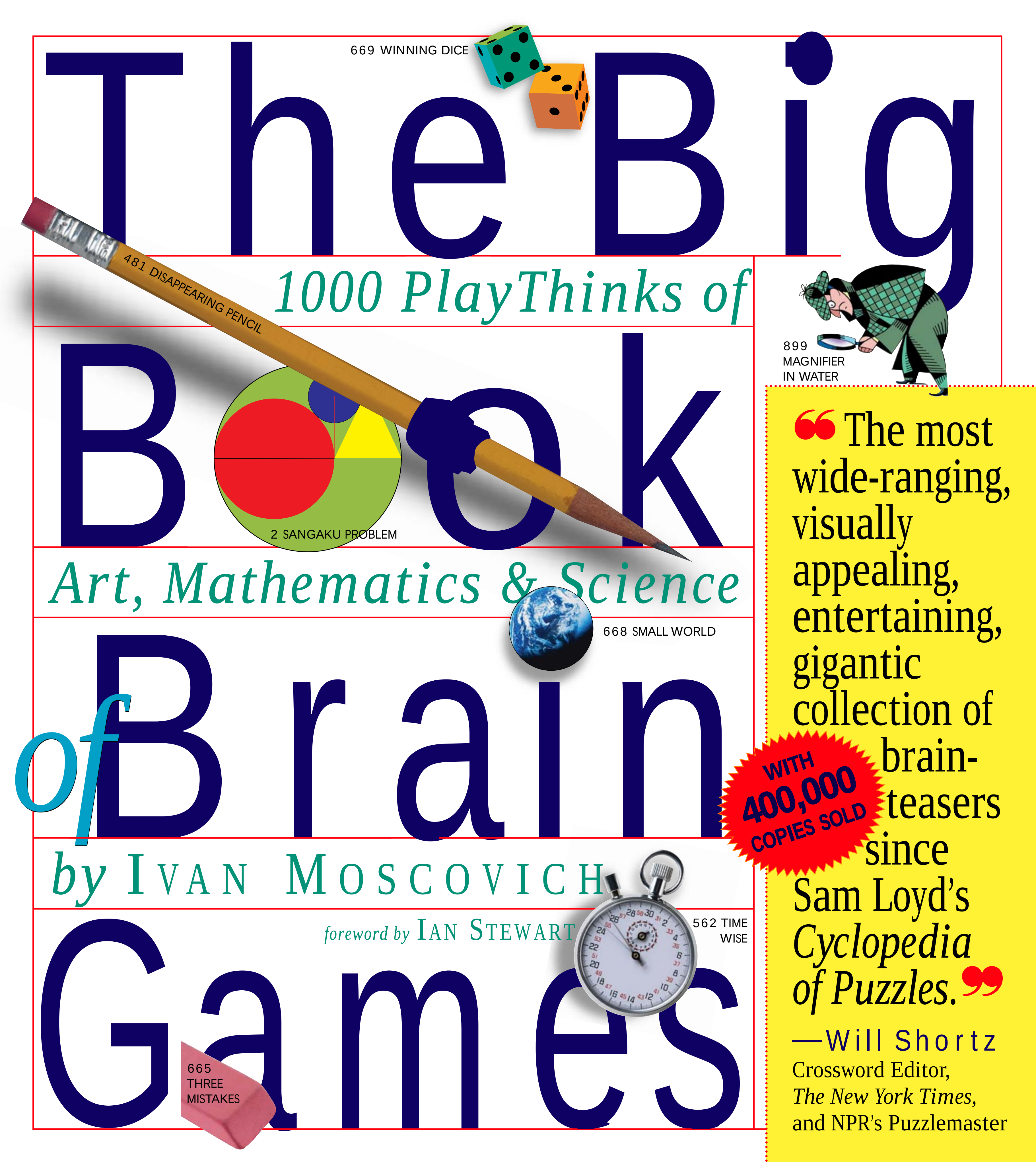
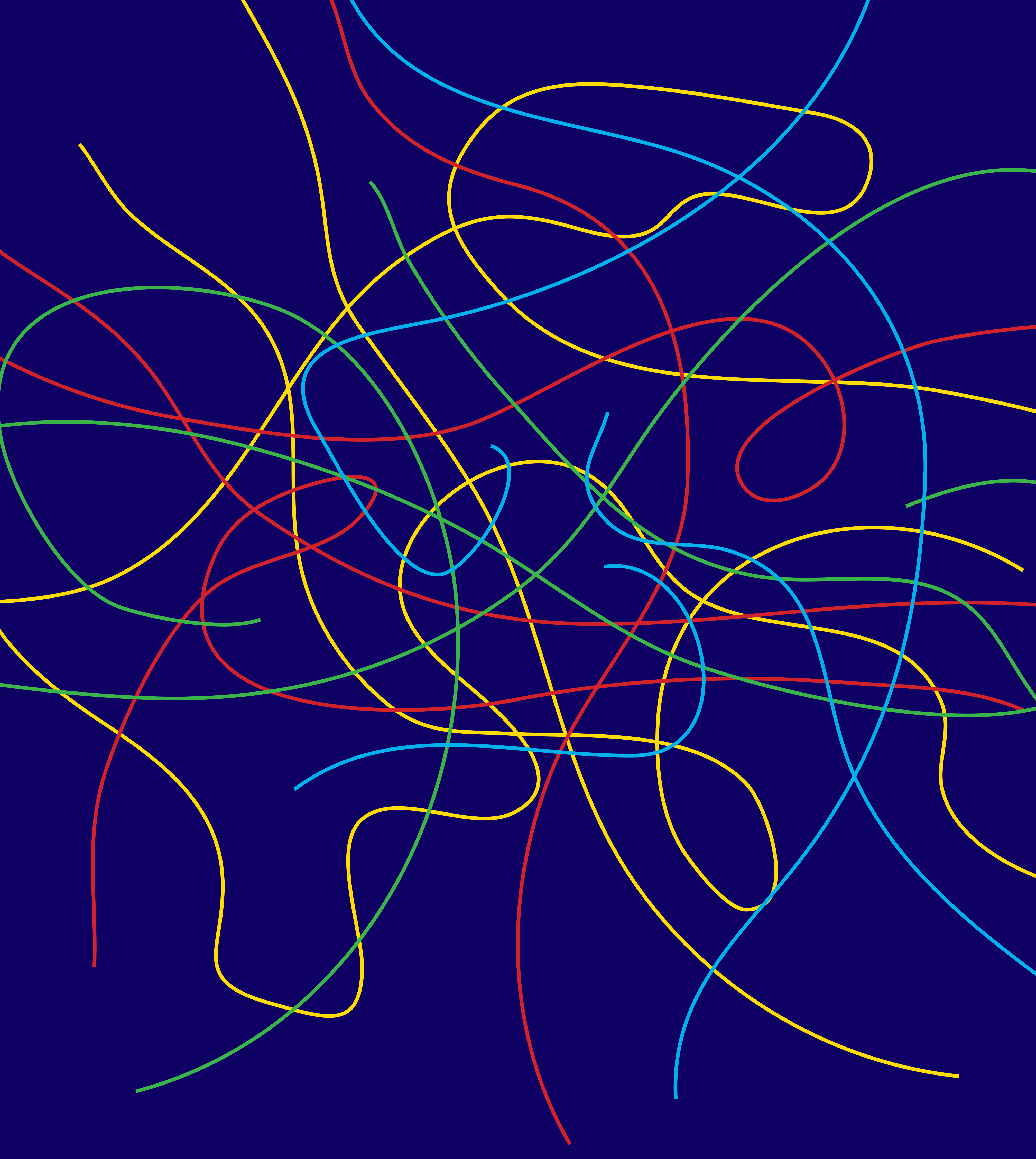
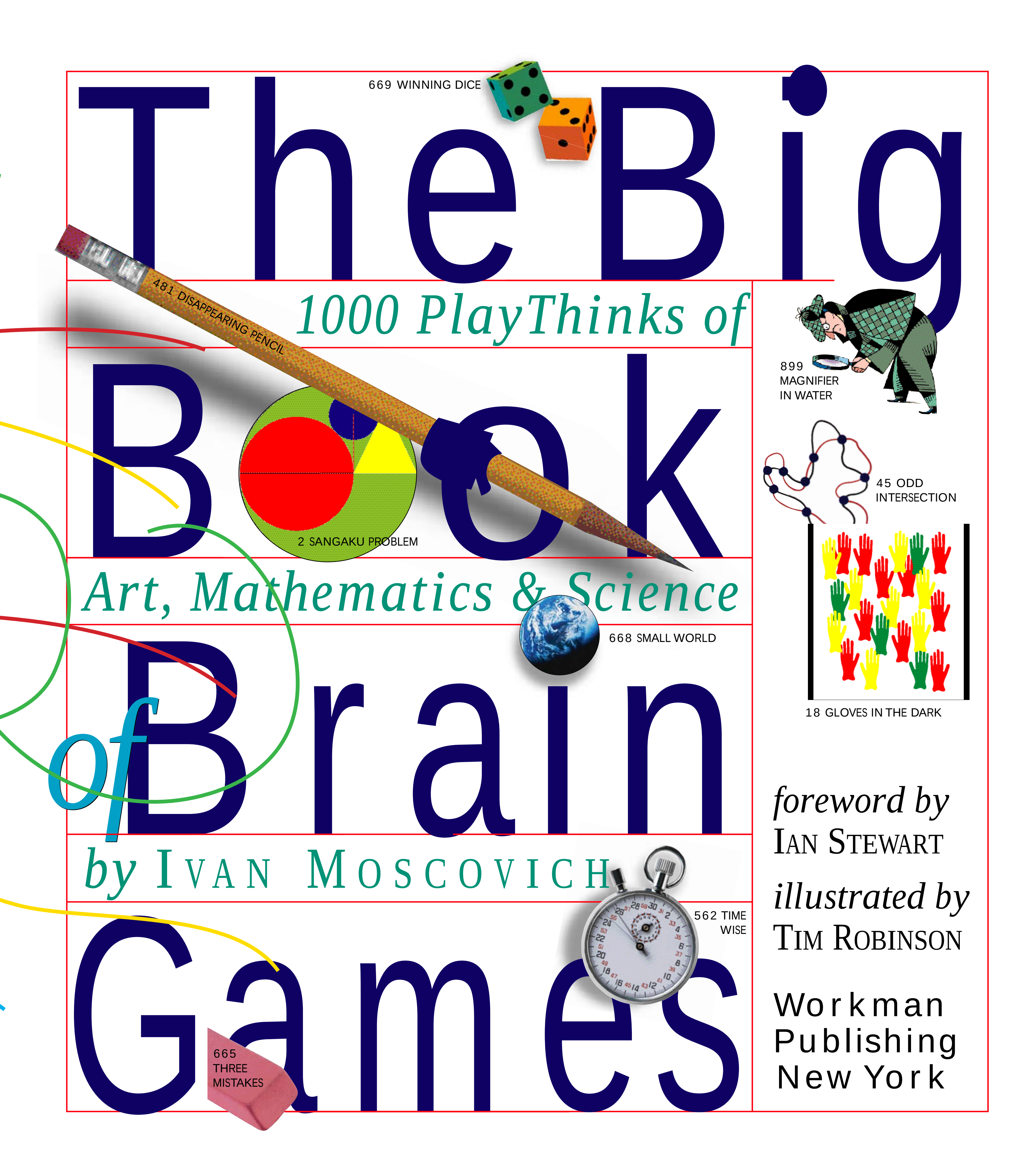

The Big
1000 PlayThinks of
Book
Art, Mathematics & Science
of Brain
by Ivan Moscovich
foreword by Ian Stewart
Games
669 WINNING DICE
481 DISAPPEARING PENCIL
2 SANGAKU PROBLEM
899 MAGNIFIER IN WATER
668 SMALL WORLD
562 TIME WISE
665 THREE MISTAKES
WITH 400,000 COPIES SOLD
"The most wide-ranging, visually appealing, entertaining, gigantic collection of brain-teasers since Sam Loyd's Cyclopedia of Puzzles."
—Will Shortz
Crossword Editor, The New York Times, and NPR's Puzzlemaster

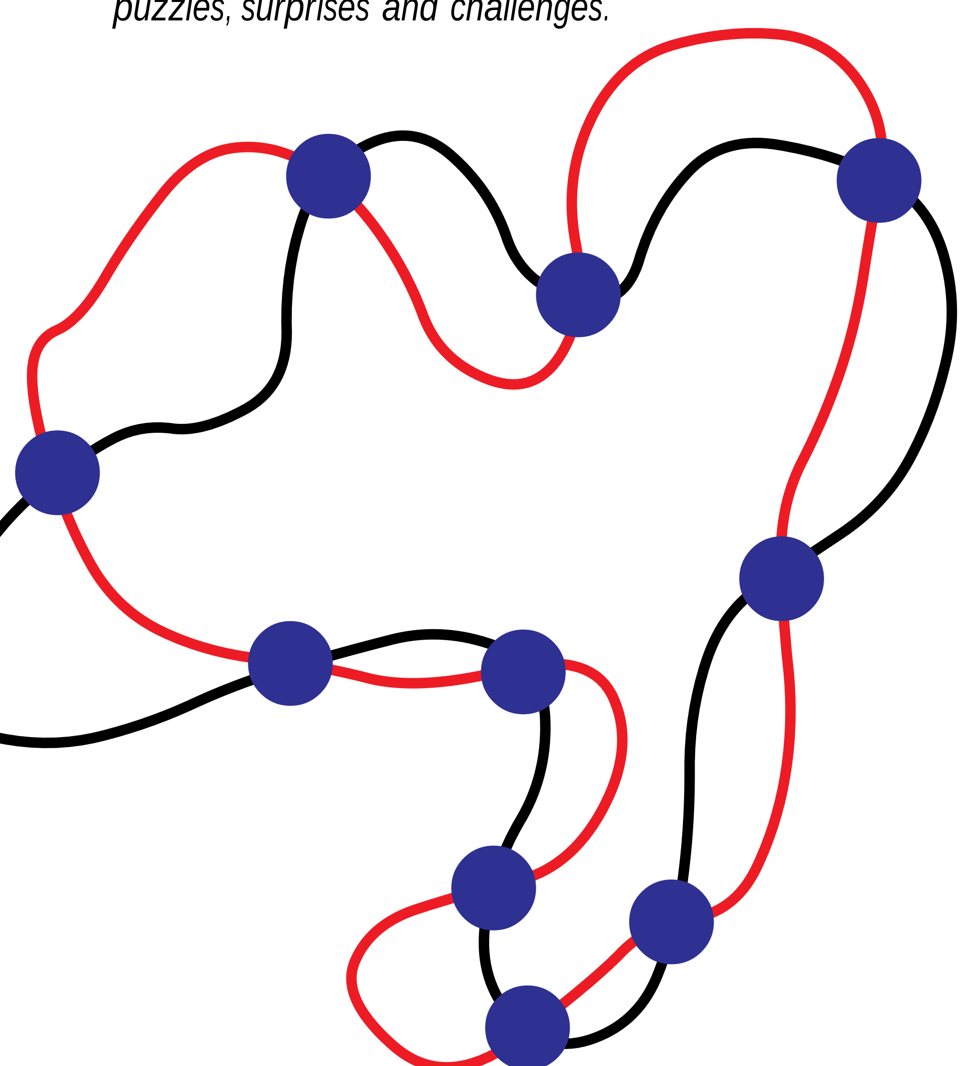
The Big Book
of
Brain Games





The Big
1000 PlayThinks of
Book
Art, Mathematics & Science
of Brain
by Ivan Moscovich
Games
foreword by
Ian Stewart
illustrated by
Tim Robinson
Workman Publishing
New York
669 WINNING DICE
481 DISAPPEARING PENCIL
899 MAGNIFIER IN WATER
2 SANGAKU PROBLEM
45 ODD INTERSECTION
668 SMALL WORLD
18 GLOVES IN THE DARK
562 TIME WISE
665 THREE MISTAKES

This book is a labor of love.
I dedicate it to my wife, Anitta,
with love and gratitude for
her infinite patience, valuable
judgment and assistance;
to my daughter, Hila, who is
my harshest but fairest critic
and continually inspires me
with new insights and ideas;
and to all those who like games,
puzzles, surprises and challenges.



Originally published in October 2001 as *1000 PlayThinks: Puzzles, Paradoxes, Illusions & Games*

Library of Congress Cataloging-in-Publication Data
is available.
ISBN-13: 978-0-7611-3466-4
ISBN-10: 0-7611-3466-2

Workman books are available at special discounts when purchased in bulk for premiums and sales promotions as well as for fund-raising or educational use. Special editions can also be created to specification. For details, contact the Special Sales Director at the address below.

Typesetting by Barbara Peragine

Workman Publishing Company, Inc.
708 Broadway
New York, NY 10003-9555
www.workman.com

First Printing April 2006

10 9 8 7 6 5 4 3 2 1

PlayThink 88, "Lost in Caves," from "The Road Coloring Problem," by Daniel Ullman. Reprinted by permission from The Mathematical Association of America, *The Lighter Side of Mathematics,* edited by Richard K. Guy and Robert E. Woodrow, page 105.

PlayThink 342, "Sharing Cakes"; PlayThink 161, "Multi-Distance Set"; PlayThink 339, "Japanese Temple Problem from 1844," from *Which Way Did the Bicycle Go?,* by Joseph D.E. Konhauser, Dan Velleman and Stan Wagon. Reprinted by permission from The Mathematical Association of America, *Which Way Did the Bicycle Go?,* pages 62, 68 and 107.

Thanks to Greg Frederickson, for permission to use several of his polygon transformation dissections, "Heptagon Magic," "Pentagonal Star," "Nonagon Magic" and "Twelve-Pointed Star" (PlayThinks 42, 479, 478 and 483); to Richard Hess, for the idea behind "Measuring Globe" (PlayThink 810); to Ian Stewart, for the illustration for "Goats and Peg-Boards" (PlayThink 309); and to the late Mel Stover, for his geometrical vanishing illusion, "Disappearing Pencil" (PlayThink 481).

Photo credits: PlayThink 585, "Jekyll and Hyde" (page 213), courtesy of Photofest; PlayThink 907 "Archimedes's Mirrors" (page 310), courtesy of the New York Public Library Picture Collection.

ACKNOWLEDGMENTS

First and foremost, I would like to thank Martin Gardner for Everything. His work, personality and friendship have been my inspiration since the mid-fifties, when I read his first "Mathematical Games" column in the first issue of *Scientific American.* His immense contribution to the popularization of recreational mathematics (and mathematics in general) has created an environment of creativity. Without him, there would have been many fewer International Puzzle Parties and mathematical exhibitions, and certainly no Gatherings for Gardner, an event like no other.

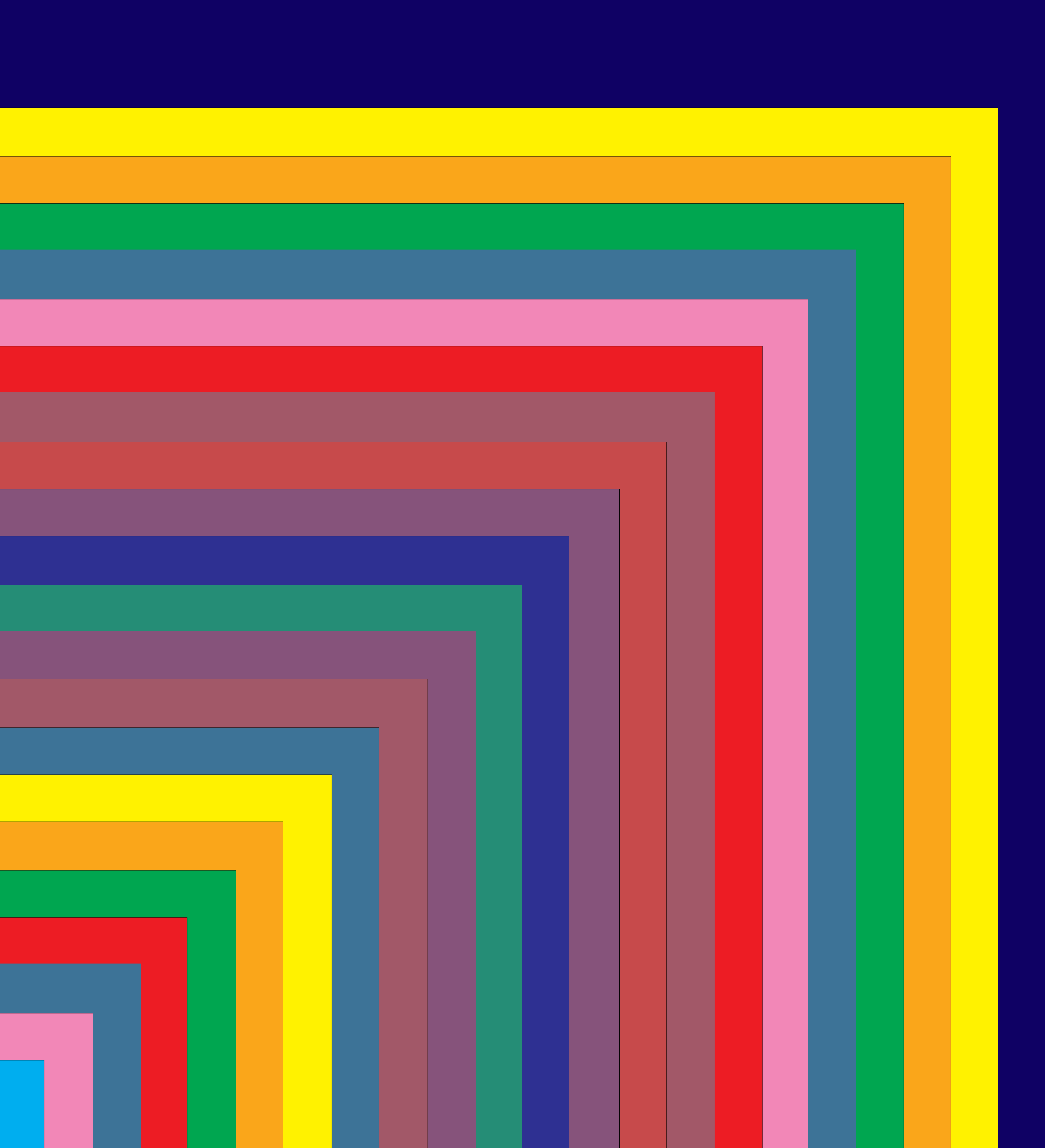
Over the last forty years or so, these conventions of like-minded souls have allowed me to meet "Martin's People," a diverse group of mathematicians, scientists, puzzle collectors, magicians and inventors unified by a fascination with mind games and a love of recreational mathematics. They have provided me with endless hours of enjoyment and intellectual enrichment and, very often, precious friendship. My appreciation and thanks to all of them, mentioning just a few: Paul Erdös, my famous relative, who provided the first sparks; David Singmaster, with whom I dreamed of a very special puzzle museum; Ian Stewart for his early help; John Horton Conway, Solomon Golomb, Frank Harary, Raymond Smullyan, Edward de Bono, Richard Gregory, Victor Serebriakoff, Nick Baxter; Greg Frederickson, for his beautiful dissections; Al Seckel, Jacques Haubrich, Lee Sallows, Jerry Slocum, Nob Yoshigahara, James Dalgety, Mel Stover, Mark Setteducati, Bob Neale, Tim Rowett, Scott Morris, Will Shortz, Bill Ritchie, Richard Hess and many, many others.

I owe a debt of gratitude to the work of pioneers—Sam Loyd, Henry Dudeney, many others—whose early books provided so much inspiration. In a way, *The Big Book of Brain Games* is a visual synthesis of the whole of recreational mathematics.

Finally, thanks to Peter Workman, for his enthusiastic, ego-boosting reaction to the first crude color dummy of *PlayThinks,* which I so timidly presented to him; to Sally Kovalchick, who got things started, and to Susan Bolotin, who finished them; to Nick Baxter and Jeffrey Winters for their help with math, science and language; and to others at Workman, all so professional, including (but certainly not limited to) Paul Hanson, Elizabeth Johnsboen, Malcolm Felder, Patrick Borelli, Janet Parker, Eric Ford, Mike Murphy, Barbara Peragine, Anne Cherry and Kelli Bagley.

I. M.

CONTENTS



FOREWORD

I wrote the "Mathematical Recreations" column of *Scientific American* for ten years, and it was in that capacity, as a gamesman (and not as the mathematician I am), that I first encountered Ivan Moscovich. It was 1984, and I was helping to write the text for his book *Ivan Moscovich's Super-Games*. I was immediately struck by his trademarks: cheerful, attractive graphics, and puzzles that are genuinely fun to work on and—with luck and hard work—to solve.

Puzzles, like many things in the realm of the intellect, are deceptively simple. They belong, so it seems, to a fantasy world full of shapes made from matchsticks, weird tiles meant to be arranged in ridiculous ways and odd numerical curiosities. Real life, we say, is not like that. The problems we encounter in our daily lives are more subtle, less clearly defined, less artificial.

Nonsense.

I don't mean that real-life problems aren't subtle; I don't mean that when we run into them they come to us with a logical plan. And I don't mean they're artificial—at least, not any more artificial than the peculiar world humanity has built for itself and fondly imagines is the natural order of things. No, what I mean is this: even simple puzzles are more subtle, less clearly defined and less artificial than they appear.

Lurking within every good puzzle is a general message about how to think when you are confronted with a problem. Even if the puzzle itself is posed in a simplified world, the way that you have to think to solve it is often useful in more significant areas of human activity. It's great. You can enjoy yourself building fences to separate four cats who live on a square grid (even though no self-respecting cat would sit still while you fenced it in) and at the same time refine your understanding of "area." You can roll dice and brush up on statistics. Or you can amuse yourself with a few coins and discover the deep mathematics of "even and odd."

Speaking of mathematics: If ever there was an area of human activity where apparently simple puzzles could open up the hidden depths of the universe, mathematics is it. For instance, one of the current frontiers of mathematical research is knot theory. On the surface, this is about how you decide whether a knot in one piece of string can be rearranged until it forms what looks like a different knot in another piece of string. Who could possibly use such a theory? Who would need it? Boy Scouts? Fishermen?

The answer is that a lot of things can be knotted—not just string. Knots are just the simplest examples in a vast area of mathematics with applications throughout science. Molecules of DNA are often knotted, and if you can recognize which knots arise in which circumstances, you can learn a great deal about their underlying biology and chemistry. There are knotlike objects in quantum mechanics, too, so an effective theory of knots can tell us about the fundamental nature of the universe.

Knot theory isn't confined to string any more than magnetic theory is confined to helping people find their way. Its simplicity is not a *restriction* on its applicability; rather, in mathematics, the simpler a concept is, the more fundamental it is likely to be. Think of numbers. They're simple, but we use them everywhere. And that's as it should be, since the simpler a tool is, the more uses it is likely to have.

The art of the mathematician is to derive far-reaching consequences from apparently simple material. And the people who best appreciate this started playing with puzzles as children. Puzzles help your mathematical imagination to develop; I *know* they helped mine. They help you learn to think in generalities, not just simpleminded specifics. They help you understand that by thinking about tangled lengths of string, you can make far-reaching discoveries in biology and physics.

This is why Ivan's new book, like the rest of his lifework, is so important. Because it shows you that puzzles are intimately involved in every aspect of life, art, science, culture. And because it makes mathematical thinking painless, interesting and fun.

IAN STEWART
Coventry, England

INTRODUCTION

I am a lover of games. Over the last forty years I have collected, designed and invented thousands upon thousands of them—hands-on interactive exhibits, puzzles, toys, books, you name it. One of the reasons I'm so passionate about games is that I believe they can change the way people think. They can make us more inventive, more creative, more artistic. They can allow us to see the world in new ways. They can inspire us to tackle the unknowable. They can remind us to have fun.

That's why I wrote this book.

Like so many who lived through the twentieth century, I have witnessed repeated attempts to snuff out humanity's creative spark—and not just by political tyrants. I have seen the creative impulse wither away in schools. I have seen it devalued at work. And along the way I have learned that to become fully free, our society must do more than repel dictators. We must encourage what is best—and what is most human—within ourselves.

I believe that one of the most effective ways to foster that special part in each of us is through play. Child psychologists have long known that children learn about the world through games; now it is time to extend that model to adults. We can understand the most abstract and difficult concepts if we allow ourselves the luxury of approaching them not as work, but as fun—and a form of exploration.

People have always felt the pull to explore new worlds, and now that most of the physical frontiers have been crossed, the mental ones should beckon us. Too often, though, we act as if challenges to the mind are too difficult to contemplate. We judge the effort needed to push into new mental territories as simply too great. And so we turn back.

It is at the place where self-doubt and fear threaten to derail our urge to explore that play becomes a truly important activity. Seeing hard work as fun is what keeps the amateur athlete training for the marathon, and it is what keeps a child or an adult struggling to find the answer to a puzzle. At the end of the race, the runner dwells in a place of pride. At the end of the game, the puzzle solver feels smart, successful and at one with the beauty of mathematics.

Shortly after I emigrated to Israel in 1952, I began planning one of the first science museums in which the exhibits invited the visitor to participate. That interactive concept became the model for many later museums, including the world-famous Exploratorium in San Francisco. At these museums, children and adults alike feel their minds wake up: they suddenly grasp concepts previously rejected as "too difficult" or "impossible to understand." Doing the "problem" is fun, and so they understand it.

The activities in this book, which combine entertainment and brain teasing, expand on that idea and apply it to concepts common to art, science and mathematics. Because they transcend puzzles and games in the traditional sense, I have given them a new name: PlayThinks. A PlayThink may be a visual challenge, riddle or puzzle; it may be a toy, game or illusion; it may be an art object, a conversation piece or a three-dimensional structure. Some of the puzzles are completely original, while others are novel adaptations of classic and modern challenges. Whatever its form, a PlayThink will ideally transfer you to a state of mind where pure play and problem solving coexist.

Because playing and experimenting with PlayThinks stimulate creative thinking, you may find the book slyly educational. I certainly hope so! My goal is for you to play the games, solve the problems and come away more curious, more inventive, more intuitive. Enjoy!

Ivan Moscovich
Nijmegen, the Netherlands

HOW TO USE THIS BOOK

In my experience a single presentation of a mathematical idea generally fails to produce a lasting impression. On the other hand, interactive games and puzzles can make even the most advanced concepts understandable.

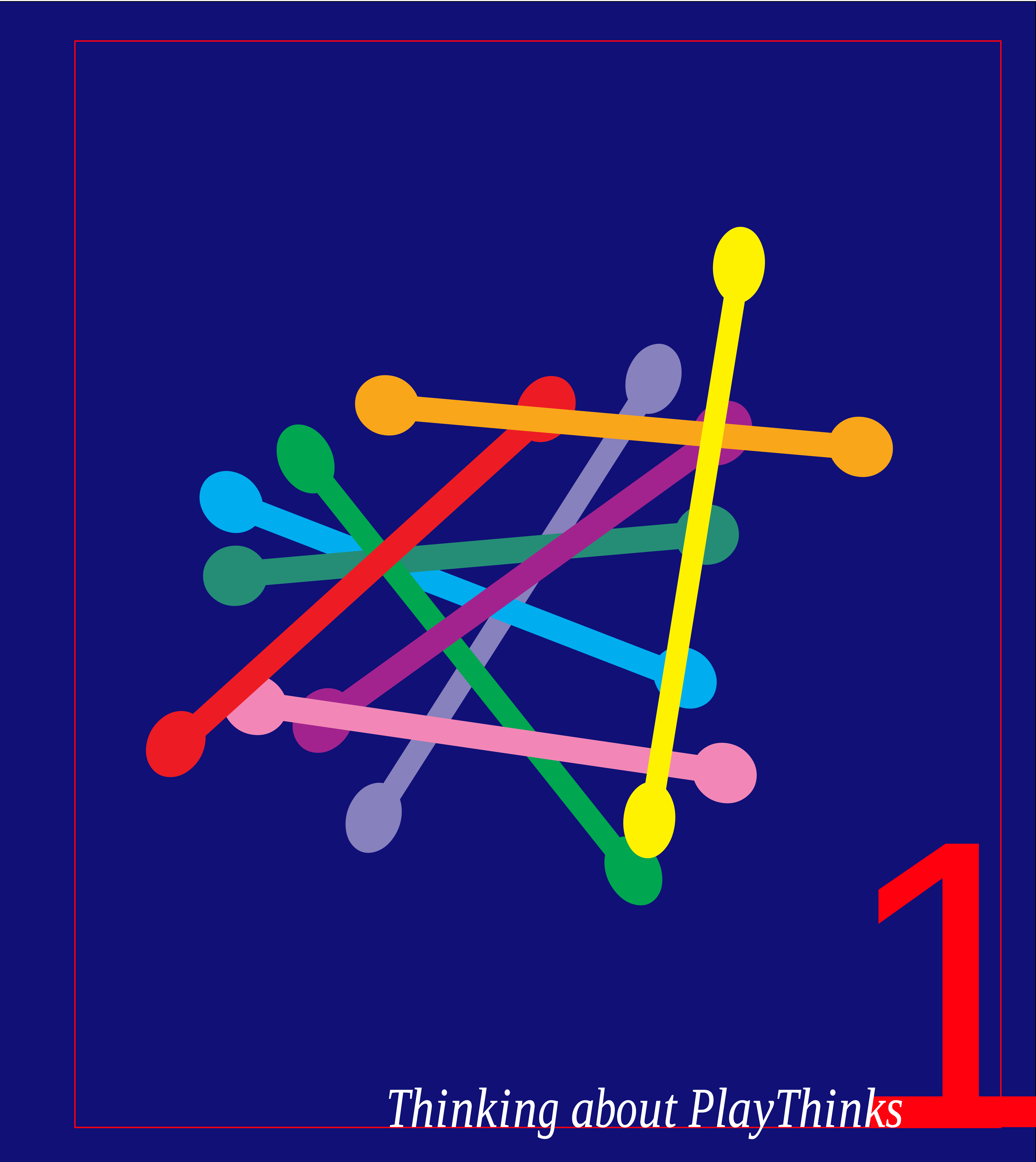
PlayThinks are designed to permit easy access to many ideas, in different contexts and at different levels. You will notice that many of them draw on the same set of ideas—probability, say, or graphing—with each one developing the concept more fully than the last. You may find that by attacking the PlayThinks in order, you can build up an understanding of a field of knowledge.

But that is far from the only way to use this book. Each PlayThink is rated in difficulty from 1 to 10. You might decide to do all the puzzles rated 1 and 2, they try the ones rated 3 and 4, and thus build up your abilities as a problem solver. (To find puzzles at your level, check the index at the back of the book.)

You might jump around in the book, first taking on the subjects that interest you most until you are ready to work your way deeper into the frontiers of what you think you don't know.

Or, using the key at the top of each puzzle as your guide, you might try all the mind puzzles (look for the 👁), then the pencil and paper puzzles (✎), and finally the more complicated ones that involve tracing or copying (🗎) and cutting (✄). You can do the solo activities when you've got a few minutes by yourself, and pull out the group games and puzzles when you're with friends. You get the idea: it's all up to you. Just don't forget to play.




1
Thinking about PlayThinks

Japanese Temple

The inspiration for PlayThinks came from *sangaku*, the Japanese temple geometry that flourished in the seventeenth, eighteenth and nineteenth centuries. In those times *sangaku* (the Japanese word for mathematical tablet) was a national pastime enjoyed by everyone from peasants to samurai nobility. People would solve geometrical proofs and puzzles, then offer the solutions to the spirits in the form of elegantly designed and executed wooden tablets. Those tablets, engraved with geometrical problems, hung under the roofs of shrines and temples. Indeed, the best *sangaku* tablets were works of art that paid homage to the spirits that guided one to the answer.

Today only a few devotees remember *sangaku*. In 1989 Hidetoshi Fukagawa and Daniel Pedoe published the first collection of *sangaku* to be translated into English; that book was later publicized in a *Scientific American* article. But more than 880 *sangaku* tablets survive. The problems typically involved geometrical constructions, often circles within circles, triangles or ellipses. The level of difficulty ranged from quite simple to impossible, though all would be considered recreational mathematics by contemporary standards. The proofs of the problems or theorems were usually not provided, just the results.

During that period, many ordinary Japanese people loved and enjoyed mathematics, and were carried away by the beauty of geometry. The authors of *sangaku* were probably teachers and their students. The tablets were crafted with loving care and were intended to be visual teaching aids for mathematicians and nonmathematicians alike.

And that defines perfectly what a PlayThink is.

I've always been fascinated by all types of puzzles and games for the mind, but the ones I like best

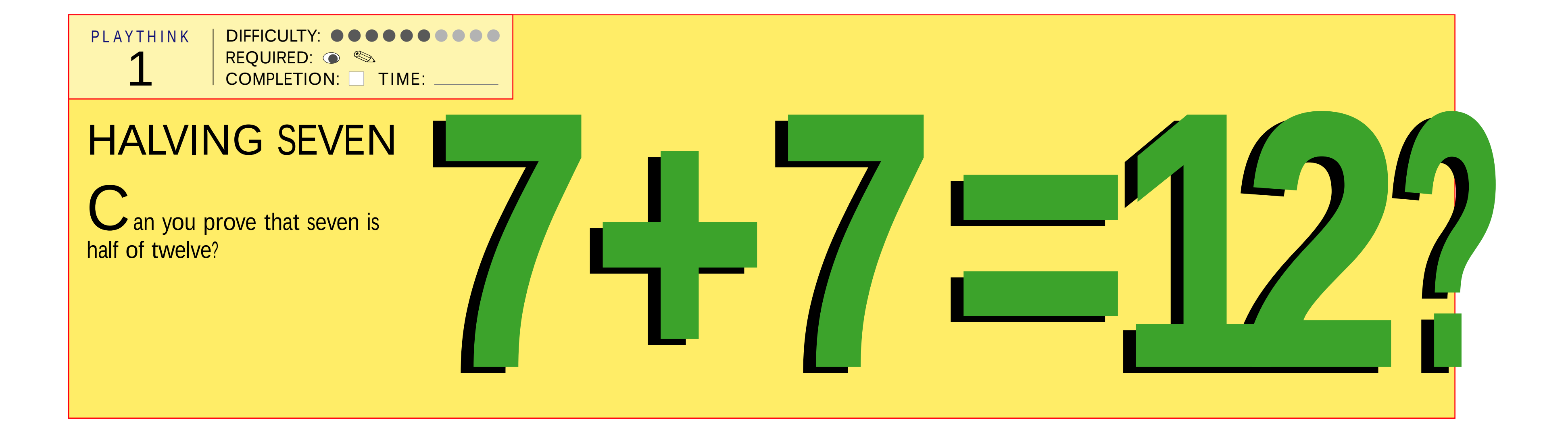
> "IMAGINATION IS MORE IMPORTANT THAN KNOWLEDGE."
>
> —ALBERT EINSTEIN

are not always the hardest. Sometimes a puzzle that is quite easy to solve is elegant or meaningful enough to make it especially satisfying. Solving puzzles has as much to do with the way you think about them as with natural ability or some impersonal measure of intelligence. Most people should be able to understand all the problems in this book, although some problems will undoubtedly seem easier than others. Thinking is what they are all about: comprehension is at least as important as visual perception or mathematical knowledge. After all, our different ways of thinking set us apart as individuals and make each of us unique.

PLAYTHINK 1 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

HALVING SEVEN

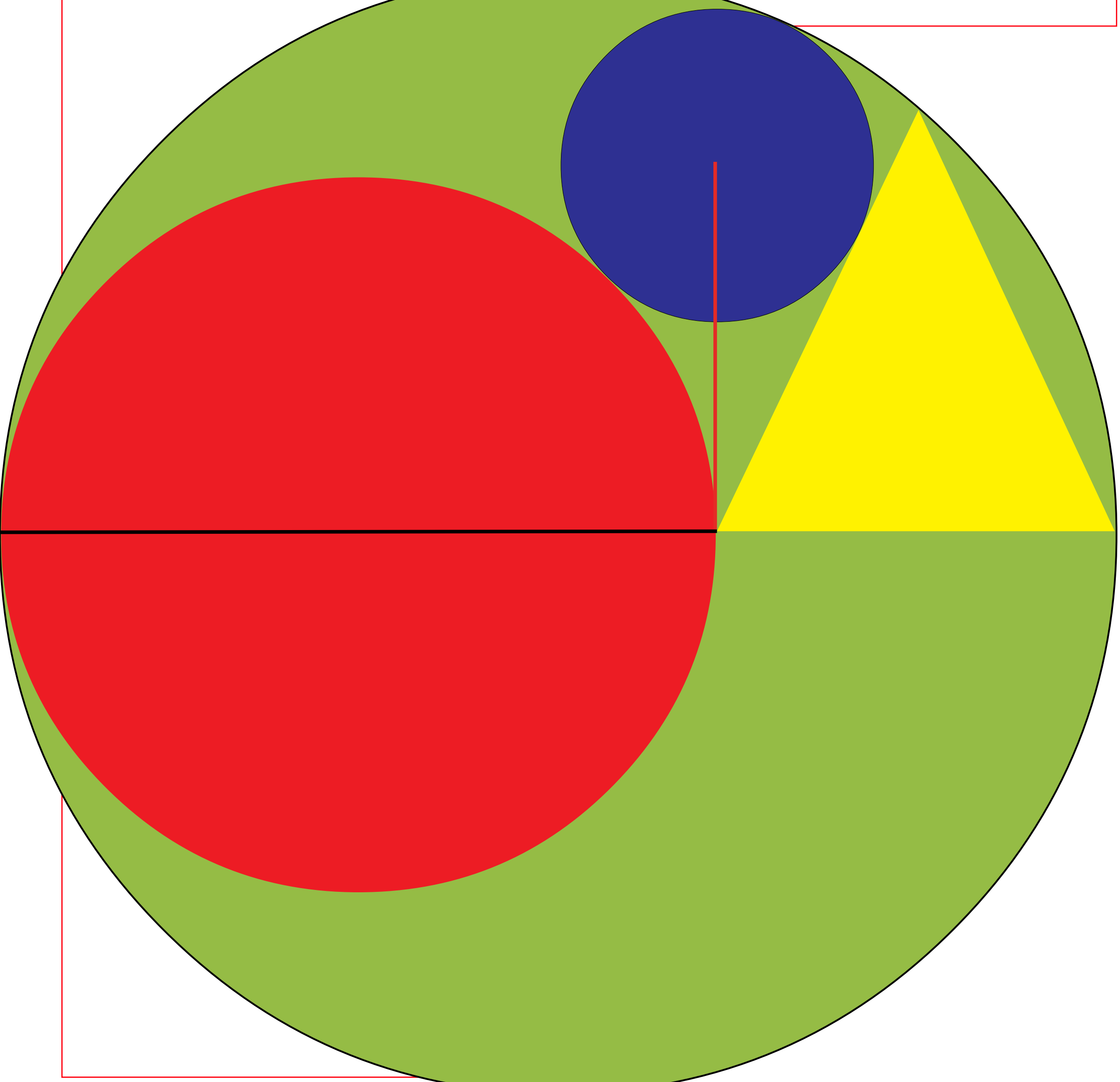
Can you prove that seven is half of twelve?



PLAYTHINK 2 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

A SANGAKU PROBLEM FROM 1803

Upon the diameter of the large green circle, place two shapes: an isosceles triangle and a smaller red circle. Position the triangle so that its base lies upon the diameter of the large circle. And position the smaller circle so that its diameter runs along the diameter of the large circle from the base of the triangle to the circumference of the large circle. Now add a third circle, inscribed so that it touches the other two circles and the triangle. If you draw a line from the center of the third circle to the point where the red circle and the triangle meet, can you prove that that line is in fact perpendicular to the diameter of the large green circle?



PLAYTHINK 3 | DIFFICULTY: ●○○○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

AHMES'S PUZZLE

Seven houses each have seven cats. Each cat kills seven mice. Each of the mice, if alive, would have eaten seven ears of wheat. Each ear of wheat produces seven measures of flour.

How many measures of flour were saved by the cats?

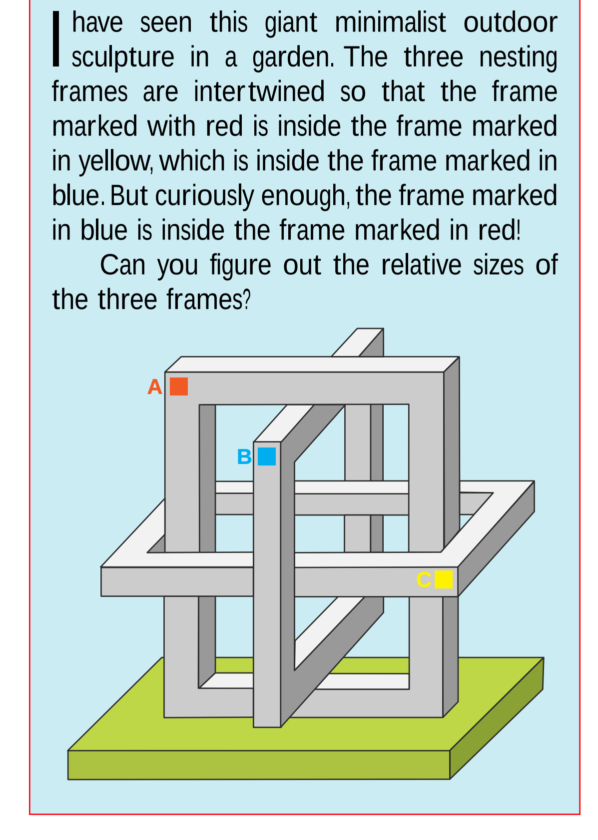


PLAYTHINK 4 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

NESTING FRAMES

I have seen this giant minimalist outdoor sculpture in a garden. The three nesting frames are intertwined so that the frame marked with red is inside the frame marked in yellow, which is inside the frame marked in blue. But curiously enough, the frame marked in blue is inside the frame marked in red!

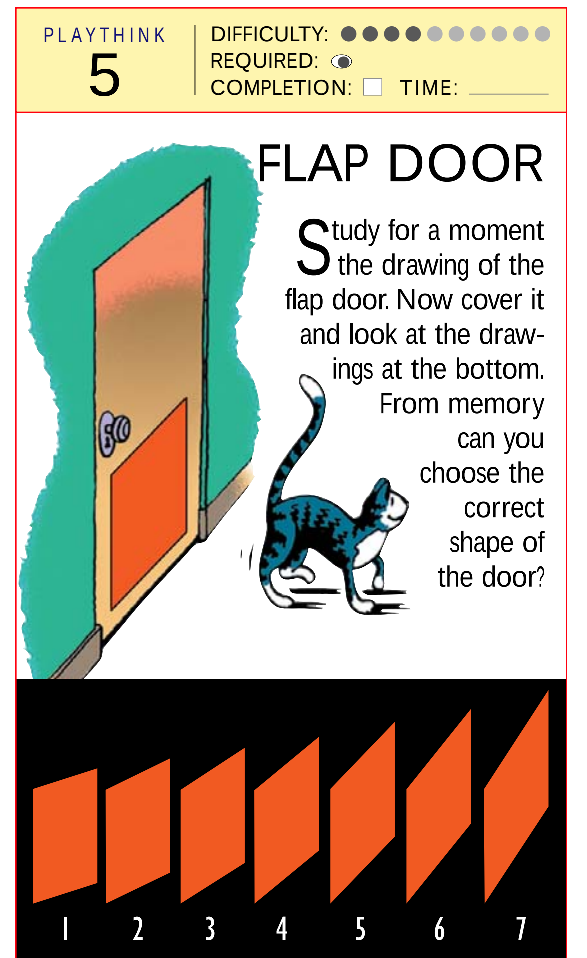
Can you figure out the relative sizes of the three frames?

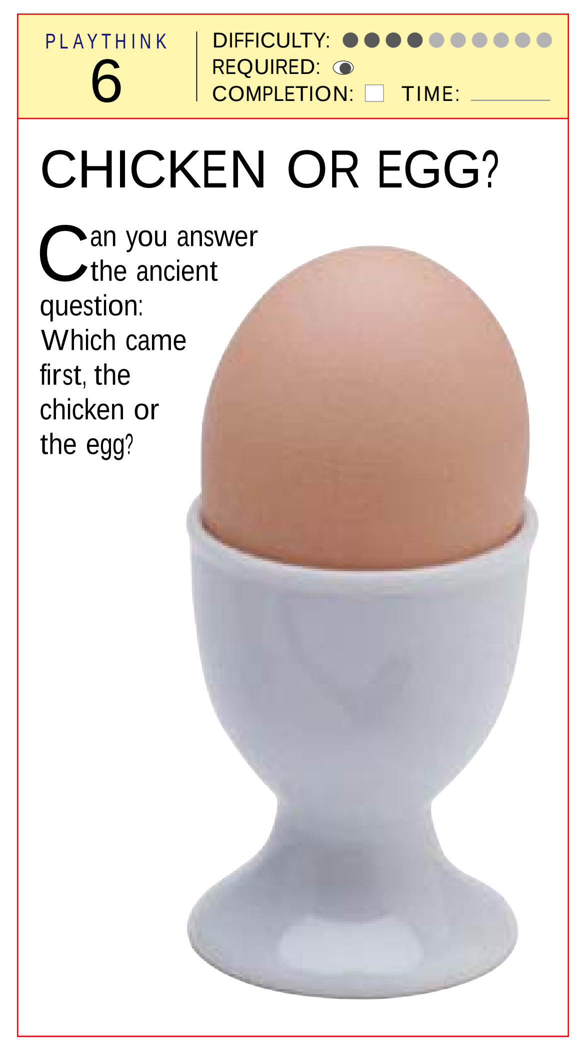


PLAYTHINK 5 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

FLAP DOOR

Study for a moment the drawing of the flap door. Now cover it and look at the drawings at the bottom. From memory can you choose the correct shape of the door?



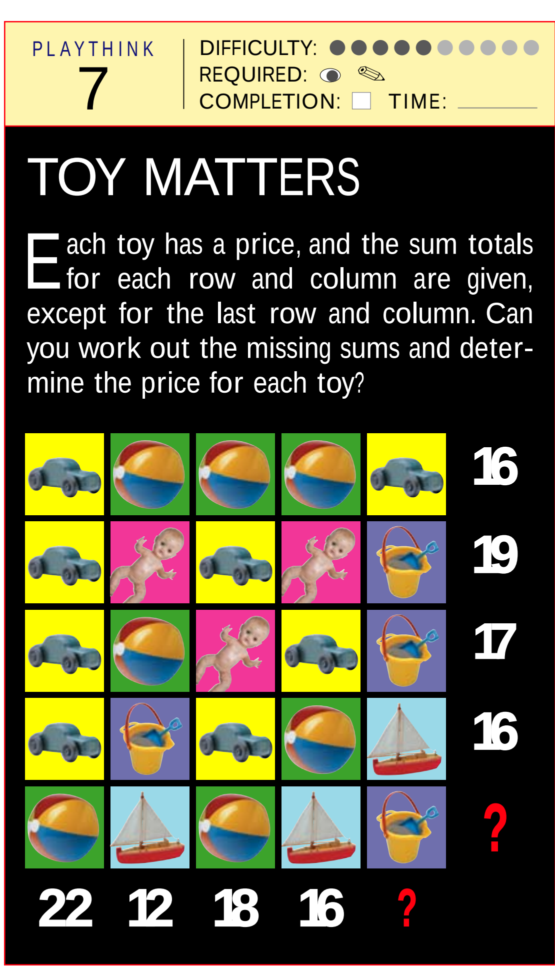


PLAYTHINK 6

DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ______

CHICKEN OR EGG?

Can you answer the ancient question: Which came first, the chicken or the egg?



PLAYTHINK 7

DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ______

TOY MATTERS

Each toy has a price, and the sum totals for each row and column are given, except for the last row and column. Can you work out the missing sums and determine the price for each toy?

PLAYTHINK 8

DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ______

CONSECUTIVE RECTANGLE SQUARES

This interesting question crops up in recreational mathematics literature: Using each of the first consecutive integers (1, 2, 3, 4, 5, 6, 7, 8, 9, 10) once each to create the dimensions of five rectangles, how many combinations of those five rectangles can be assembled into a square? In each of these four cases, place the colored rectangles at right on the grid at left.

The Beauty of Patterns

For the ancient Greeks, mathematics was the science of numbers. But this definition of mathematics has been invalid for hundreds of years. In the middle of the seventeenth century, Isaac Newton in England and Gottfried von Leibniz in Germany independently invented calculus, the study of motion and change, and touched off an explosion in mathematical activity. Contemporary mathematics comprises eighty distinct disciplines, some of which are still being split into subcategories. So today, rather than focus on numbers, many mathematicians think their field is better defined as the science of patterns.

A love affair with patterns is something that starts very early in our lives. And those patterns may take many forms—numerical, geometric, kinetic, behavioral and so on. As the science of patterns, mathematics affects every aspect of our lives; abstract patterns are the basis of thinking, of communication, of computation, of society and even of life itself.

Patterns are everywhere and everyone sees them, but mathematicians see patterns within the patterns. Yet, despite the somewhat imposing language used to describe their work, the goal of most mathematicians is to find the simplest explanations for the most complex patterns.

Part of the magic of mathematics is how a simple, amusing problem can lead to far-ranging insights. Look at PlayThink 54 (“Handshakes 2”). Figure it out? Then imagine that the people are points on a graph, and that their handshakes represent interconnecting lines. Thought of this way, the problem can lead you to picture a graph in which every point is interconnected with all the others—a useful image for, say, airline flight coordinators.

Realizing the importance of this kind of thinking, many schools are mixing more geometry, topology and probability into the math curriculum. This is all to the good: Wherever there is relationship and pattern, there is mathematics.

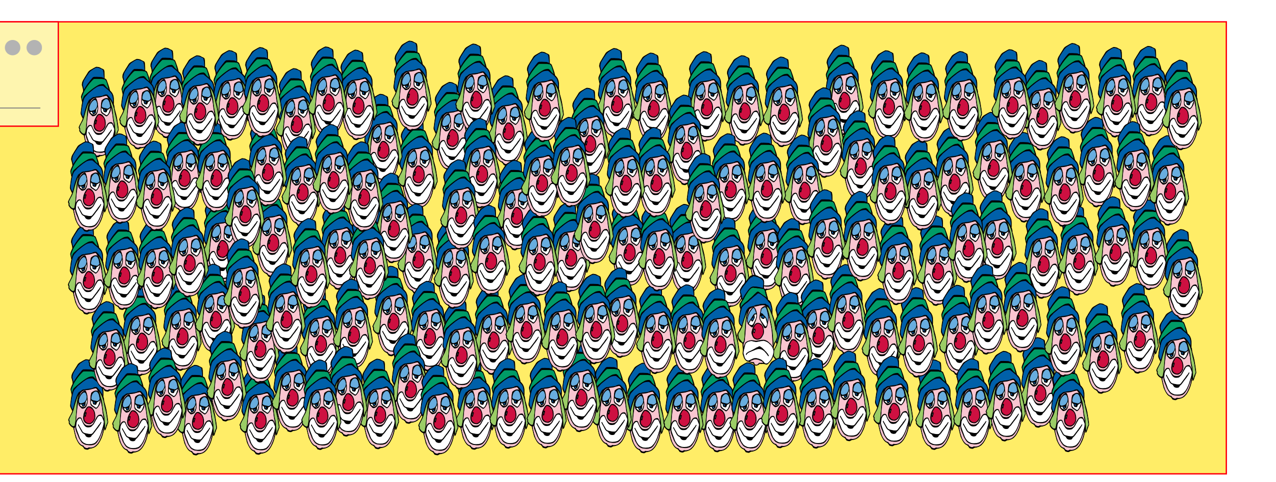
> “THERE IS AN OLD DEBATE ABOUT WHETHER YOU CREATE MATHEMATICS OR JUST DISCOVER IT. IN OTHER WORDS, ARE THE TRUTHS ALREADY THERE, EVEN IF WE DON’T YET KNOW THEM? IF YOU BELIEVE IN GOD, THE ANSWER IS OBVIOUS.”
>
> —PAUL ERDÖS

PLAYTHINK 9

DIFFICULTY: ●○○○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

SAD CLOWN

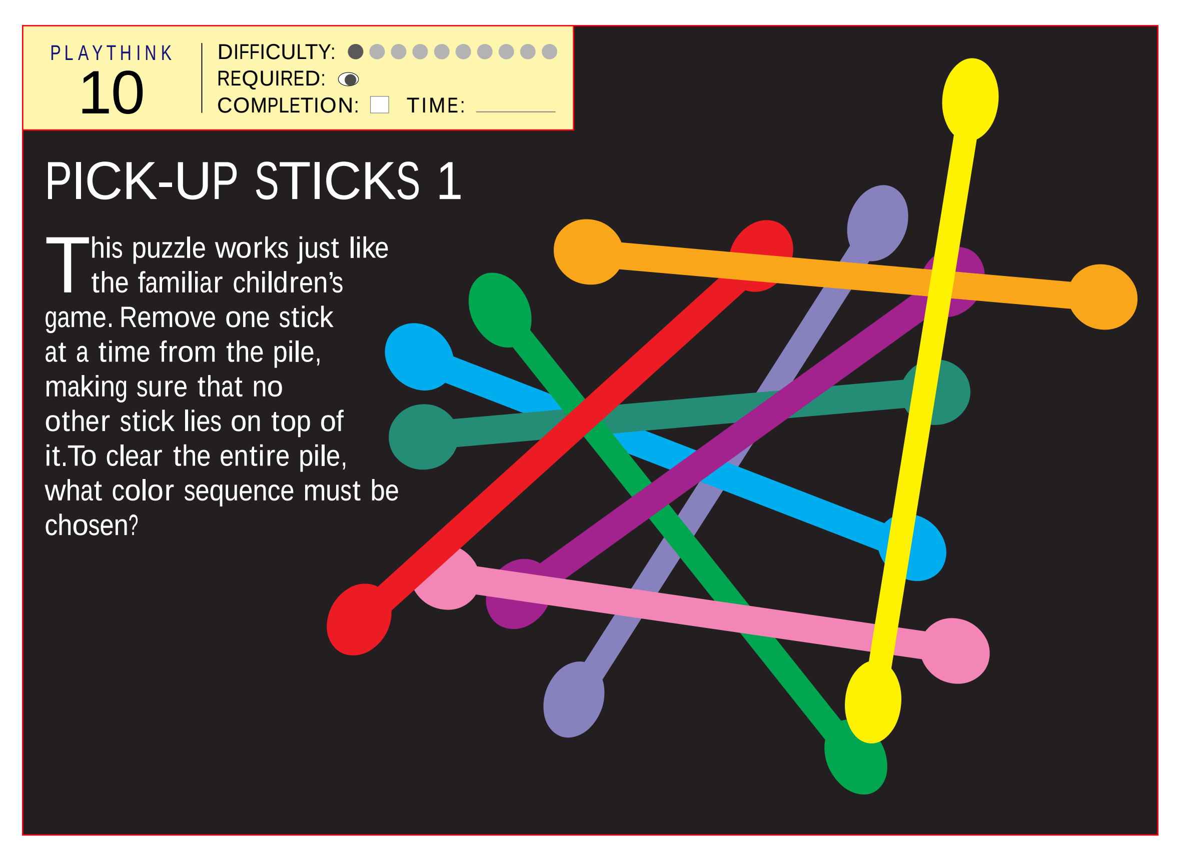
Can you find the clown with a frown?



PLAYTHINK 10 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ______

PICK-UP STICKS 1

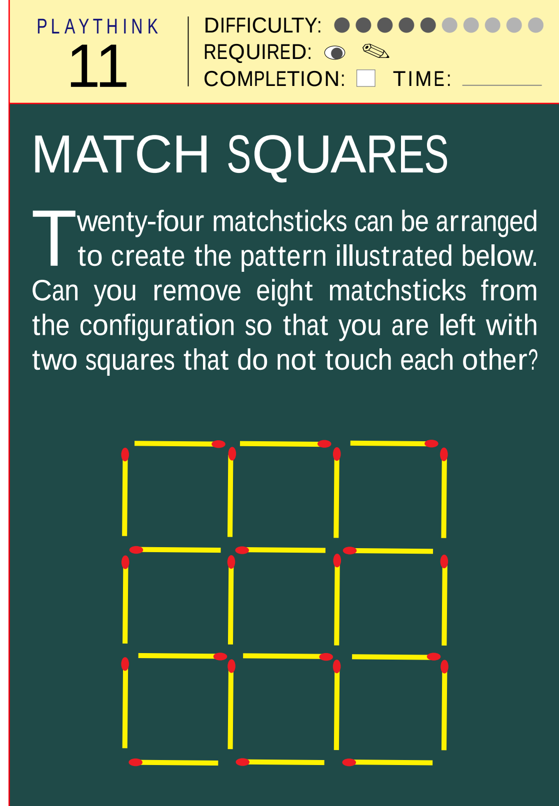
This puzzle works just like the familiar children's game. Remove one stick at a time from the pile, making sure that no other stick lies on top of it. To clear the entire pile, what color sequence must be chosen?



PLAYTHINK 11 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ______

MATCH SQUARES

Twenty-four matchsticks can be arranged to create the pattern illustrated below. Can you remove eight matchsticks from the configuration so that you are left with two squares that do not touch each other?

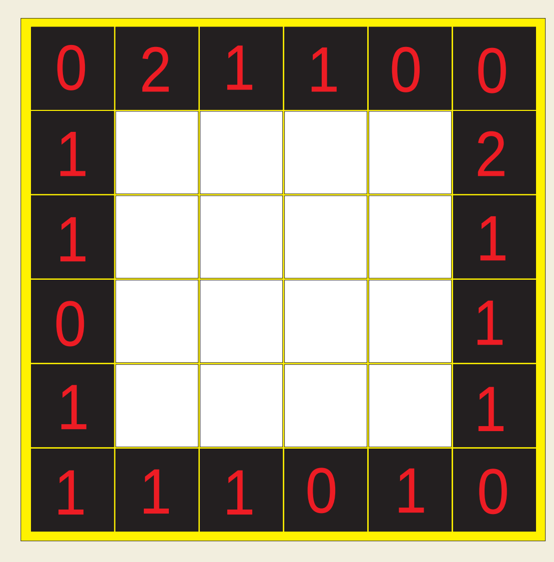


PLAYTHINK 12 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ______

ARROW NUMBER BOXES

The object of this sort of puzzle is to place arrows in the boxes according to the following rules: The arrows must point in one of the eight main compass directions (north, south, east, west, northeast, southeast, northwest, and southwest); the number of arrows pointing to each number in the outer boxes must equal the value of that number; and each box must have an arrow in it. The sample shown (upper right) is a flawed attempt at a solution, since no arrow can be placed on the blank square within the rules of the game, and one of the outer squares has no arrow pointing at it.

Can you find complete solutions for the arrow number boxes of order 4 (upper left), order 5 (lower left), and order 6 (lower right)?



0	2	1	1	0	0
1					2
1					1
0					1
1					1
1	1	1	0	1	0

2	0	4	0	2	0	3
0						1
3						0
0						0
3						2
0						0
0	0	0	2	1	2	0

0	0	1	0	1	0	0	0
0							0
0							5
2							1
1							2
3							2
0							1
0	3	1	6	2	2	2	1

Thinking as a Skill

We constantly use intuition in our everyday life. Yet until recently the scientific study of intuition was largely ignored. New research has found that intuition springs from a set of important human skills that all act together to give a so-called gut reaction. The more you use these skills, the better your intuition becomes.

PlayThinks includes problems that will sharpen your ability to recognize and perceive patterns, to stretch your imagination, to make the most of trial and error. And as you do these problems, you will improve your creativity, insight and intuition.

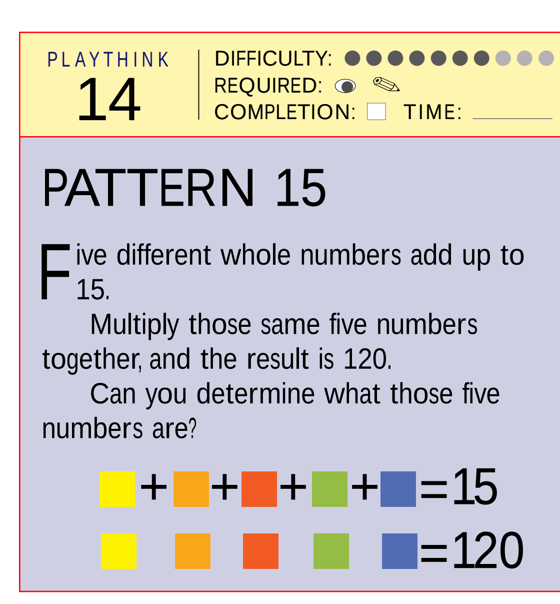
Thinking is a learnable skill, like cooking or golf. If you make even a small effort to develop it, you will see improvement. As Nob Yoshigahara, the editor of the famous *Puzzletopia Newsletter*, once said: "What jogging is to the body, thinking is to the brain. The more we do it, the better we become."

PLAYTHINK 13 — DIFFICULTY: ●●●●●●○○○○ REQUIRED: 👁 COMPLETION: ☐ TIME: ____



LOTTERY DRAW

If you draw the lucky ticket, you win the lottery jackpot. You are given the option to draw one ticket out of a box of 10, or draw ten times out of a box of 100. Which choice gives you the best odds?



PLAYTHINK 14 — DIFFICULTY: ●●●●●●●○○○ REQUIRED: 👁 ✎ COMPLETION: ☐ TIME: ____

PATTERN 15

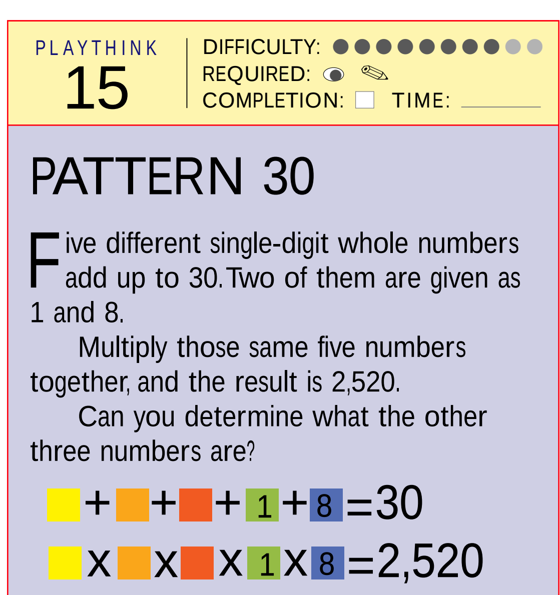
Five different whole numbers add up to 15.

Multiply those same five numbers together, and the result is 120.

Can you determine what those five numbers are?

■ + ■ + ■ + ■ + ■ = 15

■ ■ ■ ■ ■ = 120



PLAYTHINK 15 — DIFFICULTY: ●●●●●●●●○○ REQUIRED: 👁 ✎ COMPLETION: ☐ TIME: ____

PATTERN 30

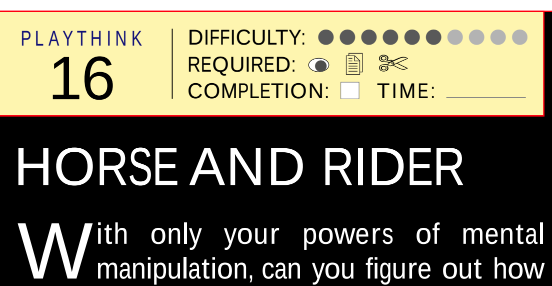
Five different single-digit whole numbers add up to 30. Two of them are given as 1 and 8.

Multiply those same five numbers together, and the result is 2,520.

Can you determine what the other three numbers are?

■ + ■ + ■ + 1 + 8 = 30

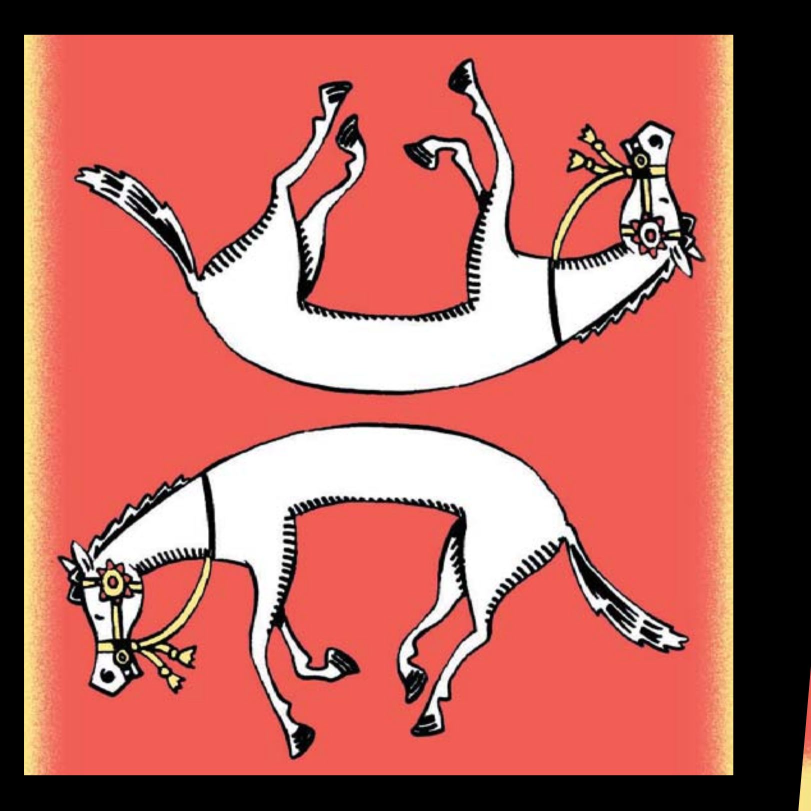
■ x ■ x ■ x 1 x 8 = 2,520

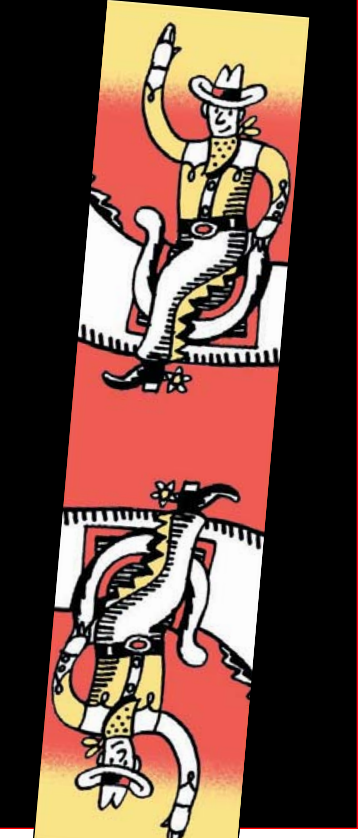


PLAYTHINK 16 — DIFFICULTY: ●●●●●●○○○○ REQUIRED: 👁 📄 ✂ COMPLETION: ☐ TIME: ____

HORSE AND RIDER

With only your powers of mental manipulation, can you figure out how to position the strip with the cowboys onto the square with the horses so that it looks as if the cowboys were riding the horses? This problem (based on the classic Trick Donkeys puzzle created by Sam Loyd) looks deceptively simple, but one soon realizes that the obvious answer is wrong. If you can't solve this in your mind, try copying and cutting out the strip to experiment with paper. Hint: The solution makes the horses look much faster.



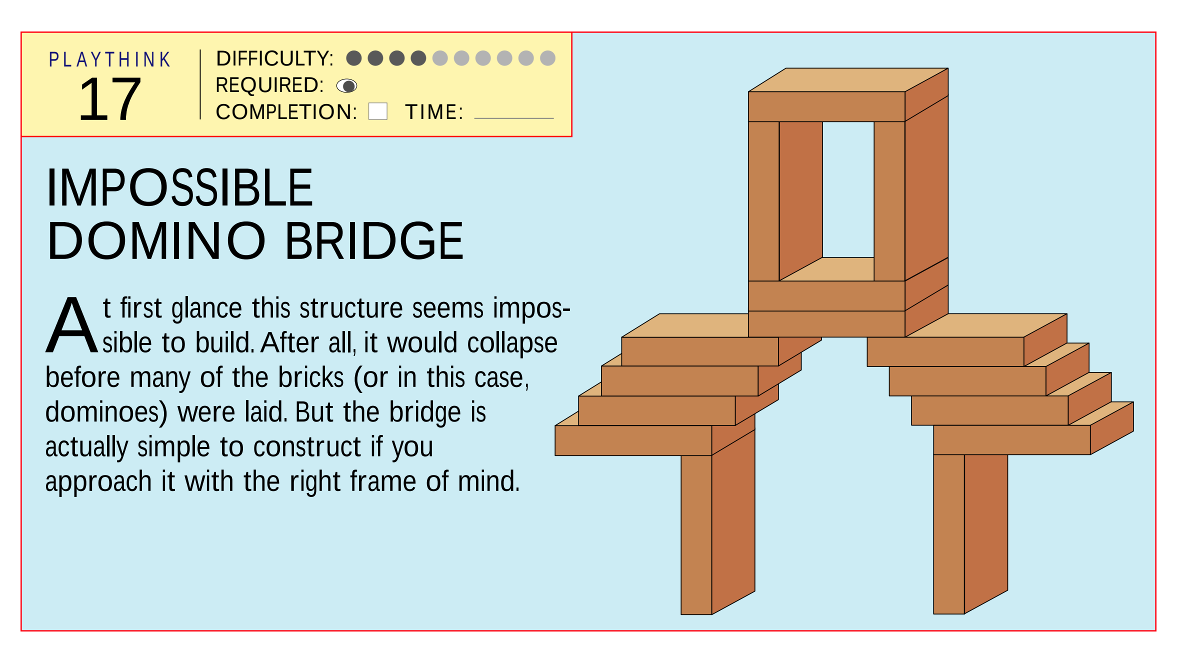


PLAYTHINK 17

DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

IMPOSSIBLE DOMINO BRIDGE

At first glance this structure seems impossible to build. After all, it would collapse before many of the bricks (or in this case, dominoes) were laid. But the bridge is actually simple to construct if you approach it with the right frame of mind.



PLAYTHINK 18

DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

GLOVES IN THE DARK

There are twenty-two gloves in a drawer: five pairs of red gloves, four pairs of yellow and two pairs of green. If the lights are out and you must select the gloves in the dark, how many must you choose to ensure that you have at least one matching pair?

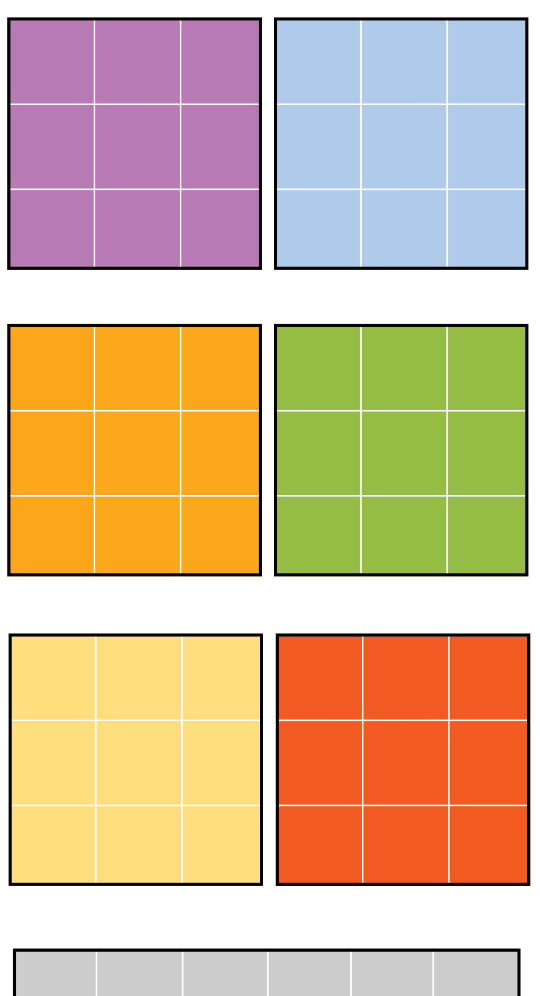


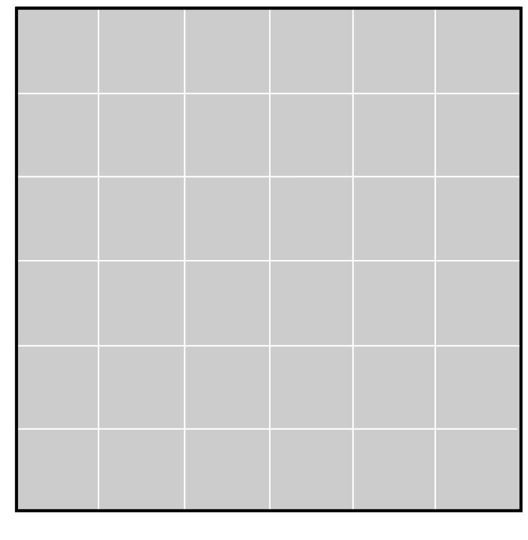
PLAYTHINK 19

DIFFICULTY: ●●●●●●●●●●
REQUIRED: ✎
COMPLETION: ☐ TIME: ______

OVERLAPPING SQUARES 2

Can you fit the six squares into the big gray square to create a pattern of eighteen squares of four different sizes formed by their outlines? The white grid lines are provided only to help in the alignment of the overlapping squares.





Getting Around Mental Blocks

Your brain works much better than you might think. It is capable of making a virtually unlimited number of synaptic connections, each of which is a pattern of thought. (The number of possible connections has been calculated, but the result is so large—1 followed by 60 million miles of typewritten zeros—it might as well be infinity.)

In spite of the vast number of possible thoughts to think, thinking can be hard work, and there is a natural human tendency to do as little of it as possible. This tendency is seen in the hit-and-run approach many people take to problem solving: they pick the first solution that comes to mind and run with it. Such an approach generally fails to take into account the full range of possible solutions. People can become trapped in their own preconceptions, not so much neglecting information that might solve the problem as simply not perceiving it.

Problem solving works best with the fewest self-imposed blinders. The greater the choice of creative concepts, the better chance there is to find an answer. If your first idea fails to solve the problem, try another. It is important to avoid the mental walls known as conceptual blocks, which can shield us from even the simplest and most obvious answer. Sometimes the conceptual block is of one's own creation, while others stem from incomplete information, emphasis on the wrong detail or deliberately misleading directions. Inventors of puzzles and magic tricks exploit such conceptual blocks to lead suggestible minds up blind alleys. But in spite of the universal tendency to suffer from blocks, most people at one time or another can tackle a problem of bewildering complexity, penetrate to its core and extract an insight of startling simplicity and elegance that solves the problem at a stroke.

> "IT ISN'T THAT THEY CAN'T SEE THE SOLUTION. IT IS THAT THEY CAN'T SEE THE PROBLEM."
>
> —G. K. CHESTERTON

The best puzzles are seldom what they seem. The solutions may demand that a common item be used in an unfamiliar way or that a conventional assumption be abandoned or that components be assembled in an unusual arrangement. The direct, head-on approach often leads nowhere, while a lengthy detour can sometimes be the fastest route to a solution. When you are faced with a mental wall, the best approach is not to tunnel through it but to walk around.

PLAYTHINK 20

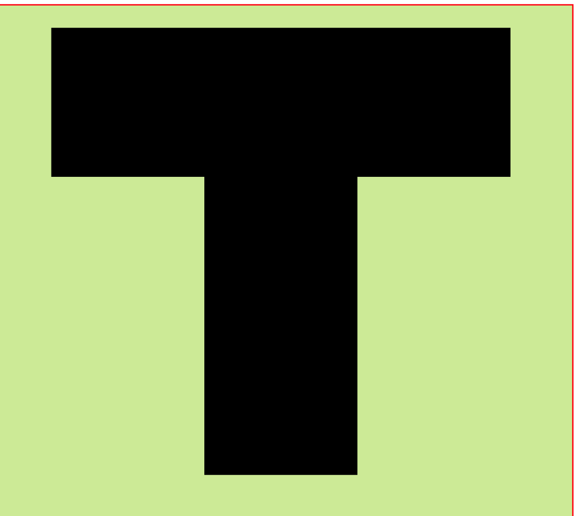
DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁 📄 ✂
COMPLETION: ☐ TIME: ______

T-PUZZLE

In this classic puzzle the four red pieces can be placed together to form a perfect capital T. Can you visualize how they fit together?

Copy and cut out the pieces to experiment with different possible solutions before you look at the answer in the back.





PLAYTHINK 21

DIFFICULTY: ●●●●●●●●○○
REQUIRED:
COMPLETION: ☐ TIME: ______

RALLY

Conventional sliding-block or sliding-disk puzzles have an empty space into which the pieces can move. Knowing how to move pieces into the space is usually the key to solving such puzzles. Rally is a sliding-disk puzzle with no empty spaces. The thirty-two disks move like a chain through two elliptical channels, one vertical and one horizontal. Each chain is composed of eighteen disks, and there are four disks common to both chains. Moving one disk in the chain moves all the other disks in its channel; by switching the movement from one channel to another, disks can be transferred from one chain to another.

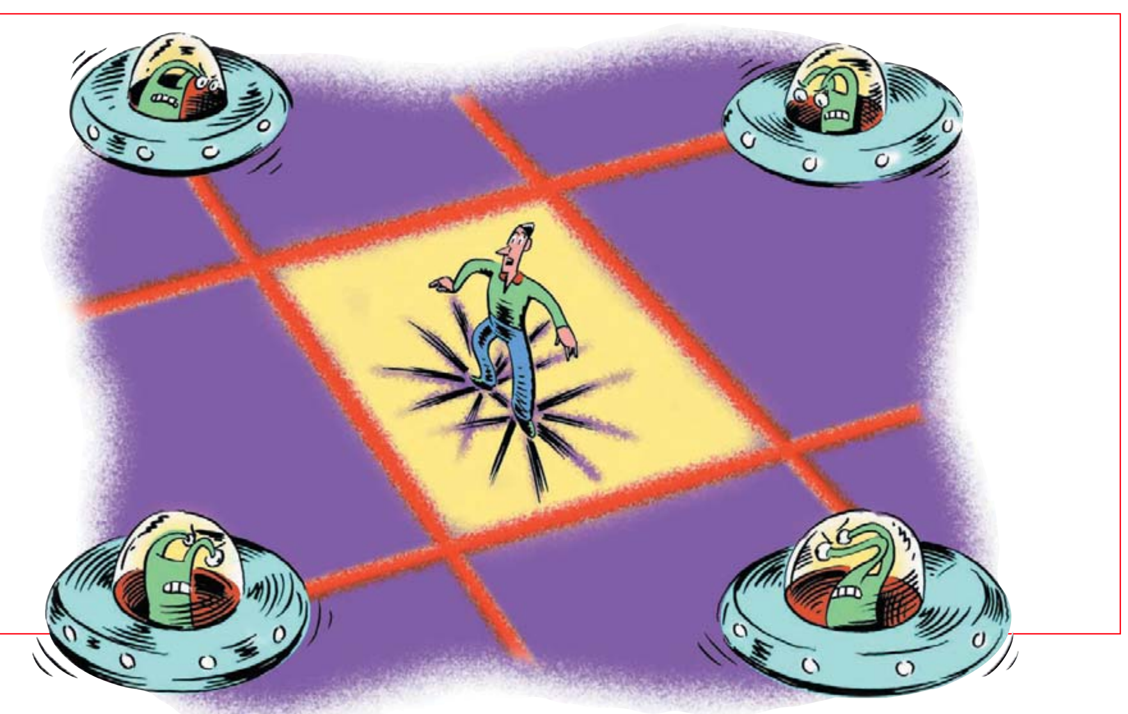
There are twelve red disks, twelve blue disks and eight yellow disks. Initially, the red disks form a square in the middle of the puzzle. How many moves will it take you to transform the red square in the middle to a blue square?

PLAYTHINK 22

DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

ALIEN ABDUCTION

Four UFOs hover above a man they plan to abduct. To catch the man, the four aliens must create a rectangular energy field around him. Each alien fires a laser randomly, either to the left of the man or to his right. Out of all the possible random combinations of the four laser shots, what is the probability that each will form a side of a rectangle around the man? (In the example shown, all rays are directed to the right of the man.)



PLAYTHINK 23
DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

INVENTOR PARADOX

Three friends talked about Ivan, but only one of them knew the truth.

"Ivan has invented hundreds of toys," Gerry said.

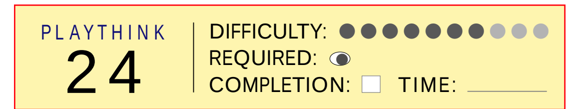
"No, he hasn't," George said. "He has invented fewer than that."

"Well, he has invented at least one toy," Anitta said.

If only one of those statements is true, can you figure out how many toys Ivan has invented?

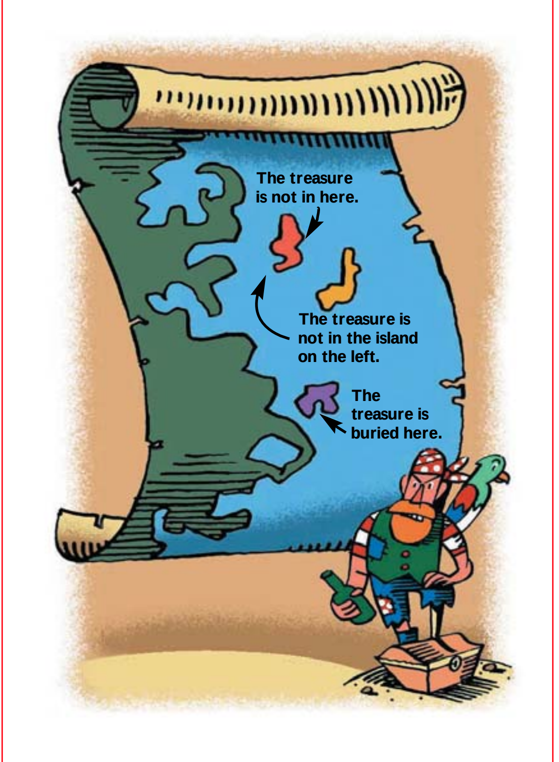
Also, Ivan is pictured on this page. Can you find him?





TREASURE ISLAND

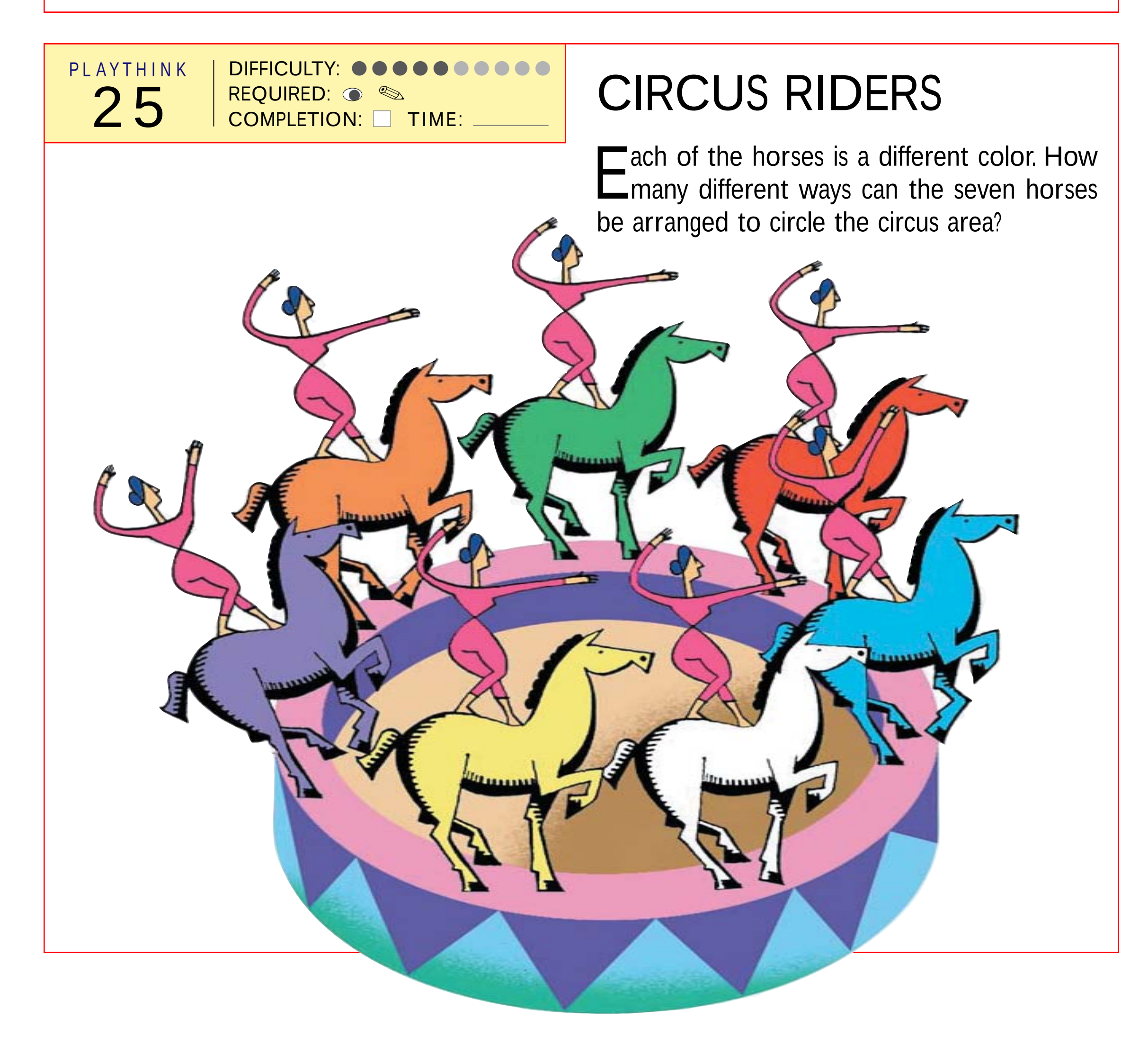
To confuse his enemies, the pirate who made this map made only one of the statements false. Can you still figure out where the treasure is buried?



PLAYTHINK 25
DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

CIRCUS RIDERS

Each of the horses is a different color. How many different ways can the seven horses be arranged to circle the circus area?

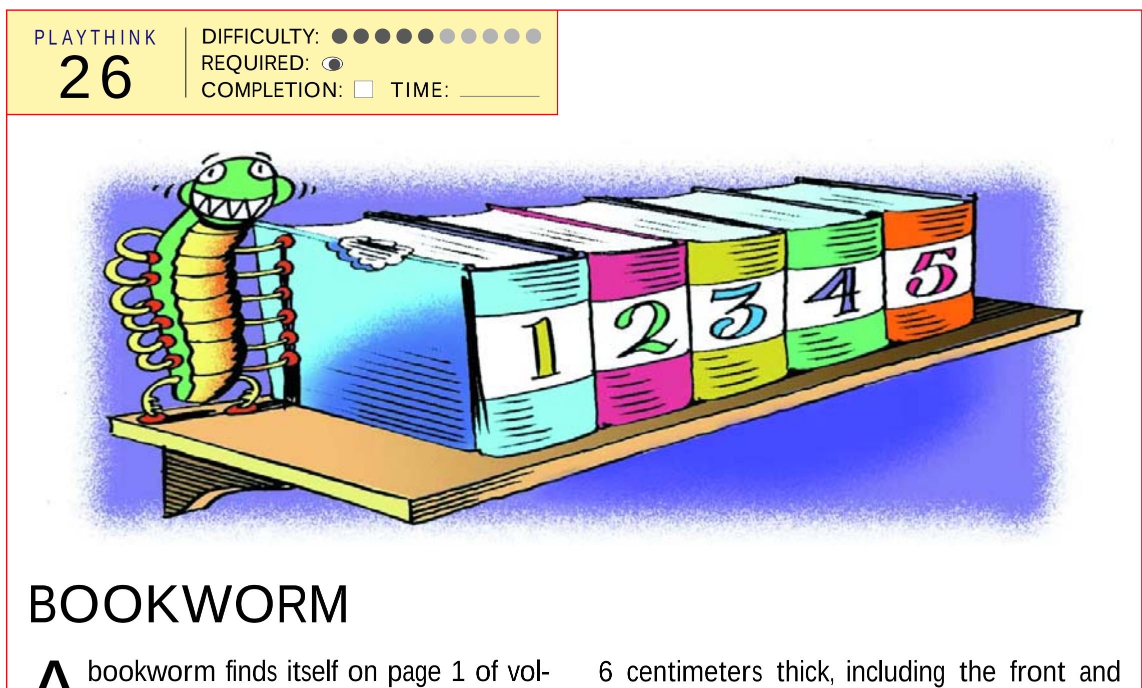


"THE SIMPLEST SCHOOLBOY IS NOW FAMILIAR WITH FACTS FOR WHICH ARCHIMEDES WOULD HAVE SACRIFICED HIS LIFE."

—ERNEST RENAN

PLAYTHINK 26

DIFFICULTY: ●●●●●○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______



BOOKWORM

A bookworm finds itself on page 1 of volume 1 and begins eating straight through to the last page of volume 5. If each book is 6 centimeters thick, including the front and back covers, which are half a centimeter each, what is the distance the bookworm travels?

PLAYTHINK 27

DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

HALLOWEEN MASK

You have five different colors of paint. How many different ways can you paint the Halloween mask if you make the eyes, nose and mouth each a different color?



PLAYTHINK 28

DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

BINARY TRANSFORMATIONS

This puzzle involves swapping pairs of rows or pairs of columns or flipping one row or column end to end to transform one pattern to another. A move constitutes one exchange of rows or columns or a reorientation (flipping) of a single row or column.

From the initial patterns on the left, create the patterns on the right. How many moves will it take?

initial configuration — PUZZLE 1 — final pattern

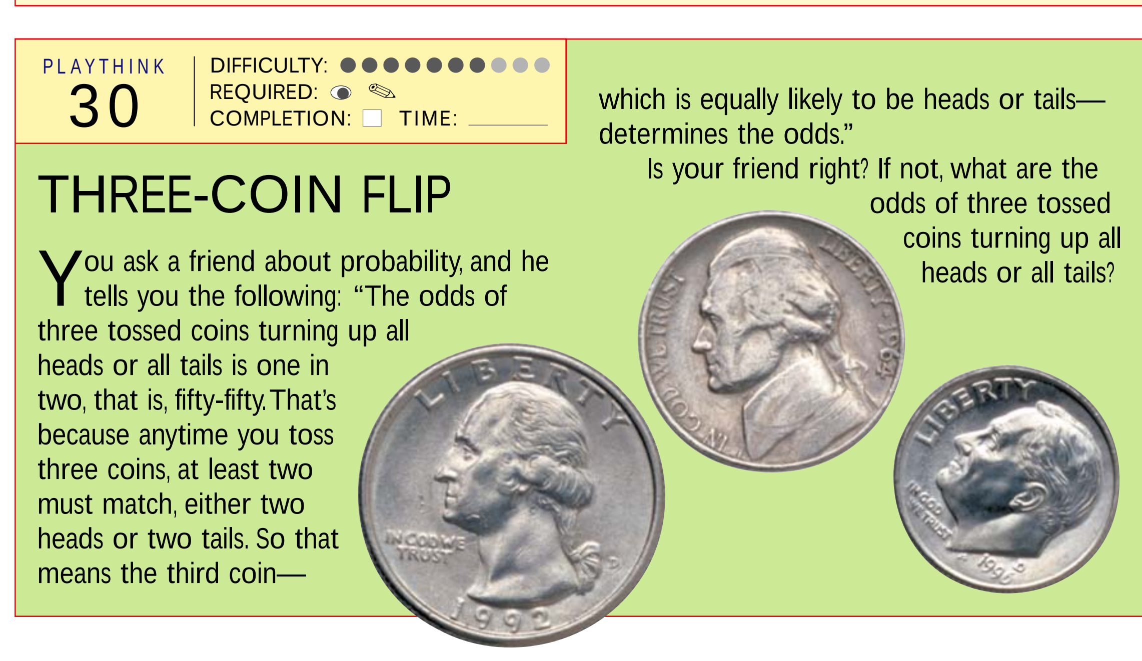
initial configuration — PUZZLE 2 — final pattern

PLAYTHINK 29 | DIFFICULTY: ●●●●●●●●○○ | REQUIRED: ✎ | COMPLETION: ☐ TIME: ____

JUMPING PEGS PUZZLE

Can you jump from point to point across the board to connect the matching numbers along the edge? Only jumps that are equal to the segments shown below are valid. To illustrate the concept, the series of jumps connecting the two points marked 5 are shown above.

The three allowable lengths:



PLAYTHINK 30 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

THREE-COIN FLIP

You ask a friend about probability, and he tells you the following: "The odds of three tossed coins turning up all heads or all tails is one in two, that is, fifty-fifty. That's because anytime you toss three coins, at least two must match, either two heads or two tails. So that means the third coin—which is equally likely to be heads or tails—determines the odds."

Is your friend right? If not, what are the odds of three tossed coins turning up all heads or all tails?



PLAYTHINK 31 | DIFFICULTY: ●●○○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

SCRAMBLED MATCHSTICKS

It takes just a couple of little twists to turn these matchsticks into a message. Can you find the word?

Why Do We Play Games?

As living, intelligent organisms, we humans possess curiosity about our environment, about one another, about ourselves—and putting that curiosity to use through an exploration of the unknown energizes us. No one knows why really, but we can feel that it's true. Likewise, playing games that engage our curiosity makes us feel more alive. Again, we don't really know why, but I think it has a lot to do with the risk of losing.

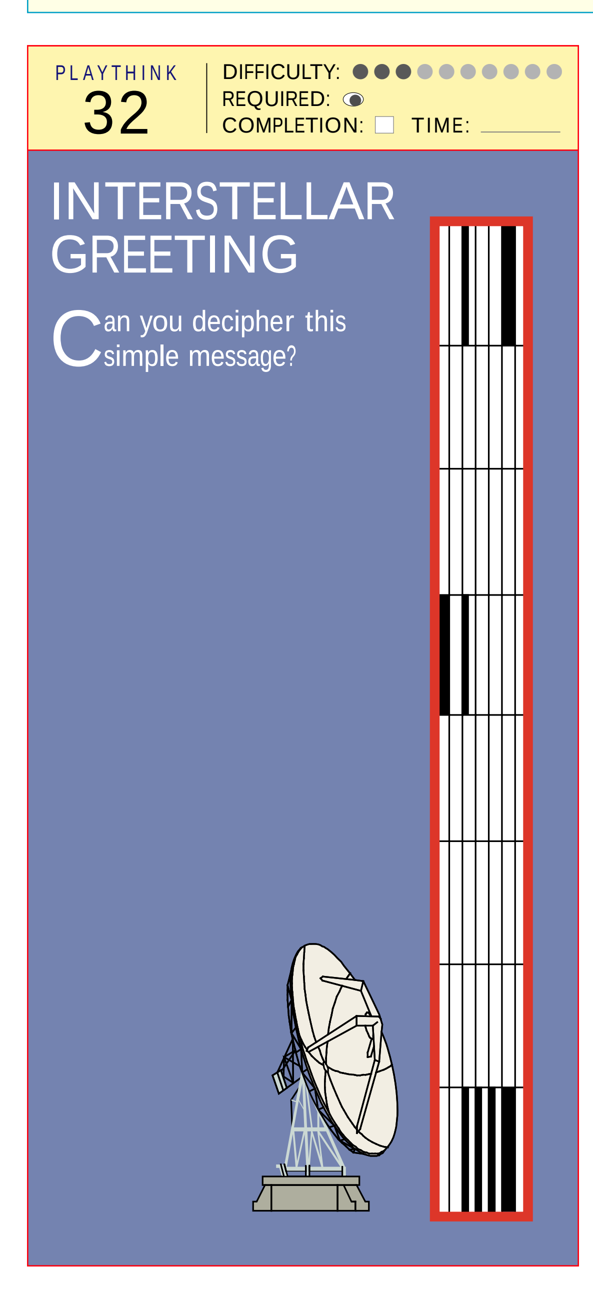
I believe that each person seeks out stimuli just slightly more complex than his or her preferred level of stimulation. What could be a better way to find stimulating uncertainties than in a game in which the outcome is never known? But games do much more than provide stimulation, ego satisfaction and fun. They help the mind develop by teaching cooperation and competition, exploration and invention. They encourage us to devise strategies for victory and, ultimately, for loss. Indeed, games duplicate, in model form, almost every human condition, aspiration and social structure. How else to explain that gaming has become one of our most potent metaphors: the money game, the marketing game, the survival game, the dating game? The meaning is always clear: games require players who want to win and know they may not.

PLAYTHINK 32

DIFFICULTY: ●●●○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

INTERSTELLAR GREETING

Can you decipher this simple message?



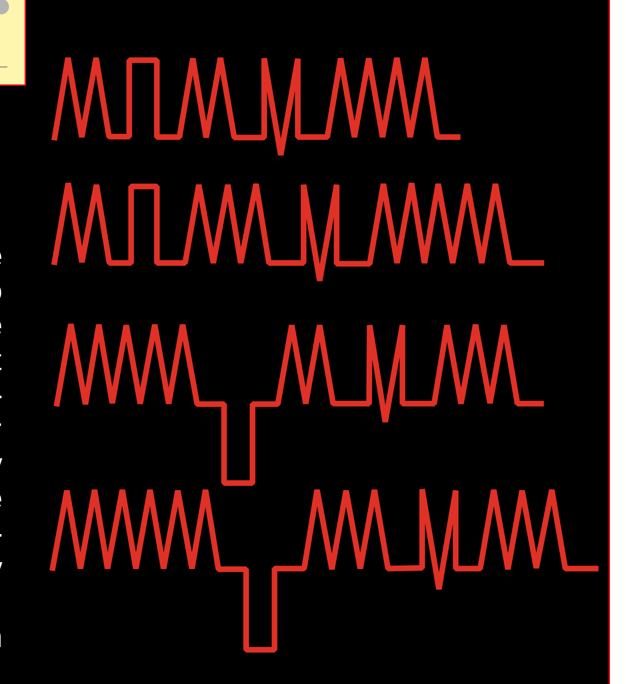
PLAYTHINK 33

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

INTERSTELLAR MESSAGE 1

Astronomers have sent messages like this one into outer space in order to establish communication with intelligent life on other planets. Researchers hope that even if such alien life forms cannot understand our written or spoken languages or make sense of images of our culture, they might use radio for communication and be adept at mathematics. For that reason messages have been sent that incorporate binary codes and simple mathematical principles.

Can you decipher the message shown below?

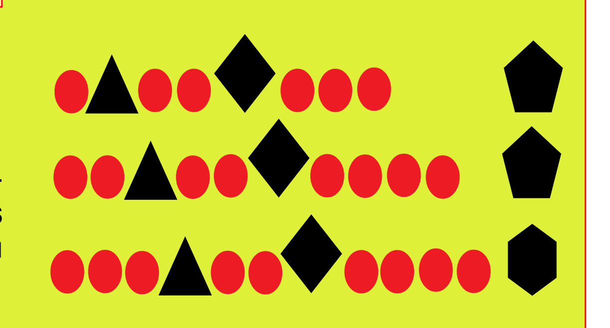


PLAYTHINK 34

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

INTERSTELLAR MESSAGE 2

Let's say the alien beings received the previous message and answered back with this series of dots and geometric figures. Can you work out the meaning of the message?



Communicating with Numbers

The most important thing a person inherits is the ability to learn a language. Language—especially written language—makes connection possible between people living in vastly different circumstances, places and times. What humans know of the past and can foretell of the future comes from language.

To get a true sense of how significant language is, consider this: is it possible to get meaning from something without the use of words or signs? Indeed, some philosophers believe that a world without language would be a world devoid of meaning.

Language is carried visually by either signs, which are written marks that stand for units of language, or symbols, which represent an object itself. In the 20,000 years since humans first scratched simple tallies on a bone, the visual aspect of language has flourished. First objects, then words were abstractly represented. By 300 B.C. the library of Alexandria contained some 750,000 papyrus scroll books, the greatest storehouse of knowledge the world had ever seen—possible only through the use of signs and symbols.

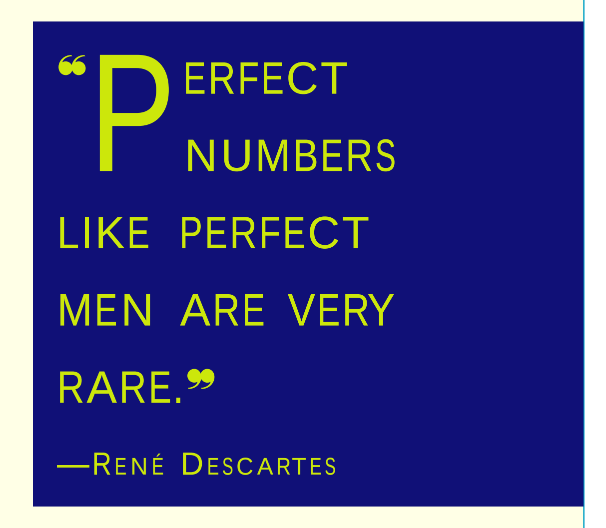
Later, technological developments such as block printing (by the Chinese) and movable type (by Johannes Gutenberg) enabled written language to reach virtually every person on the planet. Although attempts to replace the some 3,000 languages and dialects with one "invented" language, such as Esperanto, have consistently failed, the use of symbols to supplement spoken language has proliferated. Indeed, the modern world is awash in signs and symbols.

Symbolic language promotes a type of visual thinking that today's designers and communication engineers must take into account. Older ways of presenting complex ideas and more verbal forms of recalling information are quickly being rendered obsolete. Change is happening so quickly that even written language may not be the most trustworthy means of communicating with future generations. It is no exaggeration to say that anyone trying to send a message to the future—be it a memorial to a great leader or a warning about a toxic waste site—ought to look at the efforts that have been made by astronomers to communicate with intelligent life forms on other planets.

If such aliens existed, they would be unfamiliar with any human language, written or spoken. Astronomers involved with the Search for Extra-Terrestrial Intelligence, or SETI, are scanning the heavens with radio telescopes in search of a scrap of message—intentional or accidental—amid the natural noise of the stars, although no one knows what such a message might look like. Other astronomers have tried to send messages to distant stars in the form of pictographs symbolizing everything from the human form to the lightest chemical elements. But even such simple pictures would require some ingenuity to decode.

Perhaps mathematics will provide the key.

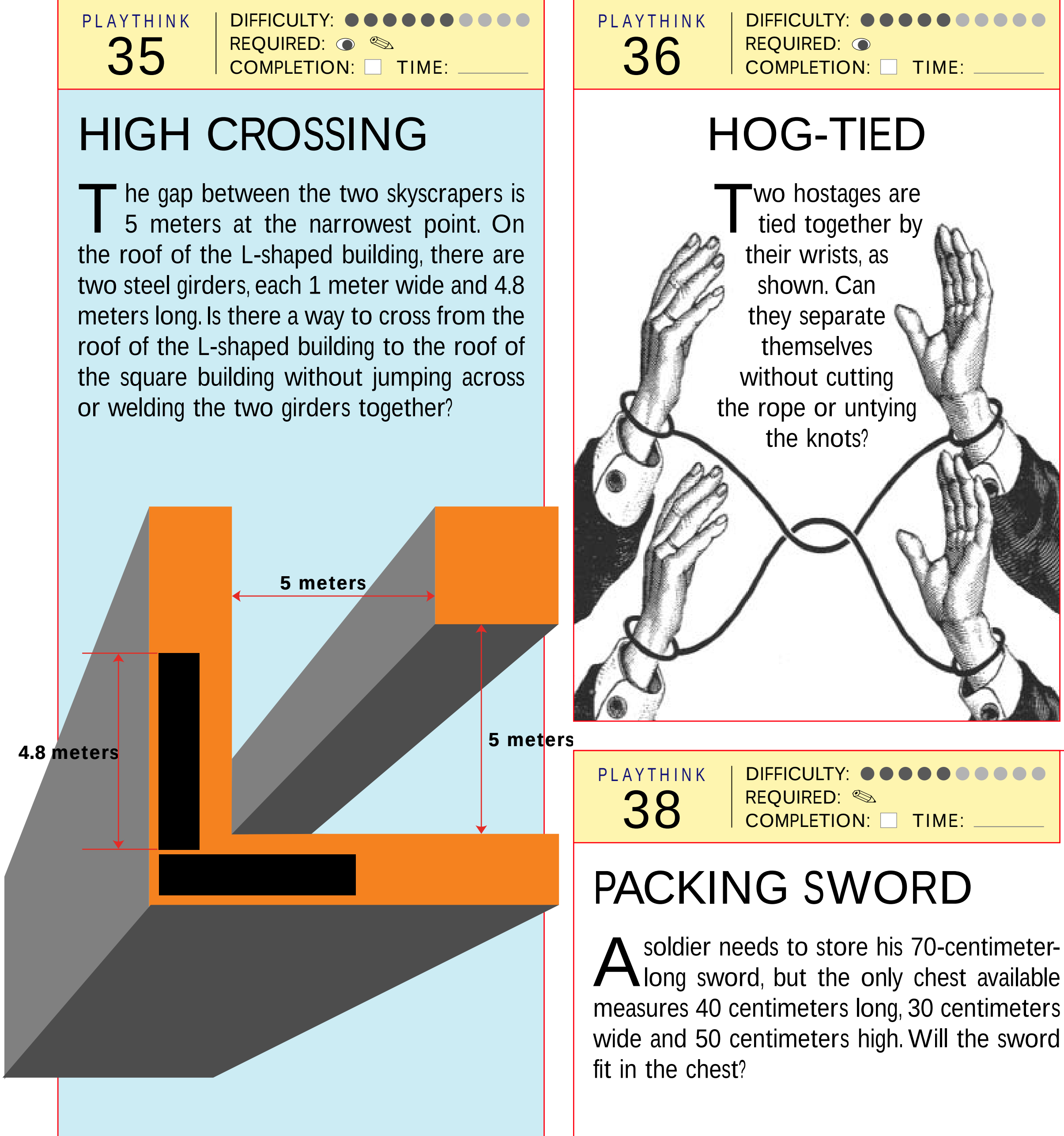
Only mathematics can be a language universal enough for both humans and extraterrestrials to understand. The interstellar greeting may not be "hello" but "one, two, three. . . ."



PLAYTHINK 35 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✏ | COMPLETION: ☐ TIME: ____

HIGH CROSSING

The gap between the two skyscrapers is 5 meters at the narrowest point. On the roof of the L-shaped building, there are two steel girders, each 1 meter wide and 4.8 meters long. Is there a way to cross from the roof of the L-shaped building to the roof of the square building without jumping across or welding the two girders together?



PLAYTHINK 36 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

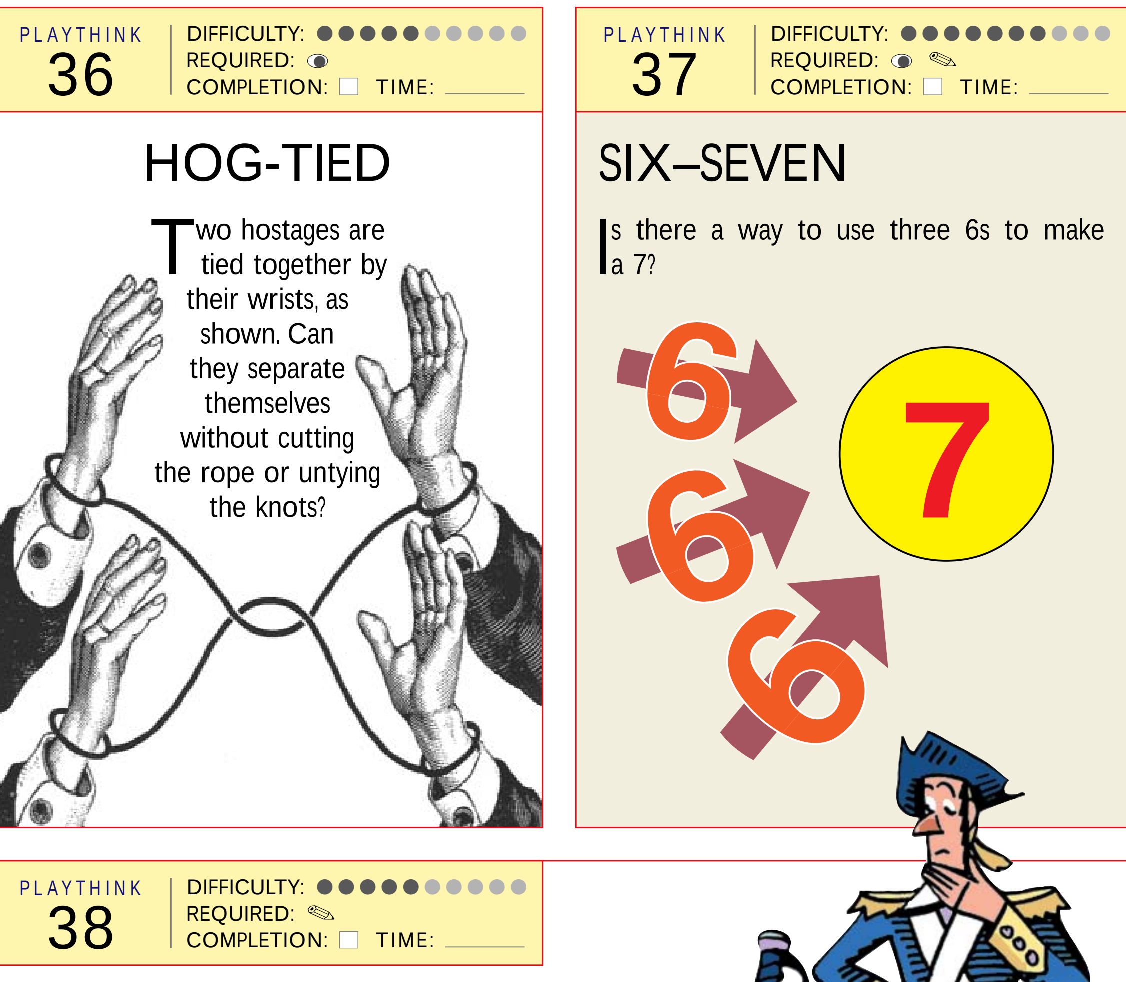
HOG-TIED

Two hostages are tied together by their wrists, as shown. Can they separate themselves without cutting the rope or untying the knots?

PLAYTHINK 37 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✏ | COMPLETION: ☐ TIME: ____

SIX–SEVEN

Is there a way to use three 6s to make a 7?



PLAYTHINK 38 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: ✏ | COMPLETION: ☐ TIME: ____

PACKING SWORD

A soldier needs to store his 70-centimeter-long sword, but the only chest available measures 40 centimeters long, 30 centimeters wide and 50 centimeters high. Will the sword fit in the chest?

PLAYTHINK 39 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✏ | COMPLETION: ☐ TIME: ____

DIVISION INTO FIVE

The colored shape is divided into four identical pieces. Can you divide the white square into five identical pieces?



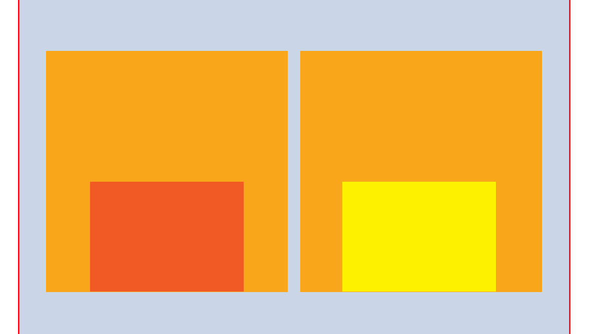
PLAYTHINK 40

DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ______

STRANGE VIEWS

The two drawings below are two views of a three-dimensional object. The drawing at left is the view from the front; the drawing on the right shows the object directly from above.

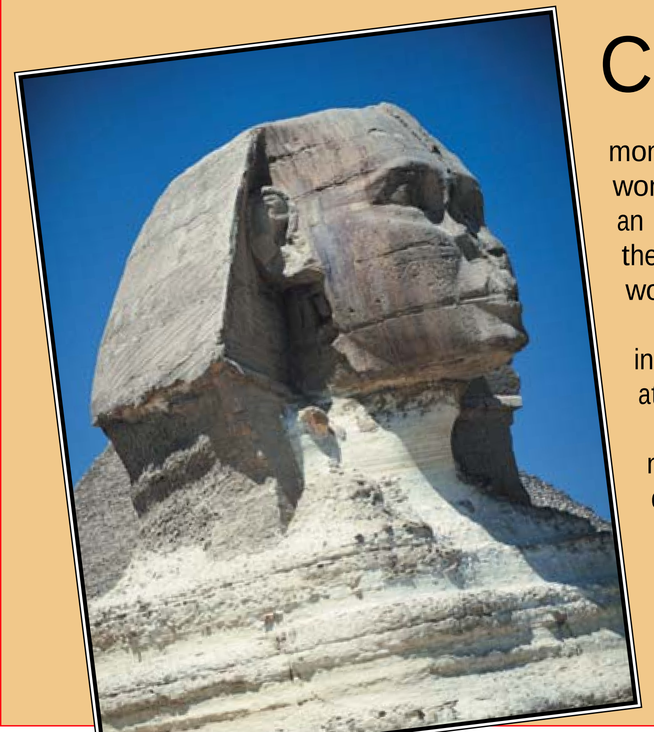
Can you work out the shape of this strange object and make a sketch of it?



PLAYTHINK 41

DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ______

RIDDLE OF THE SPHINX



Can you solve one of the greatest puzzles of antiquity?

In Greek mythology the Sphinx was a monster who possessed the head of a woman, the body of a lion and the wings of an eagle. The Sphinx guarded the gates of the city of Thebes, challenging all who would enter with this simple riddle:

"What goes on four legs in the morning, on two legs at noon, and on three legs at dusk?"

The Sphinx killed anyone who could not answer the riddle and vowed to destroy herself should anyone solve it. She had to make good on her word when Oedipus told her the answer. Can you?

PLAYTHINK 42

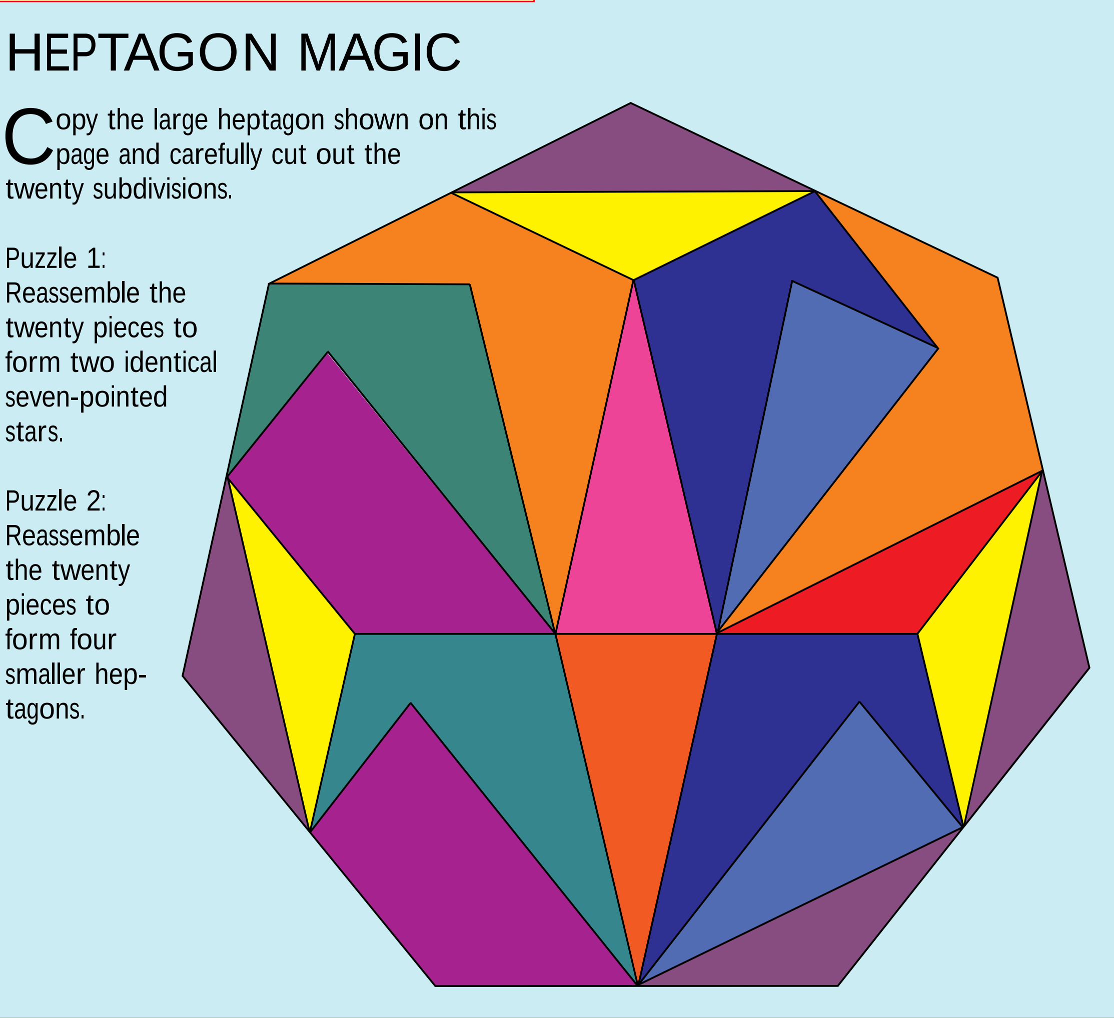
DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ______

HEPTAGON MAGIC

Copy the large heptagon shown on this page and carefully cut out the twenty subdivisions.

Puzzle 1:
Reassemble the twenty pieces to form two identical seven-pointed stars.

Puzzle 2:
Reassemble the twenty pieces to form four smaller heptagons.



PLAYTHINK 43

DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ______

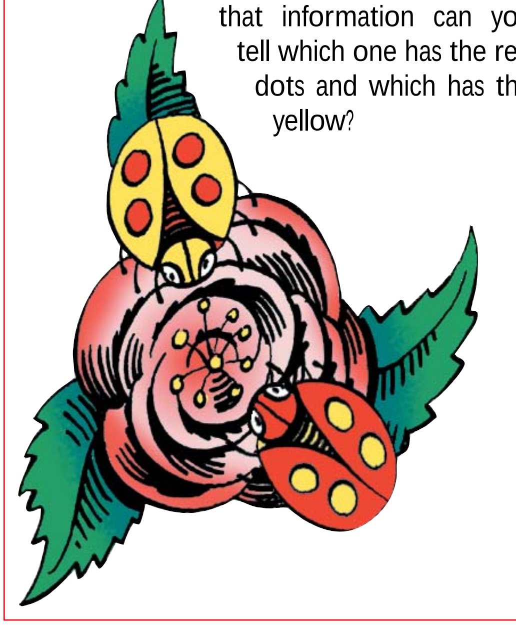
LADYBUG RENDEZVOUS

Mister Ladybug meets Miss Ladybug on the petal of a flower.

"I'm a boy," says the one with red dots.

"I'm a girl," says the one with yellow dots.

Then they both laugh because at least one of them is lying. From that information can you tell which one has the red dots and which has the yellow?



Four Stages of Problem Solving

There is no recipe for creativity. But research on the subject has identified four essential steps to creative problem solving:

Stage 1: Preparation—This requires both a short-term reading up on the problem at hand and a broader understanding that only a commitment to a well-rounded education can bring. After all, you never know where the unexpected solution to a difficult problem might lie.

Stage 2: Incubation—No one knows why getting away from a problem is useful. Some psychologists see it as a period of rest; others, as a time when you subconsciously select and discard various pieces of information. Whatever the reason, creative thinking requires some quiet, unstructured time.

Stage 3: Illumination—This is the sudden flash of insight, the proverbial light bulb glowing overhead. Some call it the "Aha!" moment.

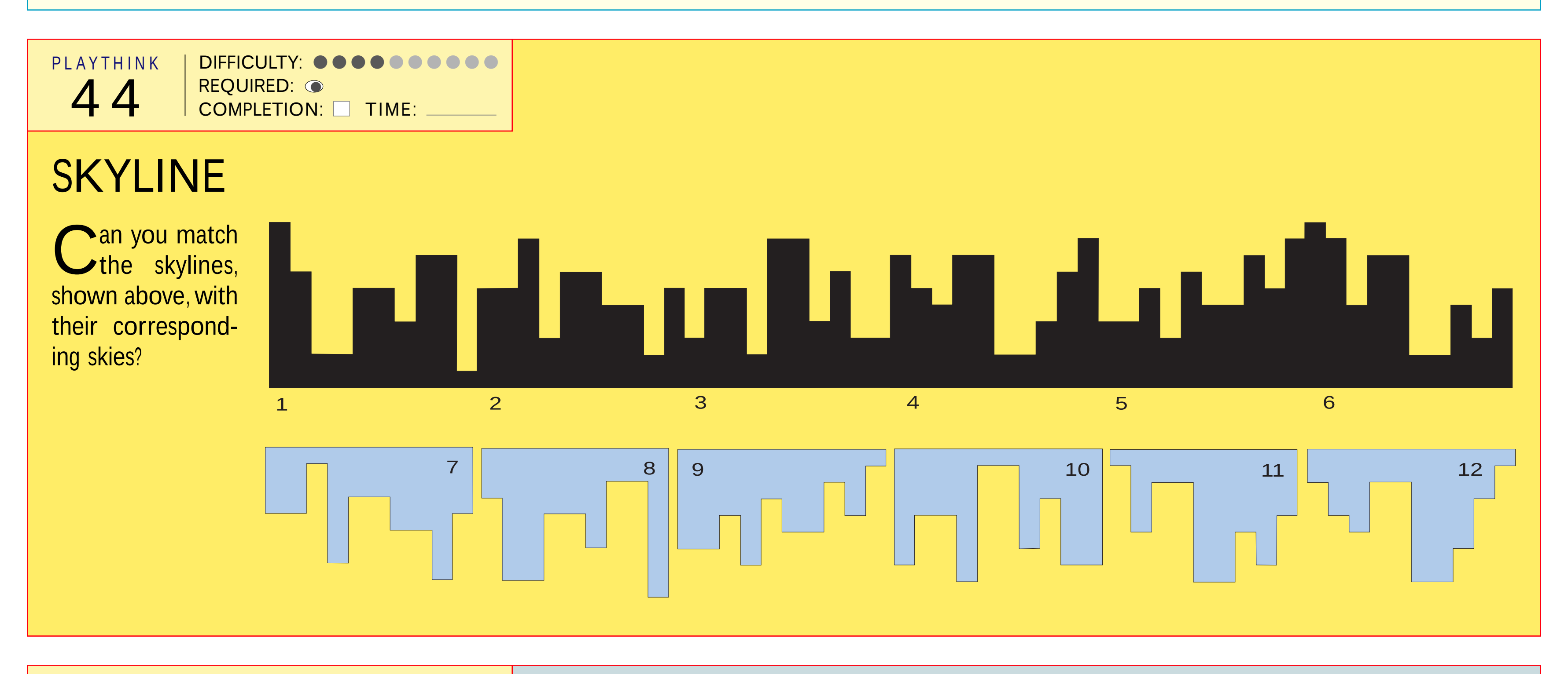
Stage 4: Elaboration—Sometimes a flash of insight is really just the flicker of a bad idea. One must always check the validity of an answer. And then comes the most important part: explaining the solution in a way that can be understood by others.

PLAYTHINK 44

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

SKYLINE

Can you match the skylines, shown above, with their corresponding skies?



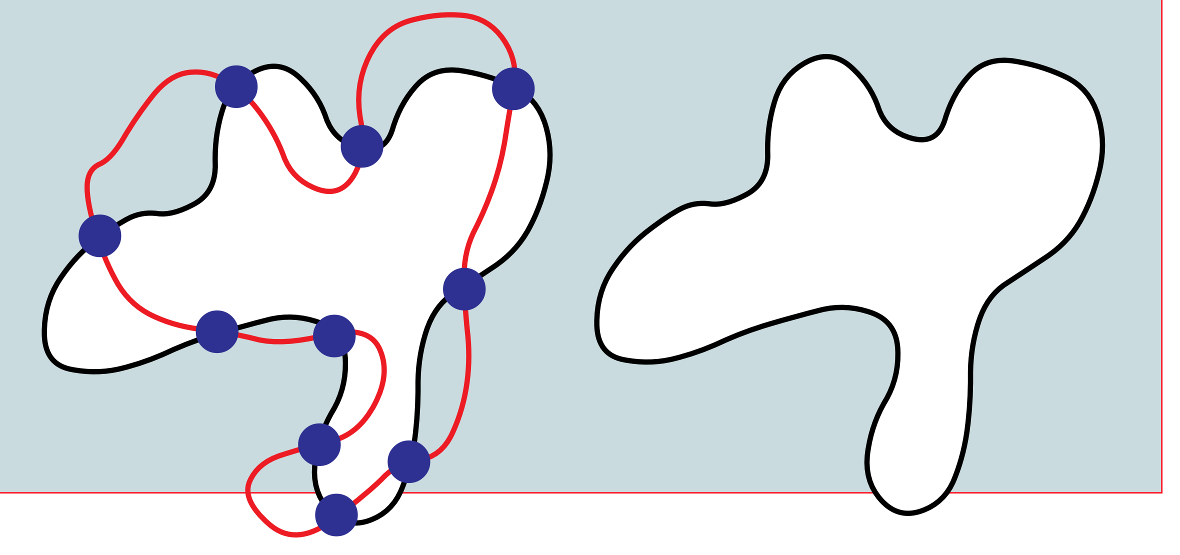
PLAYTHINK 45

DIFFICULTY: ●●○○○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

ODD INTERSECTION

The red closed line is drawn so that it crosses a black closed line from inside to outside or vice versa exactly ten times.

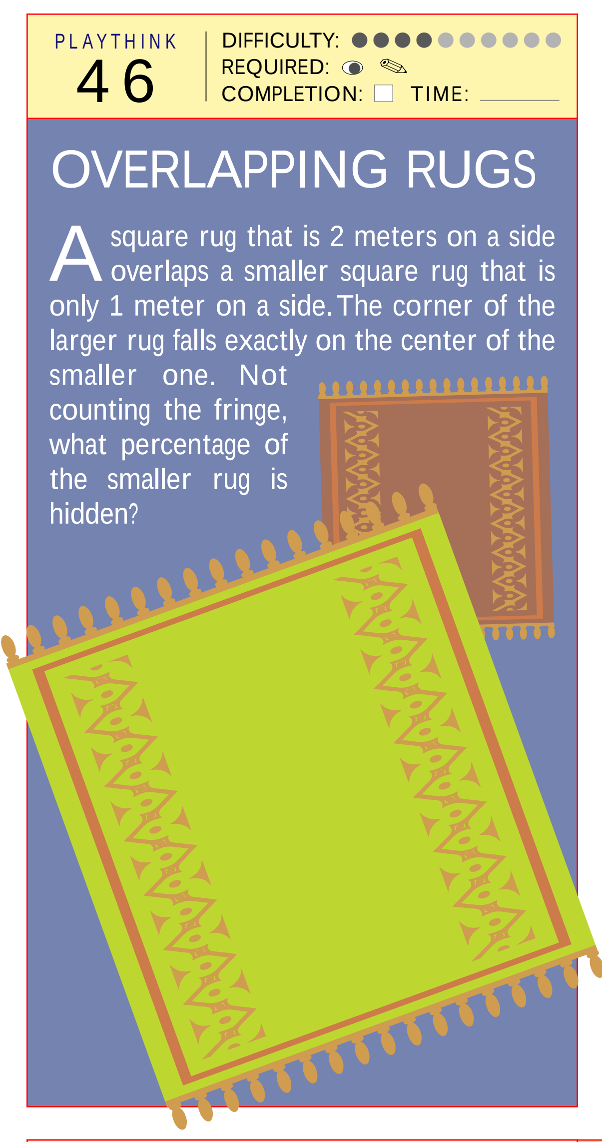
Can you draw a new red closed line over the same black line so that it makes only nine crossings?



PLAYTHINK 46
DIFFICULTY: ●●●●○○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

OVERLAPPING RUGS

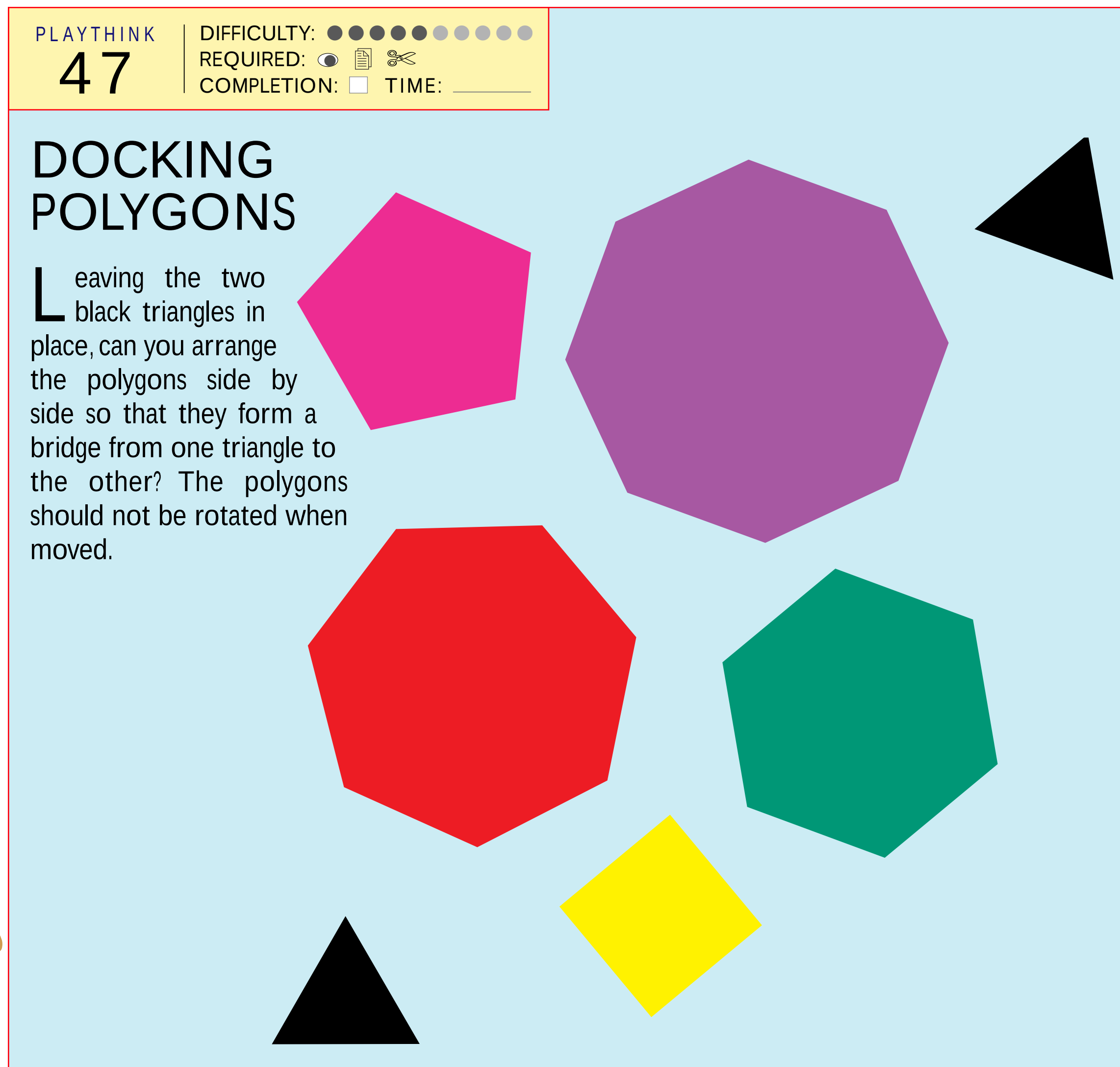
A square rug that is 2 meters on a side overlaps a smaller square rug that is only 1 meter on a side. The corner of the larger rug falls exactly on the center of the smaller one. Not counting the fringe, what percentage of the smaller rug is hidden?



PLAYTHINK 47
DIFFICULTY: ●●●●●○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

DOCKING POLYGONS

Leaving the two black triangles in place, can you arrange the polygons side by side so that they form a bridge from one triangle to the other? The polygons should not be rotated when moved.



PLAYTHINK 48
DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

MURPHY'S LAW OF SOCKS

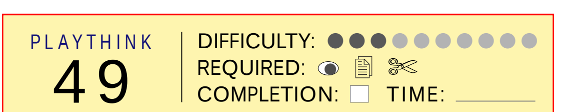
Imagine that after washing five pairs of socks you discover that two socks are missing. Which scenario is more likely:

A. The two missing socks make a complete pair and you are left with four complete pairs.

B. You are now left with three pairs of socks and two orphan socks.

Captain Edward A. Murphy stated, "Anything that can go wrong will, and at the worst possible time." Does Murphy's law rule the sock drawer?





PLAYTHINK 49
DIFFICULTY: ●●●○○○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

HOLE IN A POSTCARD

Can you make a hole in a postcard large enough for a man to step through?



PLAYTHINK 50
DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

PHONE NUMBER

A man and a woman meet at a bar. After a long conversation they agree to have dinner the next day if the man remembers to call the woman to confirm the date. The next morning the man discovers that he can remember the digits in her number—2, 3, 4, 5, 6, 7 and 8—but he has completely forgotten their order.

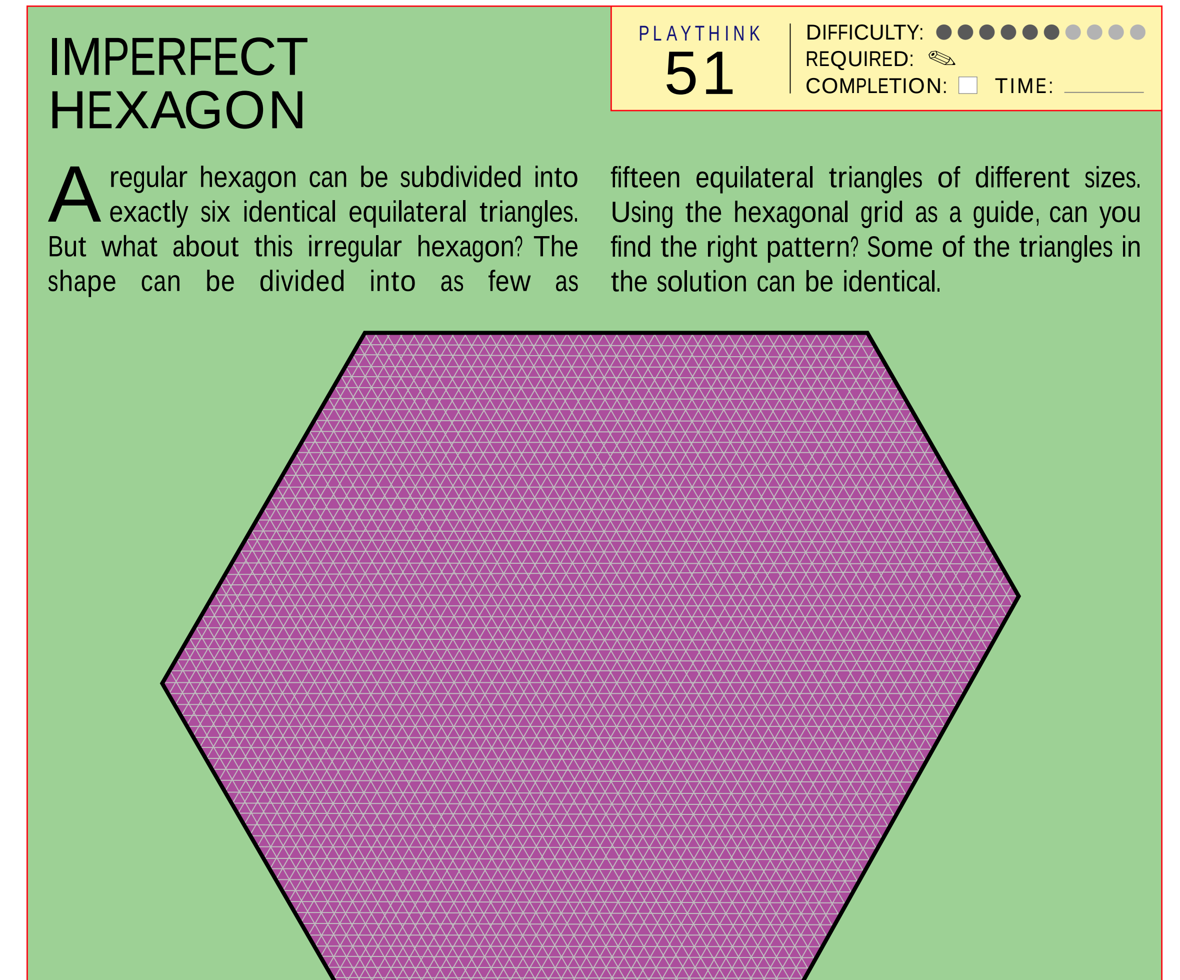
If he decides to arrange the seven digits in random order and dial every combination, what are the chances that any given phone number will be hers?



IMPERFECT HEXAGON

PLAYTHINK 51
DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

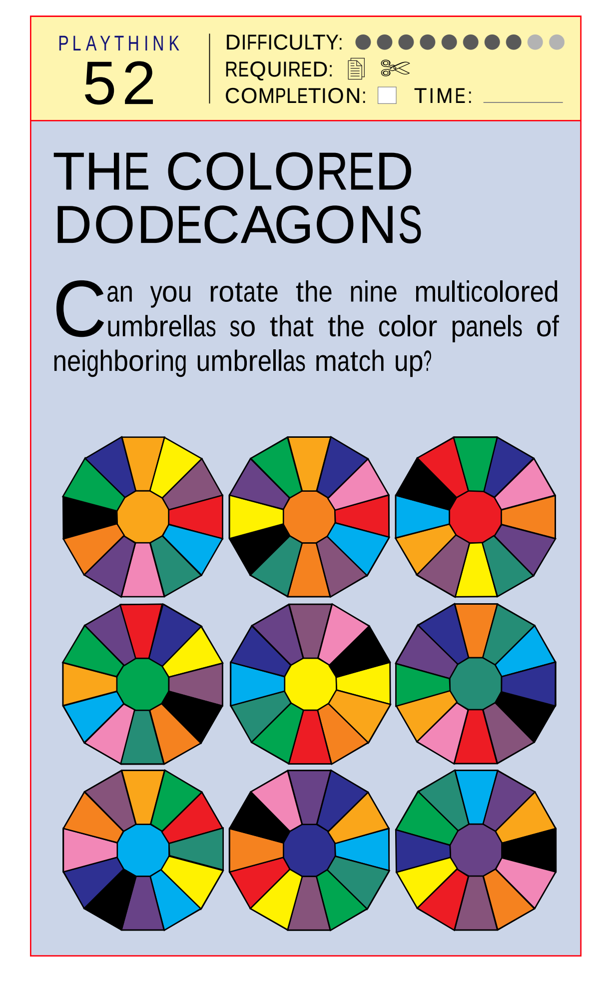
A regular hexagon can be subdivided into exactly six identical equilateral triangles. But what about this irregular hexagon? The shape can be divided into as few as fifteen equilateral triangles of different sizes. Using the hexagonal grid as a guide, can you find the right pattern? Some of the triangles in the solution can be identical.



PLAYTHINK 52
DIFFICULTY: ●●●●●●●●○○
REQUIRED:
COMPLETION: ☐ TIME: ____

THE COLORED DODECAGONS

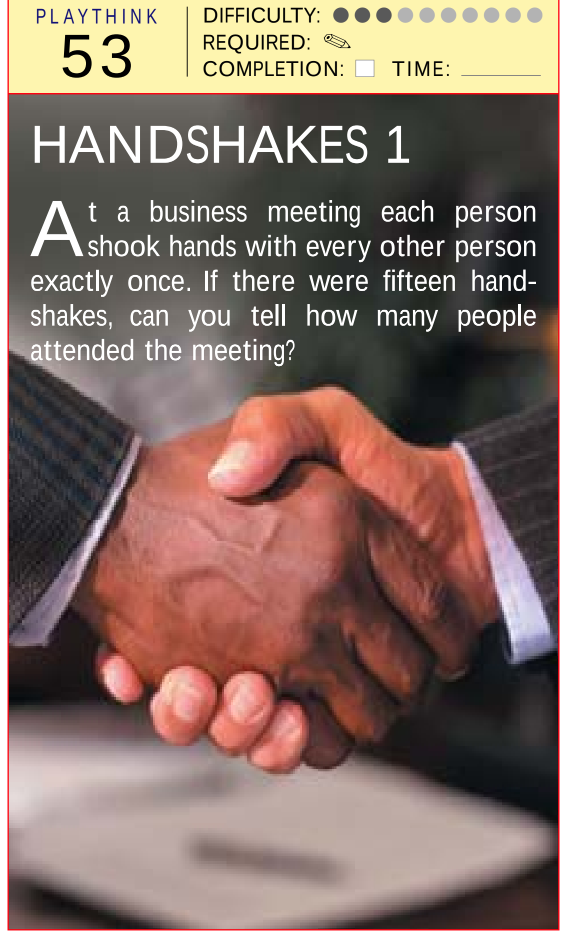
Can you rotate the nine multicolored umbrellas so that the color panels of neighboring umbrellas match up?



PLAYTHINK 53
DIFFICULTY: ●●●○○○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

HANDSHAKES 1

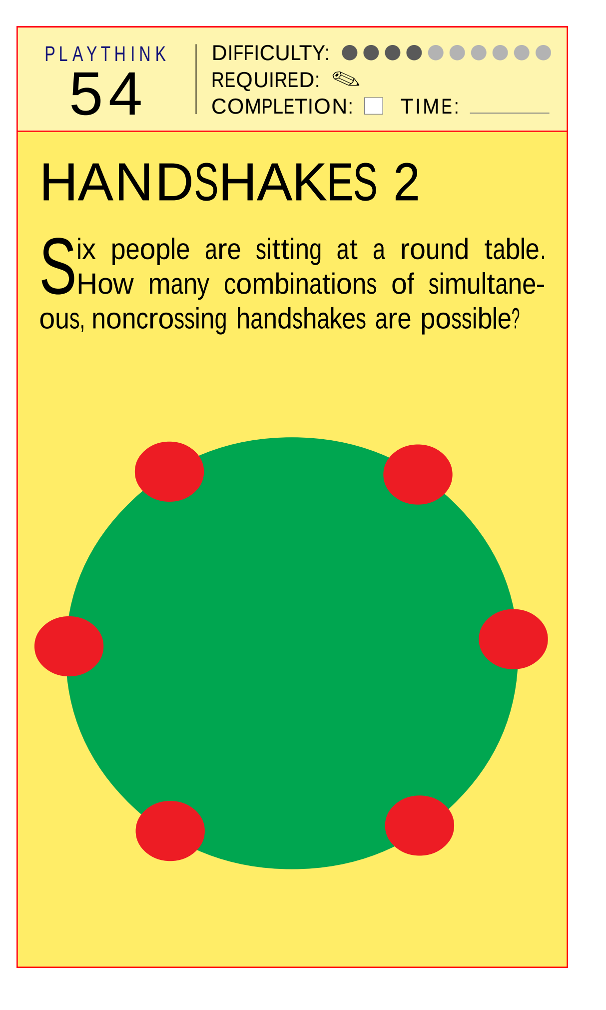
At a business meeting each person shook hands with every other person exactly once. If there were fifteen handshakes, can you tell how many people attended the meeting?



PLAYTHINK 54
DIFFICULTY: ●●●●○○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

HANDSHAKES 2

Six people are sitting at a round table. How many combinations of simultaneous, noncrossing handshakes are possible?

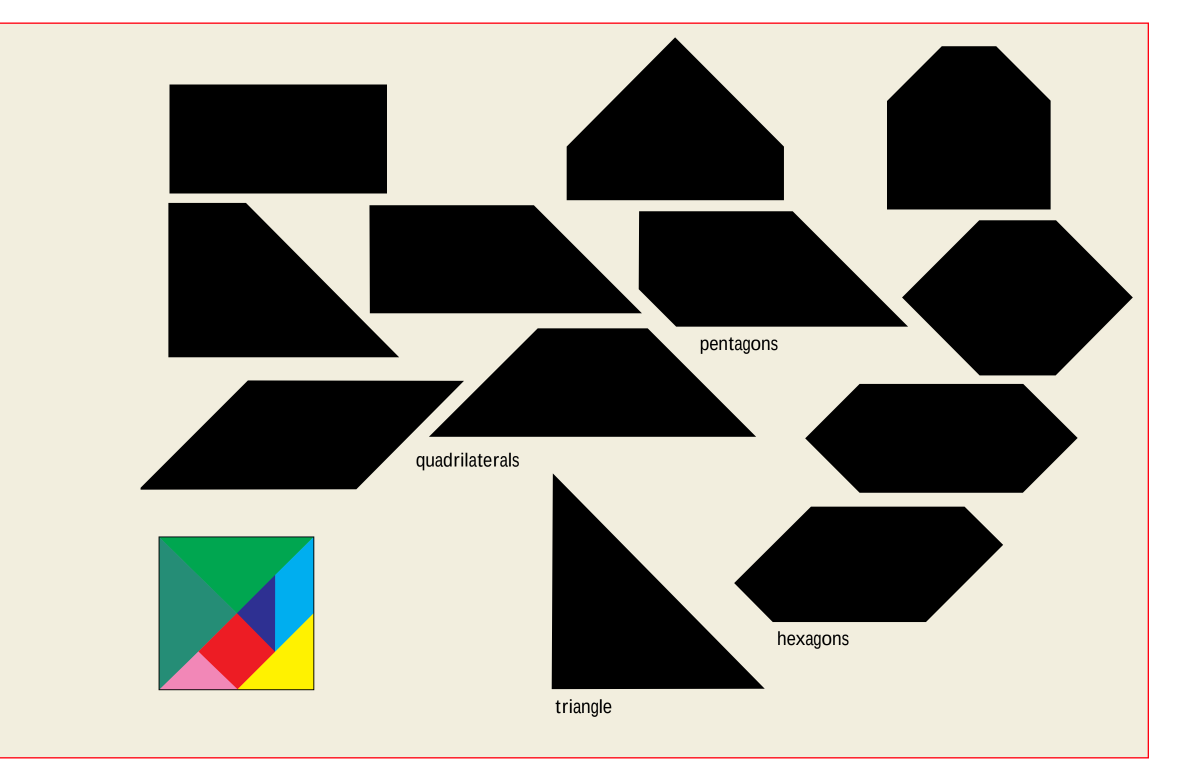


PLAYTHINK 55

DIFFICULTY: ●●●●●●●●●○
REQUIRED:
COMPLETION: ☐ TIME: ______

TANGRAM POLYGONS

A tangram is a set of seven three- and four-sided puzzle pieces that can be combined to form a number of complex shapes. In 1942 the Chinese mathematicians Fu Traing and Chuan Chih proved that the seven tangram pieces can form exactly thirteen different convex polygons: one triangle, six quadrilaterals, two pentagons and four hexagons. The thirteen polygons are shown and the tangram pieces have been placed on one of the quadrilaterals (a square) to demonstrate the principle. Can you arrange the tangram pieces to form the other twelve polygons?

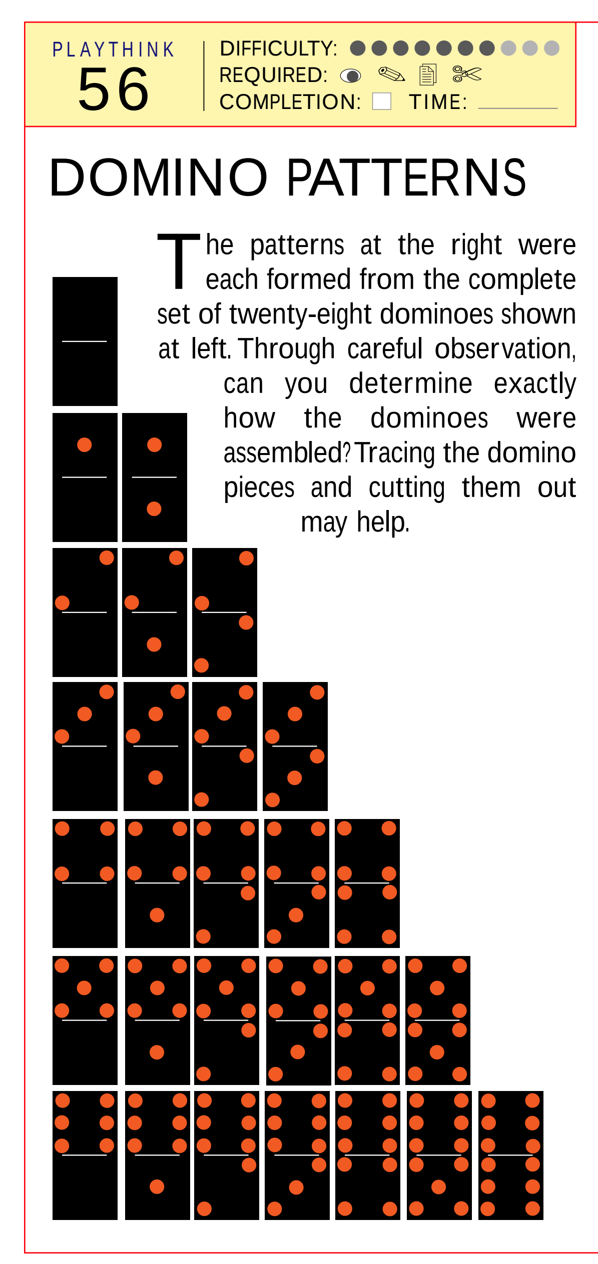


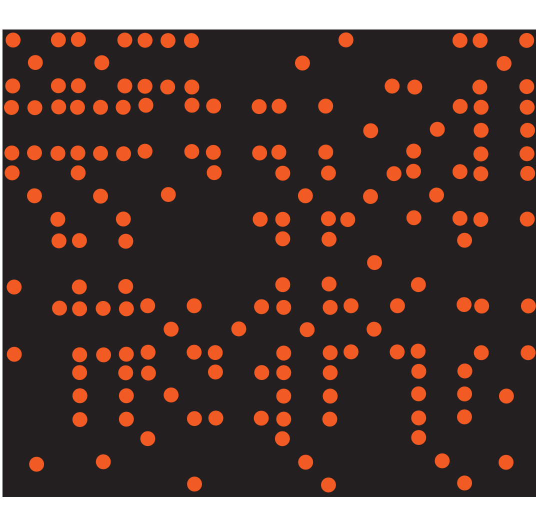
PLAYTHINK 56

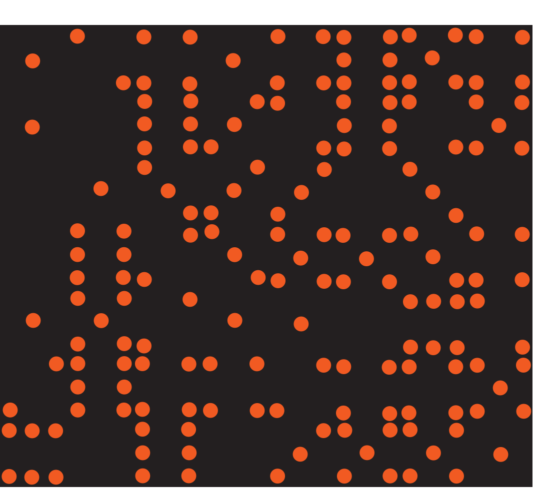
DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

DOMINO PATTERNS

The patterns at the right were each formed from the complete set of twenty-eight dominoes shown at left. Through careful observation, can you determine exactly how the dominoes were assembled? Tracing the domino pieces and cutting them out may help.







PLAYTHINK 57

DIFFICULTY: ●●●○○○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______



LAST MAN

Imagine you are the editor at a science fiction magazine and you read the following lines at the beginning of a story:

"The last man on earth sat alone in his room. Suddenly there was a knock at the door!"

Can you change one word in the first sentence to make the man's isolation before the knock at the door more complete?



PLAYTHINK 58
DIFFICULTY:
REQUIRED:
COMPLETION: TIME:

HOTEL KEYS

A porter leads eight guests to their hotel rooms, rooms 1 through 8. Unfortunately, the keys are unlabeled and the porter has mixed up their order. Using trial and error, what is the maximum number of attempts the porter must make before he opens all the doors?

PLAYTHINK 59
DIFFICULTY:
REQUIRED:
COMPLETION: TIME:

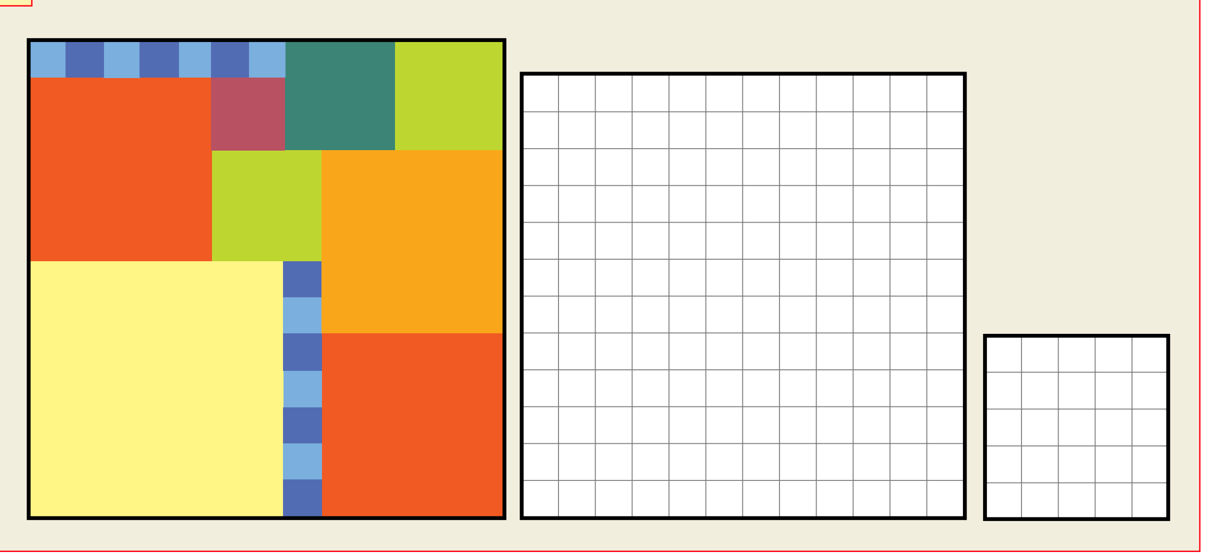
MISSING FRACTIONS

Can you determine the logic to the pattern and use that knowledge to fill in the missing squares?

PLAYTHINK 60
DIFFICULTY:
REQUIRED:
COMPLETION: TIME:

SQUARE SPLIT

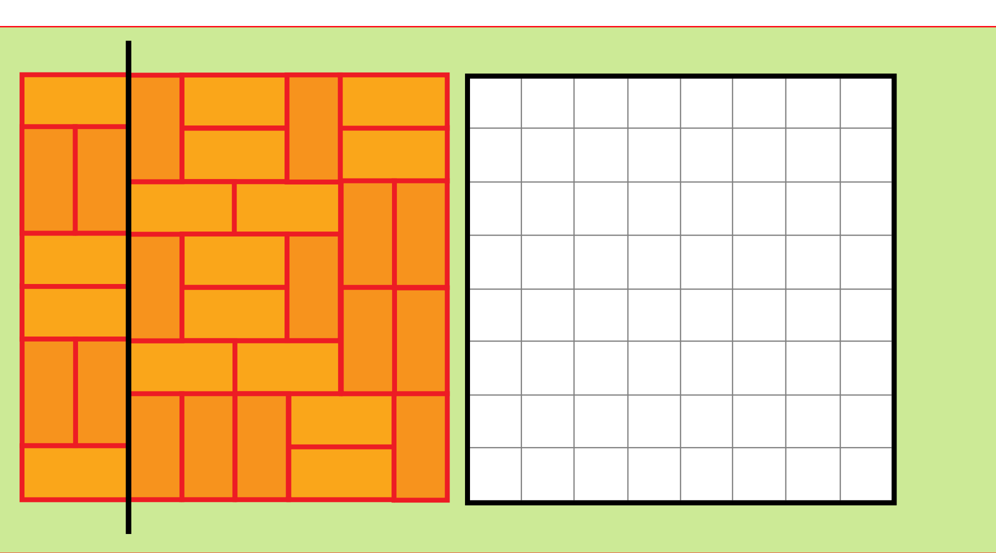
Can you rearrange the twenty-two square pieces that comprise the square on the left to make the two squares at the right?



PLAYTHINK 61
DIFFICULTY:
REQUIRED:
COMPLETION: TIME:

FAULT-FREE SQUARE

The manner in which the one-by-two bricks were packed into a square has created a so-called fault line—a straight line of edges that runs from one side to the other. To create a stronger structure, can you repack the bricks into the square so that it is free of faults?



PLAYTHINK 62

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

PENTAHEXES

Five regular hexagons can be joined side to side to create a shape called a pentahex. There are twenty-two possible pentahex shapes, eleven of which are used to form the honeycomb at left. (To make it easier to find them, each pentahex is outlined with a heavy boundary.) Can you tell which of the four pentahexes shown at the right were not used to make the honeycomb?

1

2

3

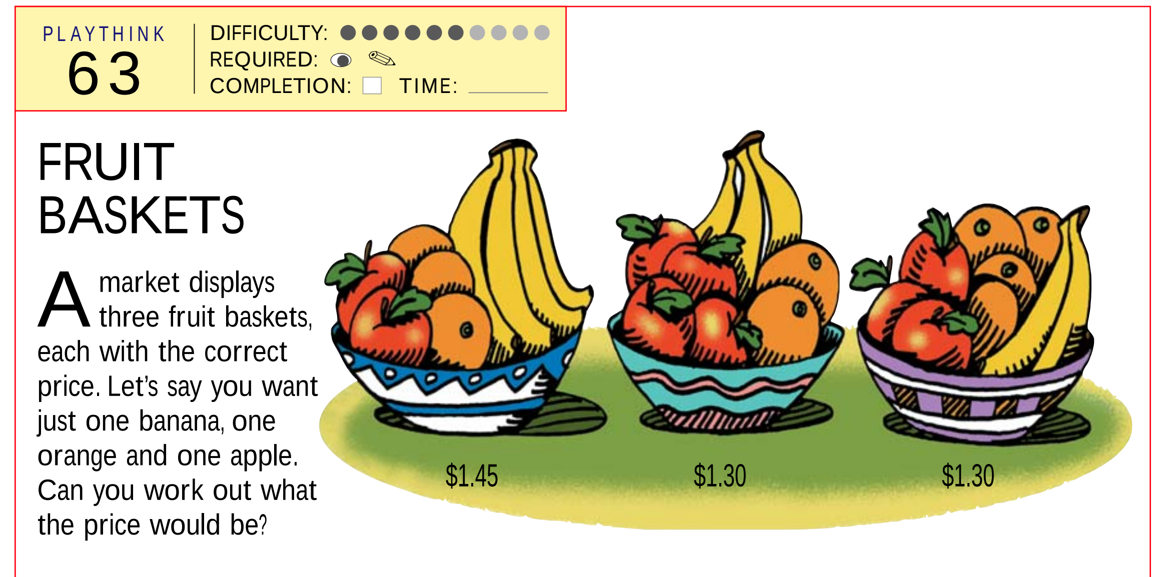
4

PLAYTHINK 63

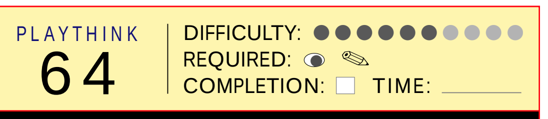
DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

FRUIT BASKETS

A market displays three fruit baskets, each with the correct price. Let's say you want just one banana, one orange and one apple. Can you work out what the price would be?



PLAYTHINK 64



DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

FAMILY REUNION

One grandfather, one grandmother, two fathers, two mothers, four children, three grandchildren, one brother, two sisters, two sons, two daughters, one father-in-law, one mother-in-law and one daughter-in-law attended a family reunion. If both halves of each relationship attended (i.e., the father *and* the son), how many people showed up?

Games vs. Puzzles

Adults can continue to delight in the science of patterns by solving puzzles (which, if well constructed, have one solution) and playing games (which can end in many ways). The boundary between the two, however, is not entirely clear-cut. Mathematicians have studied many simple games and found strategies that never fail to bring victory to one player. For example, if he or she plays properly, the first player will never lose a game of tic-tac-toe. Indeed, when fully understood, simple, well-designed games can seem very much like puzzles.

PLAYTHINK 65 | DIFFICULTY: ●●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

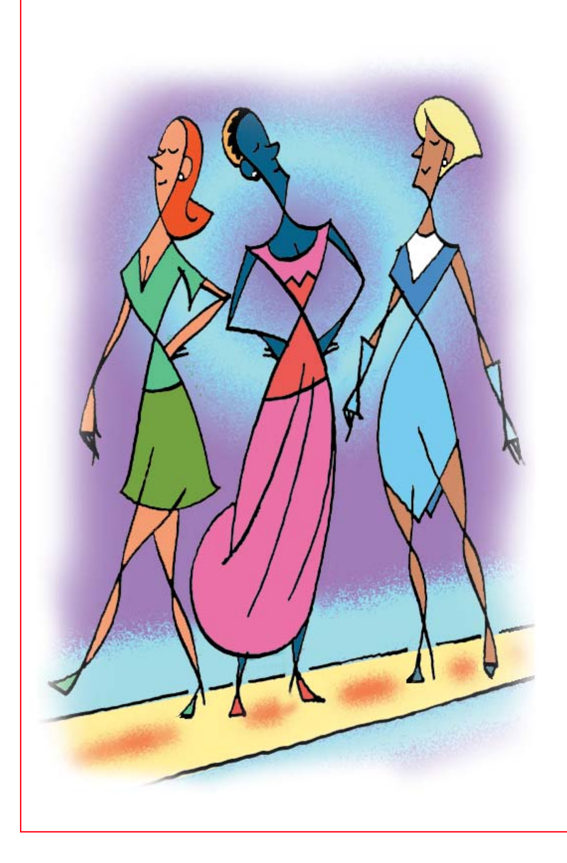
FASHION SHOW

Three models—Miss Pink, Miss Green and Miss Blue—are on the catwalk. Their dresses are solid pink, solid green and solid blue.

"It's strange," Miss Blue remarks to the others. "We are named Pink, Green and Blue, and our dresses are pink, green and blue, but none of us is wearing the dress that matches her name."

"That *is* a coincidence," says the woman in green.

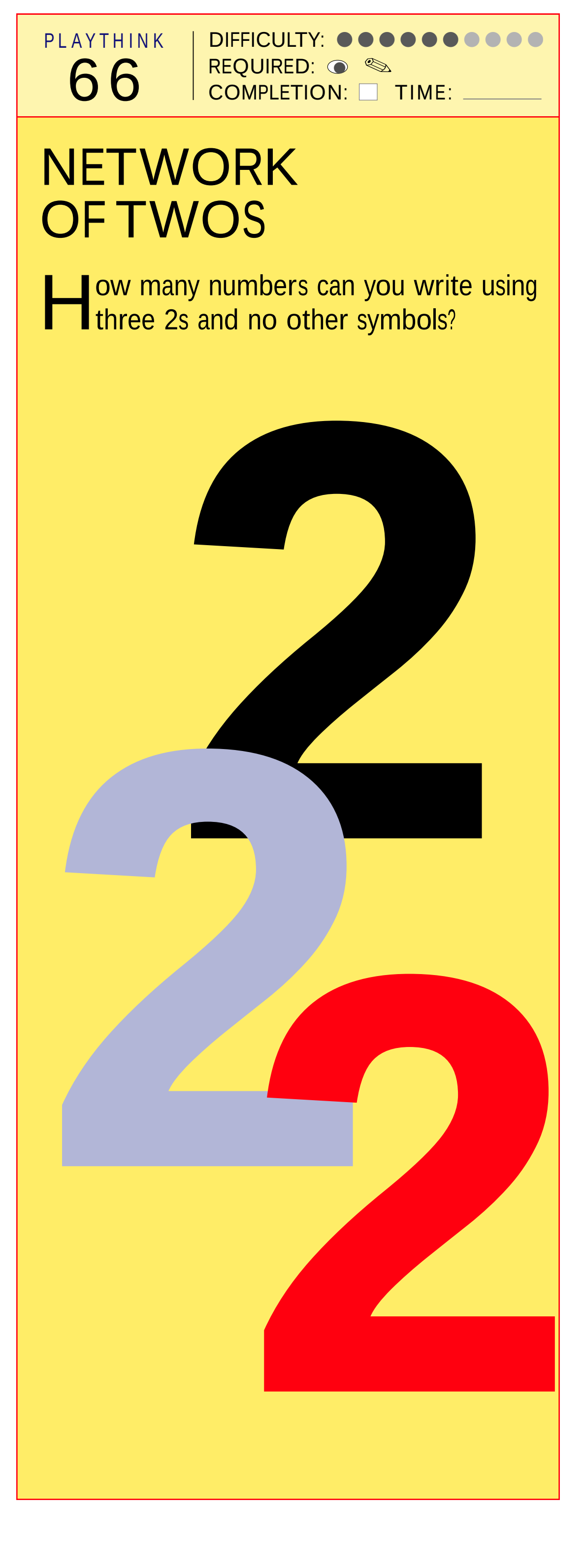
From that information, can you determine the color of each model's dress?



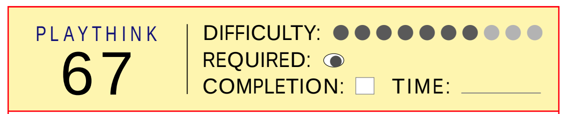
PLAYTHINK 66 | DIFFICULTY: ●●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

NETWORK OF TWOS

How many numbers can you write using three 2s and no other symbols?



PLAYTHINK 67 | DIFFICULTY: ●●●●●●●●○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____



PIGGY BANKS

Three nickels and three dimes are distributed among three piggy banks so that each bank holds two coins. Although each bank has a number of cents printed on its side, all three banks are mislabeled. Is it possible to determine how to correctly relabel the banks simply by shaking one of the banks until one of the coins drops out? If so, explain how.



PLAYTHINK 68

DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

OVERLAPPING SQUARES

Fourteen identical squares have been placed one on top of the other to form this rectangular pattern. Can you determine the order in which the squares must have been laid down? Start with the bottommost and work your way up.

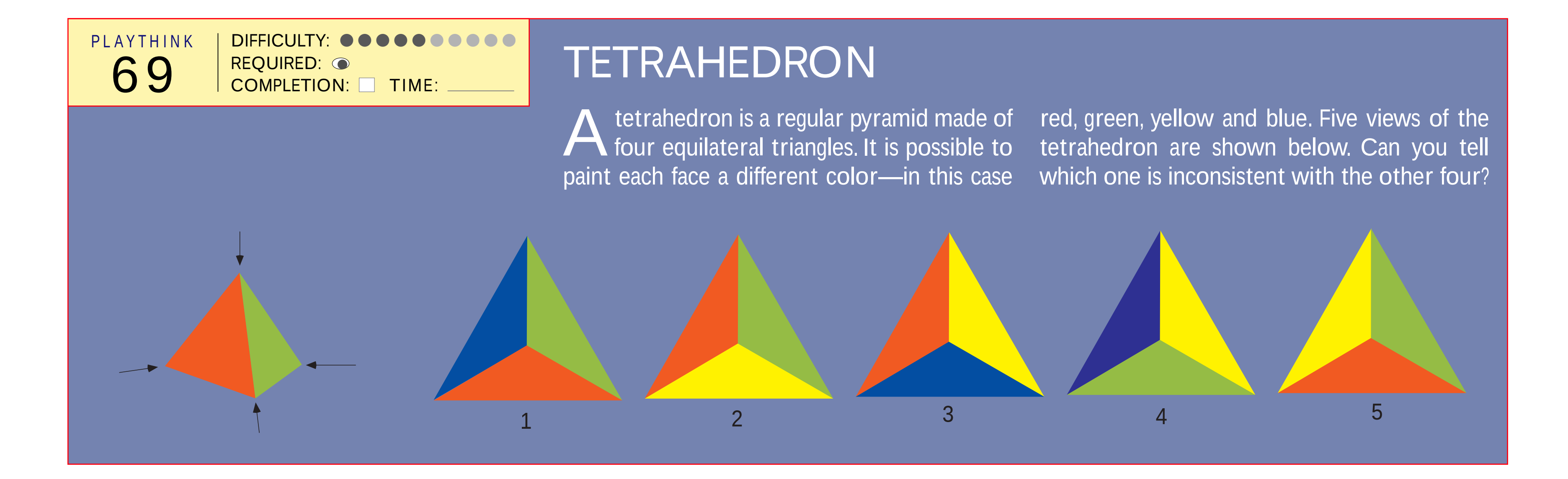
> "WHENEVER YOU LOOK AT A PIECE OF WORK AND YOU THINK THE FELLOW WAS CRAZY, THEN YOU WANT TO PAY SOME ATTENTION. . . . ONE OF YOU IS LIKELY TO BE, AND YOU HAD BETTER FIND OUT WHICH ONE. . . . IT MAKES AN AWFUL LOT OF DIFFERENCE."
>
> —CHARLES FRANKLIN KETTERING

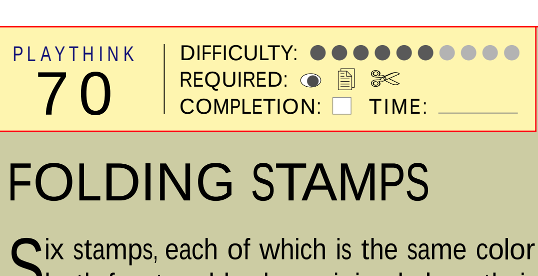
PLAYTHINK 69

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

TETRAHEDRON

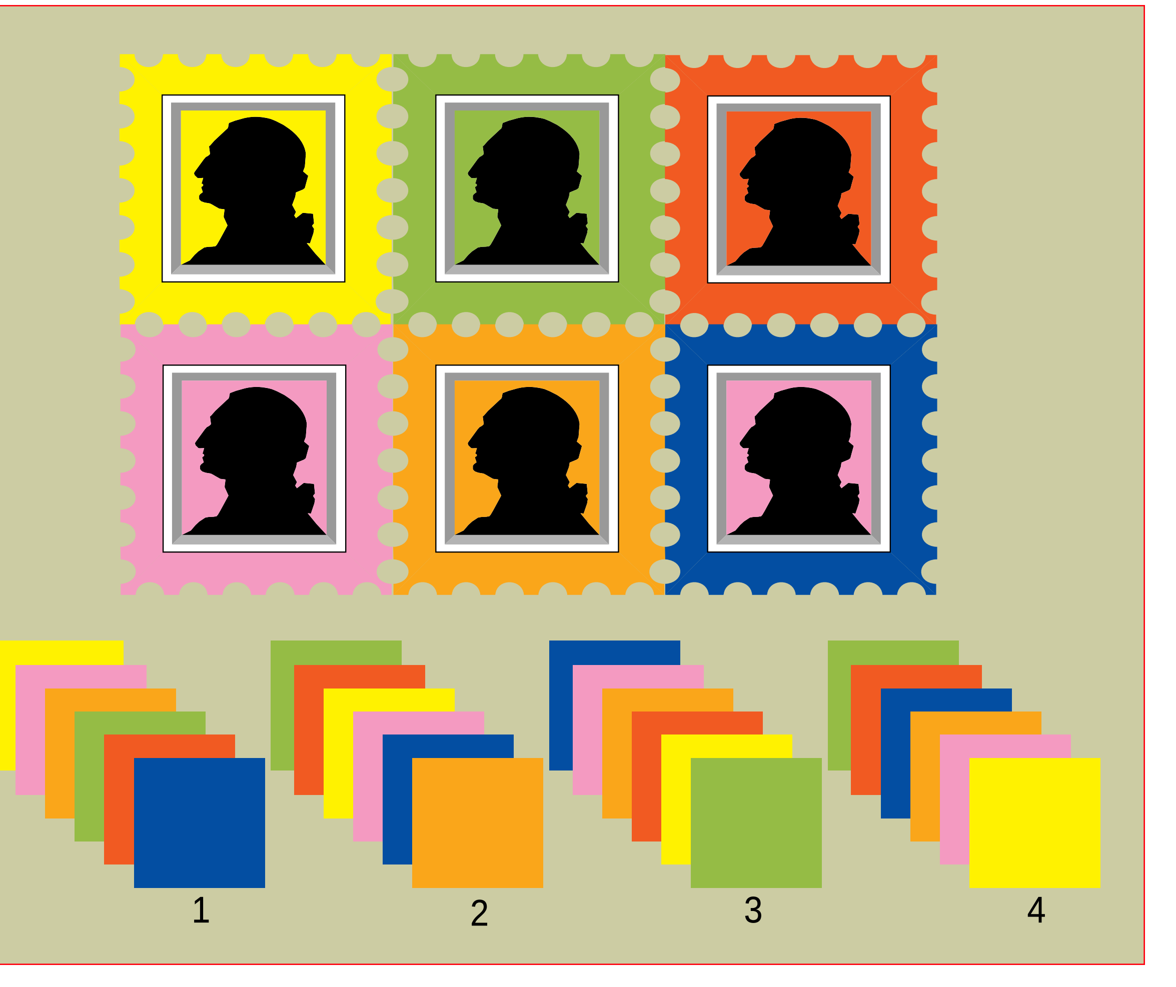
A tetrahedron is a regular pyramid made of four equilateral triangles. It is possible to paint each face a different color—in this case red, green, yellow and blue. Five views of the tetrahedron are shown below. Can you tell which one is inconsistent with the other four?

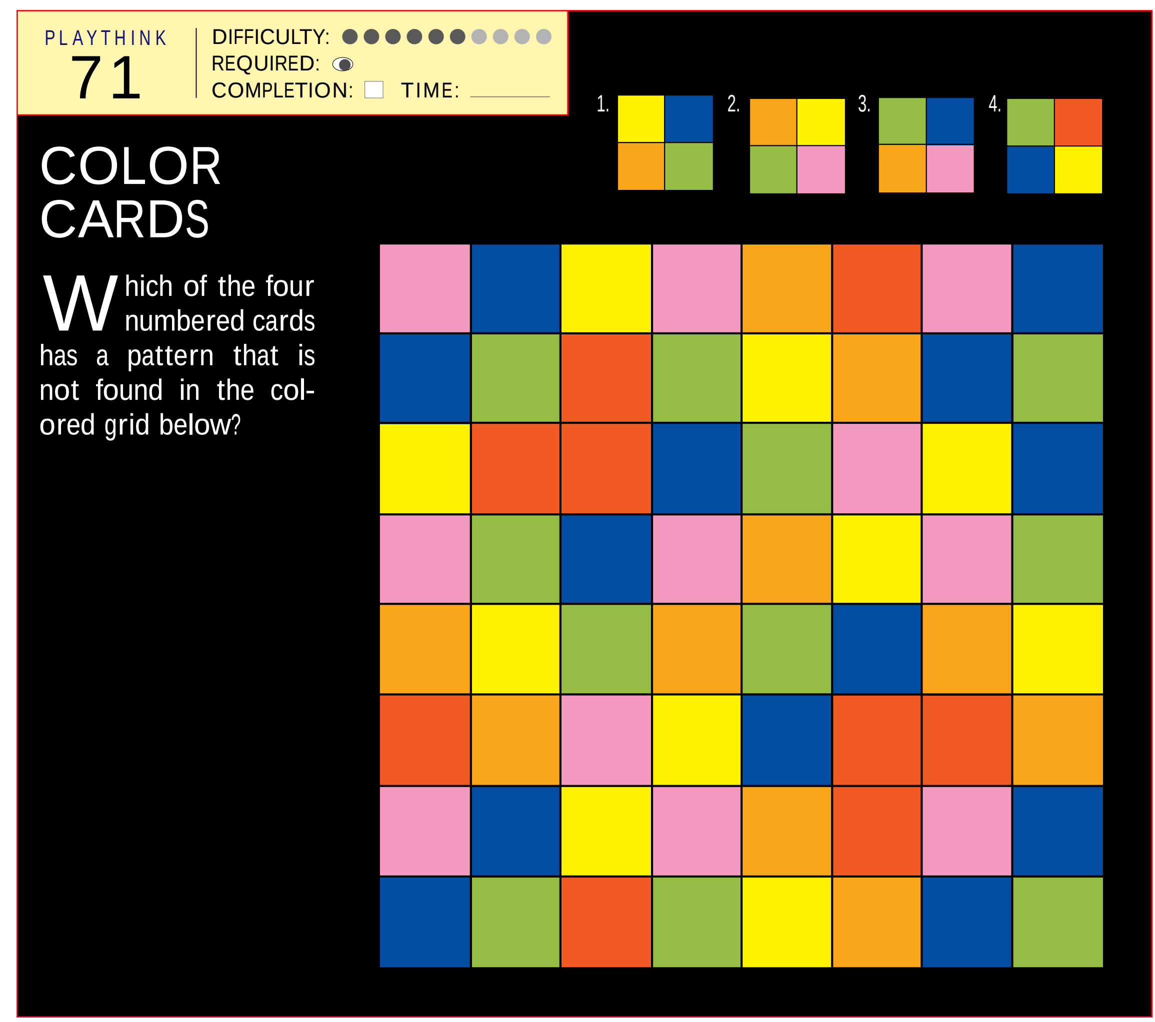




FOLDING STAMPS

Six stamps, each of which is the same color both front and back, are joined along their edges to form a two-by-three sheet. That sheet, however, can be folded along the perforations to create a stack of stamps. Of the four stacks shown, which is impossible to form by folding the sheet?





COLOR CARDS

Which of the four numbered cards has a pattern that is not found in the colored grid below?

PLAYTHINK 72
DIFFICULTY:
REQUIRED:
COMPLETION: TIME:

CRYPTOGRAM

This message has been encrypted with a simple cipher. Can you break the code to discover the three secret words?

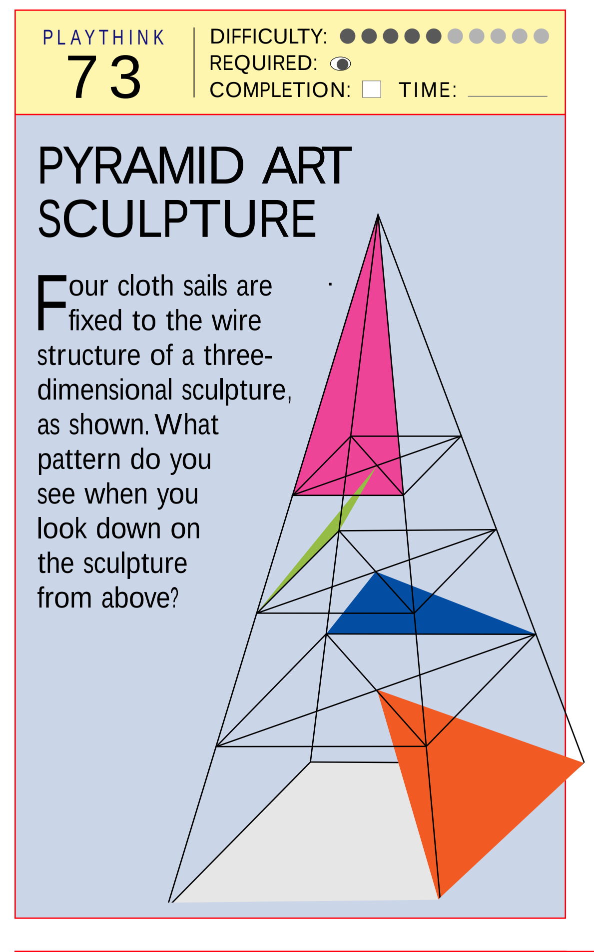
POF UIPVTBOE
QMBZUIJOLT

PLAYTHINK 73

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

PYRAMID ART SCULPTURE

Four cloth sails are fixed to the wire structure of a three-dimensional sculpture, as shown. What pattern do you see when you look down on the sculpture from above?

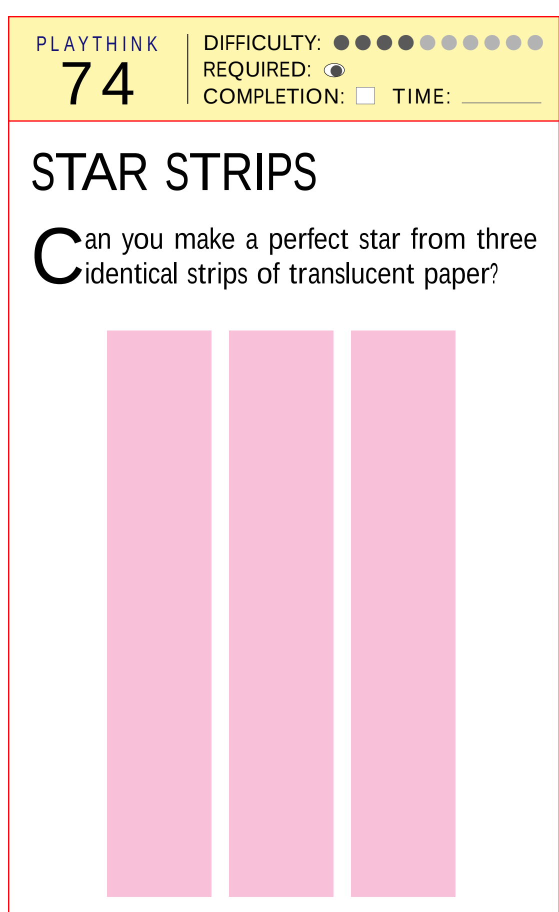


PLAYTHINK 74

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

STAR STRIPS

Can you make a perfect star from three identical strips of translucent paper?

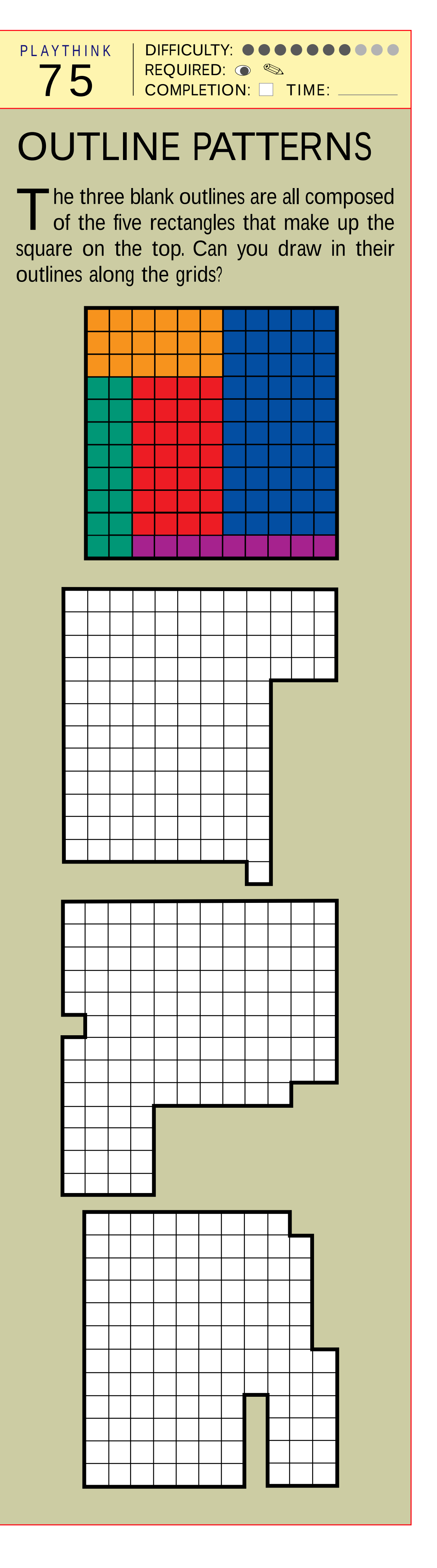


PLAYTHINK 75

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

OUTLINE PATTERNS

The three blank outlines are all composed of the five rectangles that make up the square on the top. Can you draw in their outlines along the grids?



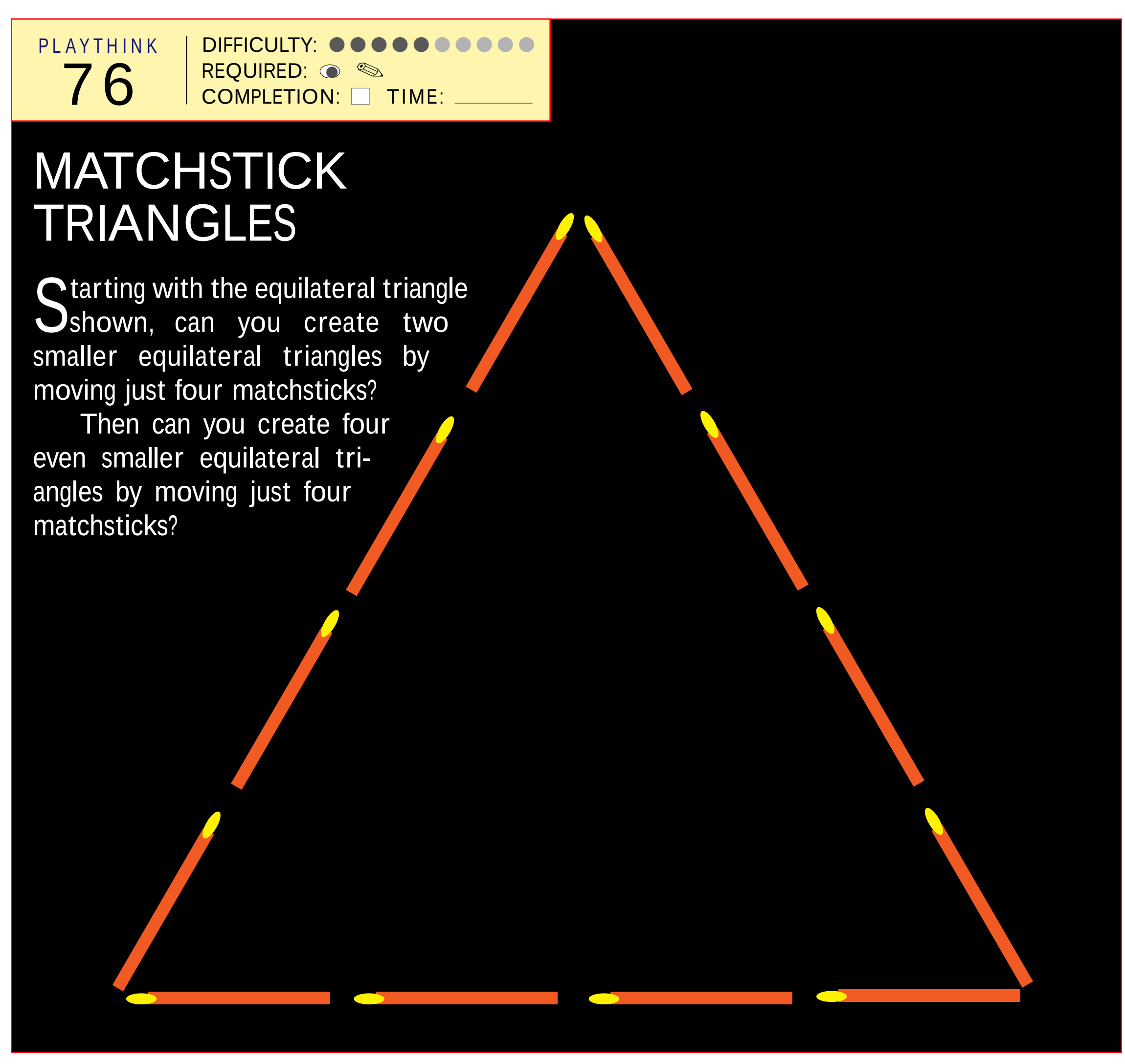
PLAYTHINK 76

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

MATCHSTICK TRIANGLES

Starting with the equilateral triangle shown, can you create two smaller equilateral triangles by moving just four matchsticks?

Then can you create four even smaller equilateral triangles by moving just four matchsticks?



PLAYTHINK 77

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

PAIRS IN ROWS AND COLUMNS

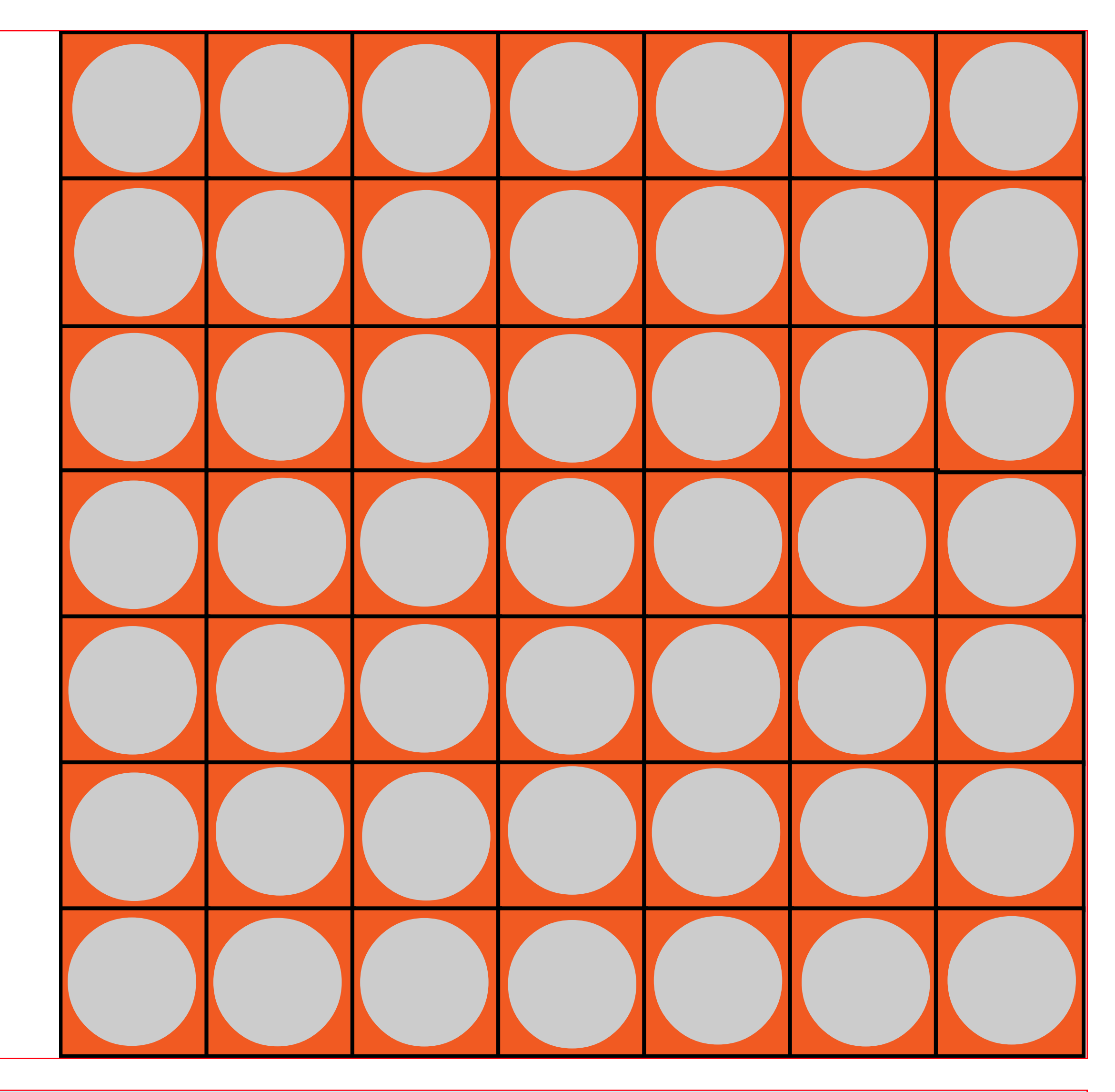
The object of this game is to place twenty-one small coins on the game board so that they satisfy the following conditions:

Each row must contain three coins.

Each column must contain three coins.

When you compare any two rows or columns, there can be only one pair of adjacent coins vertically (for rows) or horizontally (for columns).

How quickly can you win this game?

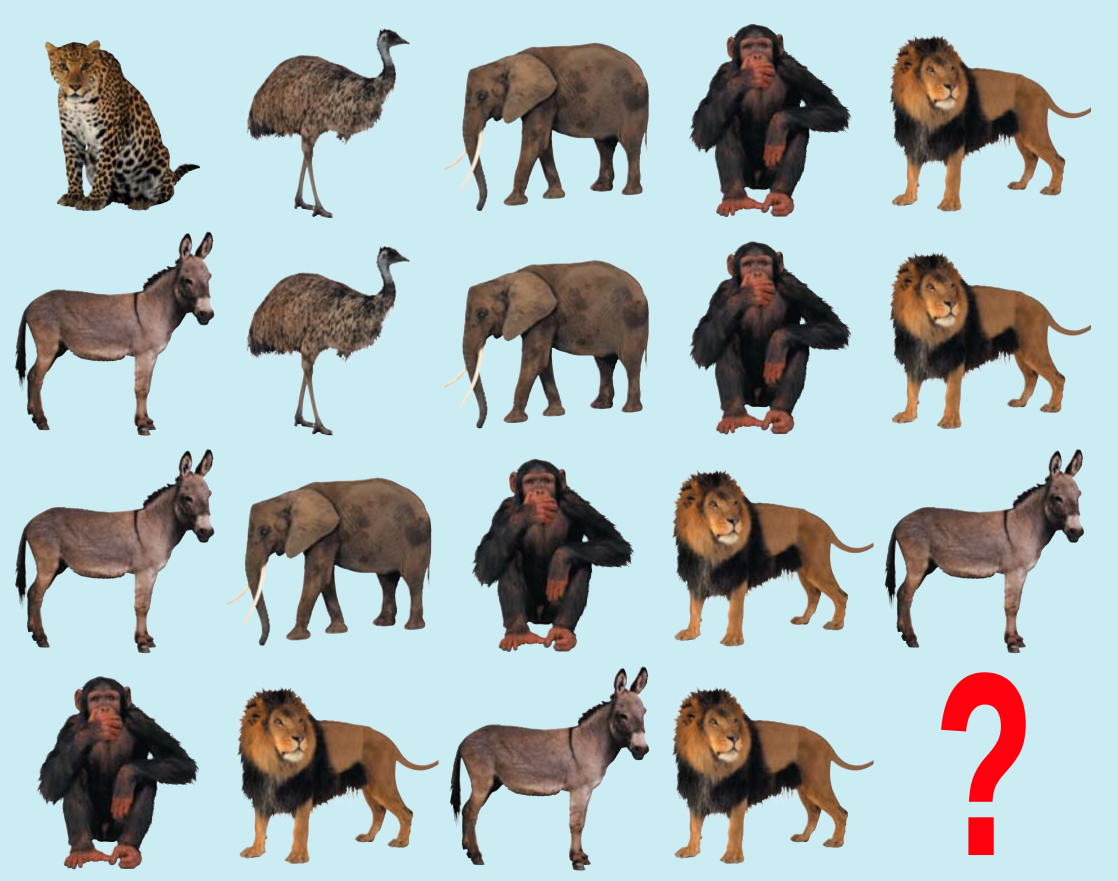


PLAYTHINK 78

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

ORDER LOGIC

Can you uncover the logic of the pattern and add the missing animal?

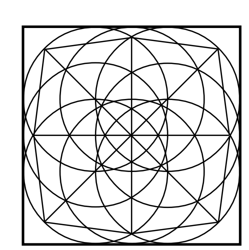


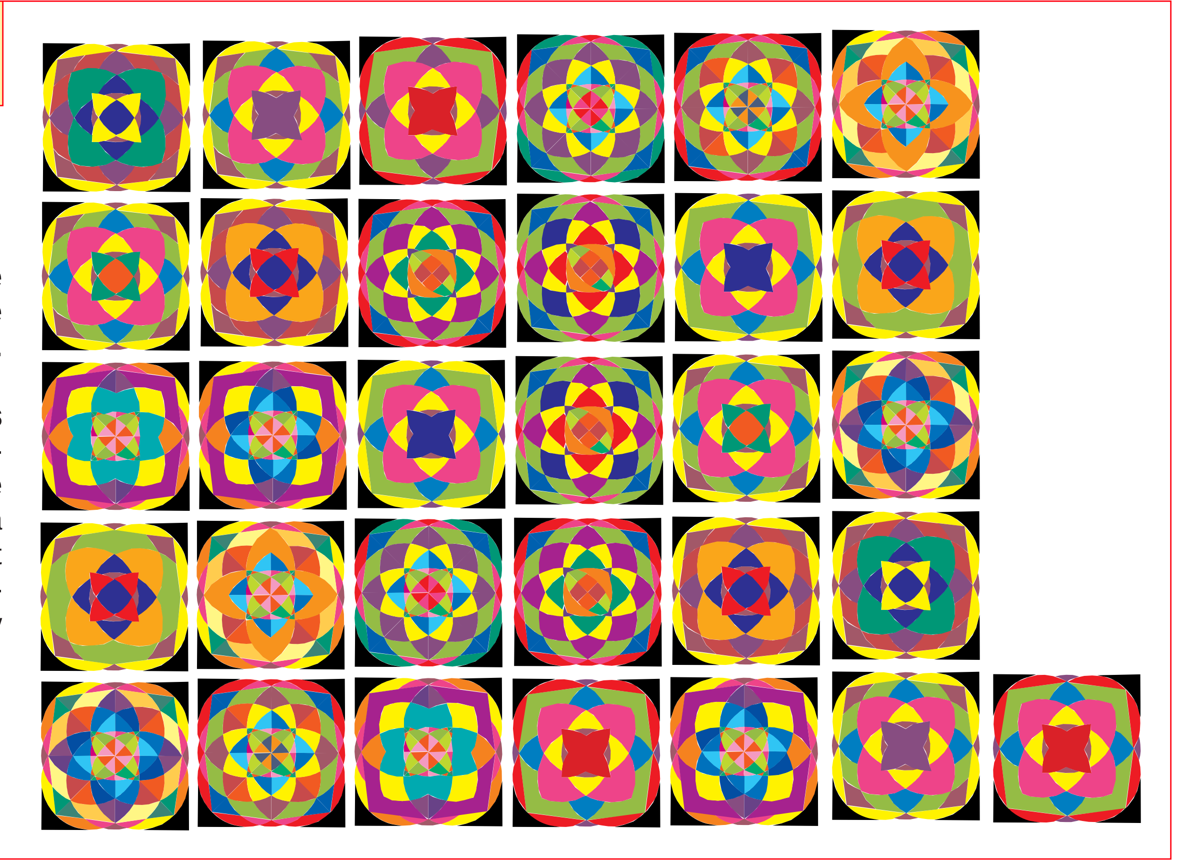
PLAYTHINK 79 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

CIRCLE ART MEMORY GAME

Pairing games have been long popular the world over. In this one, match all the like pairs of cards to discover the odd one out. How long will it take you to solve this?

One interesting note: These game cards employ simple color variations of a single pattern that can be constructed by a compass and ruler. It is similar to decorative patterns made by the ancient Greeks.

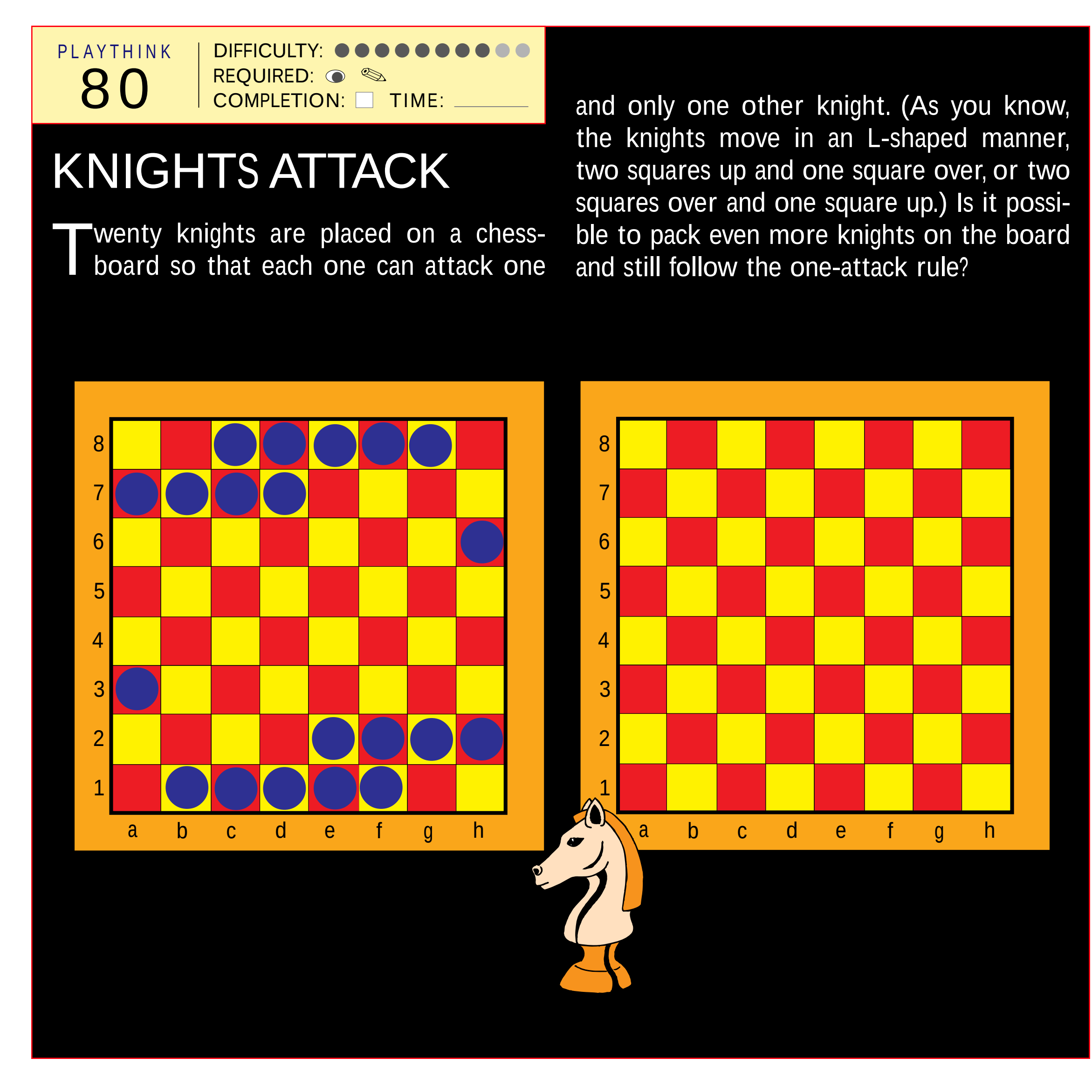




PLAYTHINK 80 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

KNIGHTS ATTACK

Twenty knights are placed on a chessboard so that each one can attack one and only one other knight. (As you know, the knights move in an L-shaped manner, two squares up and one square over, or two squares over and one square up.) Is it possible to pack even more knights on the board and still follow the one-attack rule?



PLAYTHINK 81 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

HAT MIX

Three men check their hats at the theater, but the attendant mixes up the checks as she hands them out. When the three men return after the performance to claim their hats, what are the chances that they all will have their own hats returned to them?



PLAYTHINK 82

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

WORD PATTERN

Can you find the secret message from Socrates?

AFGTRYT	SUGYUJO	SDNYTVB	MKRRDVB	UPMPLKM	SVFETVH
ATGTRHT	SEGYURO	SDEY–IB	MKSRDVB	U–OPLNM	SVLETYH
HGNDCTY	RTUIOMK	LMCZSTU	WETYUNV	OKPLMNH	SEFTCVG
–ONDNTY	REUI–GK	LOCZOTU	WDTY–KV	ONPLMOH	SWFTCLG
FJWBNMK	DEVNKOL	LPNMSGE	KERTYUN	SEFTRYV	XDCVFRE
FEWBDMK	DGVNEOL	L–AMSNE	KDRT–ON	SNFTREV	X–EVFVE
SEDCFVG	YUOPLKM	VBRHTRF	CDFRTYU	DEVBPKO	POUKJHY
SIDCFVG	YLOP–IM	VBGHTNF	COFRTRU	DAVBNKO	POCKJEY
WERTYFD	DFGYHUO	BNMKOPX	CVBNJUY	FRGVBHU	VBNJKOP
W–STYFD	DOGYCUO	BRMKAPX	CTBNJEY	FRGSBHU	VBNJKOP

PLAYTHINK 83

DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

COIN ON A CORNER

If you randomly toss a coin (smaller than a square on this game board), what is the probability of the coin landing on a corner of one of the squares?

Puzzles and Intelligence

Most of us grew up with a concept of intelligence that is driven by tests: the person who can answer the most questions is thought to be the most intelligent. But imagining that intelligence can be boiled down to a single number—the IQ—is an obsolete notion. If you find yourself having difficulty with some of these PlayThinks, don't worry that you are not "smart" enough to do the puzzles. It is all a matter of freeing up your latent creativity. With the proper mind-set, anyone can do these puzzles.

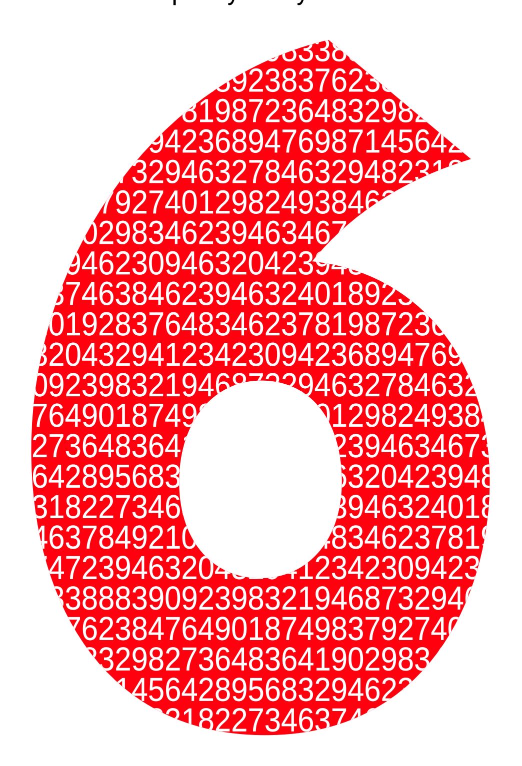
And if you find the puzzles easy, congratulations. But remember, that fact by itself does not mean you are smart. It just means you are especially attuned to this style of thinking.

> "IN THESE DAYS, A MAN WHO SAYS A THING CANNOT BE DONE IS QUITE APT TO BE INTERRUPTED BY SOME IDIOT DOING IT."
>
> —ELBERT GREEN HUBBARD

PLAYTHINK 84 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

FACTORS

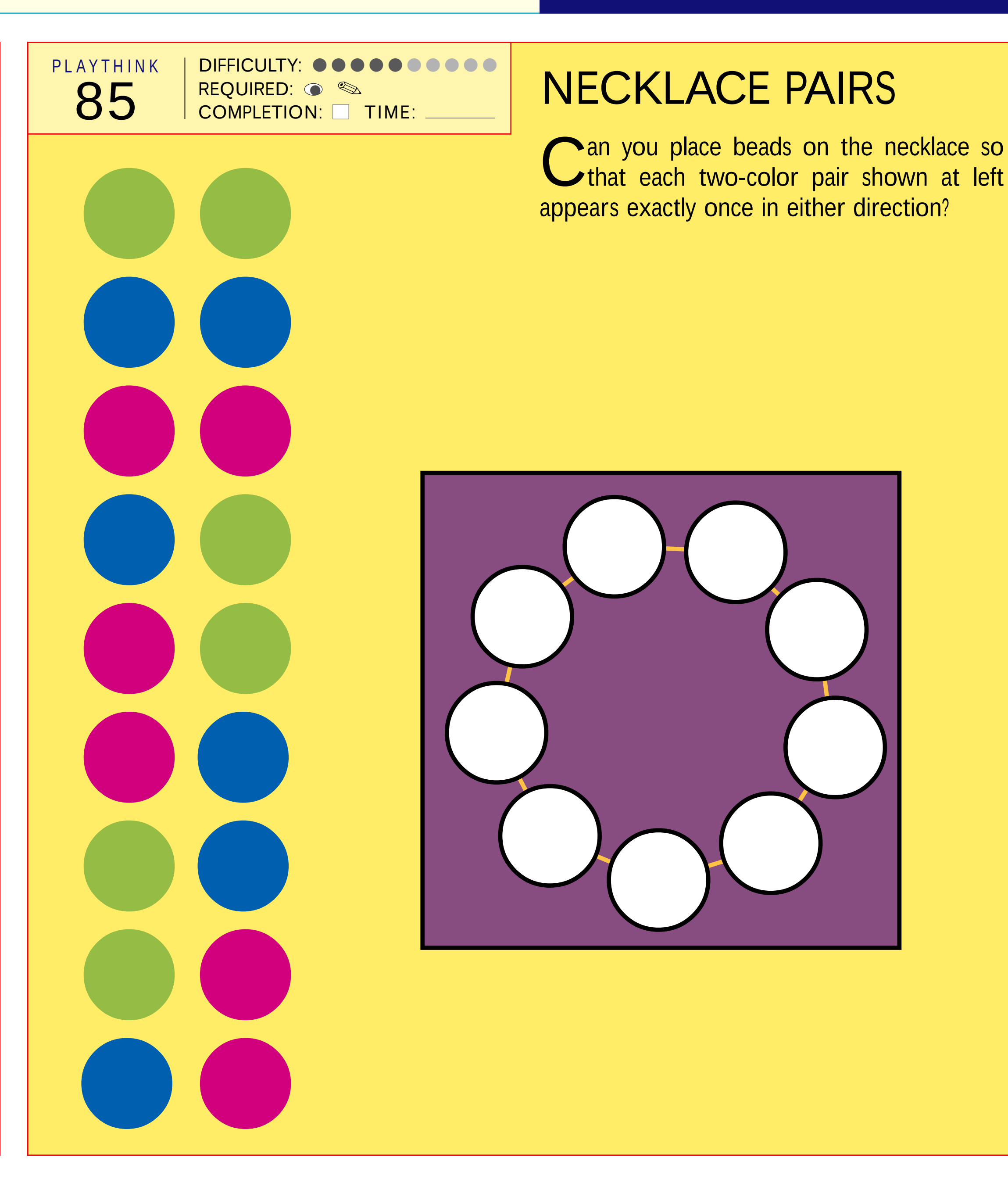
At the chalkboard the teacher demonstrates the four factors of the number 6—that is, the whole numbers that can divide into 6 and leave no remainder. (Remember, a number is always its own factor, as is 1.) Between 1 and 100, there are five numbers that have exactly twelve factors. How quickly can you find all five?



PLAYTHINK 85 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

NECKLACE PAIRS

Can you place beads on the necklace so that each two-color pair shown at left appears exactly once in either direction?

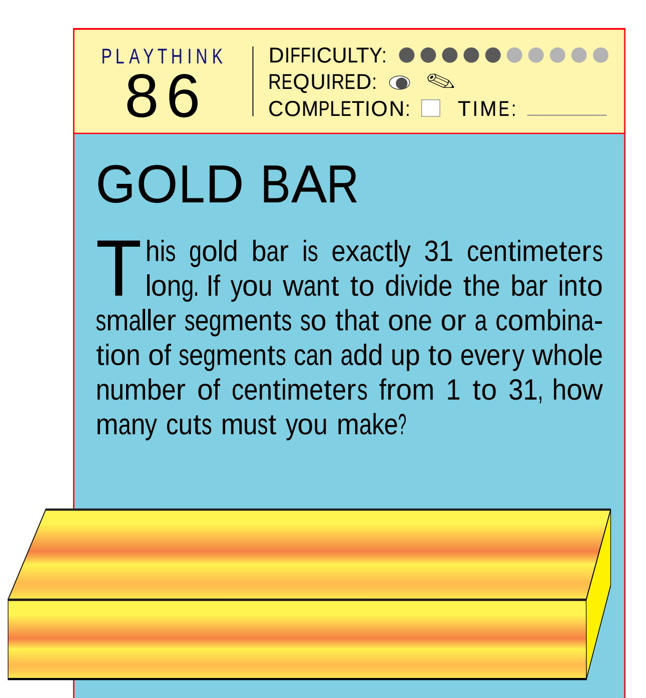


PLAYTHINK 86

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

GOLD BAR

This gold bar is exactly 31 centimeters long. If you want to divide the bar into smaller segments so that one or a combination of segments can add up to every whole number of centimeters from 1 to 31, how many cuts must you make?

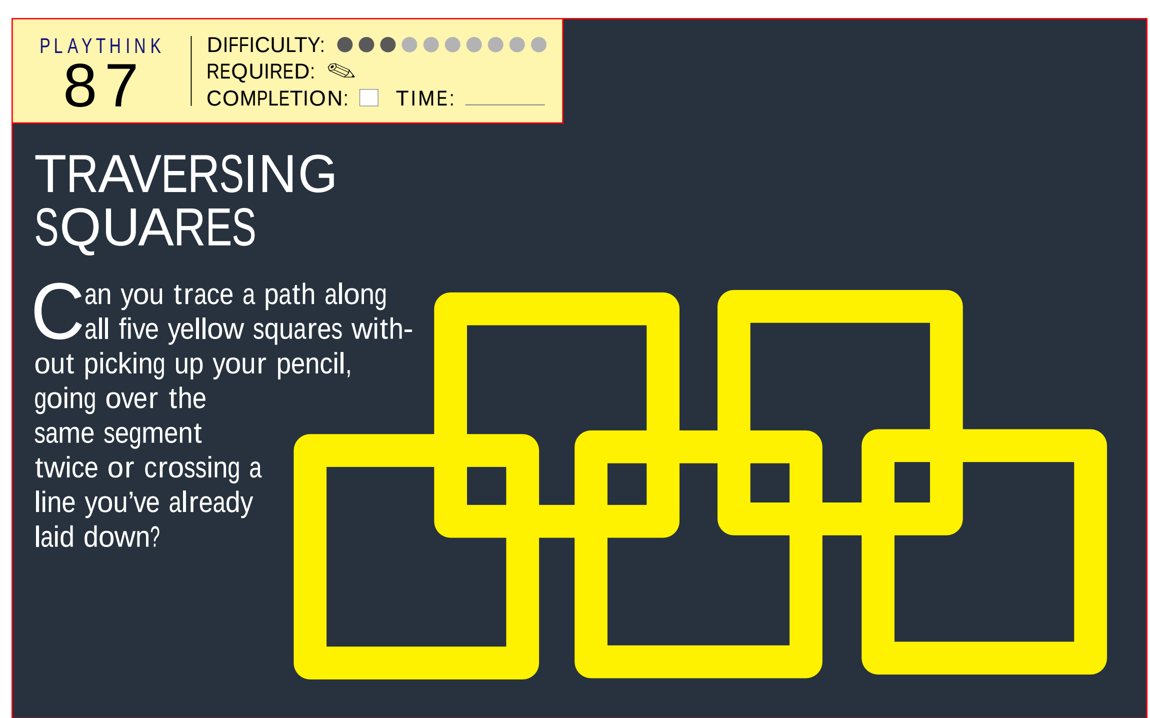


PLAYTHINK 87

DIFFICULTY: ●●●○○○○○○○
REQUIRED: ✎
COMPLETION: ☐ TIME: ______

TRAVERSING SQUARES

Can you trace a path along all five yellow squares without picking up your pencil, going over the same segment twice or crossing a line you've already laid down?



PLAYTHINK 88

DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

LOST IN CAVES

Five hapless tourists are lost in a labyrinth of caves. The path from one cave to another is marked with either a red arrow or a blue arrow. Their guide is outside and doesn't know where any of the five tourists is. Nonetheless, he shouts a sequence of red and blue arrows for the tourists to follow. Amazingly, all the tourists follow that sequence and arrive at the same cave, and the guide is there to pick them up.

Can you work out the sequence of reds and blues the guide shouted and which cave the tourists ended up in?


Geometry
2

In the Beginning . . .

There is an old debate among mathematicians: Is mathematics something they create or a truth they discover? The answer depends upon one's idea of truth. Some people believe that mathematical concepts are tools that were created in response to otherwise unsolvable questions, much the way screws were invented to hold pieces of wood together, or telephones to carry voices over long distances. Others view mathematics as a truth that exists regardless of whether anyone finds it: Mathematicians do not invent solutions to problems; they discover them.

Although the debate often divides the mathematical community, some mathematicians are so sure in their view that they give the matter little thought. As the famous Hungarian mathematician Paul Erdös declared, "If you believe in God, the answer is obvious."

To me the answer is also obvious. Mathematics was not invented. Mathematical forms existed before the advent of life on our planet or, indeed, before the earth was formed. When the sun with its solar system was just a cloud of dust and gas, galaxies, stars and other planets had developed configurations and motions based on simple geometric forms and principles. Some galaxies, for instance, possess a striking beauty derived from their form: an exact logarithmic spiral. The motions of stars, planets, comets and other bodies through space follow trajectories that can be described by geometric curves: ellipses, parabolas and hyperbolas.

And let us not forget that the first time three dinosaurs joined two other dinosaurs at a watering hole, there were five dinosaurs, whether or not anyone was there to count them.

When trying to trace the history of mathematics, it is important to keep those facts in mind. Mathematics started with the beginning of the universe itself. In many cases historians limit themselves to the dictionary definition of mathematics—an abstract science that investigates deductively the conclusions implicit in the elementary conceptions of spatial and numerical relations—and so begin their discussion with Thales, the great Greek mathematician who lived some 2,600 years ago. But although Thales helped invent the language by which we describe mathematics, humans had been using math long before that. The oldest mathematical textbook is an Egyptian papyrus scroll written by the scribe Ahmes in 1850 B.C. And even that may not be the beginning—4,000-year-old clay bricks found in the Tigris River valley bear numbers inscribed by Babylonian priests.

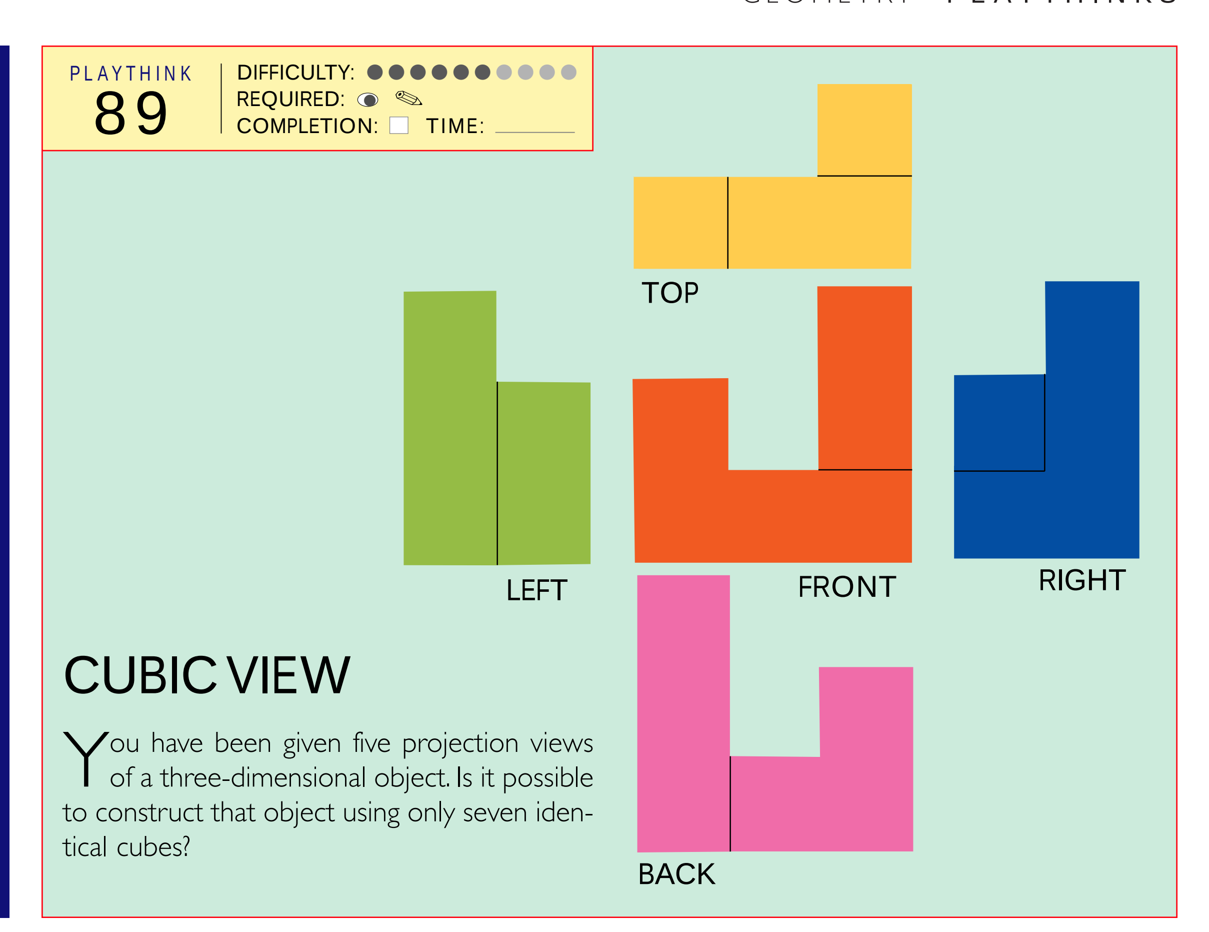
Even our prehistoric, cave-dwelling ancestors had a good grasp of many mathematical concepts. Prehistoric art, which reduced the complex shapes found in nature to simple, abstract forms, paved the way for geometry. And a hunt that produced fewer killed animals than there were hunters—so that the leaders needed to figure out how to divide the spoils—helped develop the concepts of division and inequality. The constant northern stars provided a reliable clue to direction, and finger counting progressed into arithmetic.

Some mathematics, notably anything that depends upon the use of the base ten number system, is undeniably a human invention. But most mathematics does not rely upon that sort of human ingenuity. It was a truth that existed before it was discovered. Take, for example, the Pythagorean theorem: although it is forever linked to the great Greek mathematician Pythagoras, it has been independently discovered several times, by various civilizations, throughout history. If our present society were to disappear, the Pythagorean theorem would eventually be discovered again. And if there is some other form of intelligent life on some distant planet, it, too, has probably discovered that the sum of the squares of the lengths of the sides of a right triangle is equal to the square of the hypotenuse.

> "I HAVE AN INFAMOUSLY LOW CAPACITY FOR VISUALIZING RELATIONSHIPS, WHICH MADE THE STUDY OF GEOMETRY AND ALL SUBJECTS DERIVED FROM IT IMPOSSIBLE FOR ME."
>
> —SIGMUND FREUD

PLAYTHINK 89

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______



CUBIC VIEW

You have been given five projection views of a three-dimensional object. Is it possible to construct that object using only seven identical cubes?

PLAYTHINK 90

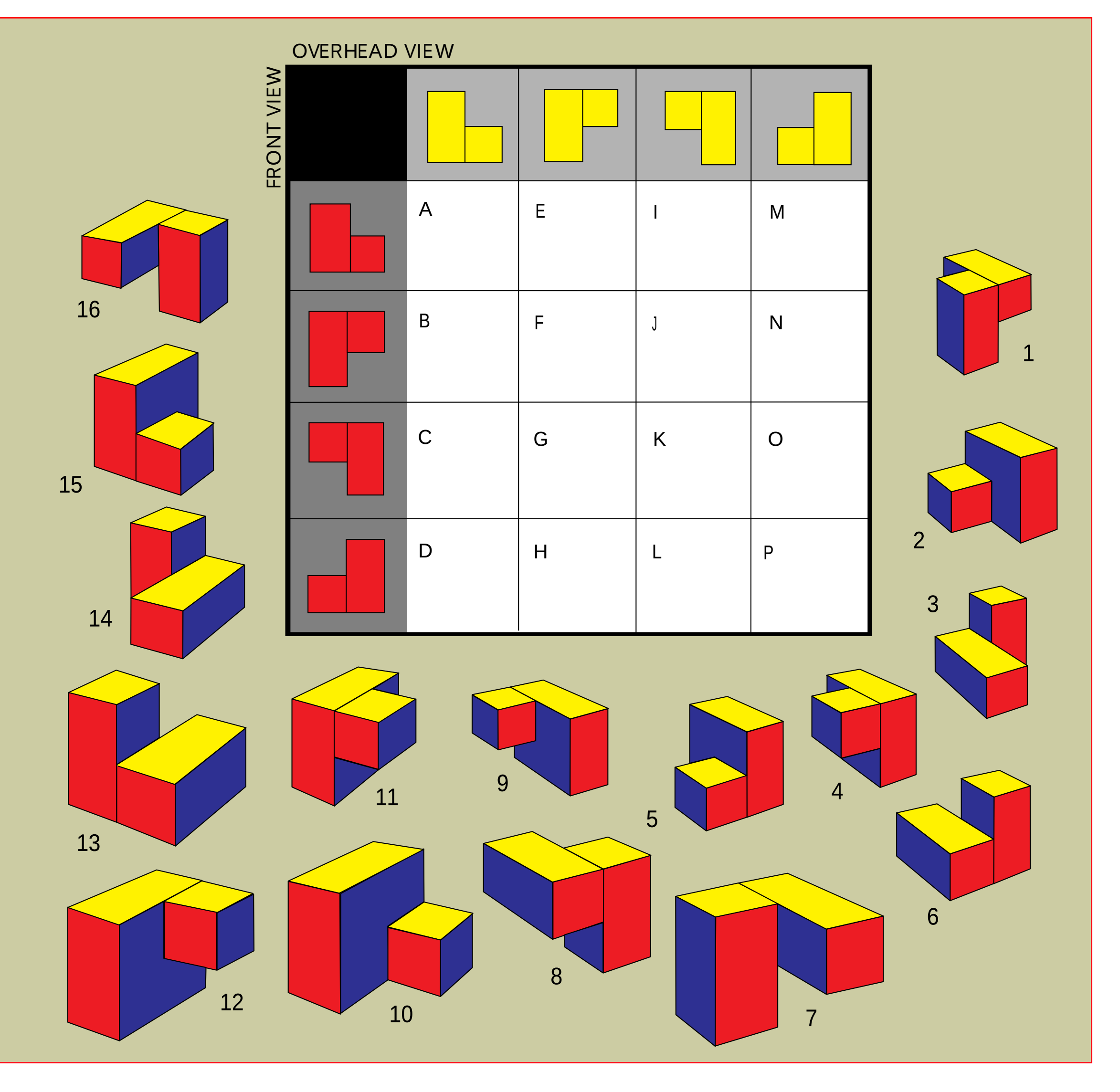
DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

MULTIVIEWS

Imagine flying over a city in an airplane. As seen from above, the buildings seem quite different from the way they look when you are standing in front of them. And yet nothing about the buildings has changed. This is the concept that architects tap into when they represent their building plans in two different ways: the plan, which represents the way the building will be laid out on the ground, and the front elevation, which is derived directly from the plans to represent the way the building will look from the front. A third type of architectural drawing, the perspective, combines those two views to create a more realistic view of the building.

This puzzle is based on the same concept. There are sixteen objects, yet seen from the front, they present only four different types of views. And seen from the top, they present four different types of views. But every object with a similar front view has a different overhead view—sixteen unique objects.

Can you match each object with its proper overhead and front views? Write your answers in the boxes provided.



Projective Geometry

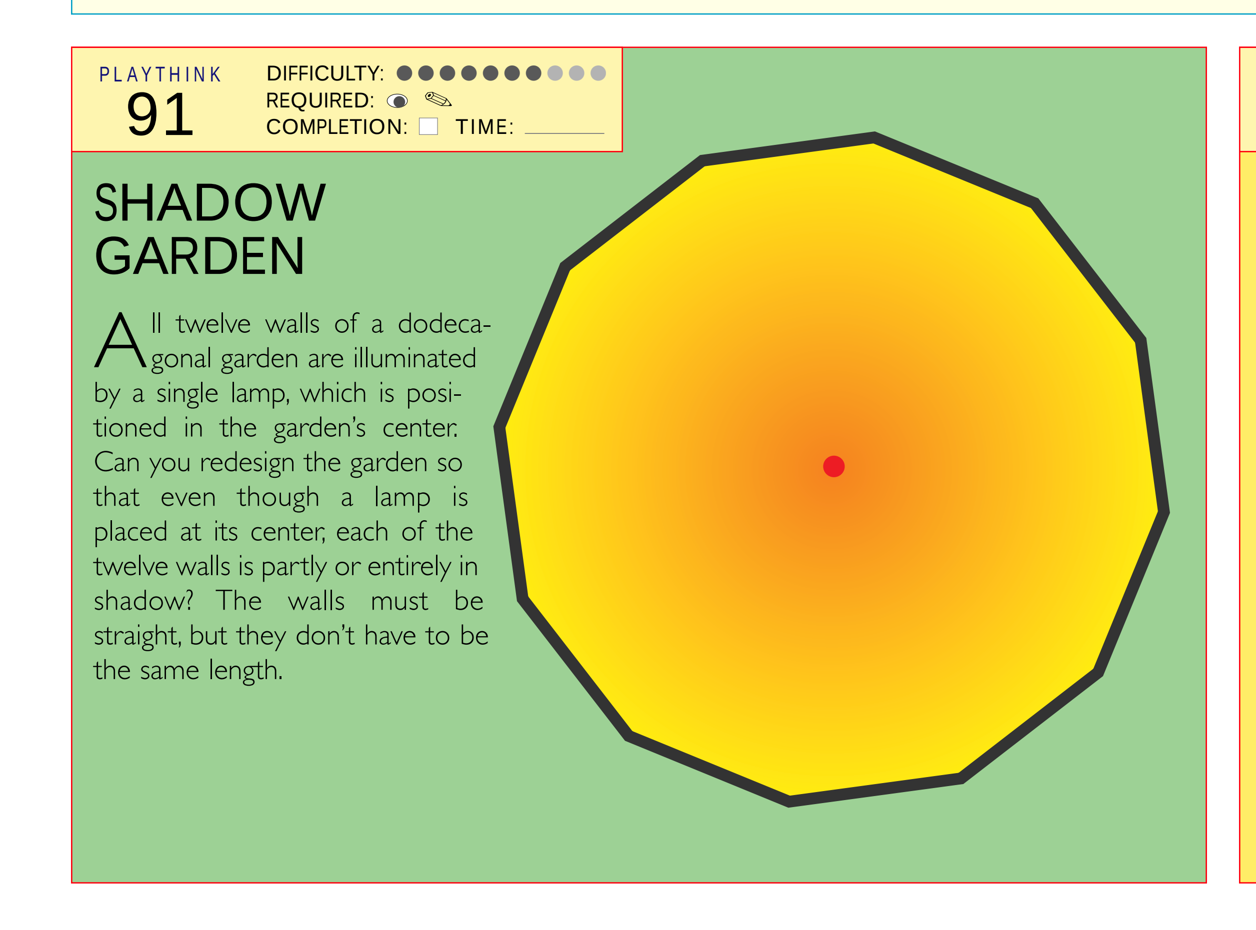
Our eyes present a distorted view of the world. The parallel tracks of a railroad should never meet, but rails in the distance do look as if they come to a point. Large things look small when they are far off, and distance can make two objects that are of equal size appear to be on radically different scales. The reverse is true as well: a thumb can obscure the largest galaxy.

Even though human perception of scale is a given, it was only during the Renaissance that painters solved the problem of representing the perspective of a three-dimensional space on a two-dimensional plane. That solution, called projection, created not only a breakthrough in art but also a new type of geometry—a form of mathematics that closely approaches the world of illusion.

Projective geometry studies what happens to shapes when they are distorted in special ways. Although the results can be startling, projective transformations preserve many of the geometric properties of the objects being projected. That's what enables three-dimensional objects to be recognizable in their two-dimensional form.

Maps are projections. The Flemish cartographer Gerardus Mercator employed projective geometry to produce the first modern map of the world in 1569. The so-called Mercator projection was made from the center of the earth; the surface was projected onto an imaginary cylinder tangent to the equator. Although the resulting map was quite useful for navigation, Mercator's projection distorted the areas near the poles. That's why Greenland, which has an area comparable to Mexico, appears on a Mercator map to be the same size as South America.

These days we see the uses of projective geometry all around us. Photographs are images of projections, as are many mechanical and architectural drawings. And video games in realistic 3-D are possible because sophisticated computer programs can calculate the projection of imaginary three-dimensional objects.

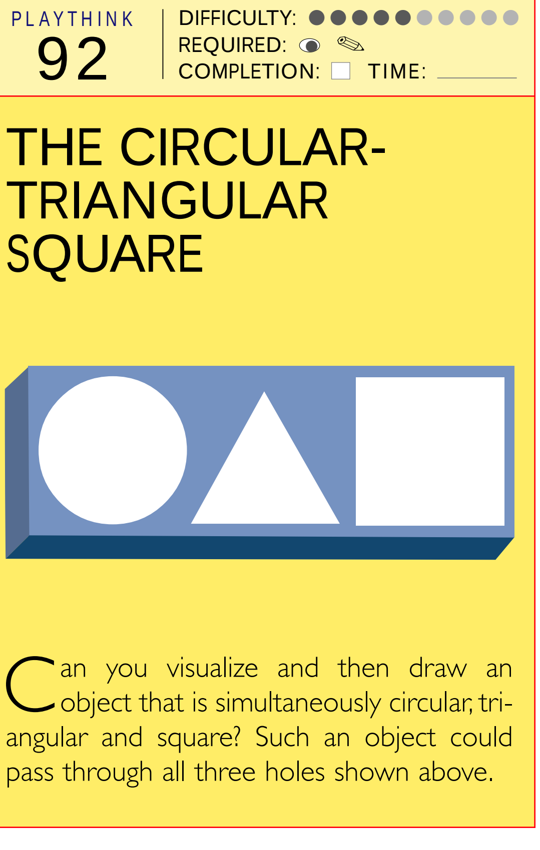


PLAYTHINK 91

DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

SHADOW GARDEN

All twelve walls of a dodecagonal garden are illuminated by a single lamp, which is positioned in the garden's center. Can you redesign the garden so that even though a lamp is placed at its center, each of the twelve walls is partly or entirely in shadow? The walls must be straight, but they don't have to be the same length.



PLAYTHINK 92

DIFFICULTY: ●●●●●○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

THE CIRCULAR-TRIANGULAR SQUARE

Can you visualize and then draw an object that is simultaneously circular, triangular and square? Such an object could pass through all three holes shown above.

PLAYTHINK 93

DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ______

CUBES IN PERSPECTIVE

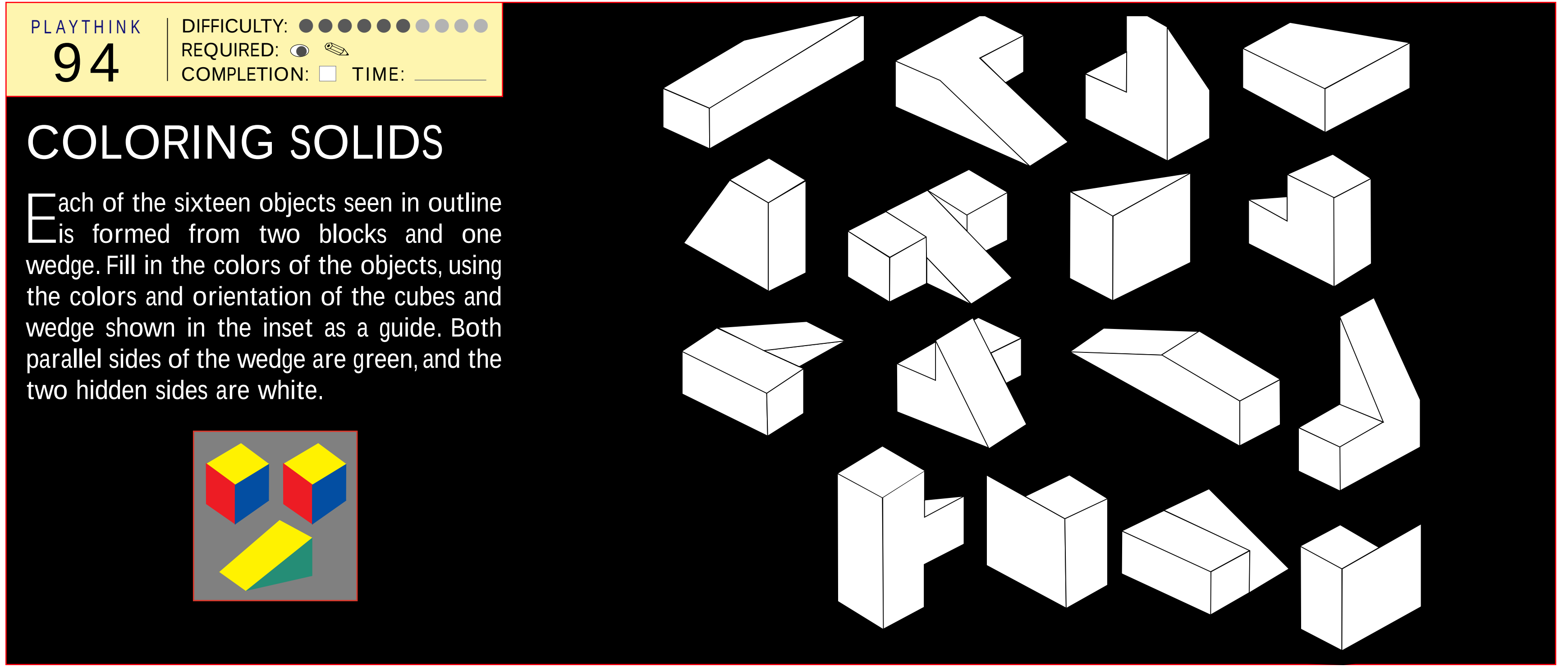
These eight colored figures represent cubes that have been drawn by tracing along the lines in the center grid. Through careful observation, can you retrace the lines to re-create each figure?

PLAYTHINK 94

DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ______

COLORING SOLIDS

Each of the sixteen objects seen in outline is formed from two blocks and one wedge. Fill in the colors of the objects, using the colors and orientation of the cubes and wedge shown in the inset as a guide. Both parallel sides of the wedge are green, and the two hidden sides are white.



"GEOMETRY ENLIGHTENS THE INTELLECT AND SETS ONE'S MIND RIGHT. ALL ITS PROOFS ARE VERY CLEAR AND ORDERLY. . . . IN THIS CONVENIENT WAY, THE PERSON WHO KNOWS GEOMETRY ACQUIRES INTELLIGENCE. IT HAS BEEN ASSUMED THAT THE FOLLOWING STATEMENT WAS WRITTEN UPON PLATO'S DOOR: 'NO ONE WHO IS NOT A GEOMETRICIAN MAY ENTER OUR HOUSE.'"

—IBN KHALDUN

PLAYTHINK 95

DIFFICULTY: ●●●○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

BLUEPRINT AND SOLIDS

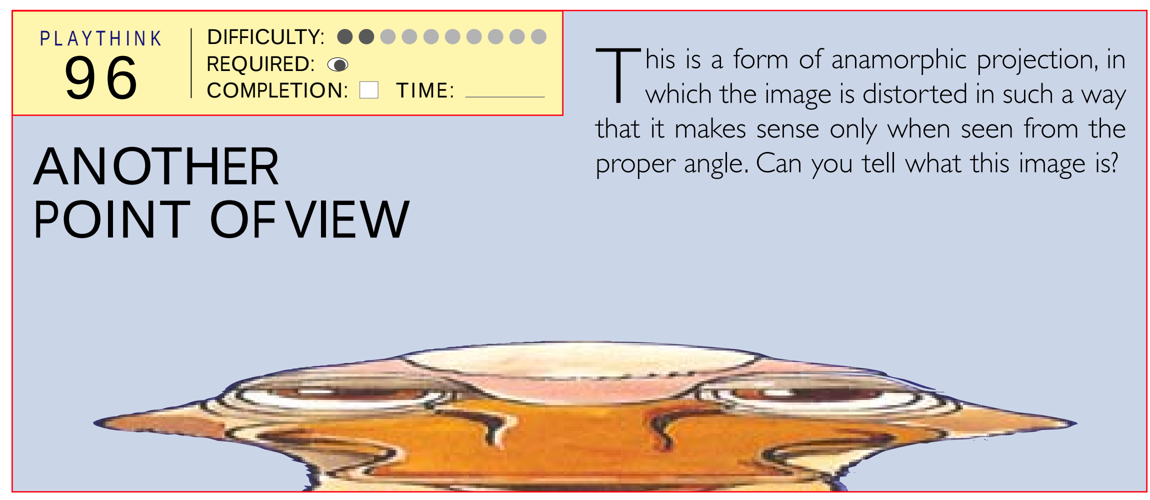
For each group of objects, can you find the one that was constructed by folding the pattern provided?

PLAYTHINK 96

DIFFICULTY: ●●○○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

ANOTHER POINT OF VIEW

This is a form of anamorphic projection, in which the image is distorted in such a way that it makes sense only when seen from the proper angle. Can you tell what this image is?



PLAYTHINK 97

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

WHAT'S IN THE SQUARE?

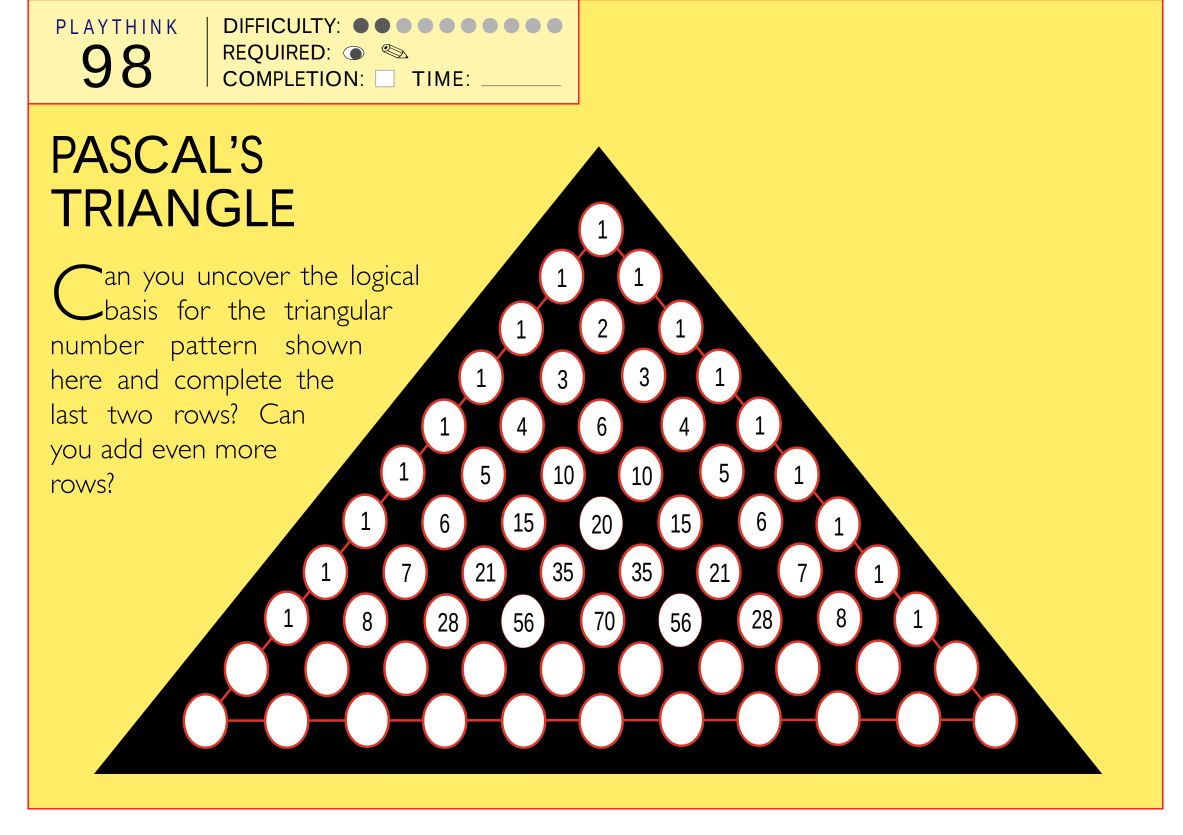
An artist has constructed a sculpture from colored plastic sheets attached to a three-dimensional wire frame. From most perspectives the sculpture looks chaotic. But can you envision what it looks like if seen from the red point on the left of the image? You can use the empty frame at right as a guide to solving the puzzle.

PLAYTHINK 98

DIFFICULTY: ●●○○○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

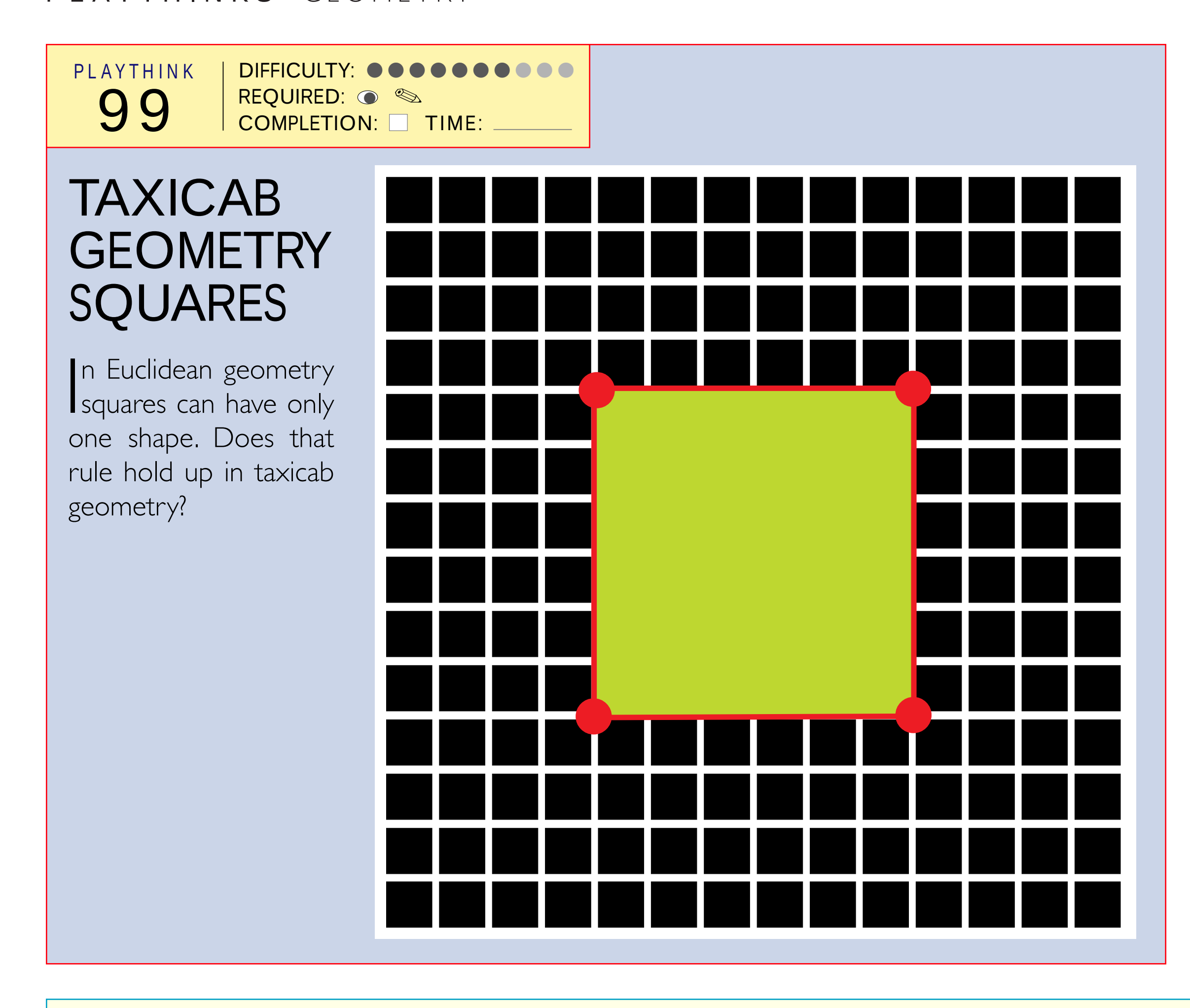
PASCAL'S TRIANGLE

Can you uncover the logical basis for the triangular number pattern shown here and complete the last two rows? Can you add even more rows?



"*UBI MATERIA, IBI GEOMETRIA.* (WHERE THERE IS MATTER, THERE IS GEOMETRY.)"

—JOHANNES KEPLER

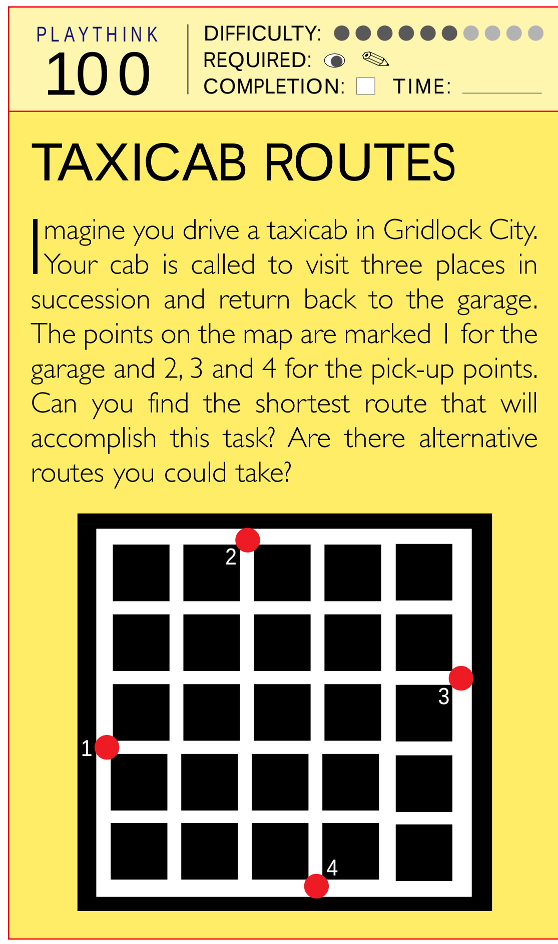


PLAYTHINK 99

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

TAXICAB GEOMETRY SQUARES

In Euclidean geometry squares can have only one shape. Does that rule hold up in taxicab geometry?



PLAYTHINK 100

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

TAXICAB ROUTES

Imagine you drive a taxicab in Gridlock City. Your cab is called to visit three places in succession and return back to the garage. The points on the map are marked 1 for the garage and 2, 3 and 4 for the pick-up points. Can you find the shortest route that will accomplish this task? Are there alternative routes you could take?

Early Geometry

Early humans learned how to build structures more efficiently by simple trial and error. And when the ancient Egyptians added a great deal of ingenuity to the mix, they accomplished wondrous feats of architecture and engineering—and in the process developed the first form of geometry.

Later in antiquity, Greek geometers were absorbed in the study of simple forms—the circle, the square, the triangle. Armed with only compass and ruler, they set out to find geometric truths; by 350 B.C. Euclid had compiled a set of rules concerning space and shapes that dominated geometry for 2,000 years.

Although early Greek geometers made huge theoretical advances, the mathematician Eratosthenes, who lived in Alexandria, Egypt, in the third century B.C., accomplished perhaps the greatest practical achievement. He learned that on a day in midsummer in the town of Syene (near present-day Aswan), the reflection of the noonday sun was visible on the water of a deep well. For that to occur, the sun had to be directly overhead, with its rays pointed directly toward the center of the earth. On the same day the noonday sun cast shadows in Alexandria that measured 7.5 degrees, or about one part in fifty of a full circle. Eratosthenes knew that sunbeams travel in parallel straight lines and so deduced that the difference in the angles was caused by the curvature of the earth. Once Eratosthenes found the distance north to south between Alexandria and Syene, which is about 480 miles, he multiplied that distance by fifty to determine the circumference of the circle that passes through those two towns and the North and South Poles—in other words, the circumference of the earth. His estimate, about 24,000 miles, was remarkably accurate.

Taxicab Geometry

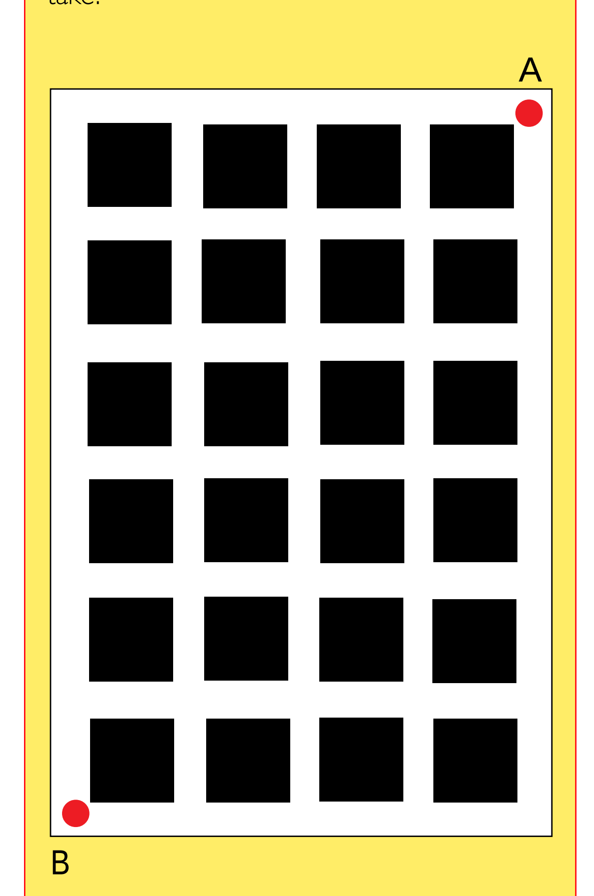
Understanding non-Euclidean geometries can be a formidable task. One approachable non-Euclidean geometry is the so-called taxicab geometry, which you can explore with a city map or even ordinary graph paper. Imagine Gridlock City, in which the streets run either north-south or east-west. (Many cities established in the nineteenth century possess just that sort of grid.) To get around Gridlock City by taxicab, one must measure distances not "as the crow flies" but "as the cab drives"—along the lines of the square grid. Taxicab distances are in general longer than ordinary distances except when you drive from one end of a street to the other.

If Gridlock City is made up of straight lines on a plane, how can it be non-Euclidean? One of Euclid's axioms states that the shortest distance between two points is a straight line. Is that the case in Gridlock City? In fact, the shortest path in most cases is a series of short lines, since travel is restricted to the street grid. You must drive around blocks, not through them.

PLAYTHINK 101 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

GRIDLOCK CITY

A man who lives at the top right corner of this city district works in the bottom left corner. What is the shortest path to his office? How many different routes can he take?



PLAYTHINK 102 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

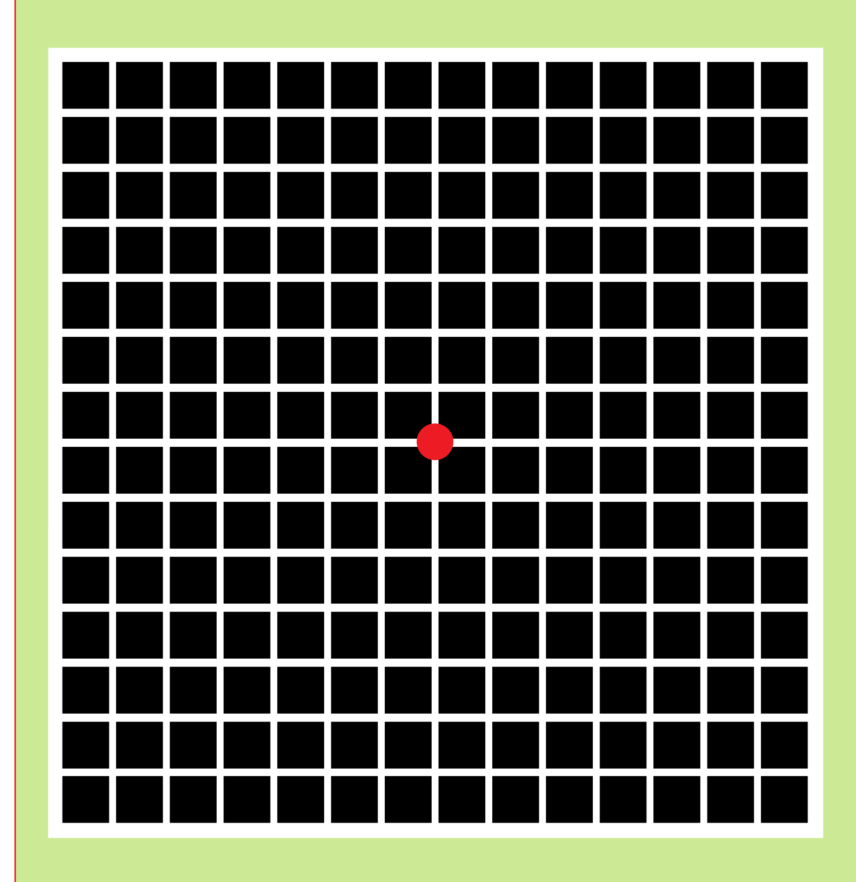
TAXICAB GEOMETRY CIRCLES

In Gridlock City you can move around only in blocks. Does that mean it is impossible to have a circle?

By definition a circle is a shape in which all points are equidistant from a fixed point. Suppose that there are six blocks to a kilometer in Gridlock City and you travel a kilometer by taxi from the center of the city. Where do you end up?

You could travel six blocks due east and stop. Or you could go five blocks east and one block north, or four blocks east and two blocks north. All those points lie on the "taxicab circle" of radius 1 kilometer.

Can you plot the shape of such a circle?



PLAYTHINK 103

DIFFICULTY: ●○○○○○○○○○
REQUIRED: 👁 📄 ✂
COMPLETION: ☐ TIME: ______

FACE IT: THE PUZZLE OF VANISHING FACES

Copy and cut out the thirty-six tiles and place them on a six-by-six game board. In the configuration shown here, there are twelve complete faces—five smiling, seven frowning. Can you rearrange the tiles so that you add a thirteenth face and make nine frowning faces and four smiling ones? Can you change the mood so that there are nine smiling faces and only four frowning ones? Or nine smiling faces and only three frowning ones?

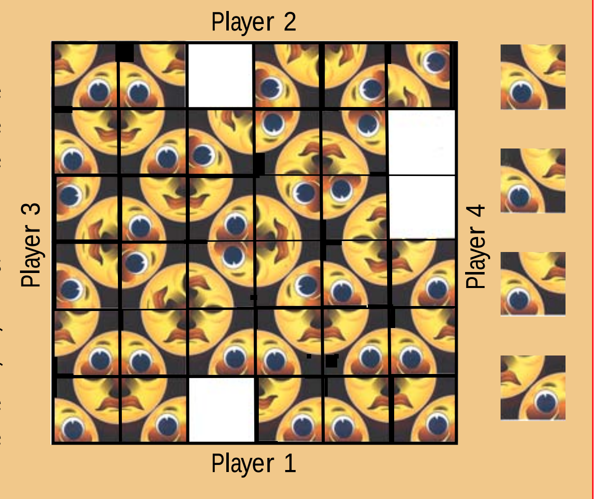
PLAYTHINK 104

DIFFICULTY: ●○○○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

FACE IT: THE GAME OF VANISHING FACES

The "Face It" puzzle tiles lend themselves to making a simple but subtly rewarding game. The object of the game is for each player to form smiling faces looking in his or her direction. Up to four persons may play, each sitting on a different side of the board. The tiles are mixed and placed face down. Players take turns selecting a tile and placing it on the board alongside one or more tiles already played. Each tile played must create a proper face match with one of the tiles next to it. The scores are tallied at the end of play. Each smile facing a player counts as one point; each frown facing the player costs a point.

In the sample game shown here, play has ended because no more pieces can be placed on the board. There is no single winner. Player 1 is faced by two smiles and three frowns, for a score of minus one. All the other players are faced by one smiling face, for a score of one a piece. Note that some of the faces are mixed or incomplete and thus don't count toward the final scores.



Worlds of Two Dimensions

Astrophysicists say that the universe possesses four dimensions—three of space and one of time—and some recent theories have suggested that there may be even more dimensions exerting an influence at the subatomic scale.

How can we begin to understand hypothetical higher dimensions? By getting outside our normal system. In this case, try to imagine a world that has only ***two*** dimensions.

In 1884 Edwin A. Abbott, an English clergyman and popularizer of science, made a beautiful attempt to describe a world made up of only two dimensions. In his satirical novel, called ***Flatland***, the characters are basic geometric figures gliding over the surface of an infinite two-dimensional plane—a vast tabletop. Apart from a negligible thickness, Flatlanders have no perception of the third or any higher dimension.

Although Abbott did not describe any of the physical laws or technological innovations of Flatland, his book spawned sequels that tackle those issues. One such book, ***An Episode of Flatland***, written by Charles Howard Hinton in 1885, cleverly extends Abbott's original.

The action in Hinton's book takes place on the apparently two-dimensional planet Astria. Astria is simply a giant circle, and its inhabitants live on the circumference, forever facing in one direction. All males face east, and all females face west. To see what is behind him, an Astrian must bend over backward, stand on his or her head or use a mirror.

Astria is divided between two nations, the civilized Unaeans in the east and the barbaric Scythians in the west. When the two nations go to war, the Scythians have an enormous advantage: they can strike the Unaeans from the back. The unfortunate and helpless Unaeans are driven to a narrow region bordering the great ocean. Facing complete extinction, the Unaeans are saved by a scientific advance: their astronomers have discovered that the planet is round. A group of Unaeans cross the ocean and carry out a surprise attack on the Scythians, who have never before been attacked from the rear. The Unaeans are thus able to defeat their foes.

Over the course of the book, Hinton fleshed out the details of his world. Houses in Astria can have only one opening. A tube or pipe is impossible. Ropes cannot be knotted, although levers, hooks and pendulums can be used.

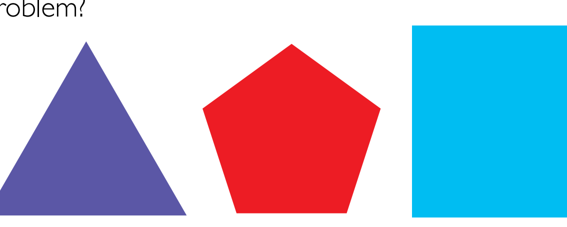
In Alexander Dewdney's 1984 book, ***The Planiverse***, the ideas of ***Flatland*** are taken to their contemporary conclusion. Dewdney, a computer scientist at the University of Western Ontario, lays down the complete theoretical base for a possible two-dimensional world in a beautiful synthesis of science, art and mathematics.

PLAYTHINK 105 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ______

FLATLAND HIERARCHY

In ***Flatland***, Edwin Abbott describes a society of geometric shapes subject to a strict hierarchy. Ladies are sharp straight lines; soldiers and workmen are isosceles triangles; middle-class people are equilateral triangles; professionals are either squares or pentagons; the wealthy are hexagons; and the top of the class system—the high priests—are circles.

Of course, since ladies are one-dimensional lines, they are invisible from some directions and may be hazardous to run into. How do you think the Flatlanders avoid this problem?



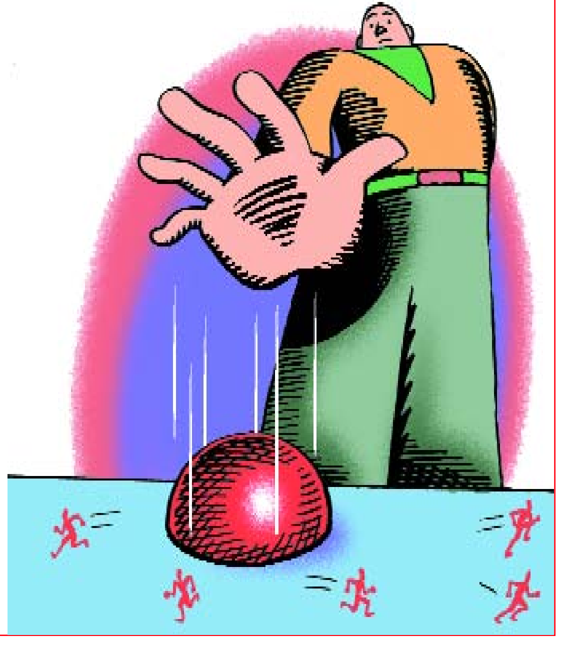


PLAYTHINK 106 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

FLATLAND CATASTROPHE

The senses of Flatlanders are limited to two dimensions. So if someone were to observe them from a point just "above" their world, the Flatlanders would have no way of seeing that observer.

But what if you tossed a ball through the two-dimensional plane of Flatland? Would the Flatlanders perceive the event as some sort of astronomical catastrophe? Can you describe exactly what they would see?



"GEOMETRIA EST ARCHETYPUS PULCHRITUDINIS MUNDI. (GEOMETRY IS THE ARCHETYPE OF THE BEAUTY OF THE WORLD.)"

—JOHANNES KEPLER

PLAYTHINK 107 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

FLATLAND PLAYPEN

The two toddlers on the facing pages of this book will cry until they can play together. How can you make them happy without removing one from the crib or the other from the high chair?

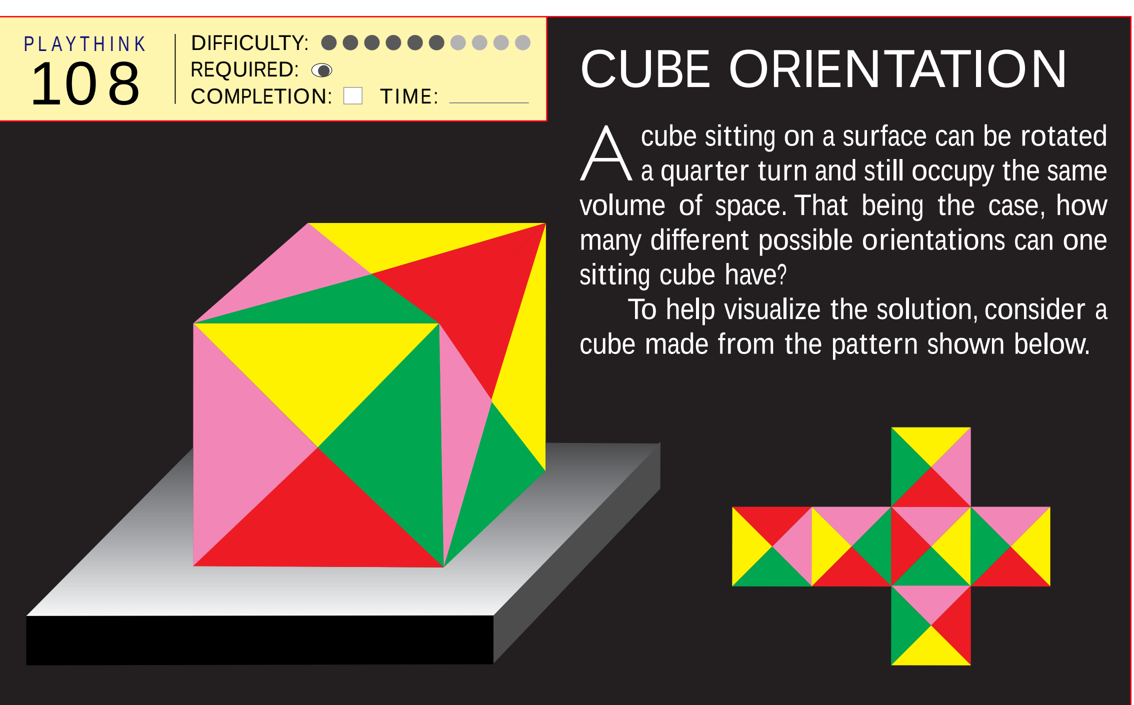


PLAYTHINK 108 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

CUBE ORIENTATION

A cube sitting on a surface can be rotated a quarter turn and still occupy the same volume of space. That being the case, how many different possible orientations can one sitting cube have?

To help visualize the solution, consider a cube made from the pattern shown below.

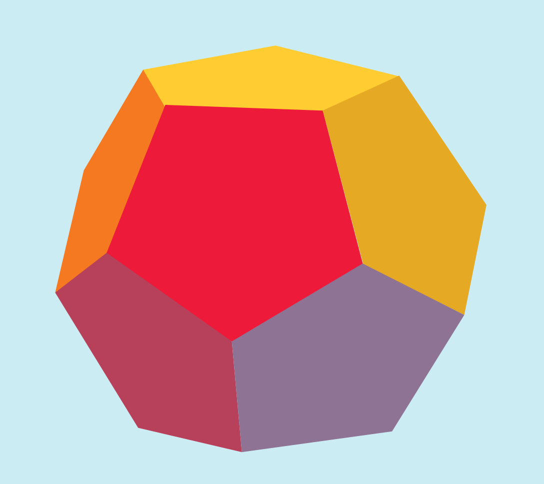


PLAYTHINK 109 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

DODECAHEDRON ORIENTATION

A dodecahedron is a regular polyhedron made up of twelve pentagonal sides. When the ancient Pythagoreans discovered the dodecahedron, they guarded it as a great secret—and anyone who disclosed its existence was punished by death.

If a dodecahedron that sits on a surface makes a 72-degree turn, it will occupy the same volume of space. That being the case, how many different possible orientations can one sitting dodecahedron have?



Symmetry

Objects that possess symmetry—the ability to undergo certain geometric transformations without changing form—are found throughout nature. The most perfect natural examples of symmetry are in the arrangements of atoms and molecules in crystals; a common example is the snowflake, which possesses many axes of symmetry. Biological creatures also display a remarkable amount of symmetry. Fivefold or pentagonal symmetry is found in many marine flowers and animals, such as the sea star, or starfish, which has five, ten or even twenty-three symmetric arms.

We human beings, who are roughly symmetric about one axis, the spine, display bilateral symmetry—the most common form of symmetry in nature. (Biologists believe we are programmed to recognize symmetry, which is why we judge highly symmetrical faces and bodies to be more beautiful than asymmetrical ones.)

Objects that look the same as they are rotated about an axis have rotational symmetry; an equilateral triangle, for instance, will appear identical in three different positions as it rotates around a point at its center. Objects with lateral symmetry can be reflected on either side of a line or axis without appearing different.

We can easily make symmetrical patterns by folding and cutting paper or by using plane mirrors—what child hasn't made snowflakes or paper dolls that way?—but symmetry is also an enormously important mathematical tool. Scientists could never have determined the structure of viruses and molecules without a full understanding of symmetry; neither could they have built the standard model of particle physics.

> "SYMMETRY IS ONE IDEA BY WHICH MAN THROUGH THE AGES HAS TRIED TO COMPREHEND AND CREATE ORDER, BEAUTY AND PERFECTION."
>
> —HERMANN WEYL

PLAYTHINK 110

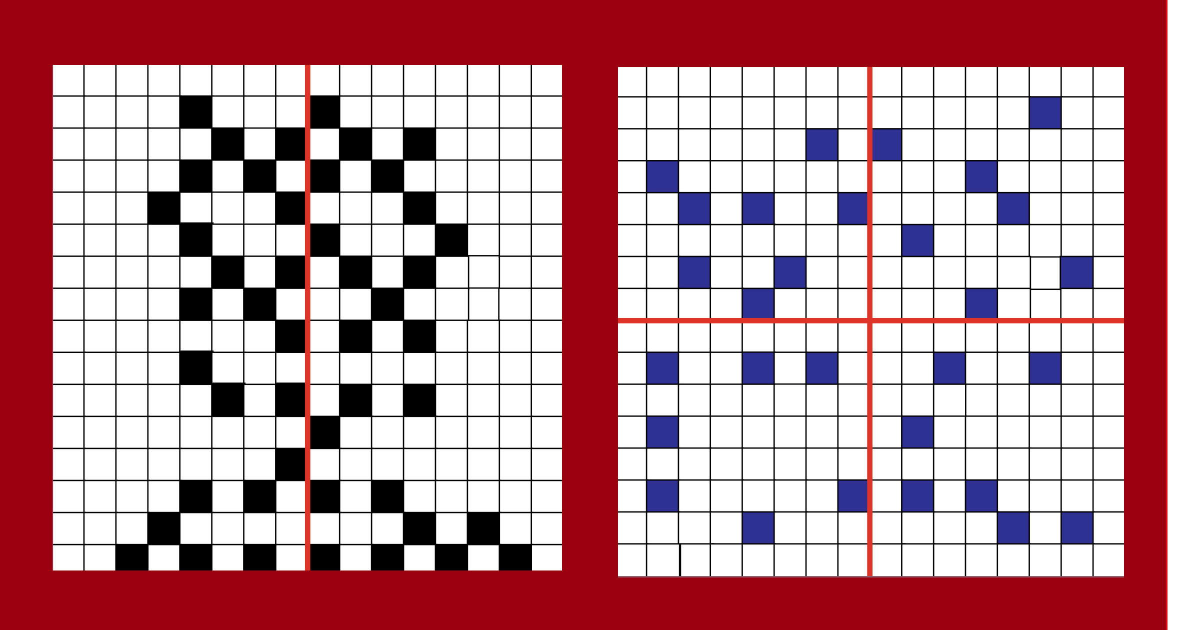
DIFFICULTY: ●○○○○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

SYMMETRY SQUARES

Both of these images are symmetrical—but some of the squares have been erased.

In the near image, by carefully observing the position of the black squares in relation to the red line, which is the axis of vertical symmetry, you should be able to fill in the rest of the picture.

In the far image, by carefully observing the position of the blue squares in relation to the two red lines, which are the axes of vertical and horizontal symmetry, you should be able to fill in the rest of the picture.



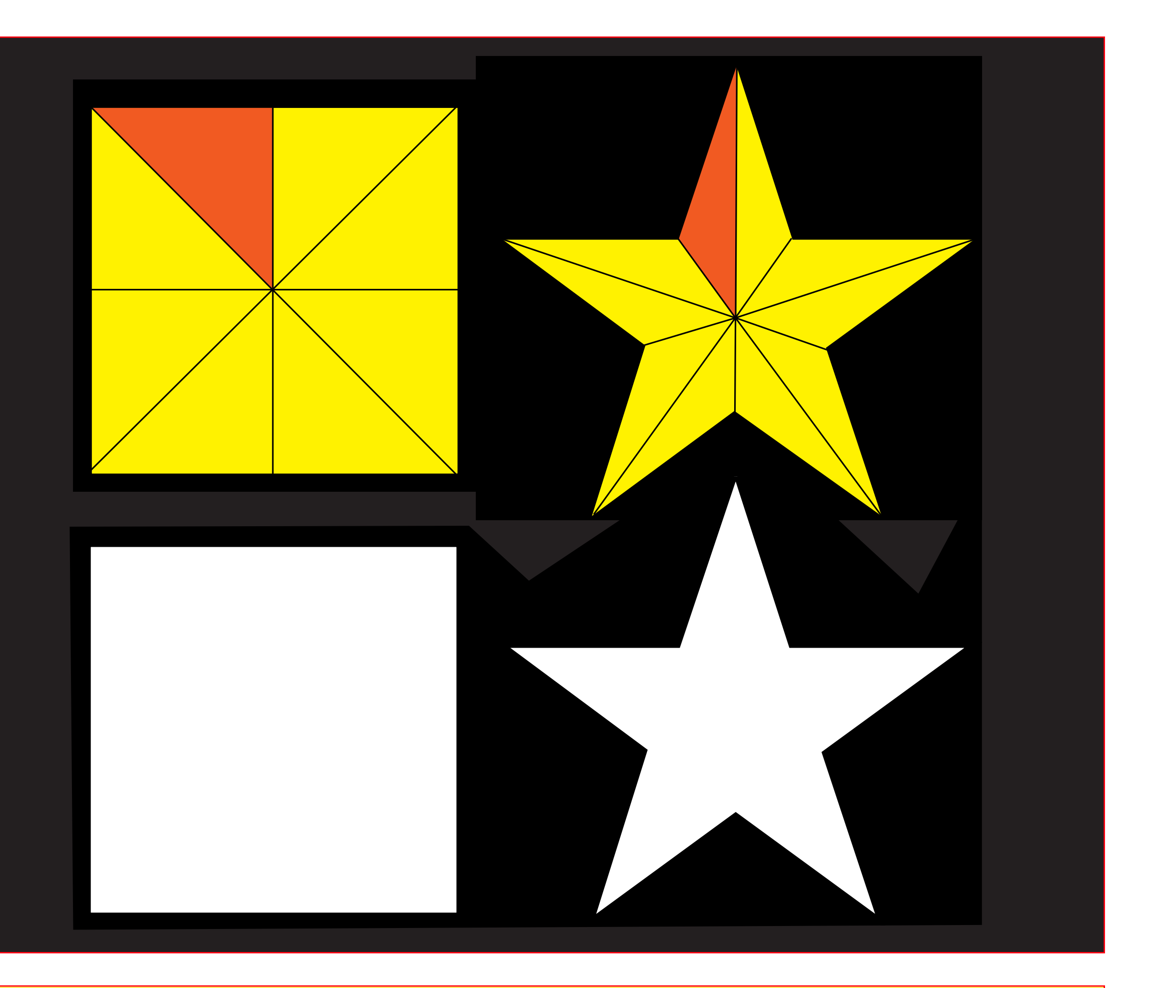
PLAYTHINK 111

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎ 📄 ✂
COMPLETION: ☐ TIME: ____

SYMMETRY OF THE SQUARE AND STAR

Cut out a square and a star and color them as shown on both sides, making sure that the red areas are red both front and back and that the yellow areas are yellow both front and back.

How many different ways can you place the square and the star in their respective outlines at right? Mathematicians call this sort of movement a transformation.

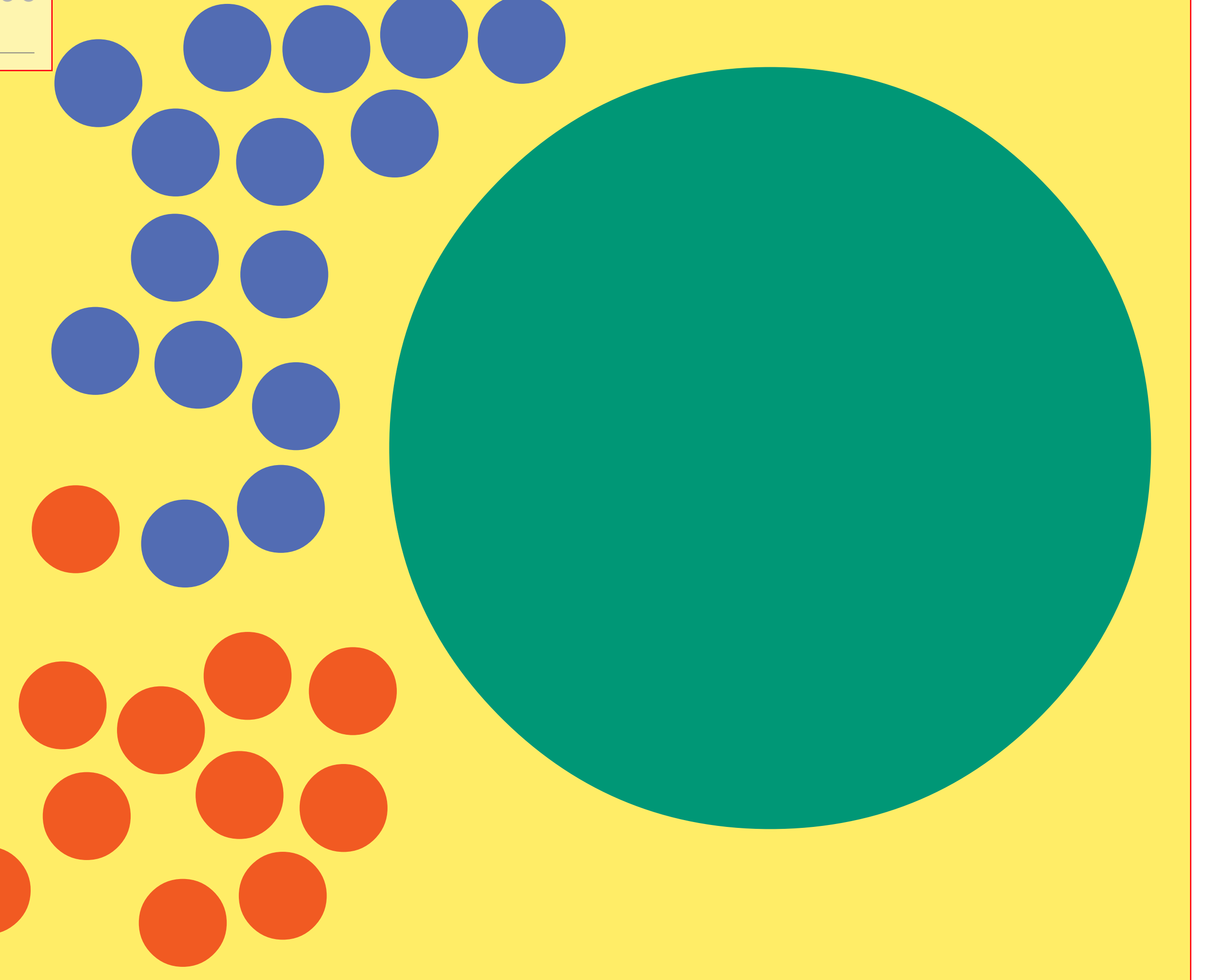


PLAYTHINK 112

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

PLACING COINS

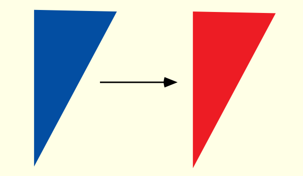
Two players take turns placing identical coins on a perfectly round table. The first player who cannot put a coin on the table without overlapping an existing coin loses. Can you devise a strategy so that one of the players will always win, no matter how large the table is?



Isometries of the Plane

A transformation of the plane is a movement of its points. There are many types of transformations, but the most important are the rigid motions, or isometries, which move figures but do not change their size or shape. (Note that an isometry that leaves the object looking the same is called a symmetry.) There are four basic types of isometries of the plane:

Translation

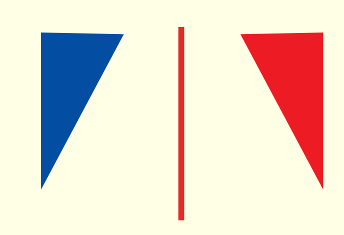


The red and blue triangles are congruent, which means they are exactly identical and that one transformation may superimpose one upon the other. In this case the blue triangle may slide onto the red one without turning. That is called translation.

Rotation

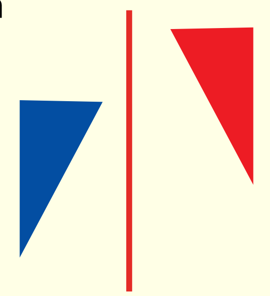
In this instance the congruent triangles can be superimposed by turning one of them around a point at one vertex. That is called rotation.

Reflection



The red and blue triangles are mirror images of each other. No motion within the plane of the triangles will allow one to be superimposed on the other. But what if you could lift one off the plane and turn it over, much like turning a page in a book? That is what occurs during reflection.

Glide Reflection

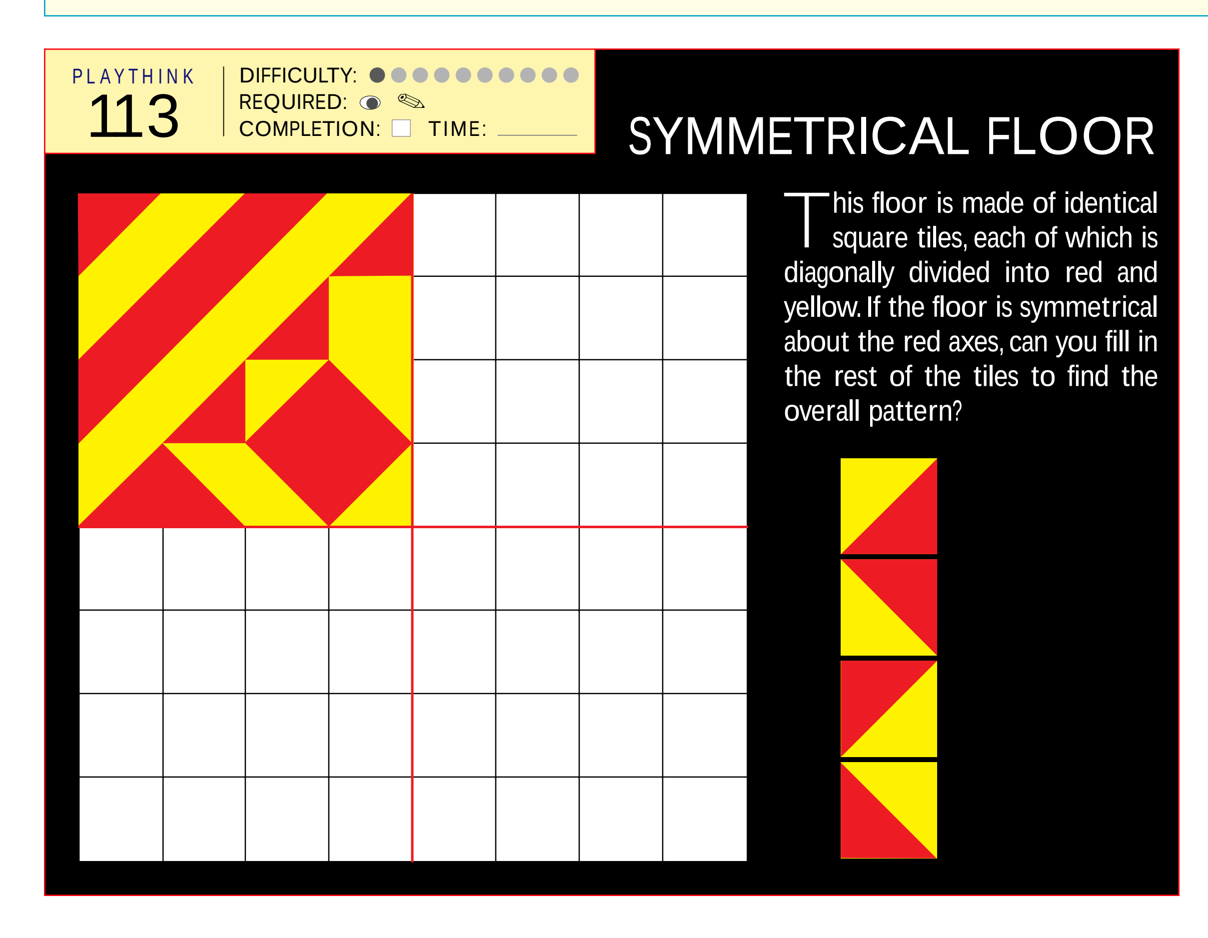


Glide reflection is simply the combination of reflection and translation.

PLAYTHINK 113 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ____

SYMMETRICAL FLOOR

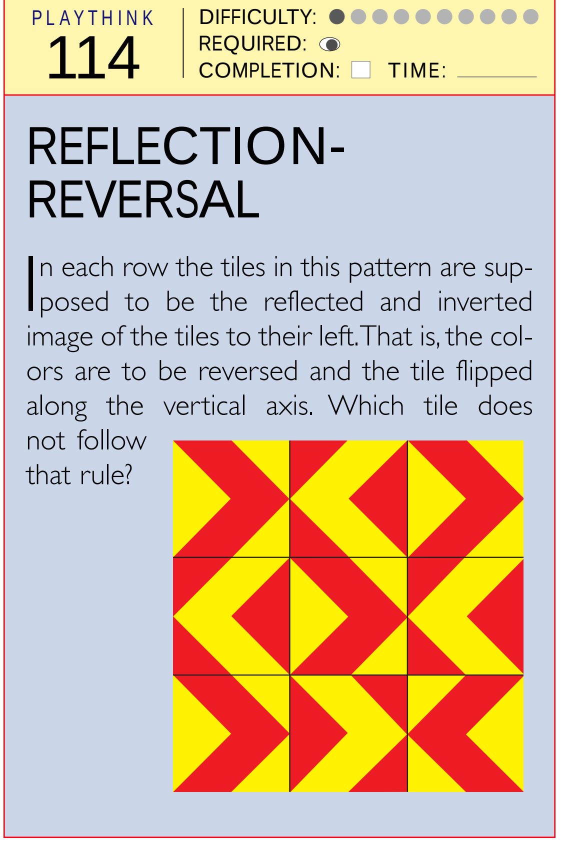
This floor is made of identical square tiles, each of which is diagonally divided into red and yellow. If the floor is symmetrical about the red axes, can you fill in the rest of the tiles to find the overall pattern?



PLAYTHINK 114 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ____

REFLECTION-REVERSAL

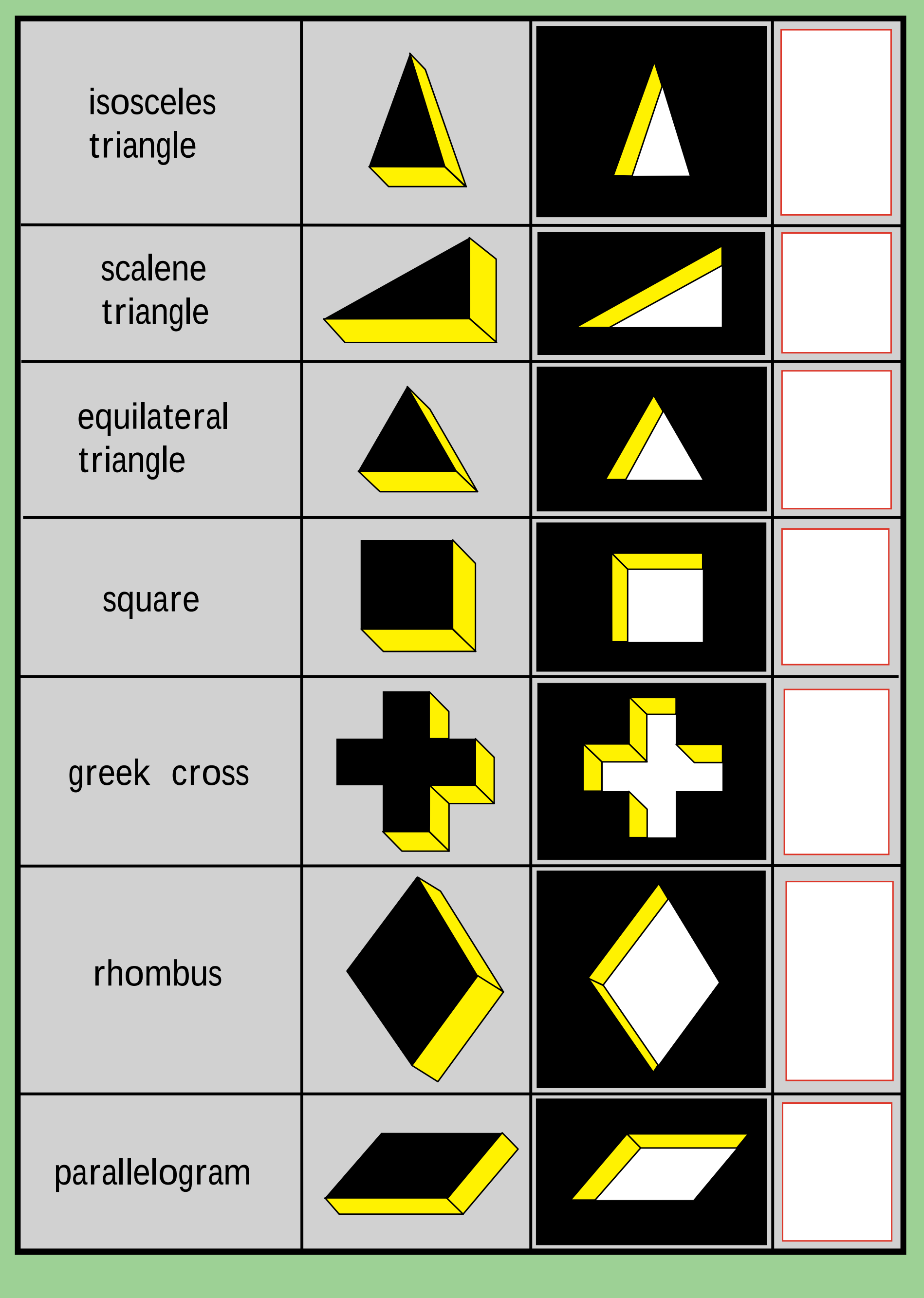
In each row the tiles in this pattern are supposed to be the reflected and inverted image of the tiles to their left. That is, the colors are to be reversed and the tile flipped along the vertical axis. Which tile does not follow that rule?



PLAYTHINK 115 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

FITTING HOLES

How many different ways can the seven flat shapes fit into the holes at right? Treat each piece like a three-dimensional object of considerable thickness that can undergo every sort of normal manipulation.



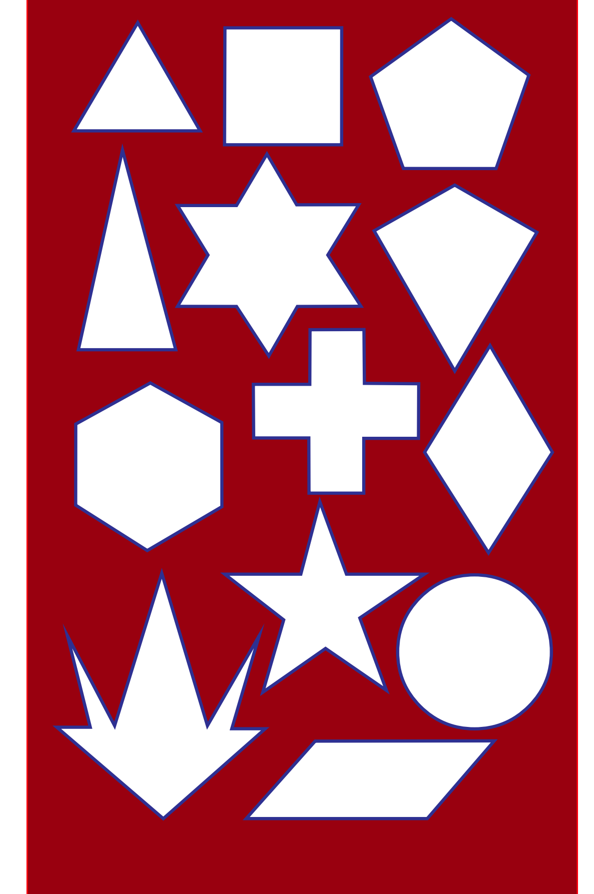
"GEOMETRY IS ONE AND ETERNAL SHINING IN THE MIND OF GOD."

—JOHANNES KEPLER

PLAYTHINK 117 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

SYMMETRY AXES

Symmetrical patterns can be found by folding and cutting paper or by using a plane mirror. For each of the thirteen shapes shown below, find and draw the symmetry axes. Are some figures not symmetrical? Which shape has the most symmetry axes?



PLAYTHINK 116 | DIFFICULTY: ●●●○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

ALPHABET 1

What do the red letters have in common? What do the blue letters have in common?



PLAYTHINK 118

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

SYMMETRY CRAFT

This image with three red lines as an indication of rotational and reflectional symmetry (the axis of which points straight up) has had most of its colored tiles removed. Even so, enough remain for you to reconstruct the original image by following the rules of symmetry. Can you color the tiles correctly?

PLAYTHINK 119

DIFFICULTY: ●●●○○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

SYMMETRY ALPHABET

Can you draw the symmetry axes for the capital letters of the alphabet? If the letter is rotationally symmetrical, draw the point of rotation. Leave asymmetrical letters unmarked.

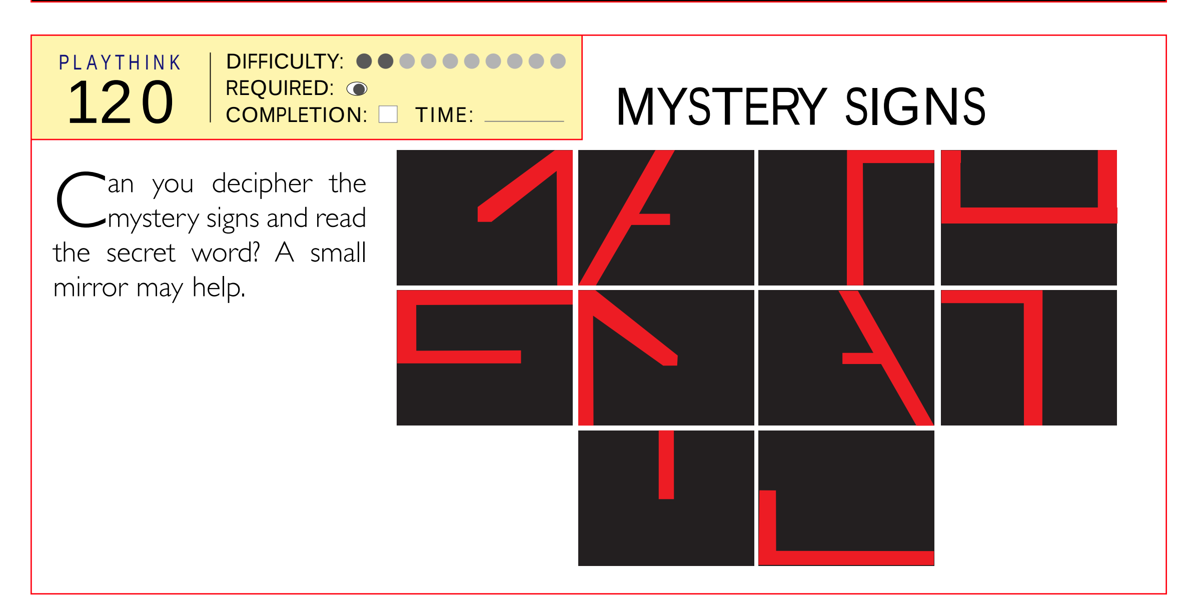
ABCD
EFGH
IJKL
MNO
PQRS
TUVW
XYZ

PLAYTHINK 120

DIFFICULTY: ●●○○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

MYSTERY SIGNS

Can you decipher the mystery signs and read the secret word? A small mirror may help.

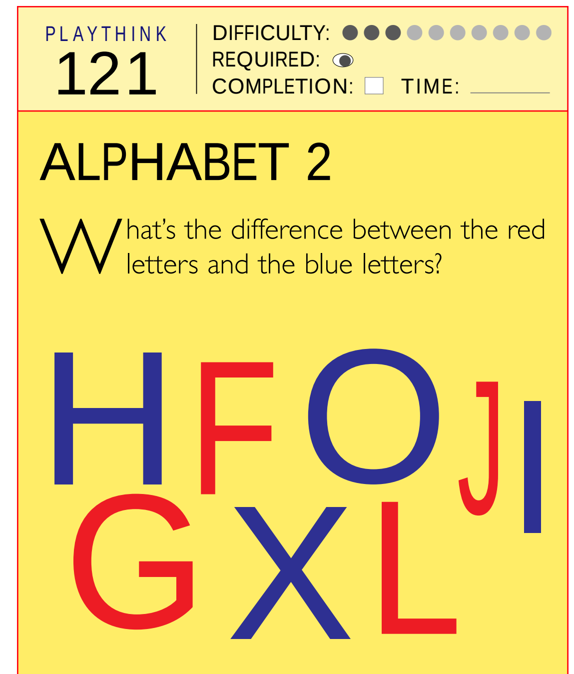


PLAYTHINK 121

DIFFICULTY: ●●●○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

ALPHABET 2

What's the difference between the red letters and the blue letters?



PLAYTHINK 122

DIFFICULTY: ●●●○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

ALPHABET 3

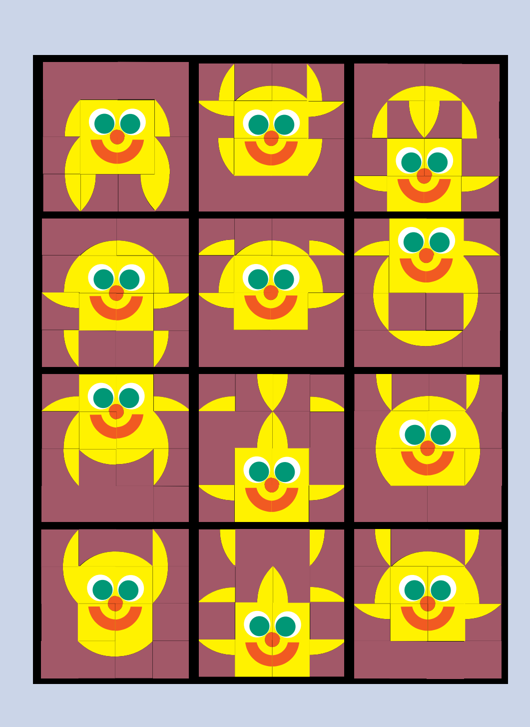
What is the difference between the red letters and the blue letters?

N P S R Z Q

PLAYTHINK 123

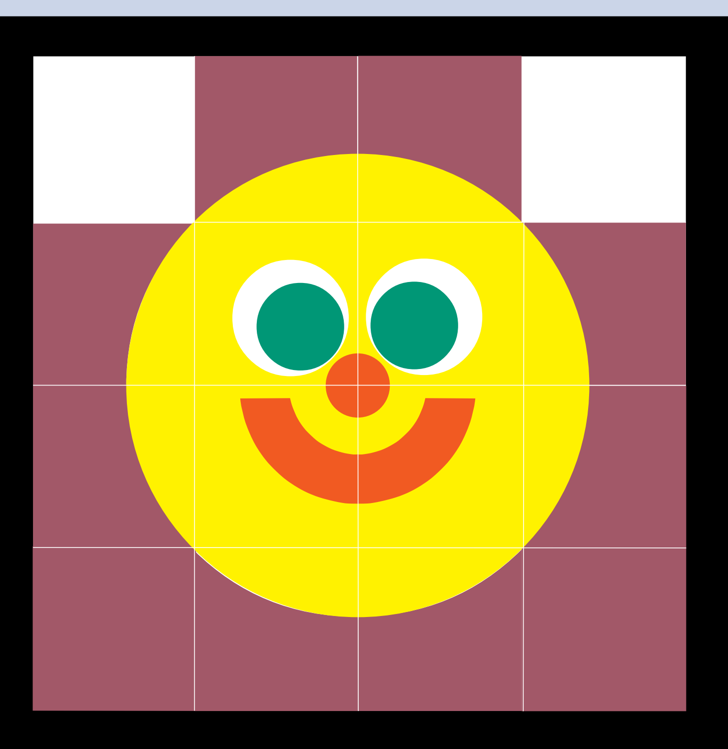
DIFFICULTY: ●○○○○○○○○○
REQUIRED: 👁 📄 ✂
COMPLETION: ☐ TIME: ______

TRANSCLOWN: GAME OF A THOUSAND FACES



Create a set of tiles like the fourteen shown below and place them in the basic starting configuration.

The object of the game is to transform the starting image into one of the twelve faces on the cards shown below left. Players take turns sliding two or more tiles horizontally or vertically into the empty spaces—taking care to preserve the symmetry of the picture at all times. No piece can leave the four-by-four grid, but several pieces may move at once, with some of the pieces filling the void left by other pieces as they move into the empty spaces.



During each turn a player may make five moves. Every time a player re-creates one of the faces on the card, that player takes the card. The winner is the player who has taken the most cards.

According to the rules of the game, which of the twelve faces is it impossible to re-create?

The Golden Rectangle

The ancient Greeks discovered a rectangle with unique properties. If you subtract from the rectangle a square with sides equal to the short side of the rectangle, you are left with a new, smaller rectangle whose sides are in the same ratio as the original. The Greeks believed that the two sides of that rectangle bore a divine relationship and called the proportion from one to another the golden ratio. That ratio, approximately 1.6180037, is often symbolized by the Greek letter ϕ, much the same way the number 3.14159 . . . is symbolized by π.

The golden ratio shows up in the growth patterns of many plants and animals. For example, the growth of the nautilus shell follows the same pattern as the logarithmic spiral formed by the golden rectangle.

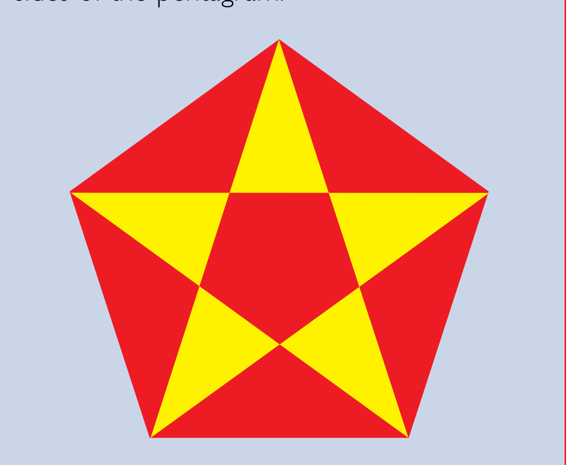
PLAYTHINK 124

DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁 ✏
COMPLETION: ☐ TIME: ______

GOLDEN TRIANGLE

Draw all the diagonals in a regular pentagon. You have created a five-pointed pentagonal star known as the pentagram. Since pentagonal symmetry is found throughout nature—in plants and in animals such as the starfish—it is sometimes called the symmetry of life.

Because the secret from which the golden rectangle and golden triangle could be created lies within the pentagram, it was the secret symbol of Pythagoras and his followers. To begin to understand its mystery, figure out the proportion of the sides of the pentagon to the sides of the pentagram.

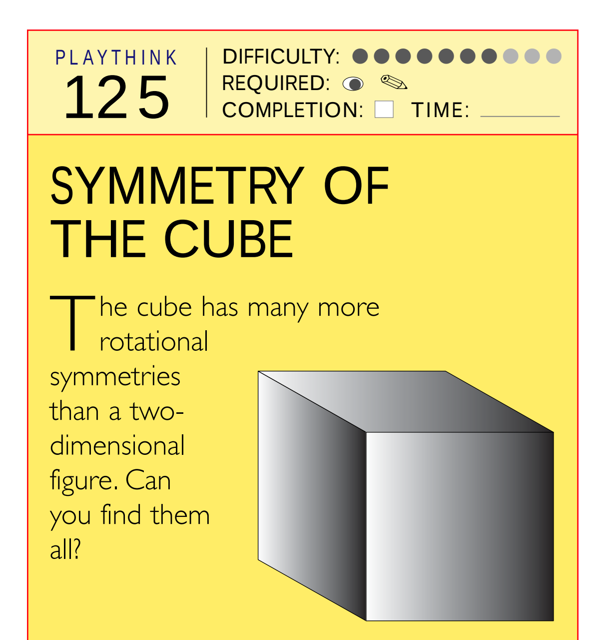


PLAYTHINK 125

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✏
COMPLETION: ☐ TIME: ______

SYMMETRY OF THE CUBE

The cube has many more rotational symmetries than a two-dimensional figure. Can you find them all?



PLAYTHINK 126

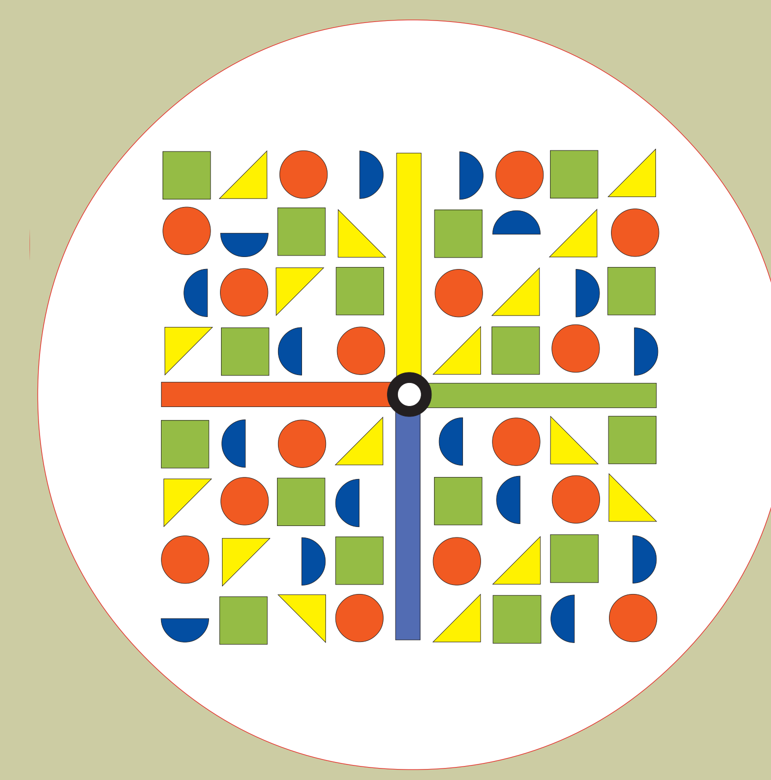
DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✏
COMPLETION: ☐ TIME: ______

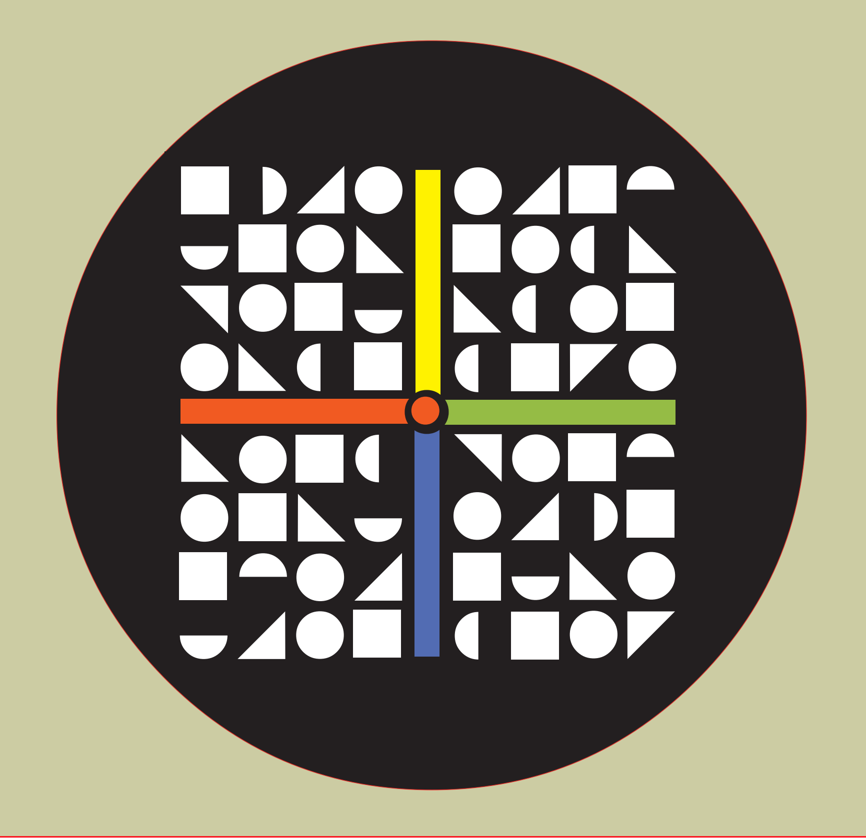
ISOMETRIX: THE SHAPE GAME

There are two boards used in this game: a stationary base with holes cut into it (bottom right) and a similar board (bottom left) that is placed atop the base and revolves around its center point. Initially there are sixty-four shapes that fit in the holes of the revolving board: sixteen squares, sixteen right isosceles triangles, sixteen circles and sixteen semicircles.

See if you can use your mind's eye to track the revolving board as it makes one complete turn. Some of the shapes will fall through the board right away; others will fall through after the first quarter turn in the clockwise direction. Still more will fall through after a full half turn. Can you fill in the table at right for the number of each type of shape that falls after each quarter turn? Can you discover which shape or shapes will stay on after a whole turn?

Number of shapes initially on top	16	16	16	16
SHAPES	■	◢	●	◗
Number of shapes initially falling through	☐	☐	☐	☐
Number of shapes falling through after 1/4 turn clockwise	☐	☐	☐	☐
Number of shapes falling through after 1/2 turn clockwise	☐	☐	☐	☐
Number of shapes falling through after 3/4 turn clockwise	☐	☐	☐	☐
Number of shapes falling through after one full turn clockwise	☐	☐	☐	☐
Number of shapes staying on top	☐	☐	☐	☐





PLAYTHINK 127

DIFFICULTY: ●●○○○○○○○○
REQUIRED: ✎ 📄 ✂
COMPLETION: ☐ TIME: ______

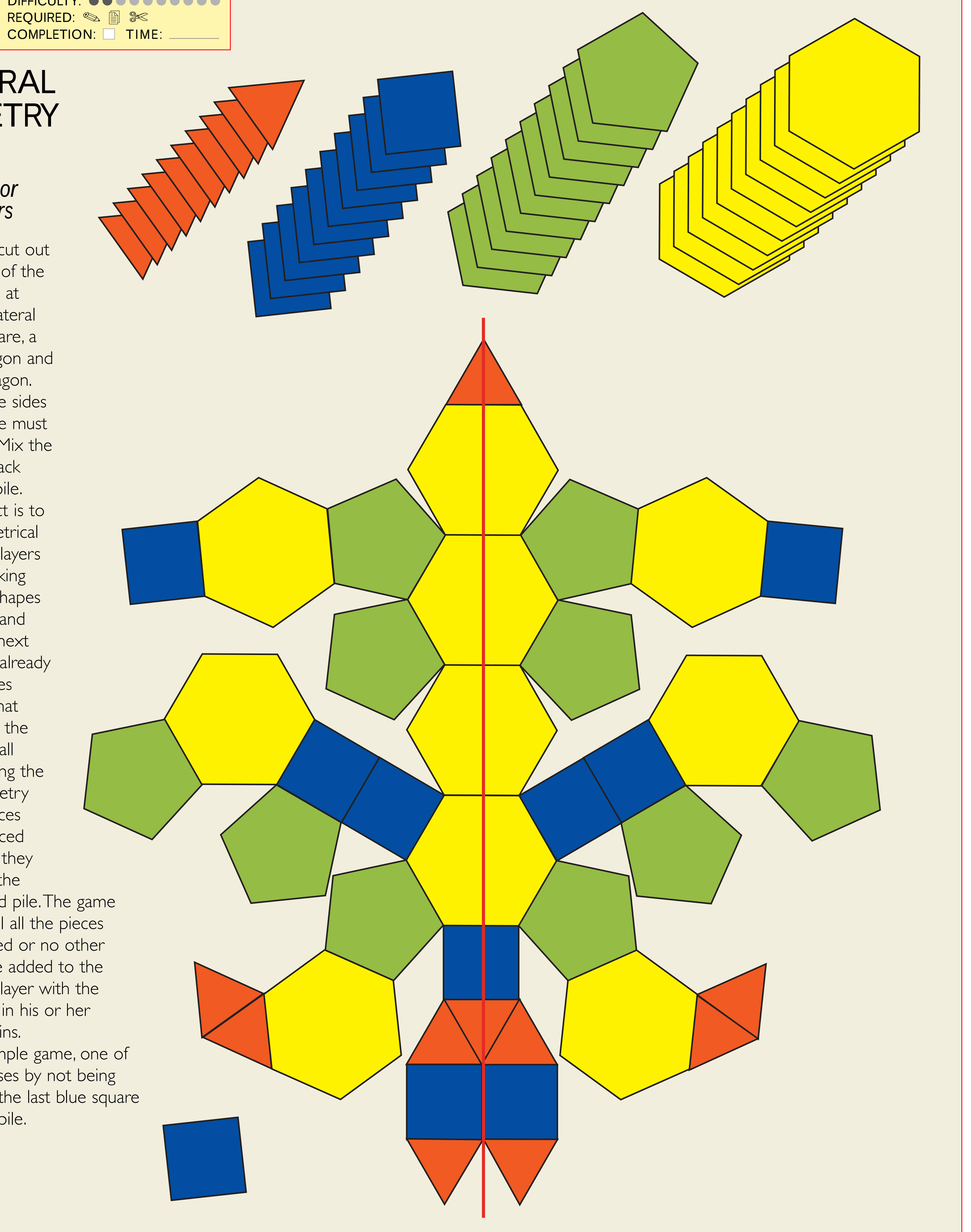
BILATERAL SYMMETRY GAME

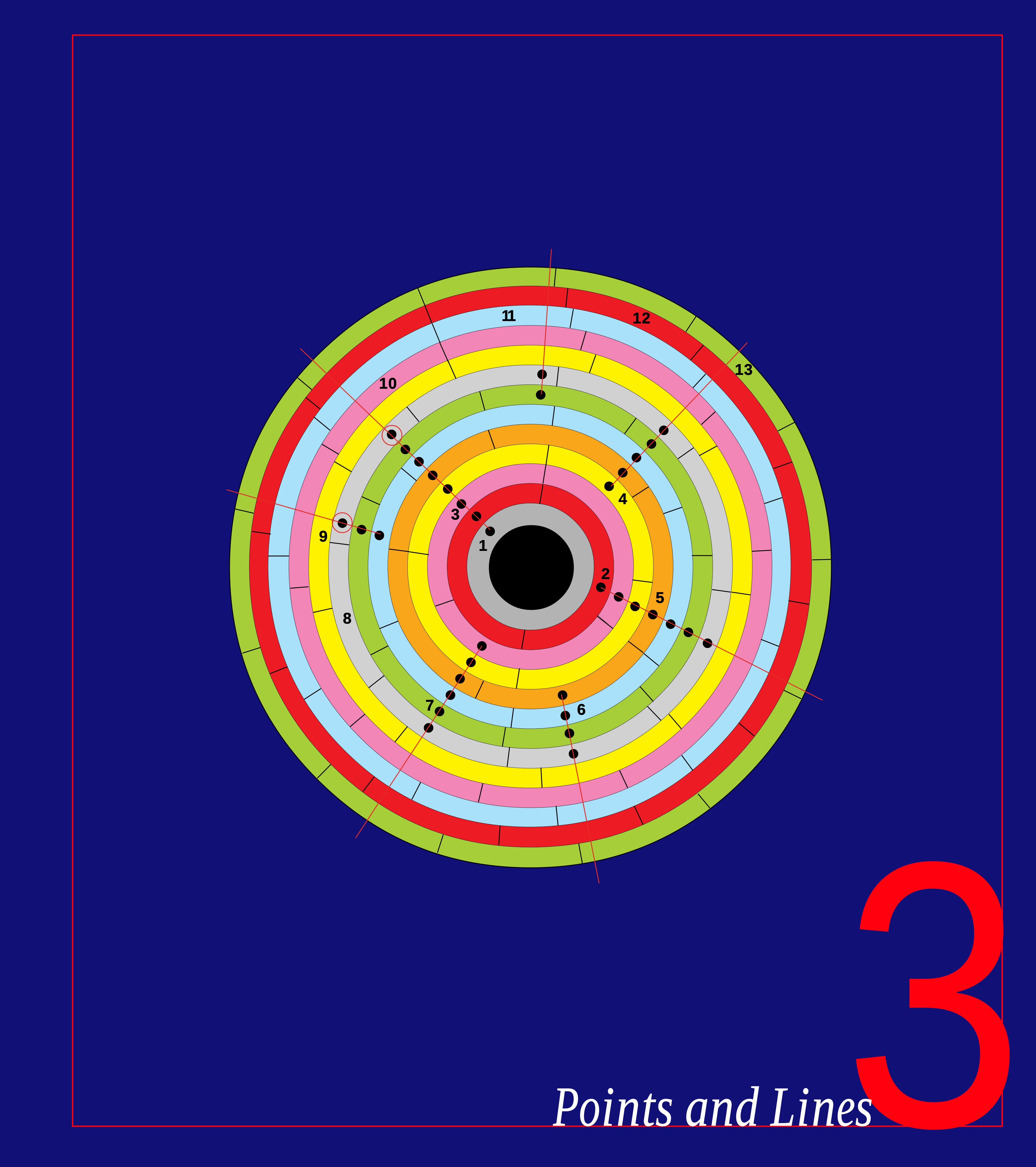
■ *For one or more players*

Copy and cut out ten each of the shapes shown at right: an equilateral triangle, a square, a regular pentagon and a regular hexagon. (Note that the sides of every shape must be identical.) Mix the shapes and stack them in one pile.

The object is to build a symmetrical pattern. The players take turns picking the top two shapes from the pile and placing them next to the pieces already laid down, sides touching, so that they preserve the pattern's overall symmetry along the vertical symmetry axis. If the pieces cannot be placed symmetrically, they must go into the player's discard pile. The game continues until all the pieces have been used or no other pieces may be added to the pattern. The player with the fewest pieces in his or her discard pile wins.

In this sample game, one of the players loses by not being able to place the last blue square in his discard pile.




1
2
3
4
5
6
7
8
9
10
11
12
13
Points and Lines
3

The Basic Tools of Geometry

Points are not just marks—they are mathematical symbols that define position. And lines are not only the fundamental elements of drawn images but also mathematical symbols that link points, indicate distance and direction and define space. Points and lines—and the relationships between them—are the basic tools of geometry.

The ancient Greeks had to turn geometry from the practical study of measuring land to the science of abstract form before they could produce mathematical proofs. They "idealized" points and lines, thereby creating an abstract world to which the laws of geometry could apply with perfect accuracy. And they understood that they could obtain real conclusions from that idealized world only by making geometry deductive—that is, based on axioms. Euclid's *Elements*, the greatest work of Greek geometry, was for centuries the ultimate textbook of human reasoning. Indeed, it was not until well into the nineteenth century that Georg Cantor took the final step of including every possible form and shape into geometry.

PLAYTHINK 128 | DIFFICULTY: ●●●●●●●●●○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

THE THIRTEEN-POINT GAME

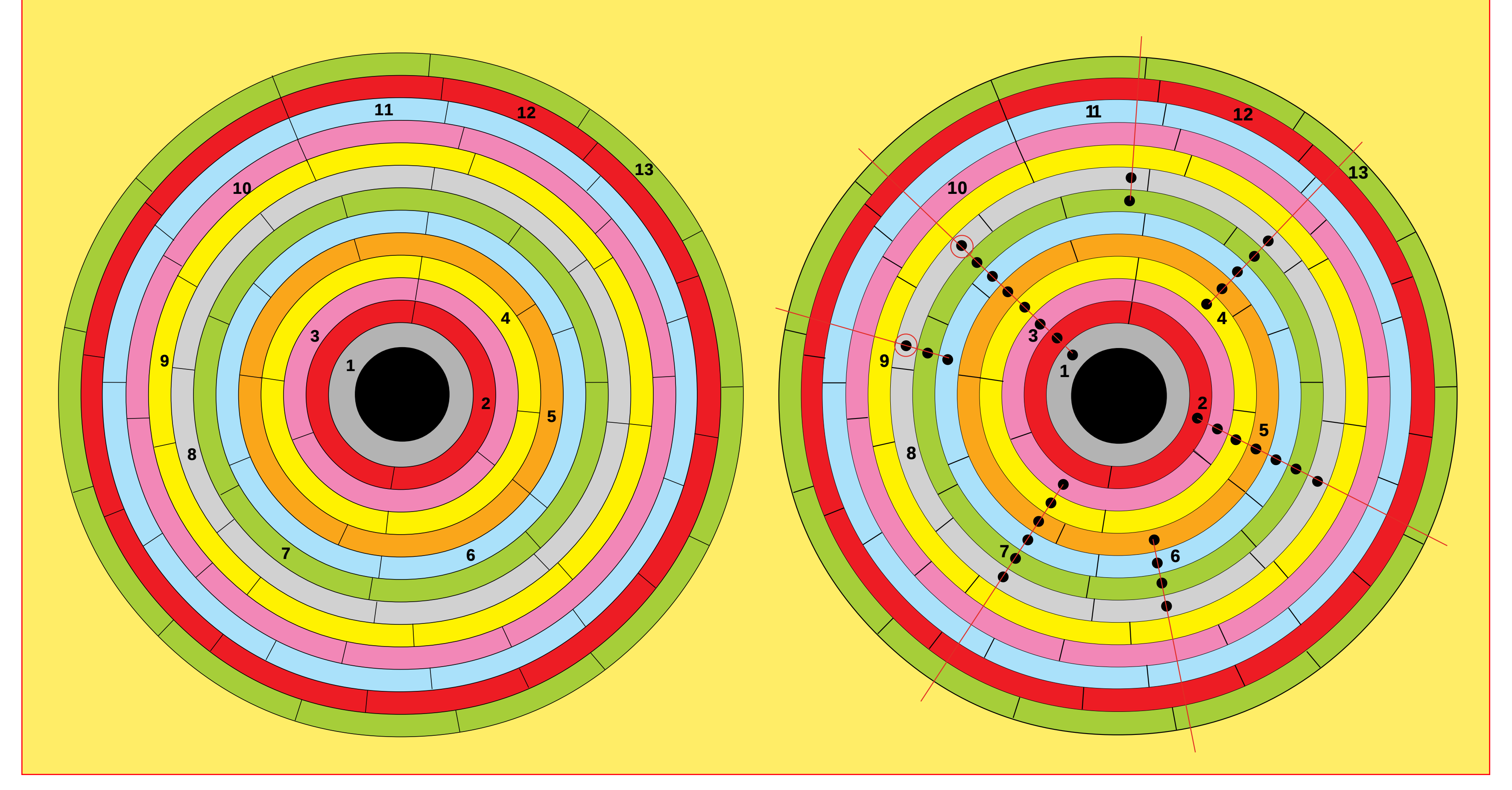
Imagine that there is a circular strip of land on which someone has planted one tree. Divide the strip in half and plant a tree somewhere on the half that doesn't have a tree. Then divide the strip into thirds and plant a tree on the third of the strip that is treeless.

How long can you keep this up? Can you divide the strip thirteen times so that each plot has its own tree?

The image shown at bottom left depicts thirteen concentric circles, but this is only for convenience. It is really the same strip of land seen after successive divisions.

A challenging two-player game can be played following this sequence: players can alternate placing marks (which symbolizes planting trees). Play continues until one player cannot plant a tree in an empty section.

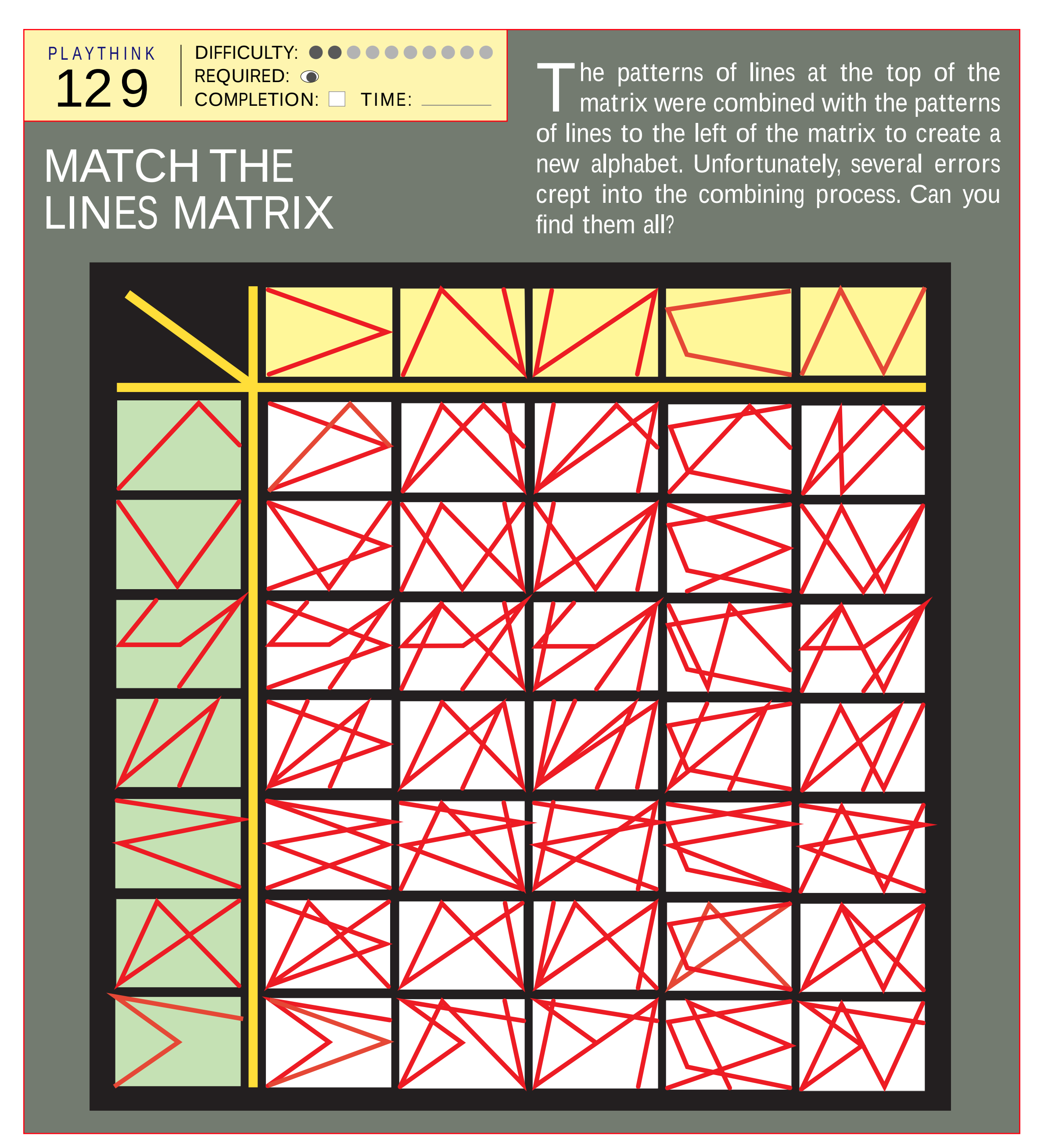
The sample game at right ended after the eighth move because two marks wound up in the same section.



PLAYTHINK 129 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

MATCH THE LINES MATRIX

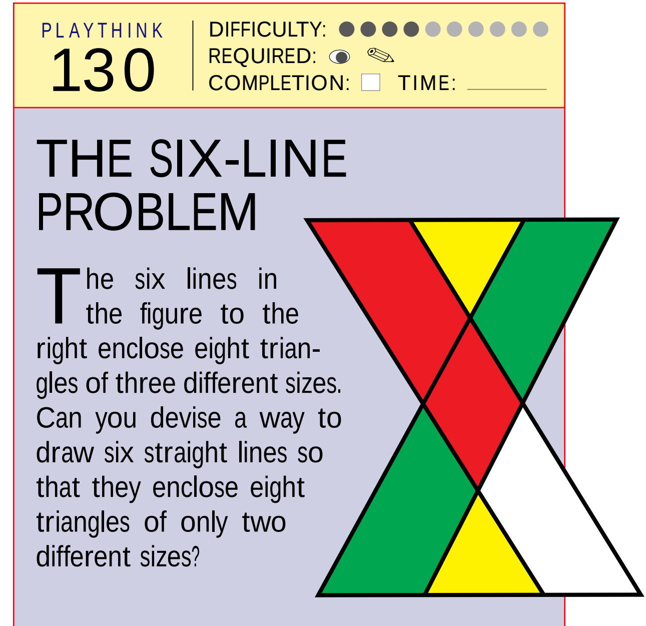
The patterns of lines at the top of the matrix were combined with the patterns of lines to the left of the matrix to create a new alphabet. Unfortunately, several errors crept into the combining process. Can you find them all?



PLAYTHINK 130 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

THE SIX-LINE PROBLEM

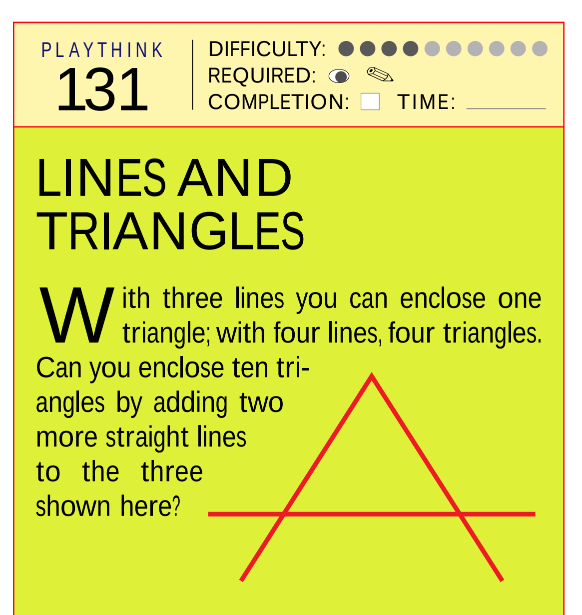
The six lines in the figure to the right enclose eight triangles of three different sizes. Can you devise a way to draw six straight lines so that they enclose eight triangles of only two different sizes?



PLAYTHINK 131 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

LINES AND TRIANGLES

With three lines you can enclose one triangle; with four lines, four triangles. Can you enclose ten triangles by adding two more straight lines to the three shown here?



Dimensions

All of geometry begins with the point, which indicates a position on a two-dimensional plane or in three-dimensional space. The point, which is the intersection of two or more lines, is a pure abstraction. You must imagine that it is there.

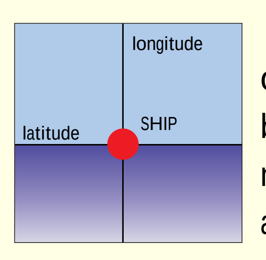
The most fundamental concept in geometry is the idea of dimension.

The position of a car on a road can be indicated by a single number, its distance from some location—a milestone.

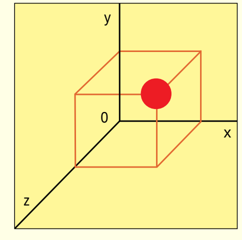
TOWN

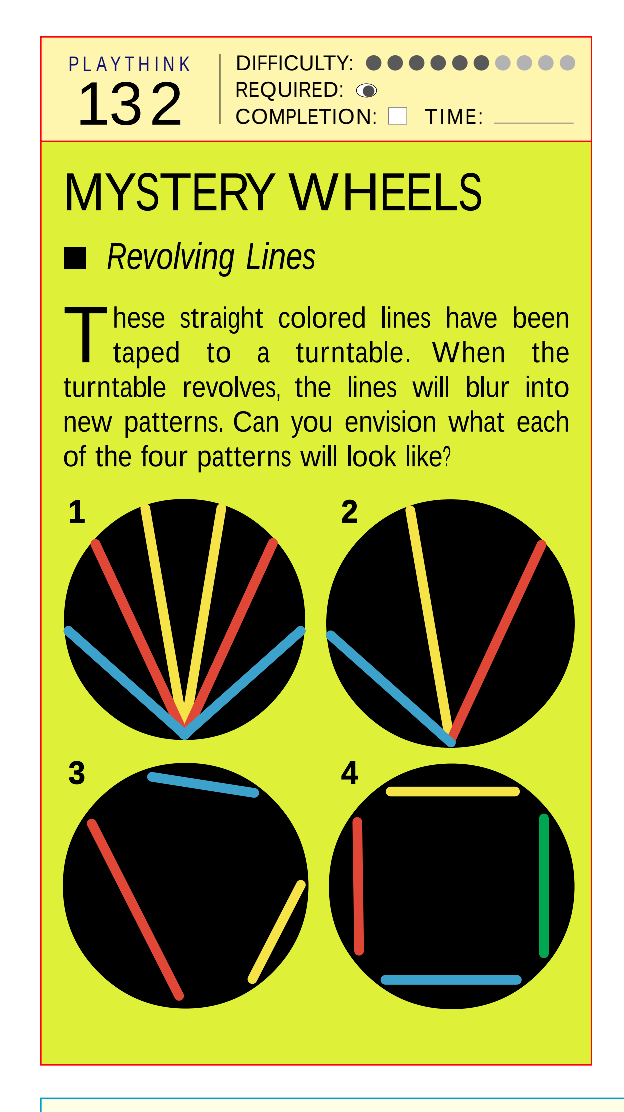
CAR

The location of a ship at sea can be determined by noting its latitude and longitude. Two dimensions, two numbers.



The position of a point in a room can be pinned down with three numbers, or coordinates—say, the distance from two of the walls and its height off the floor. Three-dimensional coordinates are usually given as *x*, *y*, and *z*.

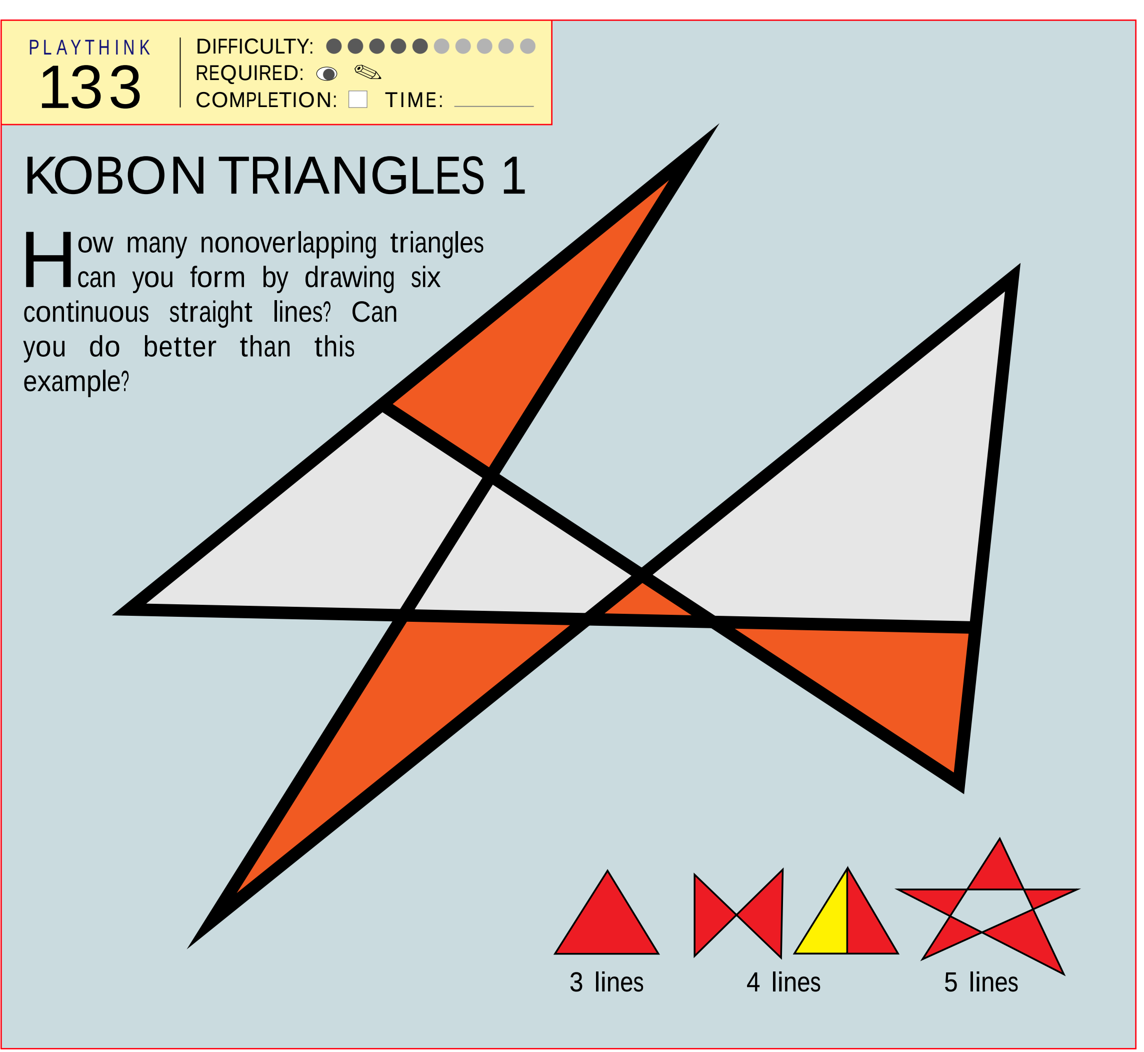




MYSTERY WHEELS

■ *Revolving Lines*

These straight colored lines have been taped to a turntable. When the turntable revolves, the lines will blur into new patterns. Can you envision what each of the four patterns will look like?



KOBON TRIANGLES 1

How many nonoverlapping triangles can you form by drawing six continuous straight lines? Can you do better than this example?

The Point

At some point something must have come from nothing.

Inside the white box at right there is a point. Can you see it?

No, there's no printing error. Just because you cannot see the point doesn't mean it isn't there. The point is an imaginary object, a purely abstract idea.

A point has no dimension and occupies no space. If a plane exists in two dimensions and a line in one dimension, then a point is a zero-dimensional object. Since it is difficult to refer to something you can't see, the point is usually represented by a dot, which is a small circle on a plane or a small sphere in three-dimensional space.

So a point is "nothing," but it is the fundamental particle from which all of geometry is built. Can it be said that geometry is built on an imaginary foundation?

Now that you have been introduced to the point, we can start constructing the beautiful and playful structure of geometry. For instance, it is now clear that within the white box there is not simply one point but an infinite number of points. That observation will soon be very important.

> "AT SOME POINT SOMETHING MUST HAVE COME FROM NOTHING."

PLAYTHINK 134

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

INSIDE-OUTSIDE

The black line forms one continuous loop. Can you determine which dots are inside the loop and which dots are outside? There is an easy way than simply following the loop around.

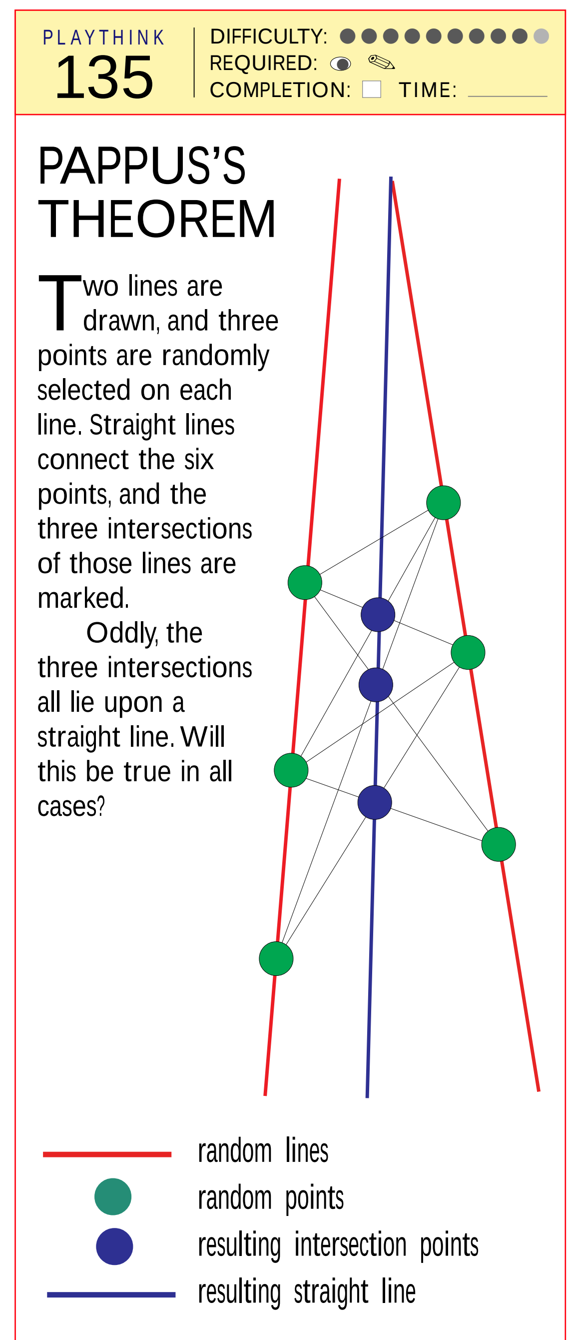
PLAYTHINK 135

DIFFICULTY: ●●●●●●●●●○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

PAPPUS'S THEOREM

Two lines are drawn, and three points are randomly selected on each line. Straight lines connect the six points, and the three intersections of those lines are marked.

Oddly, the three intersections all lie upon a straight line. Will this be true in all cases?



PLAYTHINK 136

DIFFICULTY: ●●○○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

CONVEX OR SIMPLE?

A convex polygon is one in which every point in the interior can be connected to any point on the perimeter with a straight line that does not cross the perimeter. A simple polygon is one in which no lines or sides cross each other. Working with that basic information, can you figure out how many convex polygons are shown in the drawing at right?

One of the lines or polygons depicted is different from all the others. Can you tell which one it is?



The Eighteen-Point Problem

Mathematicians sometimes invent seemingly simple, trivial-looking problems that prove much more difficult to solve than anyone dare think. One such conundrum is the eighteen-point paradox, first mentioned by Martin Gardner in his "Mathematical Games" section in *Scientific American* magazine.

The object is to distribute eighteen points along a line according to some simple rules. Lines, of course, comprise a multitude of points—indeed, an infinite number of points are on a line. So you might imagine that with sufficient foresight, one could place an infinite number of points on a line. That intuition, however, turns out to be wrong.

The rules of the game are quite simple: Place a point anywhere on the line. Now place a second point so that each of the two points lies on a different half of the line.

Then place a third point so that each of the three points is in a different third of the line. At this stage it becomes clear that the first two points cannot be just anywhere; the points must be placed carefully so that when the third point is added, each will be in a different third of the line.

The game follows a predictable pattern—place the fourth point so that all are on different quarters, the fifth so that all are on separate fifths, and so on. You can proceed with this process as carefully as you wish, but it turns out, astonishingly, that you cannot go beyond seventeen points. The eighteenth point will always violate the rules of the game.

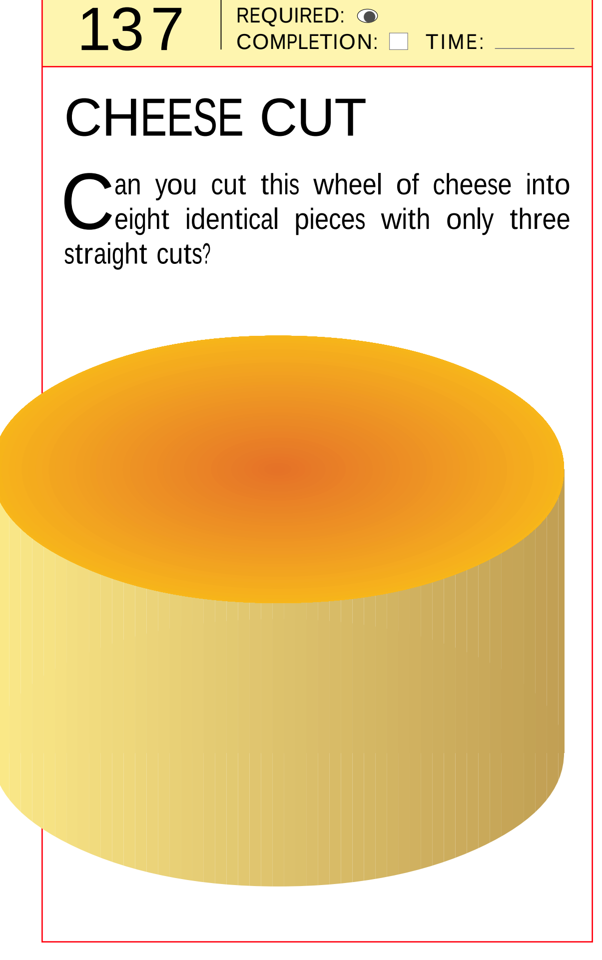
Even when you choose the locations of your points very carefully, placing ten points is a good result. You are doing even better if you can solve the version of this paradox in "The Thirteen-Point Game" (page 54).

PLAYTHINK 137

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

CHEESE CUT

Can you cut this wheel of cheese into eight identical pieces with only three straight cuts?

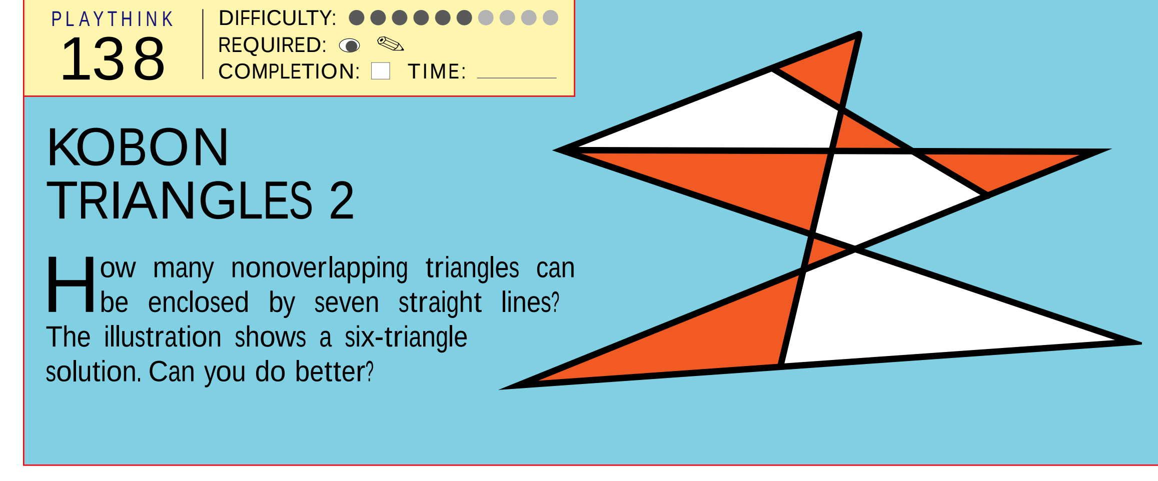


PLAYTHINK 138

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

KOBON TRIANGLES 2

How many nonoverlapping triangles can be enclosed by seven straight lines? The illustration shows a six-triangle solution. Can you do better?

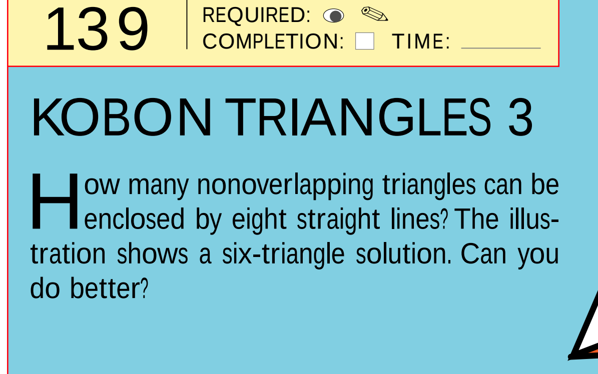


PLAYTHINK 139

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

KOBON TRIANGLES 3

How many nonoverlapping triangles can be enclosed by eight straight lines? The illustration shows a six-triangle solution. Can you do better?

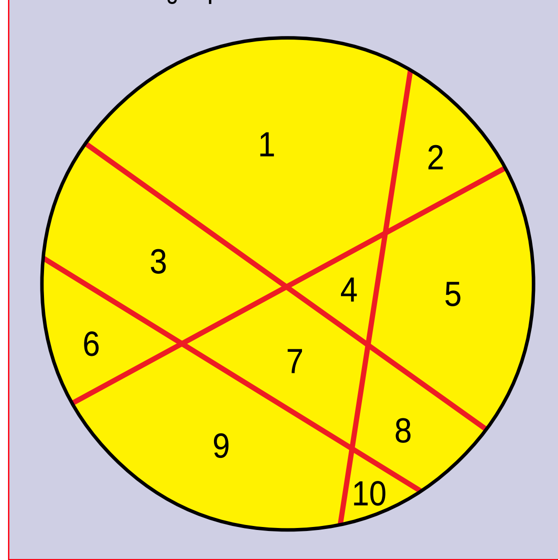


PLAYTHINK 140 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: | COMPLETION: ☐ TIME: ____

GREAT DIVIDE 1

Four straight cuts can divide a cake into ten pieces, as shown below. Is it possible to go one better and divide the cake into eleven pieces?

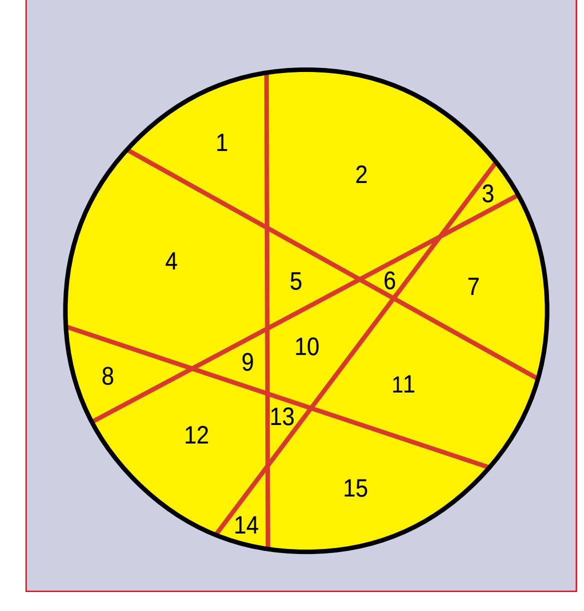
Can you determine the general rule for finding the greatest number of regions that can be formed by a given number of straight cuts in a single plane?



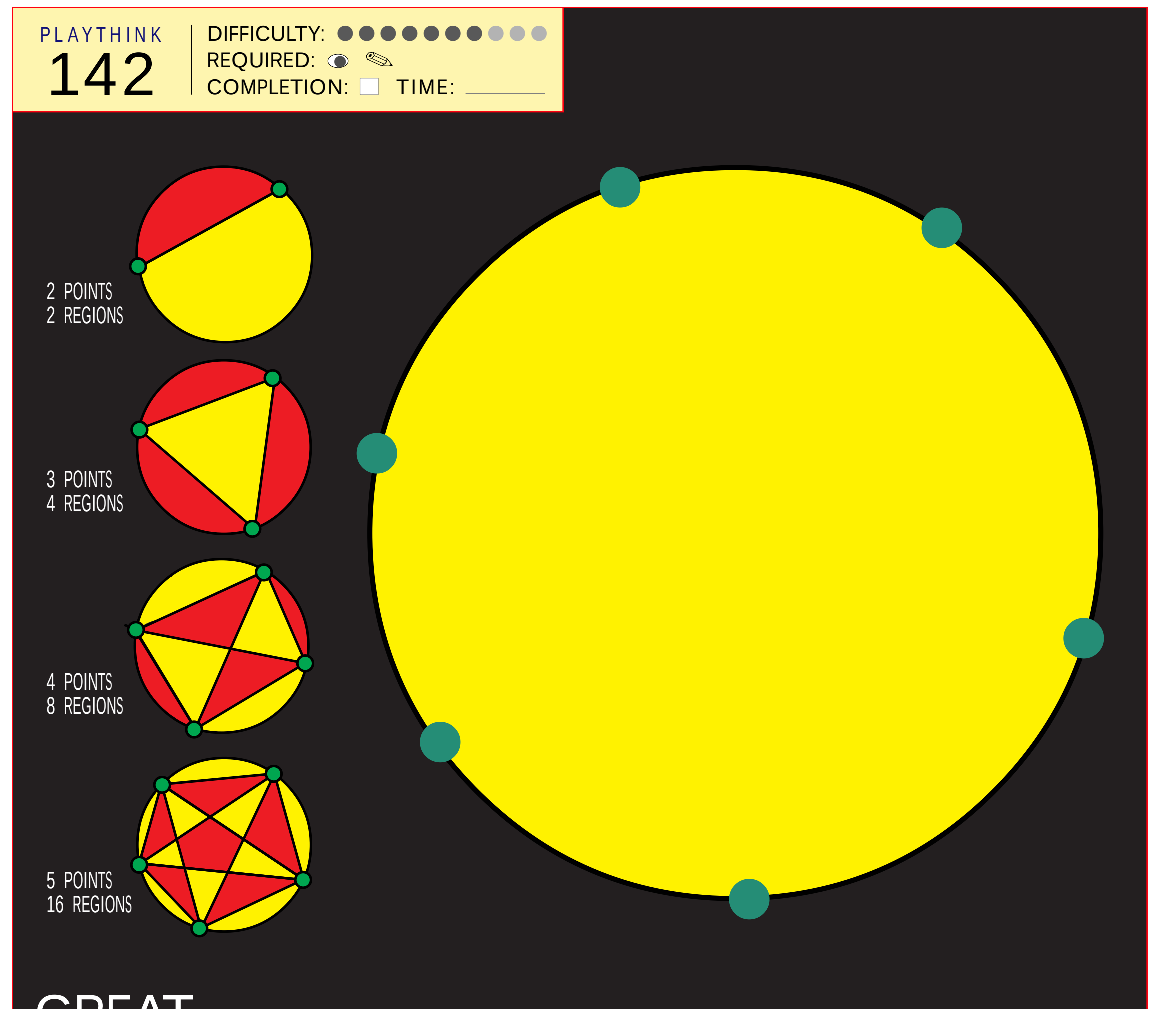
PLAYTHINK 141 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: | COMPLETION: ☐ TIME: ____

GREAT DIVIDE 2

Five straight cuts are used to cut a cake into fifteen pieces. Can you cut the cake into sixteen pieces with only five cuts?



PLAYTHINK 142 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: | COMPLETION: ☐ TIME: ____



GREAT DIVIDE 3

The circle above has six points on its circumference. Join all the points with straight lines and count the number of regions those cords have partitioned.

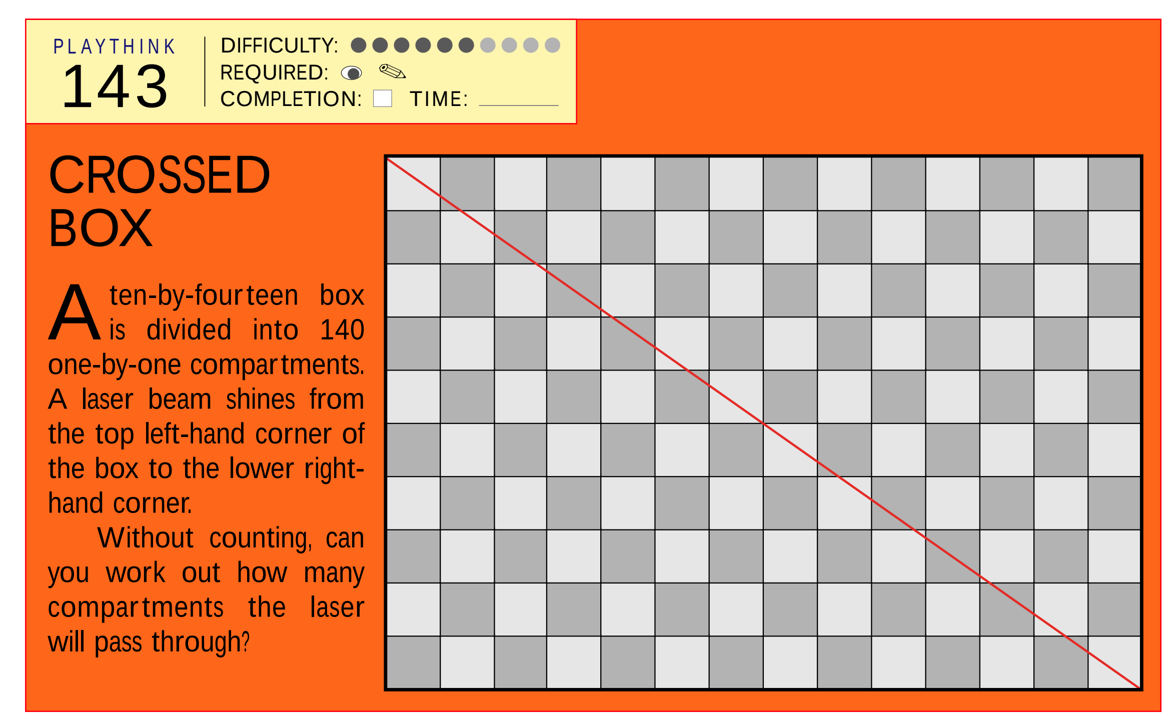
But before you count, looking at other cases may help you estimate the answer. Illustrated are the solutions for two, three, four and five randomly selected points on the circle. Based on that simple doubling series, what would your estimate be for the problem of six points?

PLAYTHINK 143 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: | COMPLETION: ☐ TIME: ____

CROSSED BOX

A ten-by-fourteen box is divided into 140 one-by-one compartments. A laser beam shines from the top left-hand corner of the box to the lower right-hand corner.

Without counting, can you work out how many compartments the laser will pass through?

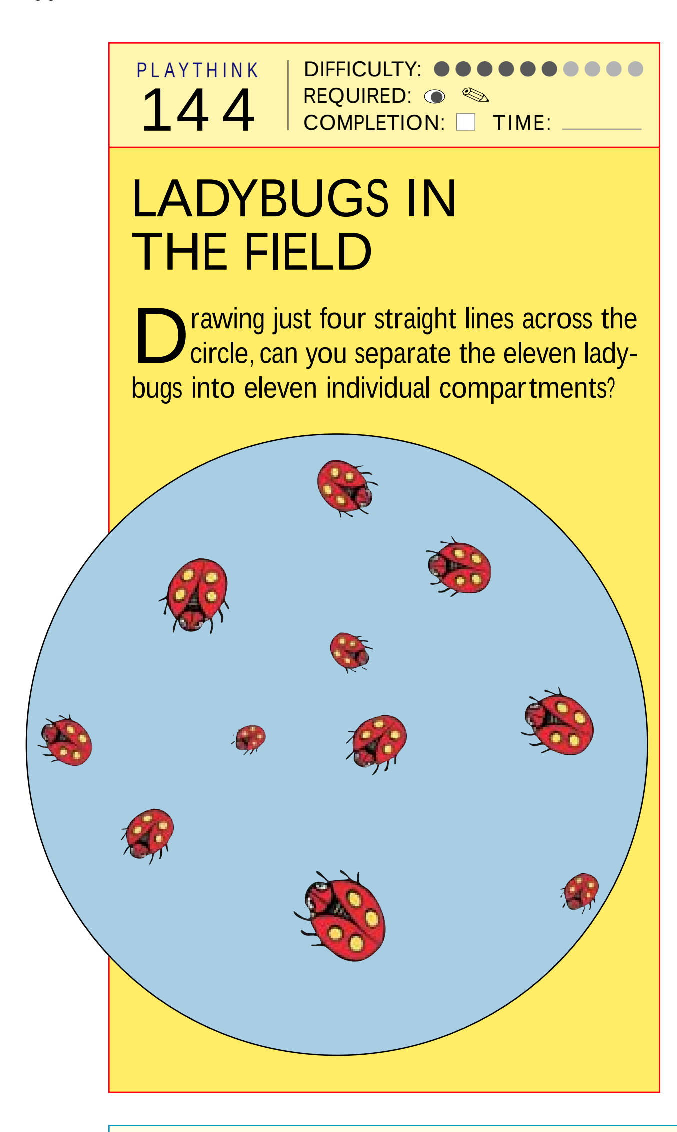


PLAYTHINK 144

DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

LADYBUGS IN THE FIELD

Drawing just four straight lines across the circle, can you separate the eleven ladybugs into eleven individual compartments?



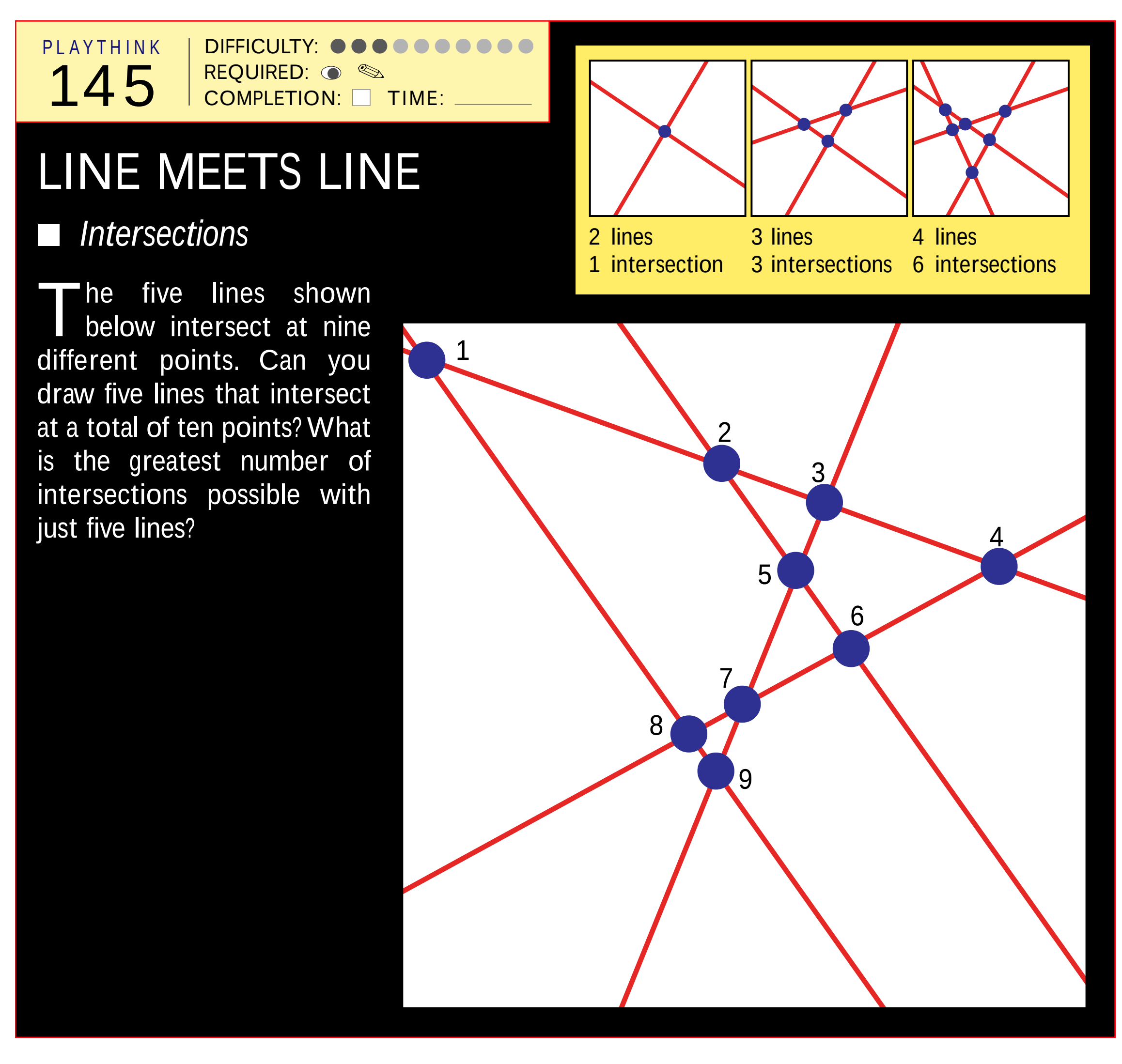
PLAYTHINK 145

DIFFICULTY: ●●●○○○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

LINE MEETS LINE

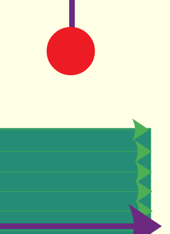
■ *Intersections*

The five lines shown below intersect at nine different points. Can you draw five lines that intersect at a total of ten points? What is the greatest number of intersections possible with just five lines?

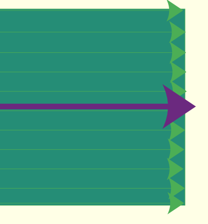


Pappus's Theorem: The Power of the Moving Point

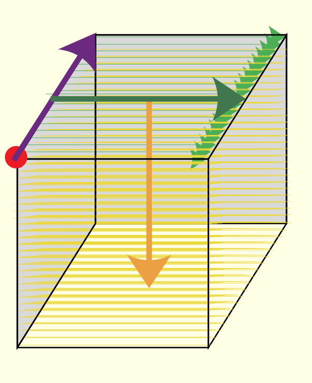
The great fourth-century mathematician Pappus of Alexandria first recognized that space could be filled with a moving point. A point moving in one dimension produces a straight line. That line moving in a direction perpendicular to the point defines a rectangle. And that rectangle moving in a direction at right angles to the point and the line creates a rectangular prism. This same concept can be extended to include points that move along curves to define complex areas and volumes. What's more, Pappus's theorem is the basis for the scanning mechanism that produces television images.



The dot creates a line when moving in one unit in some direction.



The line creates a square when moving one unit in a direction perpendicular to itself.



The square creates a cube when moving one unit in a direction perpendicular to itself.

Lines Through Dots

Let's see just how imaginative you are. Draw nine dots in a three-by-three square configuration. Then take your pencil and, without lifting it from the paper, draw a single line broken into no more than four straight segments that passes through all nine dots.

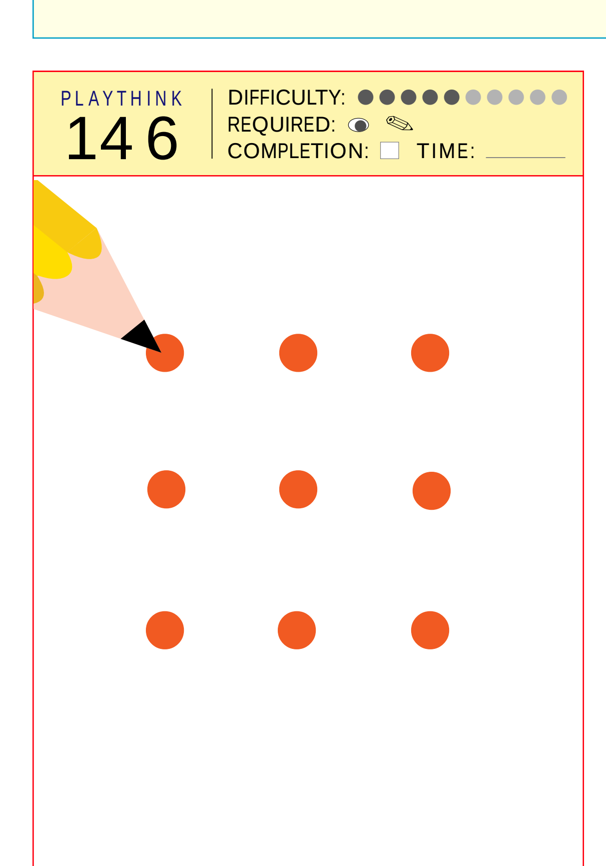
This problem appears to be impossible at first glance. Connecting eight of the nine points is easy, but connecting nine defies logic.

If you haven't discovered how to solve the problem, it may be because you have run into a conceptual block. Too often people confine themselves to a small number of possible solutions to a problem. For example, many people assume that the answer to this problem must consist of vertical and horizontal lines and that the lines must be confined to the "box" formed by the nine dots. But none of those restrictions were mentioned as part of the problem.

Diagonals and lines that extend beyond the visual boundary of the problem provide a way toward a solution.

In the 1990s business consultants and politicians often referred to the search for innovative solutions as "thinking outside the box." That was an allusion to the solution to this seemingly impossible puzzle.

PLAYTHINK 146 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

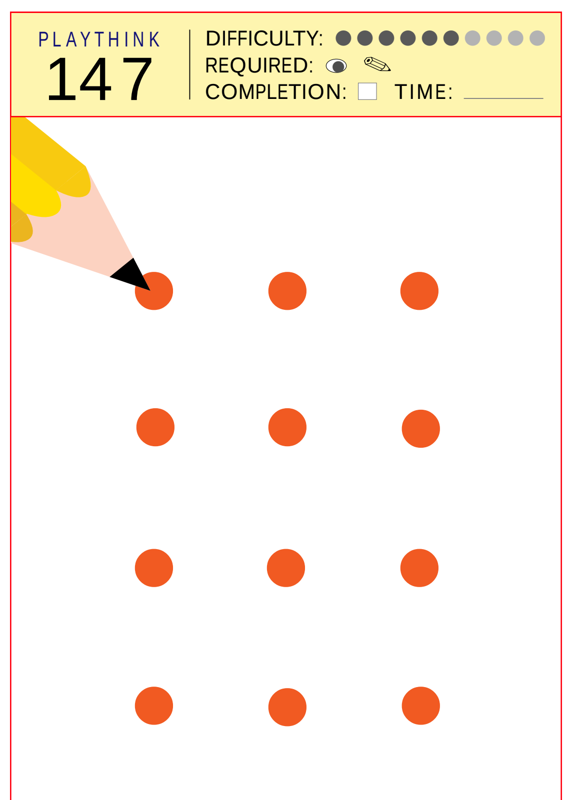


NINE-POINT PROBLEM

Can you connect the nine points with four straight lines without lifting your pencil?

Can you solve this problem using only three straight lines?

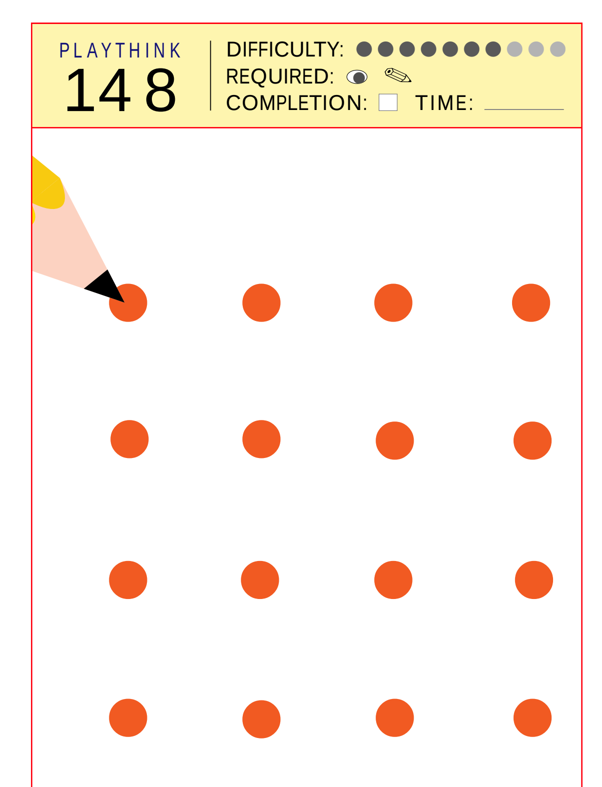
PLAYTHINK 147 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____



TWELVE-POINT PROBLEM

Can you connect these twelve dots with a series of straight lines without lifting your pencil? What is the least number of lines necessary?

PLAYTHINK 148 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____



THE SIXTEEN-POINT PROBLEM

Can these sixteen dots in a square be connected with a series of straight lines without lifting your pencil? What is the least number of lines necessary?

Coordinates

Shapes are not simply physical objects—they are also mathematical creations that can be described by numbers. And like all numbers they can be manipulated in different ways to get new results, a form of math known as geometric algebra.

The concept of geometric algebra dates back to about 300 B.C., when Euclid used a form of it for proofs in his *Elements*. The field came into its own in the mid-seventeenth century, when René Descartes and Pierre de Fermat began describing the position of points with a pair of numbers. Cartesian coordinates, named after Descartes, use axes at right angles to the point where the axes cross. In coordinates such as (2,3), the first number represents the distance along the horizontal (x) axis, and the second shows the distance along the vertical (y) axis.

With Cartesian coordinates, equations can be used to plot shapes. If an equation has two variables, the shape is two-dimensional; if it has three, the shape is three-dimensional. Cartesian coordinates can be used to analyze curves. They can also help to solve simultaneous equations; the point or points where the equations' lines cross provide the numerical solutions. These powerful tools have made geometric algebra a valuable asset to science, engineering and data analysis.

PLAYTHINK 149

DIFFICULTY: ●●○○○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

COORDINATE CRAFT

A point on a plane can be defined by the intersection of two lines, known as coordinates. Use the coordinates numbered one through twenty-four to define points on this grid. If you connect the dots in order, you will find a hidden picture.

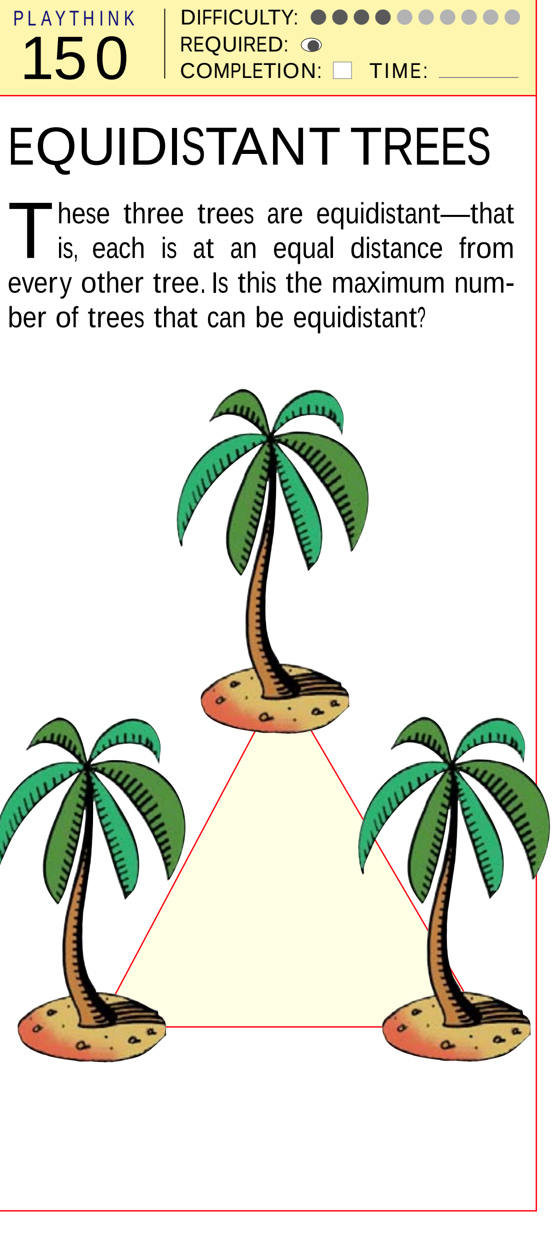
1.	9	9
2.	6	9
3.	5	10
4.	3	10
5.	2	9
6.	2	8
7.	4	7
8.	5	6
9.	1	4
10.	1	6
11.	0	3
12.	3	2
13.	4	1
14.	3	0
15.	7	0
16.	5	1
17.	4	2
18.	7	3
19.	8	5
20.	5	7
21.	6	8
22.	9	7
23.	8	8
24.	9	9

PLAYTHINK 150

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

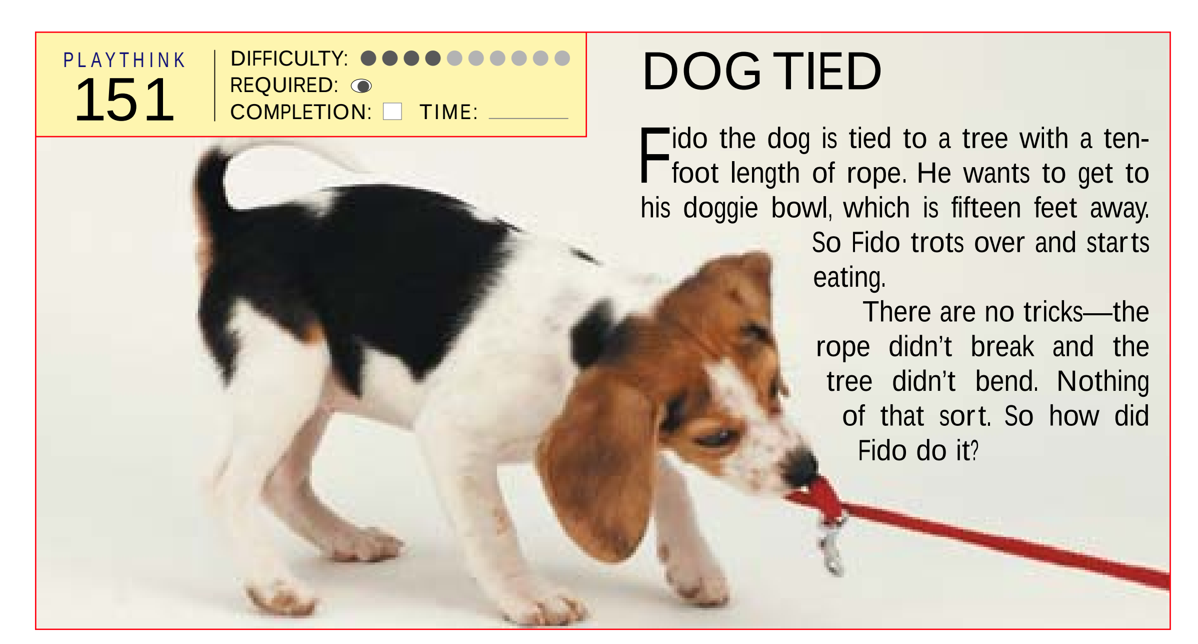
EQUIDISTANT TREES

These three trees are equidistant—that is, each is at an equal distance from every other tree. Is this the maximum number of trees that can be equidistant?



PLAYTHINK 151 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

DOG TIED



Fido the dog is tied to a tree with a ten-foot length of rope. He wants to get to his doggie bowl, which is fifteen feet away. So Fido trots over and starts eating.

There are no tricks—the rope didn't break and the tree didn't bend. Nothing of that sort. So how did Fido do it?

PLAYTHINK 152 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

LONGEST LINE

Can you find the longest line connecting two points on the two intersecting circles that passes through the point marked A? (Points A and D are the points at which the two circles intersect.)

A
B
C
D

PLAYTHINK 153 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

INTERSECT

■ *A Two-Person Game*

The object of the game is to form as many intersections as possible. Players take turns drawing lines connecting the dots along the sides of the game board; each player uses a different color pencil or pen. Each time a player's line crosses a previously drawn line, that intersection gets a dot with the color corresponding to that player.

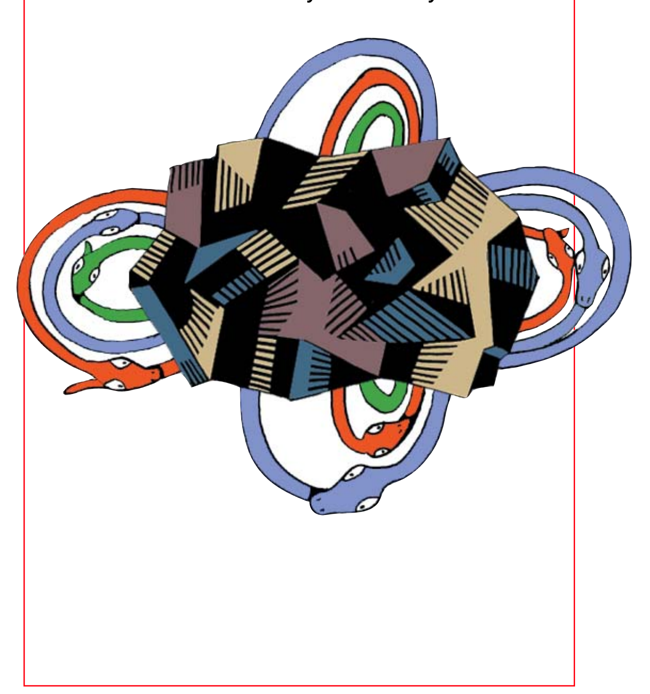
At the end of the game, the intersections are added up. Every intersection where a player crossed a line he or she drew counts as two points; every intersection where a player crossed a line drawn by the opponent counts as one point.

PLAYTHINK 154 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

SERPENTS

There are nine snakes—three red, three green and three blue—coiled in closed loops under a rock. The snakes do not touch one another, nor do their loops intersect.

Eight of the snakes are partly uncovered. Just by studying the image, can you tell what color snake is fully hidden by the rock?



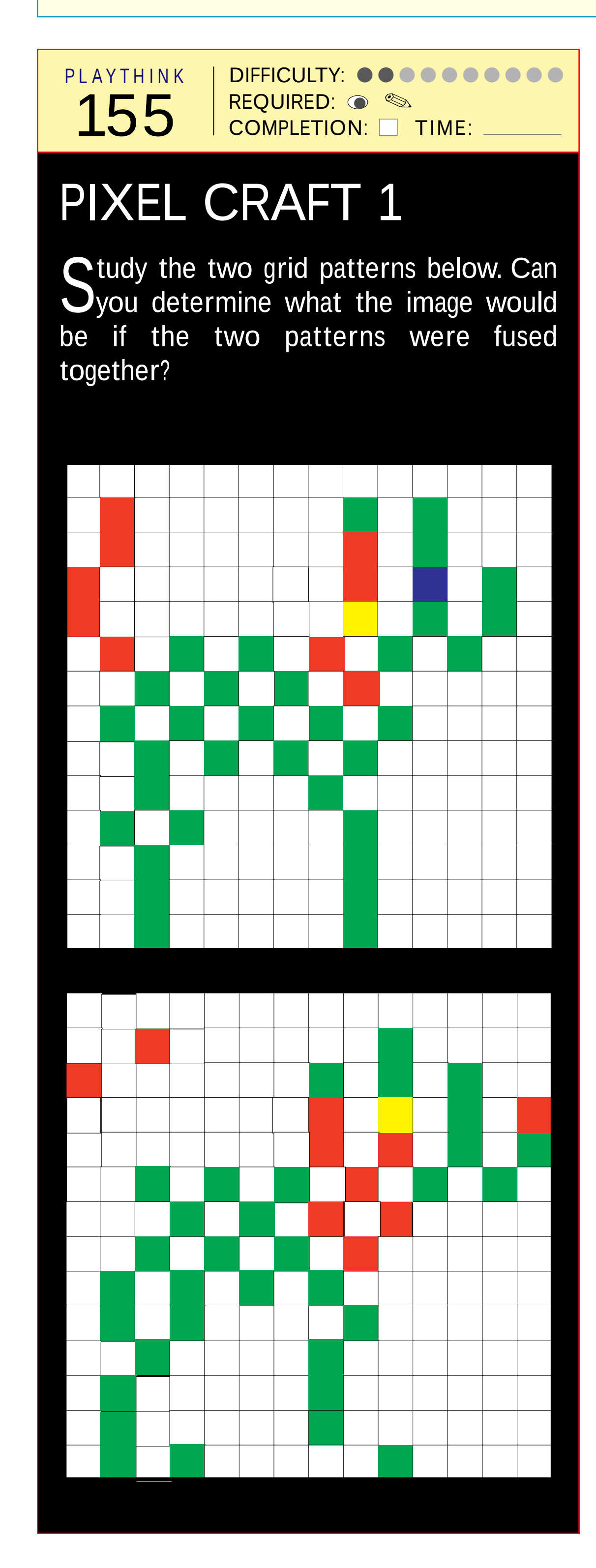
Electronic Imaging

Early in the twentieth century engineers discovered that they could deliver moving pictures to a screen by breaking up the image into very small pieces called pixels. Each pixel was encoded with information about its brightness and color, which was sent electronically to television receivers, where the pixels were combined to create a TV image. Modern computer screens employ much the same technology. If you look closely, you will see that even the most complex computer images are made of minuscule dots. That concept was simplified to produce the "Pixel Craft" puzzles below.

PLAYTHINK 155 DIFFICULTY: ●●●●●●●●●● REQUIRED: 👁 ✎ COMPLETION: ☐ TIME: ______

PIXEL CRAFT 1

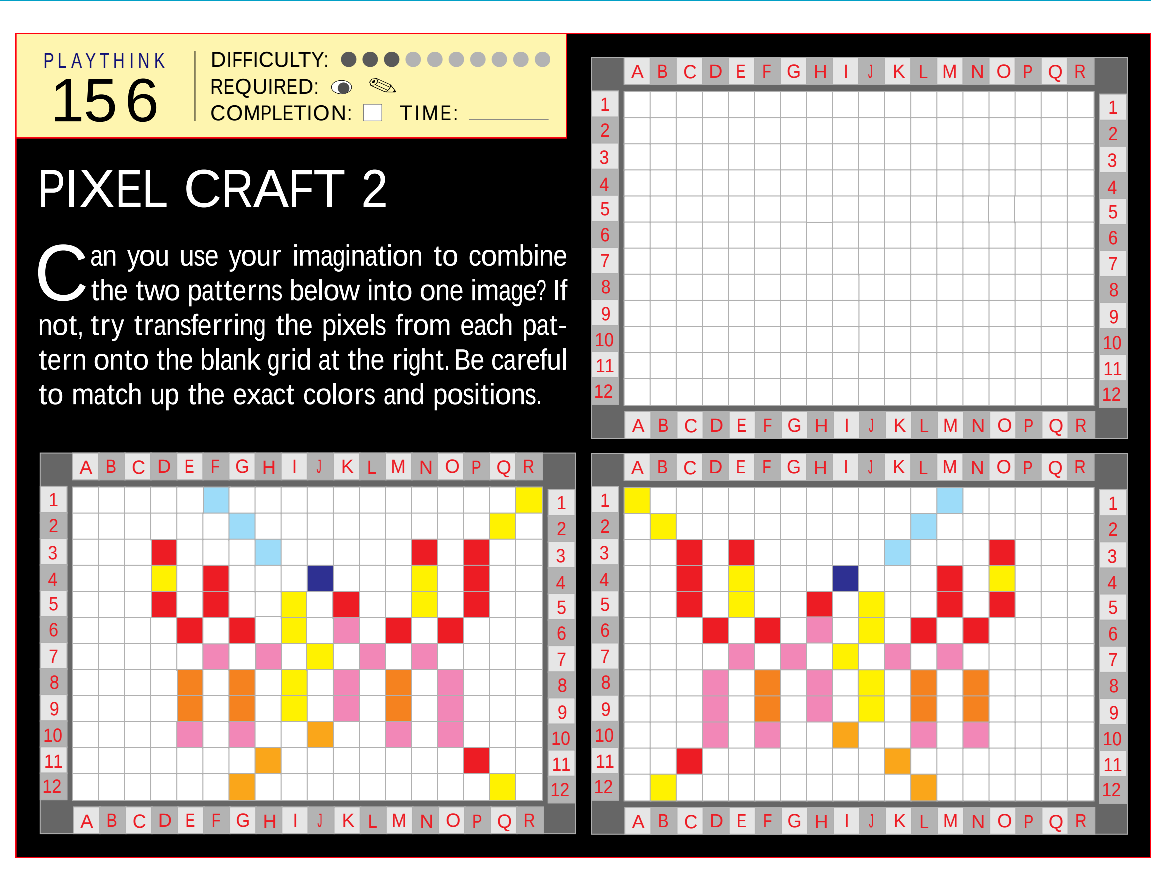
Study the two grid patterns below. Can you determine what the image would be if the two patterns were fused together?

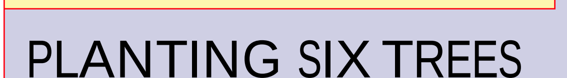


PLAYTHINK 156 DIFFICULTY: ●●●●●●●●●● REQUIRED: 👁 ✎ COMPLETION: ☐ TIME: ______

PIXEL CRAFT 2

Can you use your imagination to combine the two patterns below into one image? If not, try transferring the pixels from each pattern onto the blank grid at the right. Be careful to match up the exact colors and positions.

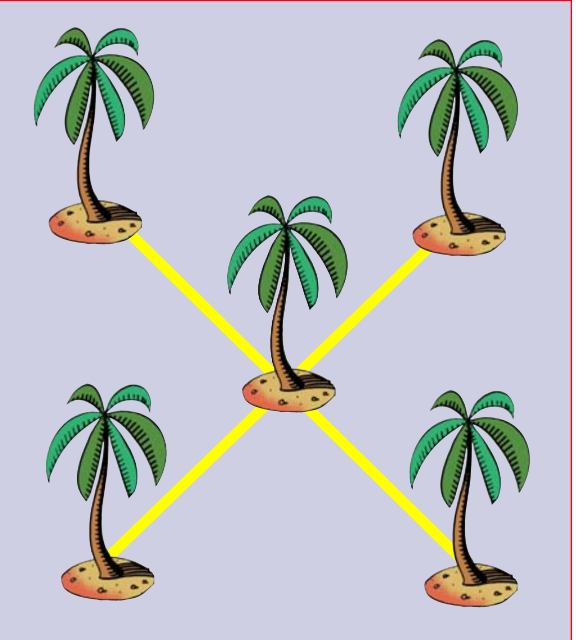




PLAYTHINK 157 DIFFICULTY: ●●●●●●●●●● REQUIRED: 👁 ✎ COMPLETION: ☐ TIME: ______

PLANTING SIX TREES

A garden is planted with five trees along six straight paths—two paths have three trees, and four paths have two trees. Can you design a new garden with six trees and four paths so that each path has exactly three trees?



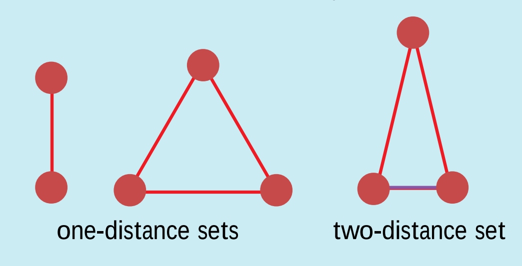
PLAYTHINK 158

DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ____

TWO-DISTANCE SETS

Points on a plane can be any distance apart. But there is a limited set of points that are exactly one or two discrete distances from every other point in the set. For example, two given points are exactly one distance from each other, and each of the three points that form the vertices of an equilateral triangle are also the same distance from the other two points. Those two sets of points are the only one-distance sets.

An isosceles triangle is an example of a two-distance set. Within a plane, how many other two-distance sets can you find?



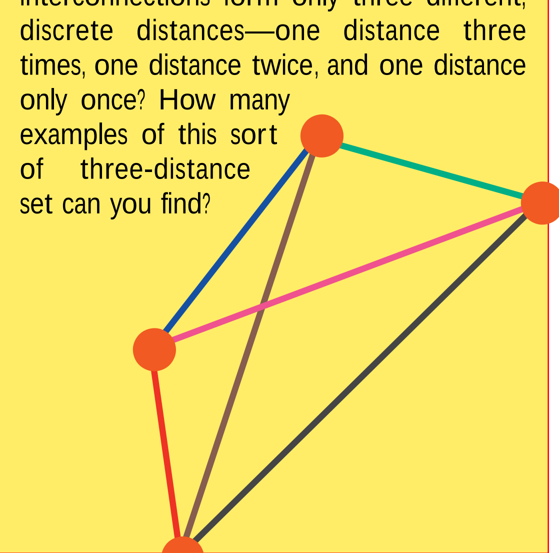
PLAYTHINK 159

DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ____

THREE-DISTANCE SETS

The four points shown below are connected by six lines, each of a different length. This is an example of a six-distance set.

Can you arrange four points so that the interconnections form only three different, discrete distances—one distance three times, one distance twice, and one distance only once? How many examples of this sort of three-distance set can you find?



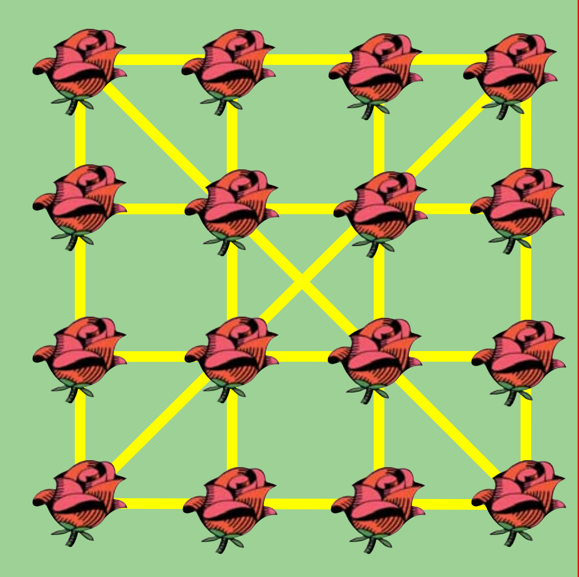
PLAYTHINK 160

DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ____

ROWS OF ROSES

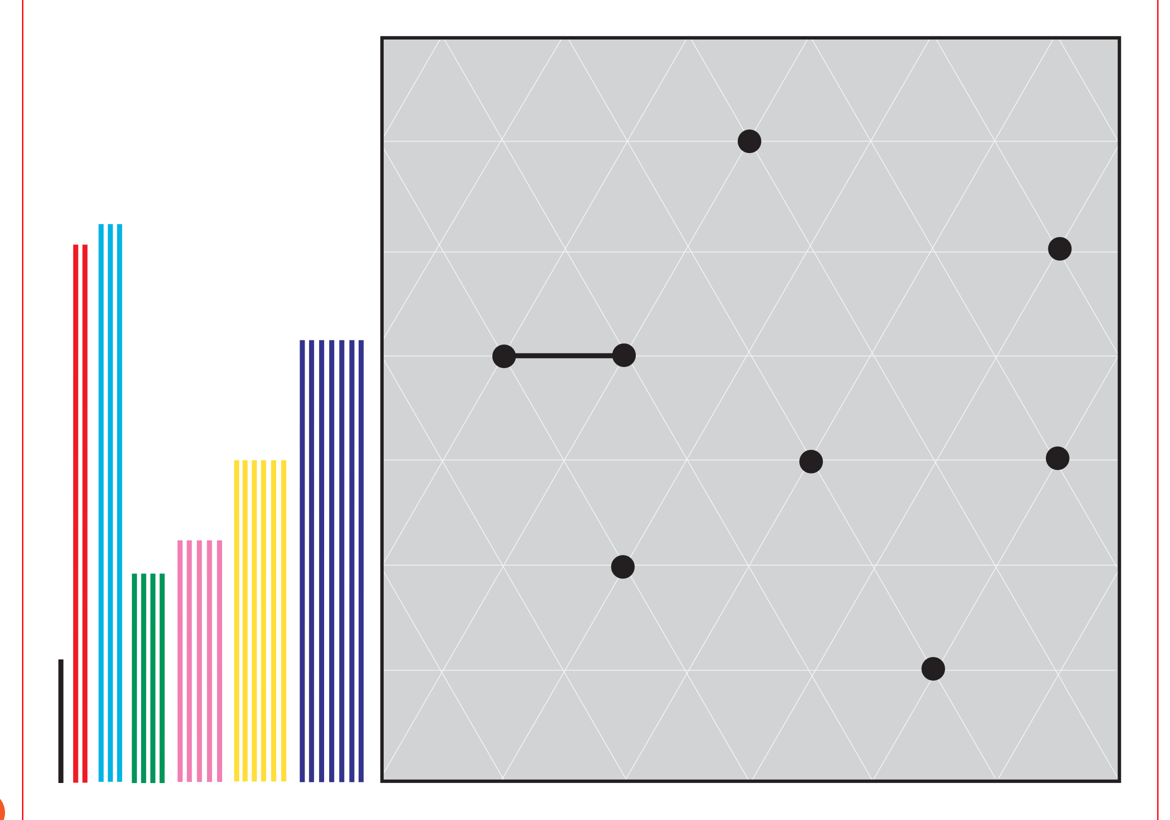
Mr. Rose wanted to plant sixteen rosebushes in his garden, and he began to plan how he wanted them laid out. At first he designed his rose garden so that there would be four rows of four roses each, which would result in ten straight lines—four vertical lines, four horizontal lines and two diagonal lines—each of which would have four bushes.

Then Mr. Rose hit on an even better plan: he would plant the sixteen bushes along fifteen straight lines with four bushes in each line. Can you figure out how he planted them?



PLAYTHINK 161

DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ____



MULTI-DISTANCE SET

Connect points on this triangular grid so that the interconnecting lines have lengths that have a special property: one length should occur only once, another should occur twice, another three times and so on.

Begin with the black segment shown. How far can you carry on?

PLAYTHINK 162 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

SQUARE THE MATCH

These puzzles involve moving matches (or any other short, straight objects of the same length, such as soda straws) to create new patterns made up of squares.

Each column directs you to the number of matches to be moved; each row tells you the number of squares that you must create. (Squares may overlap or have common corners.) Can you solve all twelve puzzles?

change 2 matches | change 3 matches | change 4 matches

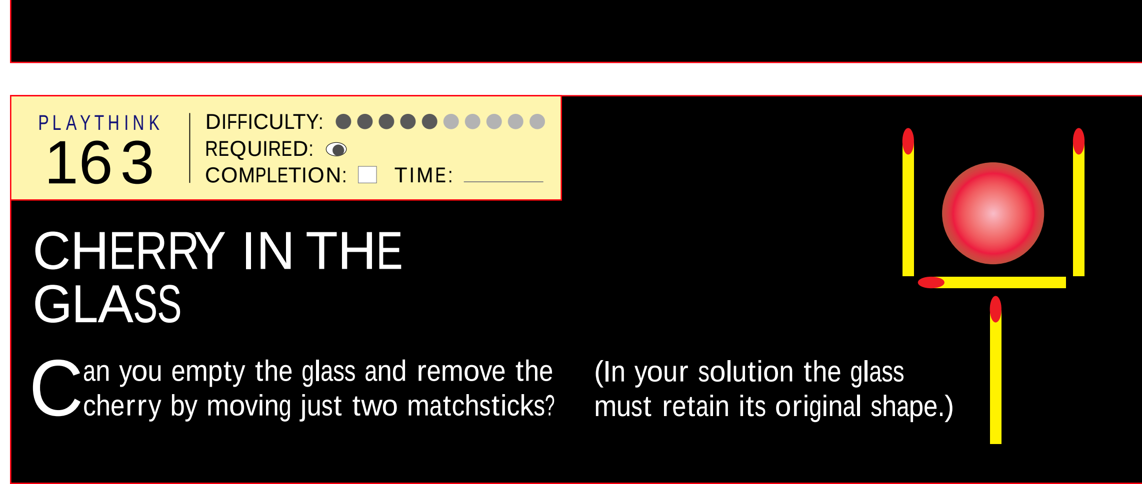
create 2 squares

create 3 squares

create 4 squares

create 5 squares

PLAYTHINK 163 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

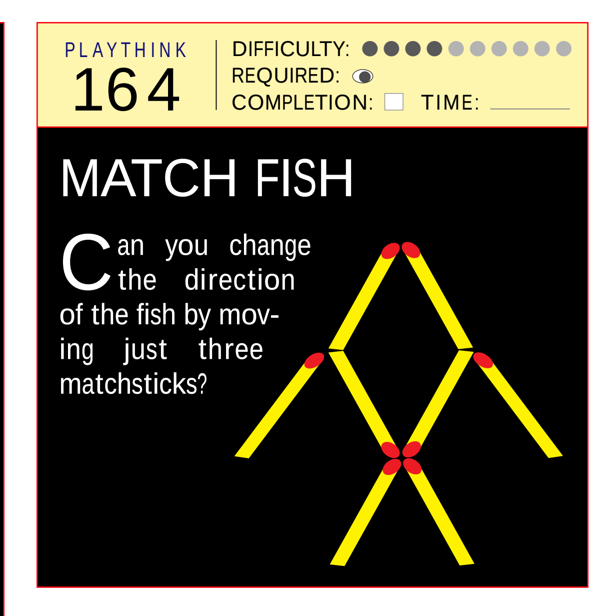


CHERRY IN THE GLASS

Can you empty the glass and remove the cherry by moving just two matchsticks? (In your solution the glass must retain its original shape.)

PLAYTHINK 164 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

MATCH FISH



Can you change the direction of the fish by moving just three matchsticks?

PLAYTHINK 165 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

MATCH POINT

In the figure below, the three matchsticks come together at a point. Can you create a figure out of matchsticks so that both ends of every matchstick are connected with exactly two other matchsticks in this way? Note that the matchsticks may meet only at their ends, and there can be no overlapping. What shape conforms to this rule and yet possesses the fewest number of matchsticks?

This problem was first posed by the German mathematician Heiko Harborth and was described by Nob Yoshigahara in his famous *Puzzletopia* newsletter. A variant of this problem demands that four matchsticks meet at every point; the best solution known requires 104 matchsticks meeting at 52 points. It has been determined that no solution exists for the variant requiring five matchsticks to meet at every point.



PLAYTHINK 166

DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

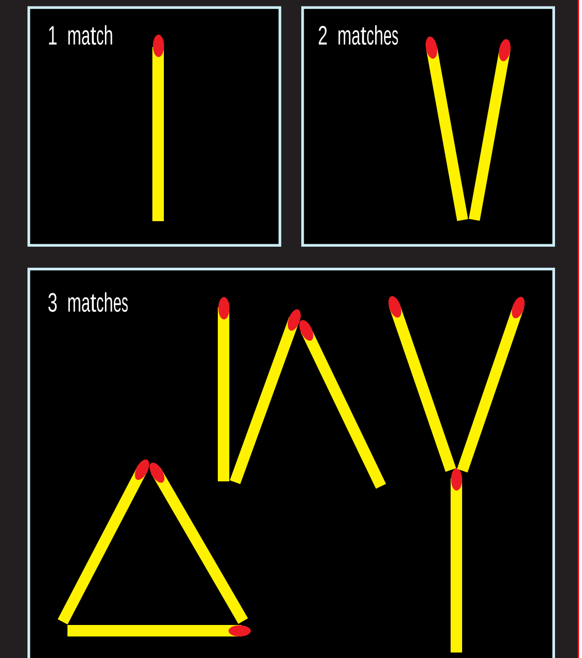
MATCH CONFIGURATIONS

This puzzle is based on an old solitaire game. How many topologically different configurations can you make with a given number of matchsticks on a flat surface? Certain restrictions apply:

1. An edge consists of a single matchstick, and the only point where two matchsticks may touch is at their ends.

2. The matchsticks must lie flat on the surface, but two figures are considered identical if one can be deformed in three-dimensional space without separating the joints (as if the figure were picked up and moved) to resemble the other.

All the possible configurations for one, two and three matchsticks are shown below. How many different configurations can you make with four matchsticks? Five matchsticks?

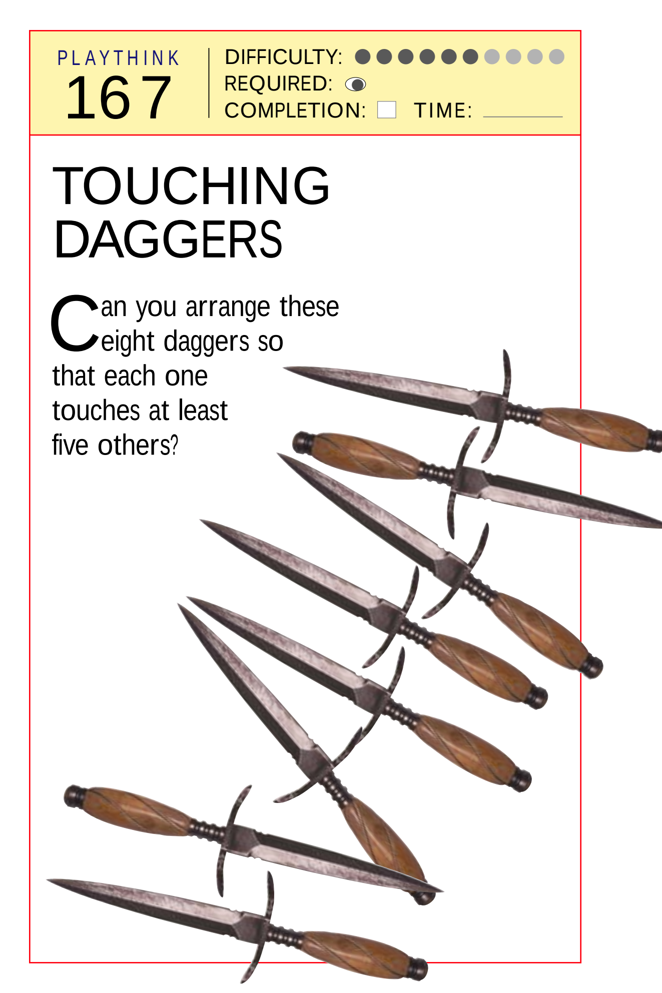


PLAYTHINK 167

DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

TOUCHING DAGGERS

Can you arrange these eight daggers so that each one touches at least five others?



Lines and Linkages

A line is the idealization of a rigid rod. Conversely, problems about linked rods are really just studies in the geometry of lines.

A linkage is a system of rods, or lines, either connected to one another by movable joints or fixed to the plane by pivots around which the rods can turn freely. If you pivot a single rod at one end, the free end moves in a circle around that point.

Circular motion is easy and natural for linkages. The trick is to construct straight-line motion in the absence of a fixed straight line. That isn't just a theoretical problem in geometry. The natural motion produced by a steam engine is rotary, and though it can be converted to straight-line motion by a piston, pistons require bearings that are subject to wear. A linkage would be a better way to harness the power of a steam engine.

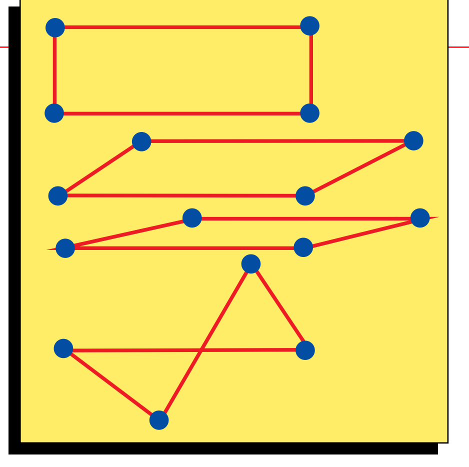
James Watt, the inventor of the steam engine, devised the first practical solution—an approximate straight-line linkage. Rather than a straight line, Watt's linkage (as it came to be known) produced a complex mathematical curve known as Bernoulli's lemniscate—an elongated figure eight—a segment of which was close enough to a straight line for Watt's purposes. Ironically, such a complex curve is more easily generated by a linkage than is a straight line.

The first mechanical device to produce exact straight-line motion was Peaucellier's linkage, invented in 1864. It is based on a general geometrical principle called inversion. Six links, four of equal length, form an inverter: if a particular point in the linkage follows one curve, then another point follows the inverse curve. Since the inverse curve to a straight line is a circle, a final, seventh, link constrains one of the points in Peaucellier's linkage to a circle. Another point is then forced to follow the inverse, the straight line.

PLAYTHINK 168
DIFFICULTY: ●●●○○○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

PARALLELOGRAM LINKAGE

Four strips are linked by four flexible joints to form the four-sided polygon known as the parallelogram. Such a four-bar linkage can transform a square or rectangle into other parallelograms, such as rhombuses and rhomboids. During the transformations shown at right, can you tell which elements and relationships change and which remain constant? Fill in the chart with your answers.

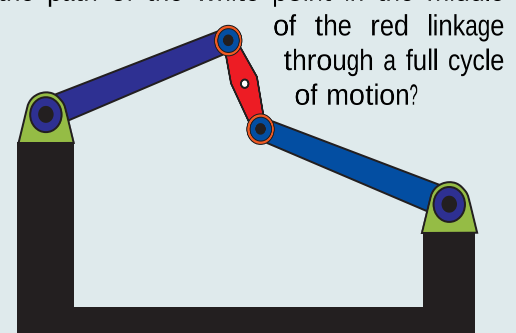


	CONSTANT	CHANGE
AREA		
PERIMETER		
SIDES		
ANGLES		

PLAYTHINK 169
DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

WATT'S LINKAGE

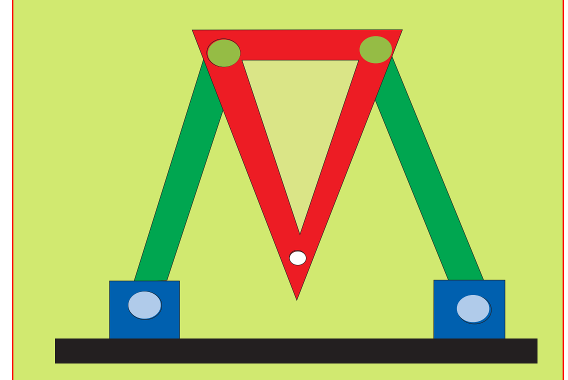
Examine the mechanical linkage shown below. The arms are anchored to the mounts on one end but may move freely on the other. And the red link connects the blue arms and constrains their motion. Given that information, can you determine the path of the white point in the middle of the red linkage through a full cycle of motion?



PLAYTHINK 170
DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

SWING TRIANGLE

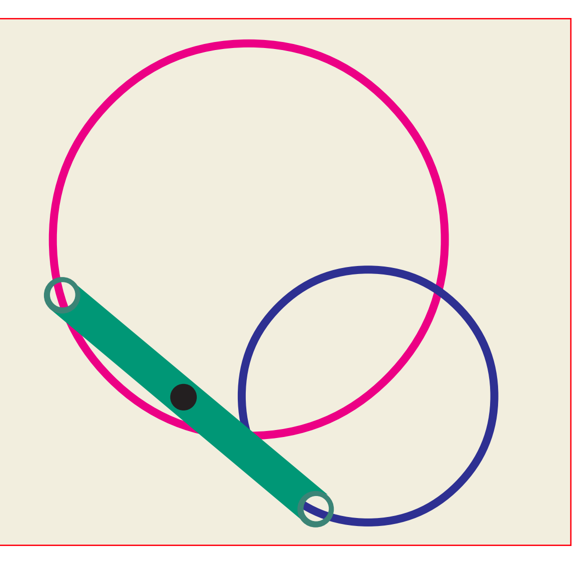
In this mechanical linkage, the green arms are anchored to the blue base but both the arms and the red triangle, though linked, are otherwise free to swing back and forth. Can you trace the path of the white dot through one full swing of the linkage?



PLAYTHINK 171
DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

MOVING ALONG CIRCLES

Imagine a straight linkage as shown here, with each end constrained to one of two intersecting circles. Can you puzzle out the path traced by the middle dot of one linkage as it moves through one full cycle? Note: It may be necessary to make this linkage yourself and trace the path with a pencil.

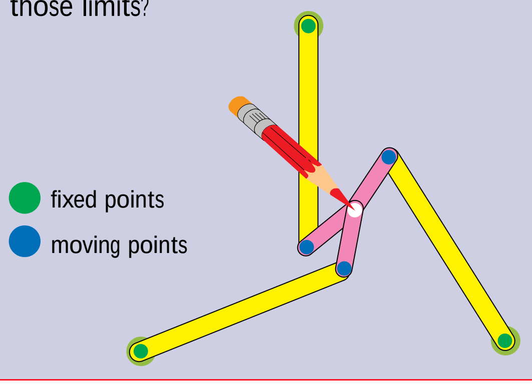


PLAYTHINK 172
DIFFICULTY: ●●●●●●●●●○
REQUIRED:
COMPLETION: ☐ TIME: ____

CRANKSHAFT

Cut six strips—three long and three short—from heavy paper or card stock. Pin the ends of the long strips to a sheet of paper so that the pinned points form the vertices of an equilateral triangle. The arms should swing freely about those points. Then attach the short strips to the free ends of the long strips so that the short strips are able to swing around the end of the long arms. Finally, attach the ends of the short strips together and punch a hole through that joint large enough for a pencil to fit through. Place a pencil through the hole.

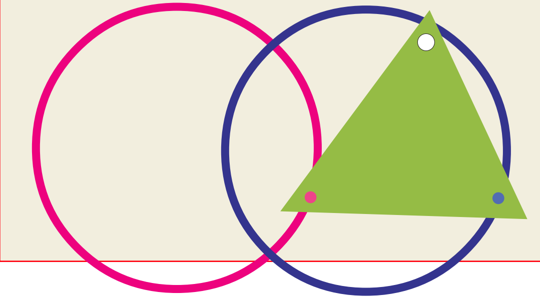
The motion of the central joint will be constrained to a certain area. Using the pencil to trace the joint's path, can you find those limits?



PLAYTHINK 173
DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

MOVING TRIANGLE

Two points of the triangle illustrated below may move along the circumferences of the intersecting circles. The third point has a hole for a pencil tip to pass through. As the triangle points follow their circular paths, the pencil traces out a complex shape. Can you determine what the shape looks like? It may be useful to construct a replica of this triangular linkage and trace out the path yourself.




4
Graphs and Networks

Graph Theory

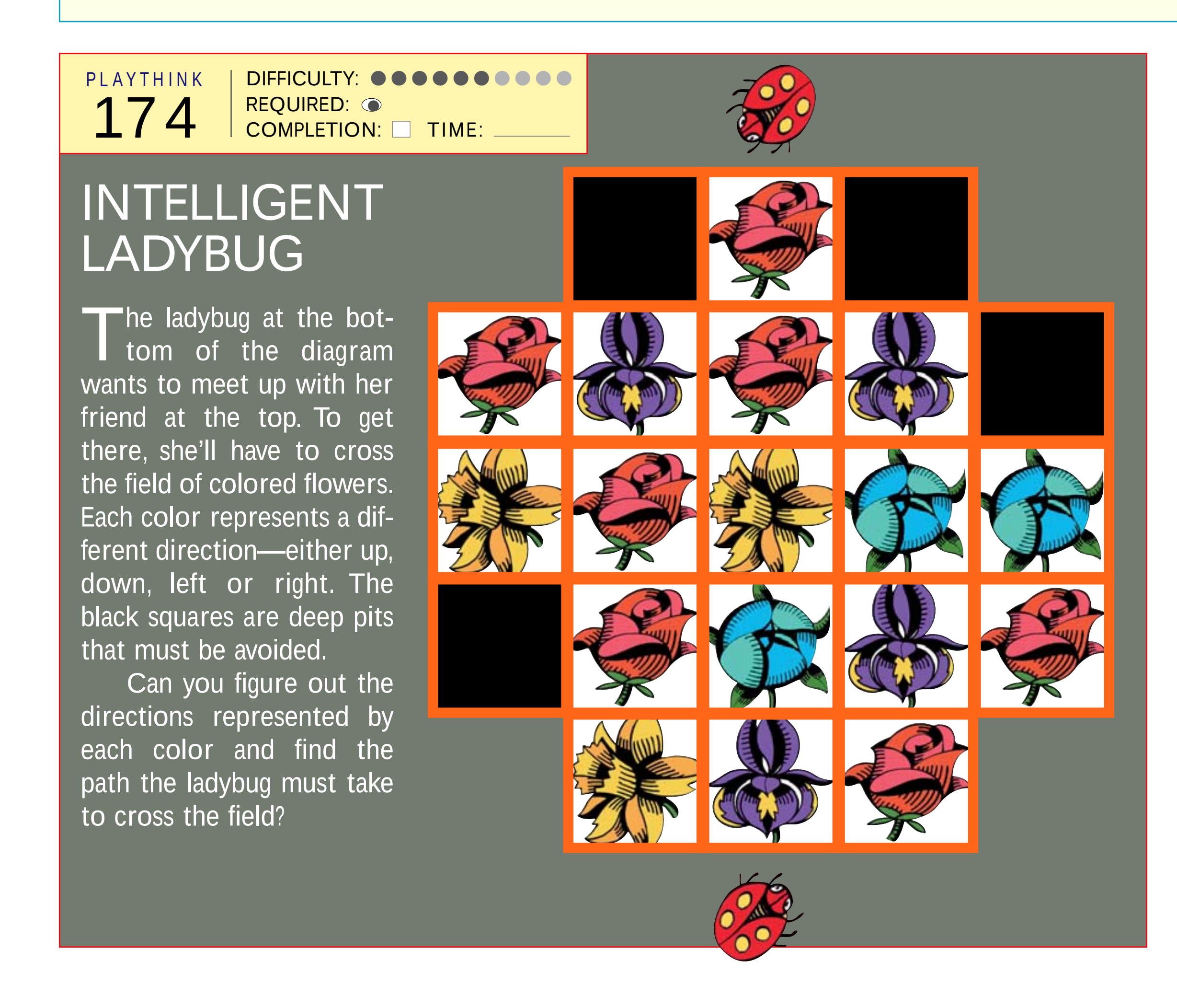
Imagine you are a traveling salesperson. You have a certain number of cities to visit in a short amount of time. Can you find the route that will allow you to visit all the cities while traveling the least distance?

Or imagine you are given a dodecahedron and are presented with the following challenge: move your finger along the edges so that it traces a path on the surface of the three-dimensional form that visits each of the vertices just once.

Those two challenges are related and are part of a field of study called graph theory. Both the real-life itinerary and the three-dimensional dodecahedron can be represented by a graph: a two-dimensional system of dots, vertices or nodes that are connected by lines or edges. Graphs embody an abstract form of a seemingly more complicated construct. For example, specific points on a graph may represent the various tasks necessary to manufacture a certain product, while the lines connecting those points show all the different orders in which those tasks may be performed. By analyzing such a graph, an engineer can find the most efficient way to order the tasks.

Two graphs are considered the same—or topologically equivalent—if the corresponding nodes are joined in a corresponding way. The exact position of the nodes or shape of the edges is unimportant; the only thing that matters is the pattern of connections.

Neither the problem of the traveling salesperson nor the puzzle involving the dodecahedron, called the Icosian Game (PlayThink 184), has a general solution; solutions for these sorts of problems must be found through trial and error. Perhaps that's part of the reason graph theory not only leads to satisfying puzzles and challenging games but is one of the most active frontiers of mathematics today.



PLAYTHINK 174 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ____

INTELLIGENT LADYBUG

The ladybug at the bottom of the diagram wants to meet up with her friend at the top. To get there, she'll have to cross the field of colored flowers. Each color represents a different direction—either up, down, left or right. The black squares are deep pits that must be avoided.

Can you figure out the directions represented by each color and find the path the ladybug must take to cross the field?



PLAYTHINK 175 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ____

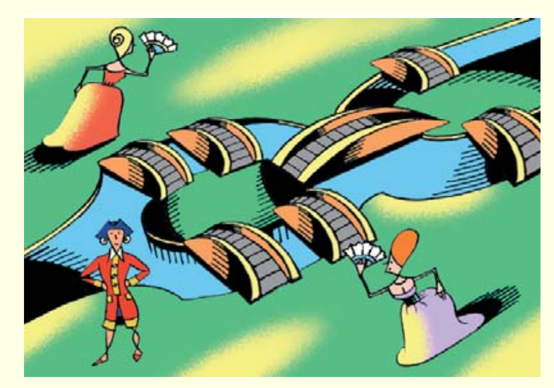
MYSTERY TRACKS

Can you figure out what could have made these tracks in the sand?

Euler's Problem

The eighteenth-century Swiss mathematician Leonhard Euler was enormously prolific, but he is best remembered for devising the solution to a recreational math problem: the Seven Bridges of Königsberg. At one time, it was fashionable to stroll through the Prussian town of Königsberg and ponder this problem: Could one cross each of the seven bridges that traversed the Pregel River and connected the various districts *once and only once*?

Although the problem itself was simple—was it possible to walk a circuit that crossed each bridge only once?—Euler found the solution by making the situation even simpler. He replaced the bridges and islands of Königsberg with lines and points.



The four landmasses (two islands and both riverbanks) became special points, or nodes, that were connected by seven lines representing the bridges. Using that abstract graph, Euler demonstrated that to complete a circuit, there would have to be a maximum of two places where an odd number of lines met, and if a return to the start was required, there would have to be no places where an odd number of lines met. The reasoning is simple, once seen: a continuous journey will enter each such junction exactly as often as it leaves except at the start and finish. Since the graph of Königsberg had four nodes, each of which possessed an odd number of lines, no solution could exist.

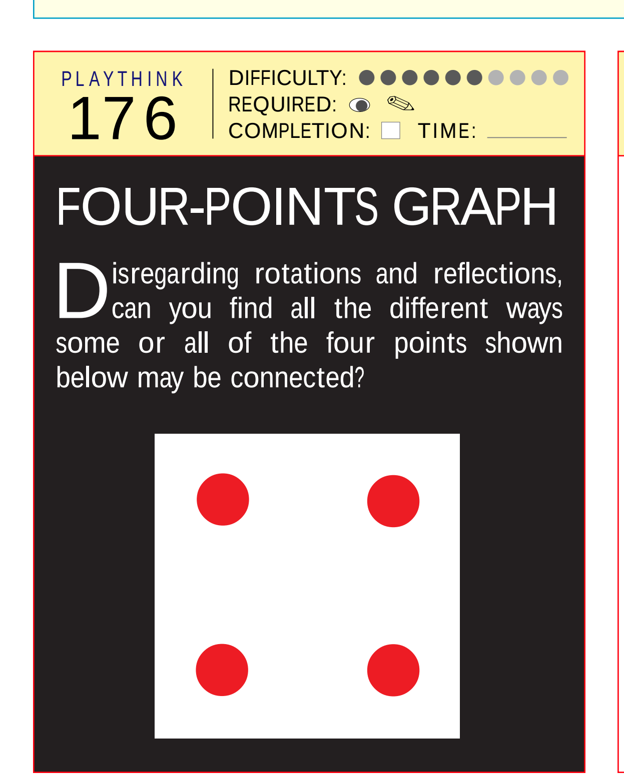
Euler's problem is really one of topology, the branch of mathematics that deals with the properties of figures that are preserved during continuous distortions. Two networks are topologically equivalent if one can be distorted to give the other, as can the city of Königsberg and Euler's graph of the city. If a network can be traversed by a single continuous path, so can any topologically equivalent network.

Euler's work on the bridges of Königsberg founded the field of graph theory. Not bad for one recreational math puzzle!

PLAYTHINK 176 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

FOUR-POINTS GRAPH

Disregarding rotations and reflections, can you find all the different ways some or all of the four points shown below may be connected?



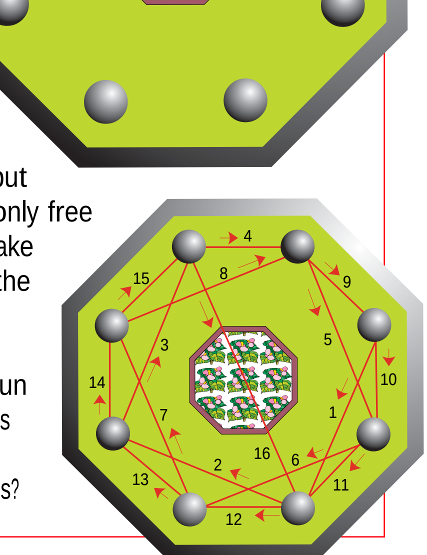
PLAYTHINK 177 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

PILLAR GAME

When I was young, I often played in a small, enclosed courtyard adorned with eight pillars near the perimeter. In the center, a low fence surrounded an octagonal flowerbed. One of my favorite games involved running in a straight line from pillar to pillar for as long as possible. I could cross my previous tracks and, if needed, hop over the fence and cut across the flowerbed. I could keep running until I had only two options: repeating a track or running in a line that ran along any side of the octagonal fence around the flowerbed. When those were my choices, I had to stop.

At right is a diagram of one of my games. In this game, I ran thirteen legs without any problem, but after that my only free move would take me alongside the fence, so the game ended.

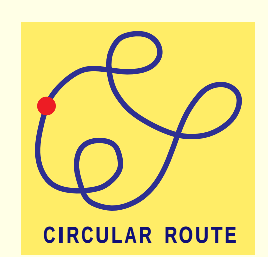
Can you run even more legs following my childhood rules?



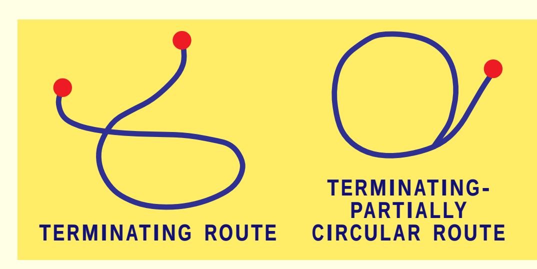
Defining Graphs and Networks

- A *route* is a path that can be drawn with one continuous line.

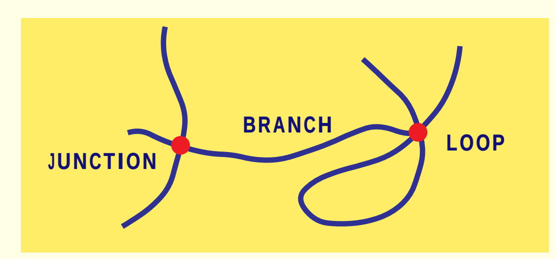
- A route is *circular* if following its entire length brings you back to the starting point.



- A route is *noncircular* if it terminates (that is, it has two end points) or if it is partially circular (it has only one end point).



- A *junction* is a point at which two or more routes meet.

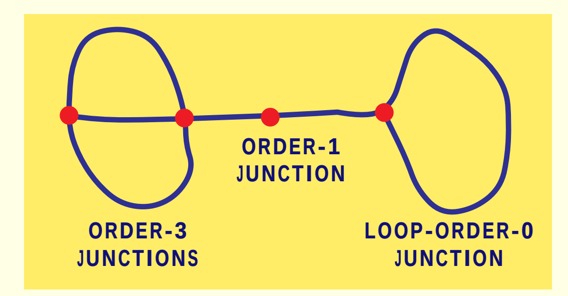


- The *power* of a junction is the number of routes leading from it.

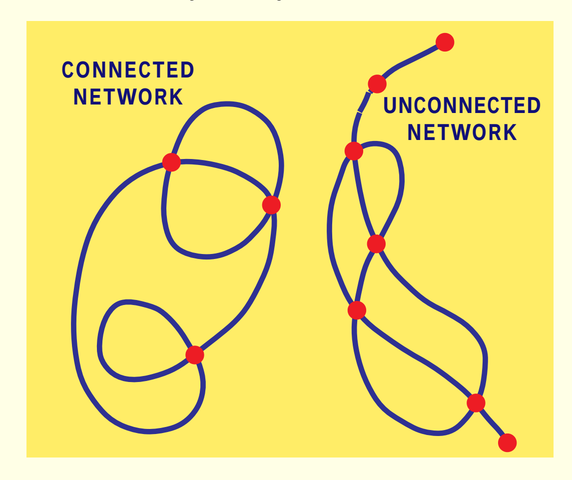
- A *branch* is a section of a route between two consecutive junctions.

- A *loop* is a section of route that begins and ends at the same junction without passing through another junction. It is a circular section. To determine the power of a junction that possesses a loop, count both arms of the loop as a separate branch.

- The *order* of a pair of junctions is the number of branches connecting them.



- A network is *connected* if there are at least two completely distinct routes between any two junctions.



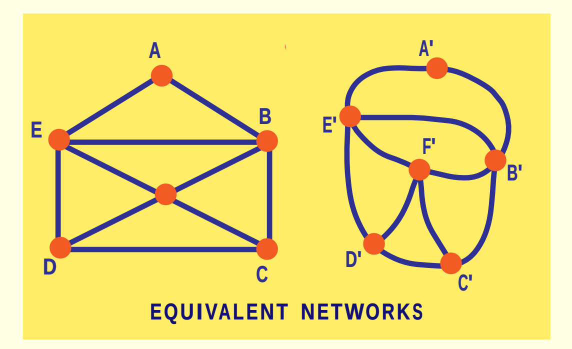
- A *region* is the space bounded by one or more branches of a network.

1 2 3

3-REGIONS NETWORK

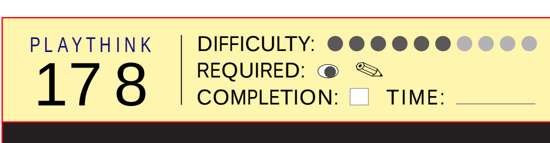
- The *rank* of a network is the minimum number of arcs needed to draw it completely if each branch is drawn just once.

- Two networks are considered *equivalent* if they have the same number of similarly powered junctions occurring in the same order.



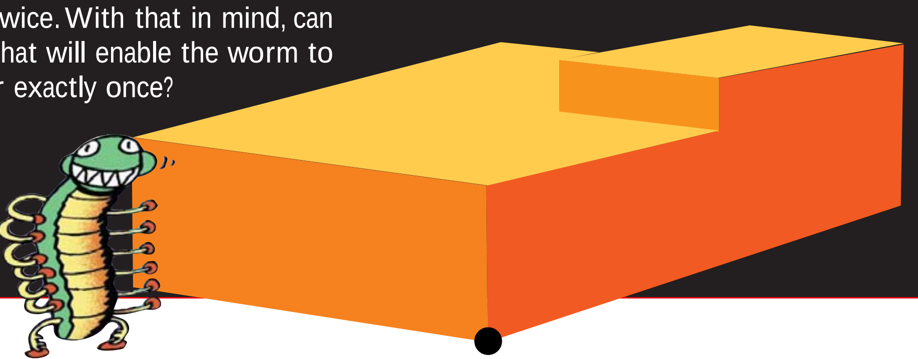
- *Tree networks* are points connected by lines that don't contain closed loops.

- The *valency* is the number of edges that meet at a given node.



3-D TRAVERSING PROBLEM

The worm can climb along the edges of the solid figure shown at right, but it is unwilling to cross its own path or cover the same territory twice. With that in mind, can you find a path that will enable the worm to visit each corner exactly once?



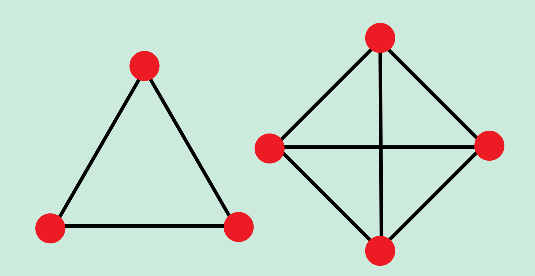
Crossing Numbers

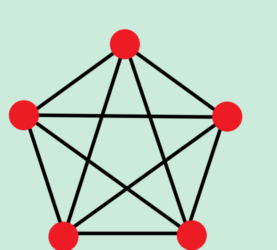
If the lines that connect the nodes of a graph were not allowed to cross, there would be serious restrictions on the kinds of graphs mathematicians could draw; in fact, a complete graph containing just five points would be impossible. But if the lines were permitted to cross, then any graph could be drawn in the plane. (You can think of the crossed lines as edges of a solid that have been projected onto the plane.) Such an "extra" meeting of two edges is called a crossing point. Topological deformation of a graph may change the number of crossings: for example, the complete graph on four nodes can be drawn as a square with its diagonals, with nodes at its corners. The intersection of the two diagonals is a crossing point. That same graph can be drawn in the plane in a way that avoids any crossings (see "Planar Graph").

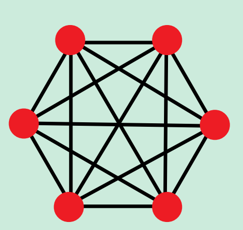
There may be dozens of ways to draw a given graph, but there is at least one that has the fewest number of crossings. That minimal number of crossings is called, naturally enough, the crossing number, and it does not change even as the graph is topologically deformed. Graphs with crossing number zero are called planar graphs. The crossing number is remarkably difficult to calculate, and it is not known in general even for complete graphs. The crossing number for the complete graph with five nodes is one (see "Complete Graphs of Five Points").

Complete Graphs

A graph is complete if there are at least two completely distinct routes between any pair of nodes (points). These are complete graphs of three to six points.

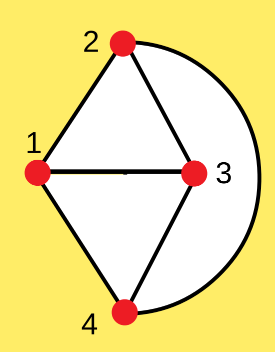


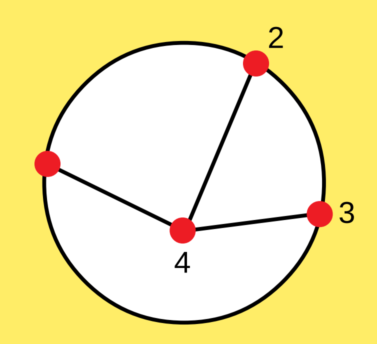




Planar Graph

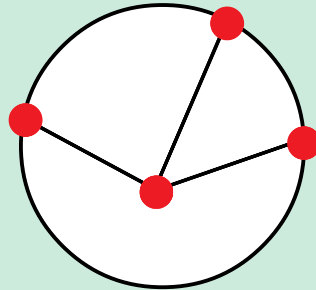
A planar graph of four points has no crossing point.



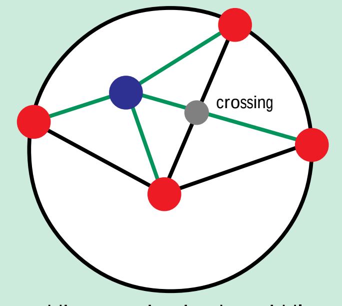


Complete Graphs of Five Points

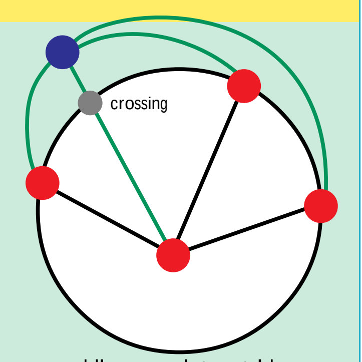
This visual proof demonstrates that a complete graph of five points must have at least one pair of crossing branches.



a four-point graph



adding a point in the middle



adding a point outside

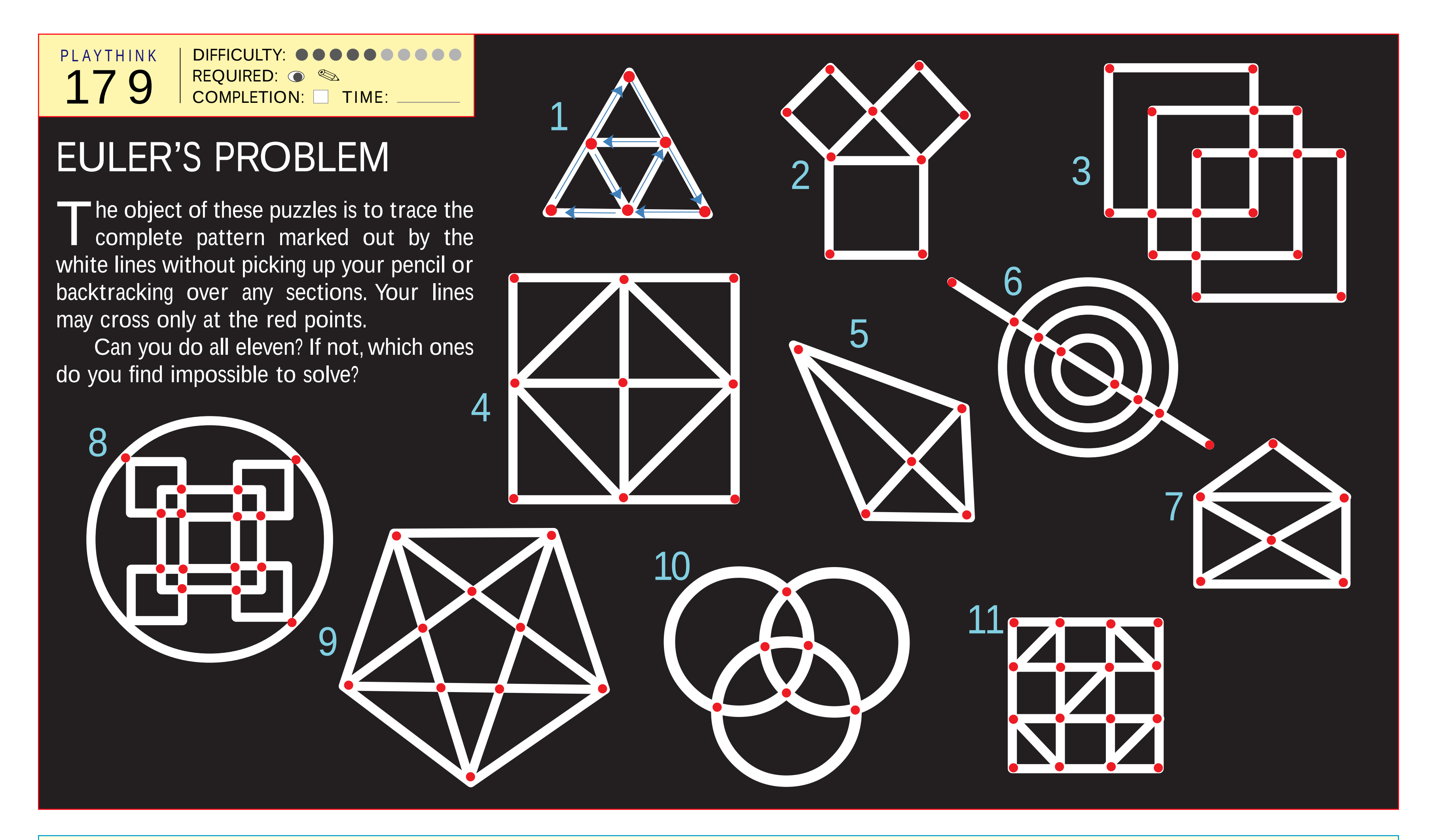
PLAYTHINK 179

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

EULER'S PROBLEM

The object of these puzzles is to trace the complete pattern marked out by the white lines without picking up your pencil or backtracking over any sections. Your lines may cross only at the red points.

Can you do all eleven? If not, which ones do you find impossible to solve?



Euler Circuits

Think of a drawing of one continuous line that returns to the point where it started—a circle, for instance. Then think of a route along a graph that covers every edge just once and ends at the same vertex. That's an Euler circuit, named after Leonhard Euler. There are two obvious questions to ask about Euler circuits: Is it possible to tell by calculation (rather than by trial and error) whether a particular graph has an Euler circuit? And how can one find the possible Euler circuit without resorting to trial and error?

Euler examined such issues by using the concepts of valence and connectedness. The valence of a vertex in a graph is the number of edges that meet at that point. And a connected graph has at least one path between each potential pair of vertices.

According to those terms, then, a graph possesses an Euler circuit if it is connected and each of its valences is even.

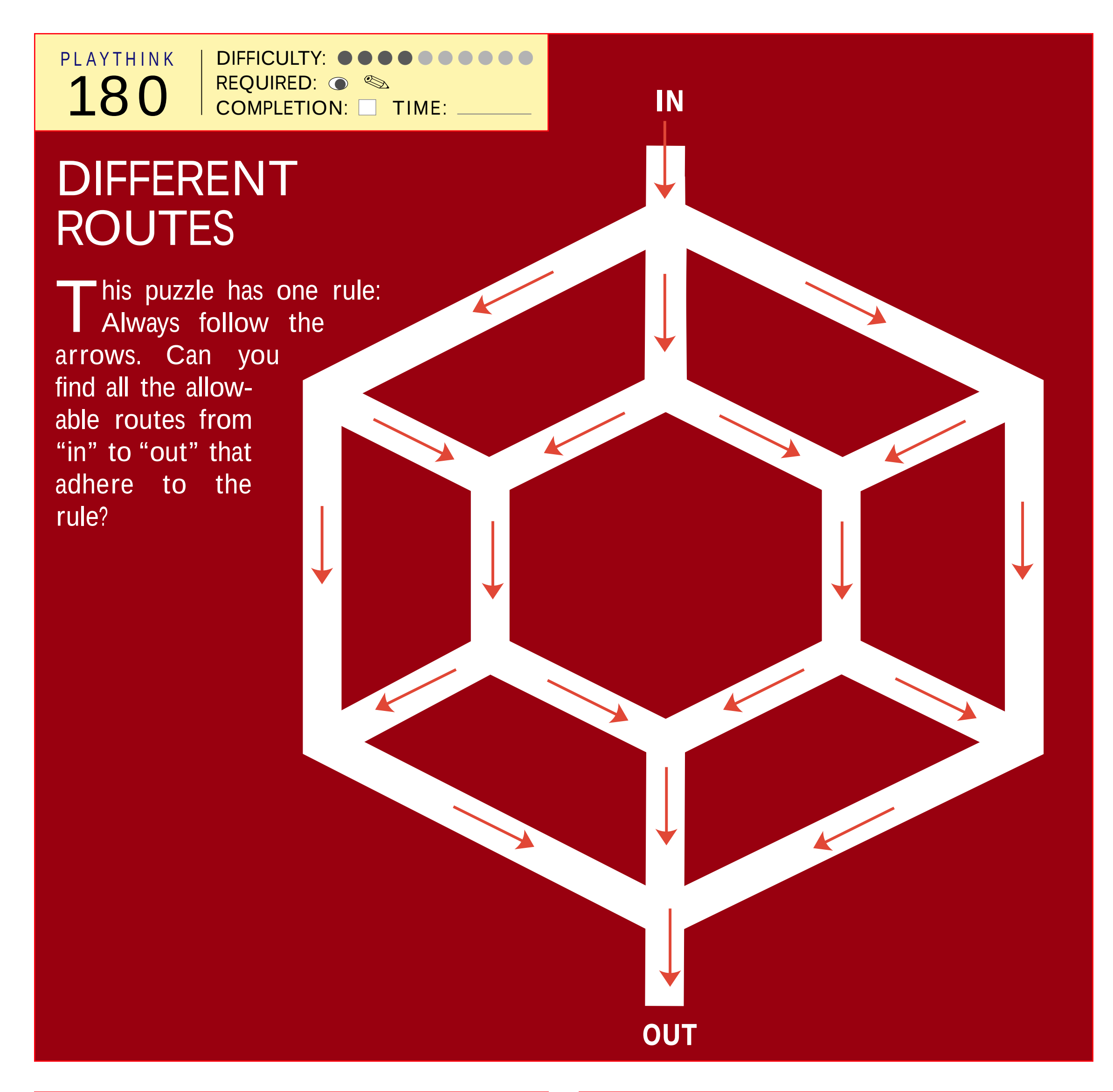
You have only to check how many lines are going into or out of every intersection point to see if an Euler circuit is possible. If there are more than two intersection points from which an odd number of lines emanate, the pattern is impossible to trace.

A slightly different take on the Euler circuit is the Hamiltonian circuit: a route along the edges of a graph that visits each vertex once and only once. Hamiltonian circuits typically run over some, but not all, of the edges of a graph. Though different, the concepts of the Euler and Hamiltonian circuits are similar, in that both forbid reuse: for Euler circuits, of edges; for Hamiltonian circuits, of vertices. Hamiltonian circuits, it turns out, are far more difficult to determine than are Euler circuits.

PLAYTHINK 180 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

DIFFERENT ROUTES

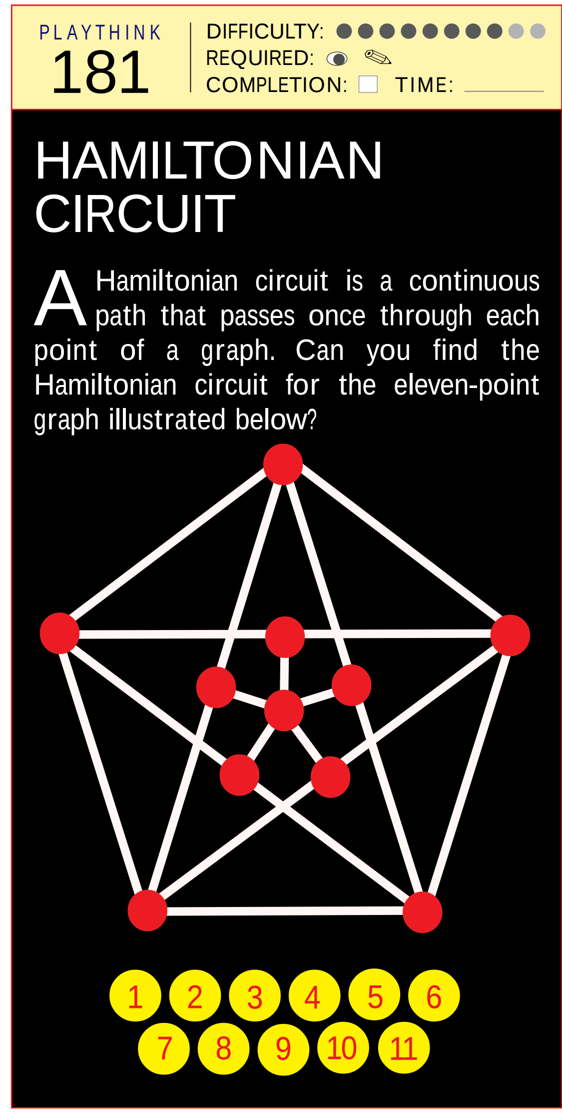
This puzzle has one rule: Always follow the arrows. Can you find all the allowable routes from "in" to "out" that adhere to the rule?



PLAYTHINK 181 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

HAMILTONIAN CIRCUIT

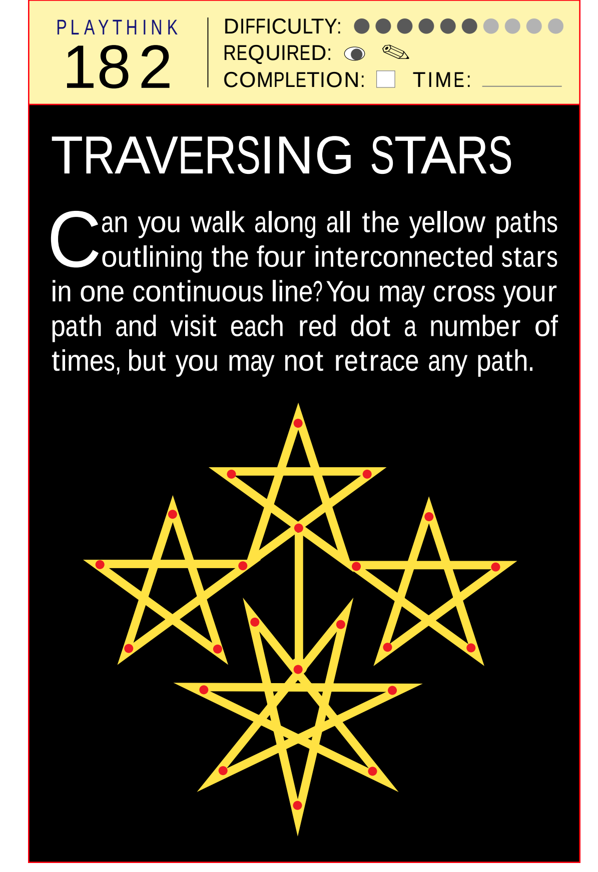
A Hamiltonian circuit is a continuous path that passes once through each point of a graph. Can you find the Hamiltonian circuit for the eleven-point graph illustrated below?



PLAYTHINK 182 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

TRAVERSING STARS

Can you walk along all the yellow paths outlining the four interconnected stars in one continuous line? You may cross your path and visit each red dot a number of times, but you may not retrace any path.

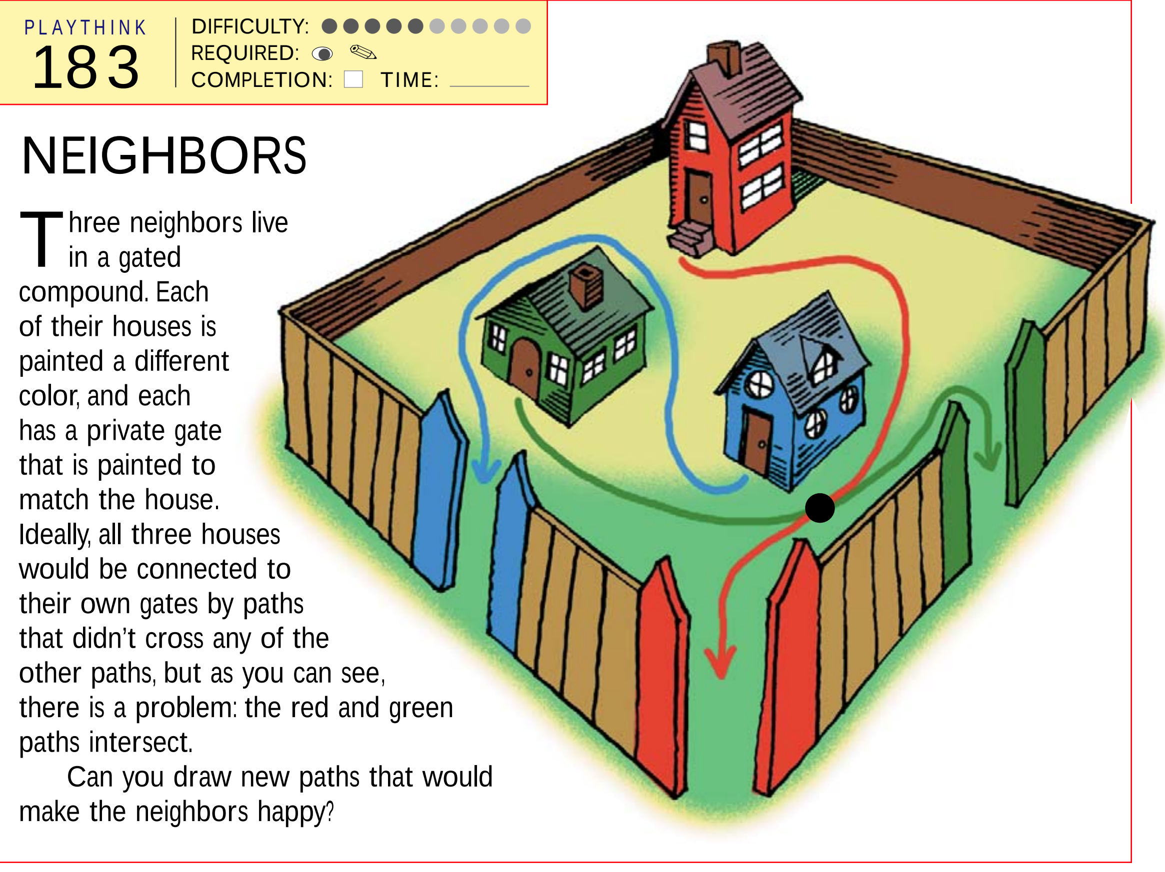


PLAYTHINK 183 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

NEIGHBORS

Three neighbors live in a gated compound. Each of their houses is painted a different color, and each has a private gate that is painted to match the house. Ideally, all three houses would be connected to their own gates by paths that didn't cross any of the other paths, but as you can see, there is a problem: the red and green paths intersect.

Can you draw new paths that would make the neighbors happy?



PLAYTHINK 184 | DIFFICULTY: ●●●●●●●●○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ______

THE ICOSIAN GAME

■ *A Journey Around a Dodecahedron*

A classic of recreational geometry, the Icosian game was invented by the mathematician W. R. Hamilton in 1859. Hamilton's original took the form of a dodecahedron, a three-dimensional solid with twelve pentagonal faces. But the game can be played on the two-dimensional diagram shown above, which is topologically equivalent to the dodecahedron.

To play, move from circle to circle following the paths marked in white. You can start at any circle you wish, but you may visit each circle only once. And you must return to where you started. To keep track of which circles you have visited, you can mark each one with consecutively numbered disks, such as those at the left.

Graphs such as this one, which project three-dimensional problems and puzzles onto the two-dimensional plane and thereby make them easier to solve, are called Schlegel diagrams.

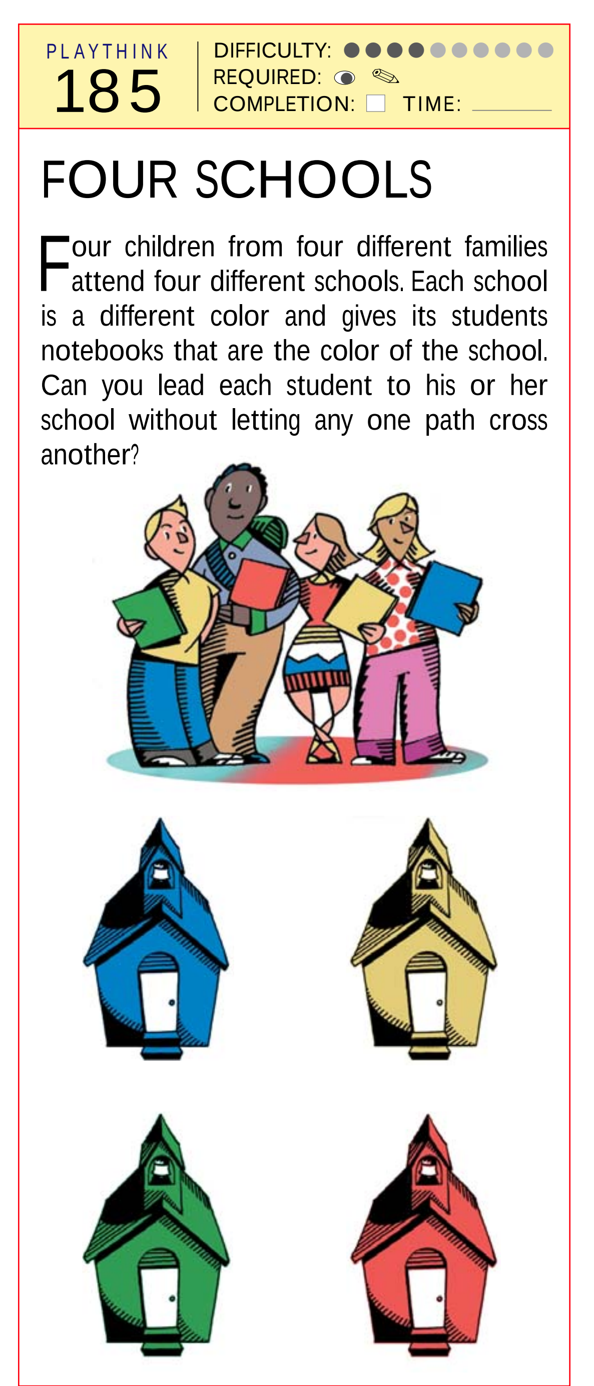
Hamilton himself devised a branch of mathematics to solve similar path-tracing problems on two-dimensional solids. He called it Icosian calculus.

Dodecahedron

PLAYTHINK 185 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ______

FOUR SCHOOLS

Four children from four different families attend four different schools. Each school is a different color and gives its students notebooks that are the color of the school. Can you lead each student to his or her school without letting any one path cross another?



Bipartite Graphs

Some graphs don't require connecting every point to every other point. One example of such a graph is a complete bipartite graph. Its nodes are divided into two sets, with *m* and *n* nodes each, and all the nodes in one set are joined to all the nodes in the other set, but no two nodes in the same set are joined. (Such a graph is a generalized version of the "Utilities" puzzles on the following page.)

Although the crossing number for certain complete bipartite graphs is known, it has not been worked out for a general *m* and *n*. For example, if m=n=7, then it is known that the crossing number is either 77, 79 or 81. But nobody knows which of these three is the correct answer.

Many other simple properties of graphs prove equally elusive. Combinatorial mathematics is still in its infancy, and it is a fertile ground for challenging puzzles and problems.

PLAYTHINK 186

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

UTILITIES 1

Before they can be occupied, these three houses must each be connected to the telephone, electricity and water lines. Nine lines are needed in total. Can you draw lines to connect each house to each utility without allowing any of the lines to intersect?



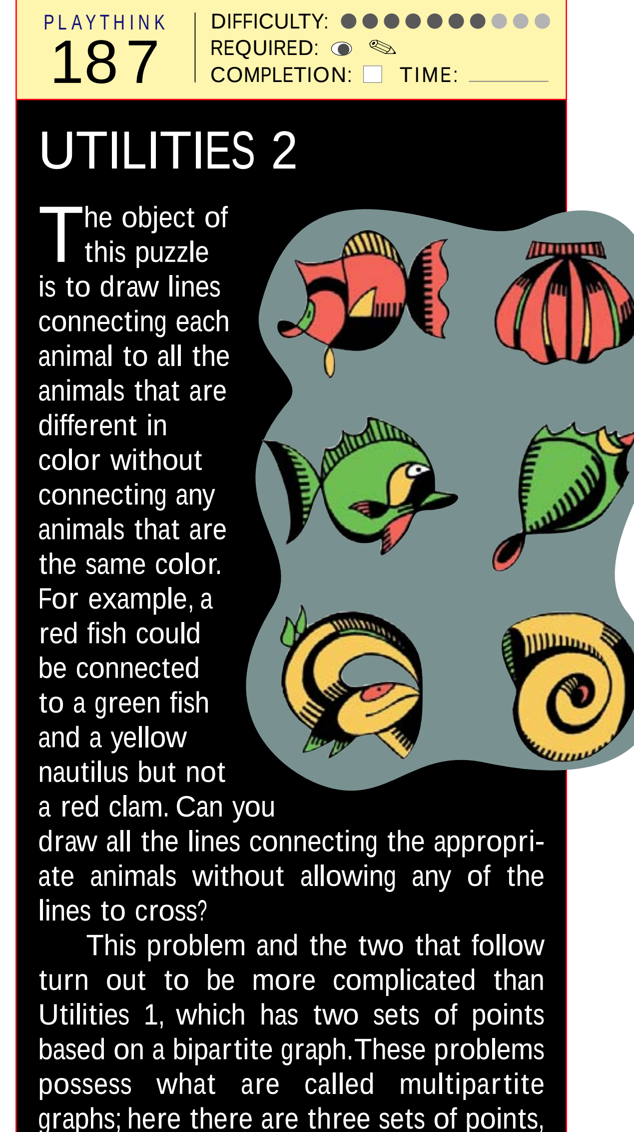
PLAYTHINK 187

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

UTILITIES 2

The object of this puzzle is to draw lines connecting each animal to all the animals that are different in color without connecting any animals that are the same color. For example, a red fish could be connected to a green fish and a yellow nautilus but not a red clam. Can you draw all the lines connecting the appropriate animals without allowing any of the lines to cross?

This problem and the two that follow turn out to be more complicated than Utilities 1, which has two sets of points based on a bipartite graph. These problems possess what are called multipartite graphs; here there are three sets of points, making for a tripartite graph.

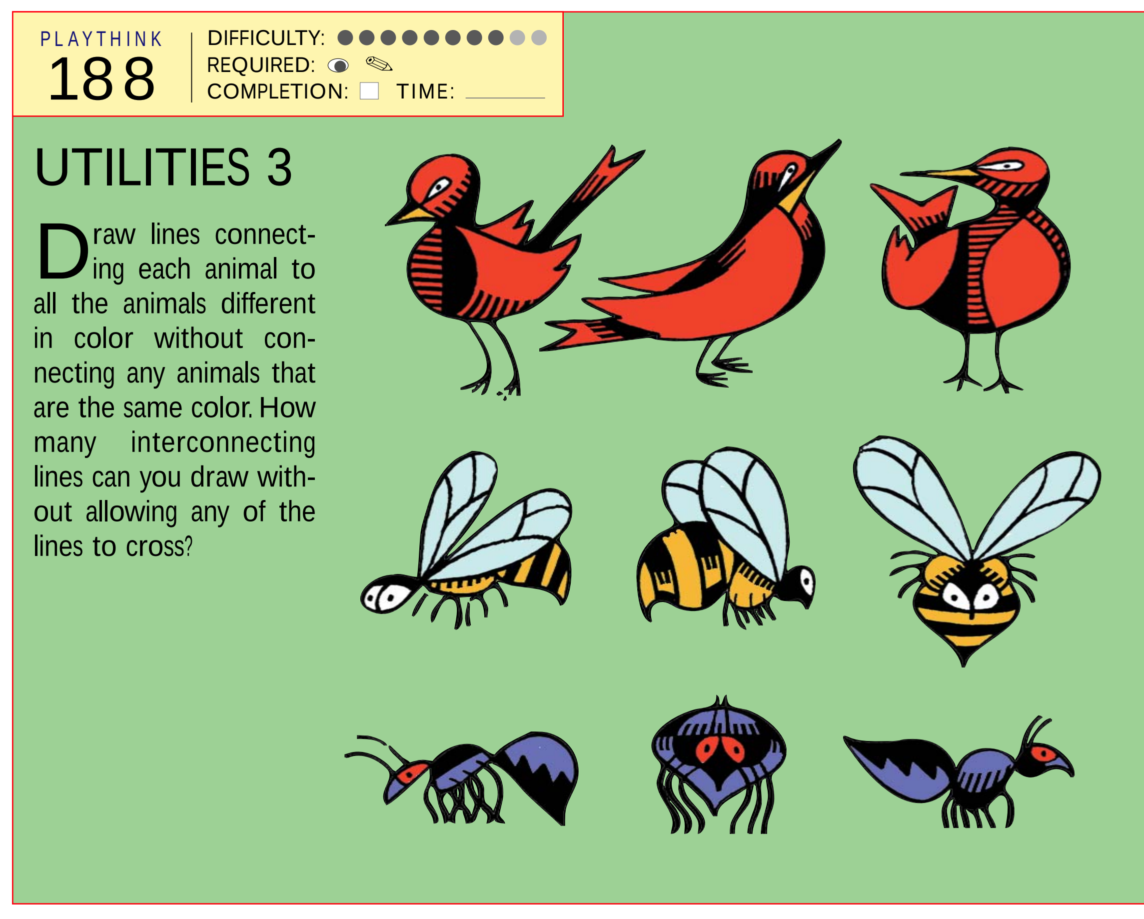


PLAYTHINK 188

DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

UTILITIES 3

Draw lines connecting each animal to all the animals different in color without connecting any animals that are the same color. How many interconnecting lines can you draw without allowing any of the lines to cross?

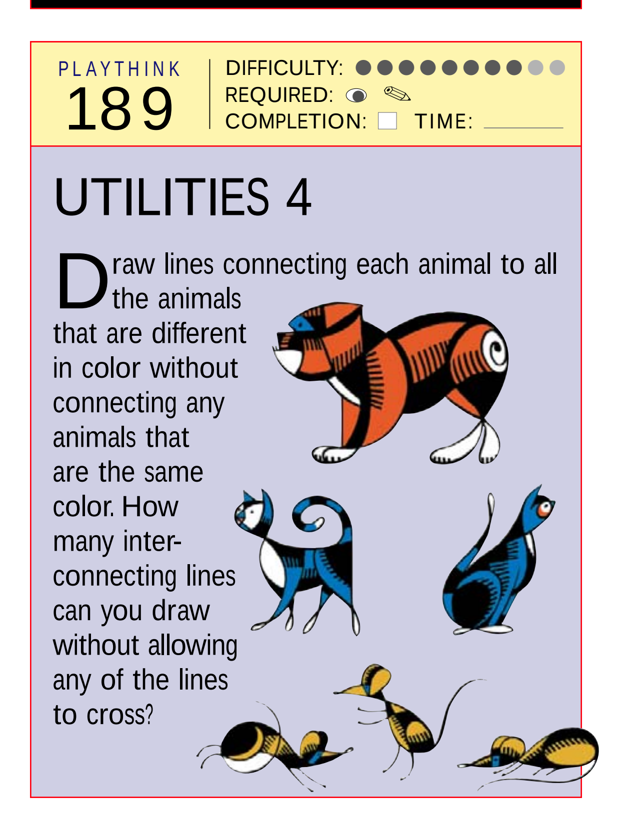


PLAYTHINK 189

DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

UTILITIES 4

Draw lines connecting each animal to all the animals that are different in color without connecting any animals that are the same color. How many interconnecting lines can you draw without allowing any of the lines to cross?

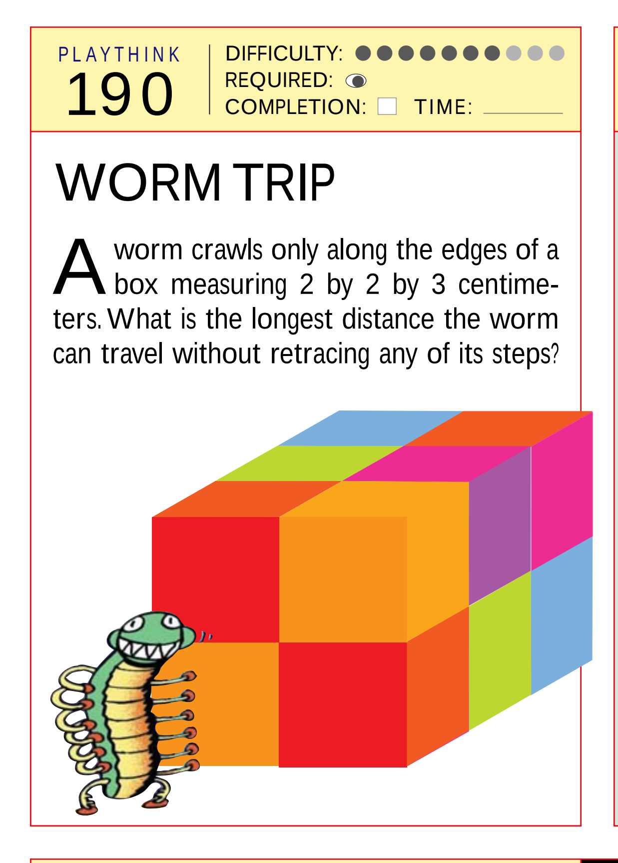


PLAYTHINK 190

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

WORM TRIP

A worm crawls only along the edges of a box measuring 2 by 2 by 3 centimeters. What is the longest distance the worm can travel without retracing any of its steps?

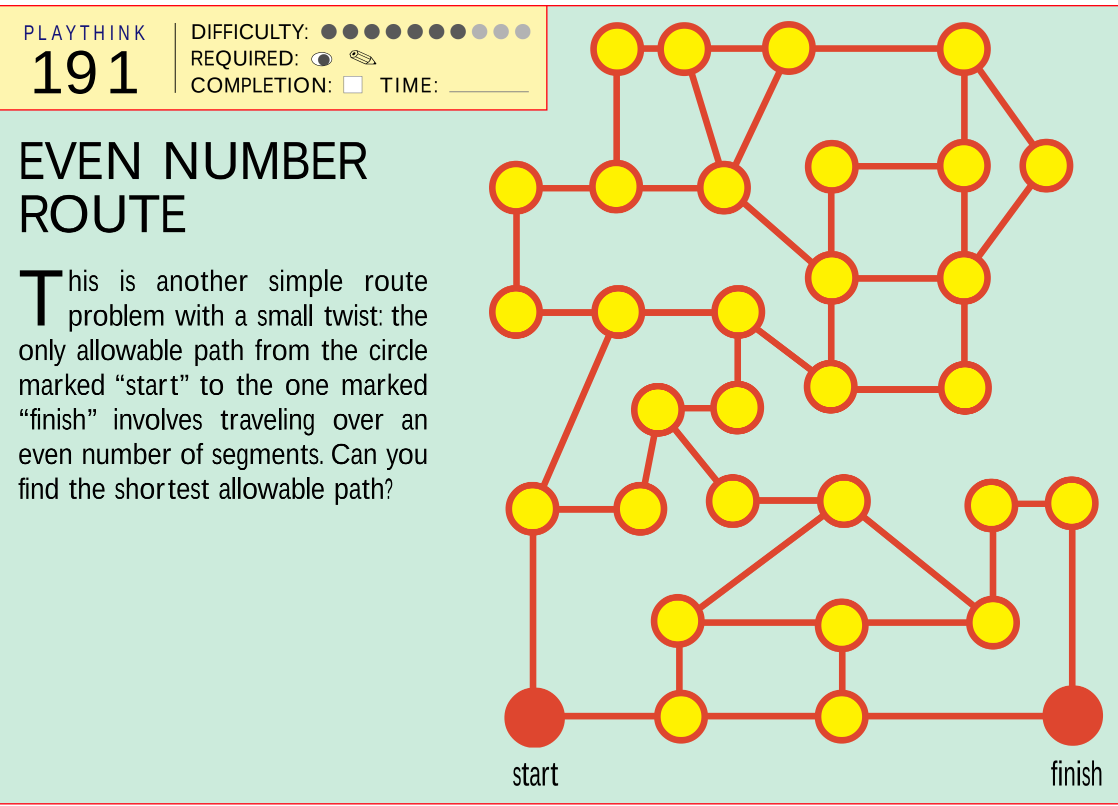


PLAYTHINK 191

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

EVEN NUMBER ROUTE

This is another simple route problem with a small twist: the only allowable path from the circle marked “start” to the one marked “finish” involves traveling over an even number of segments. Can you find the shortest allowable path?



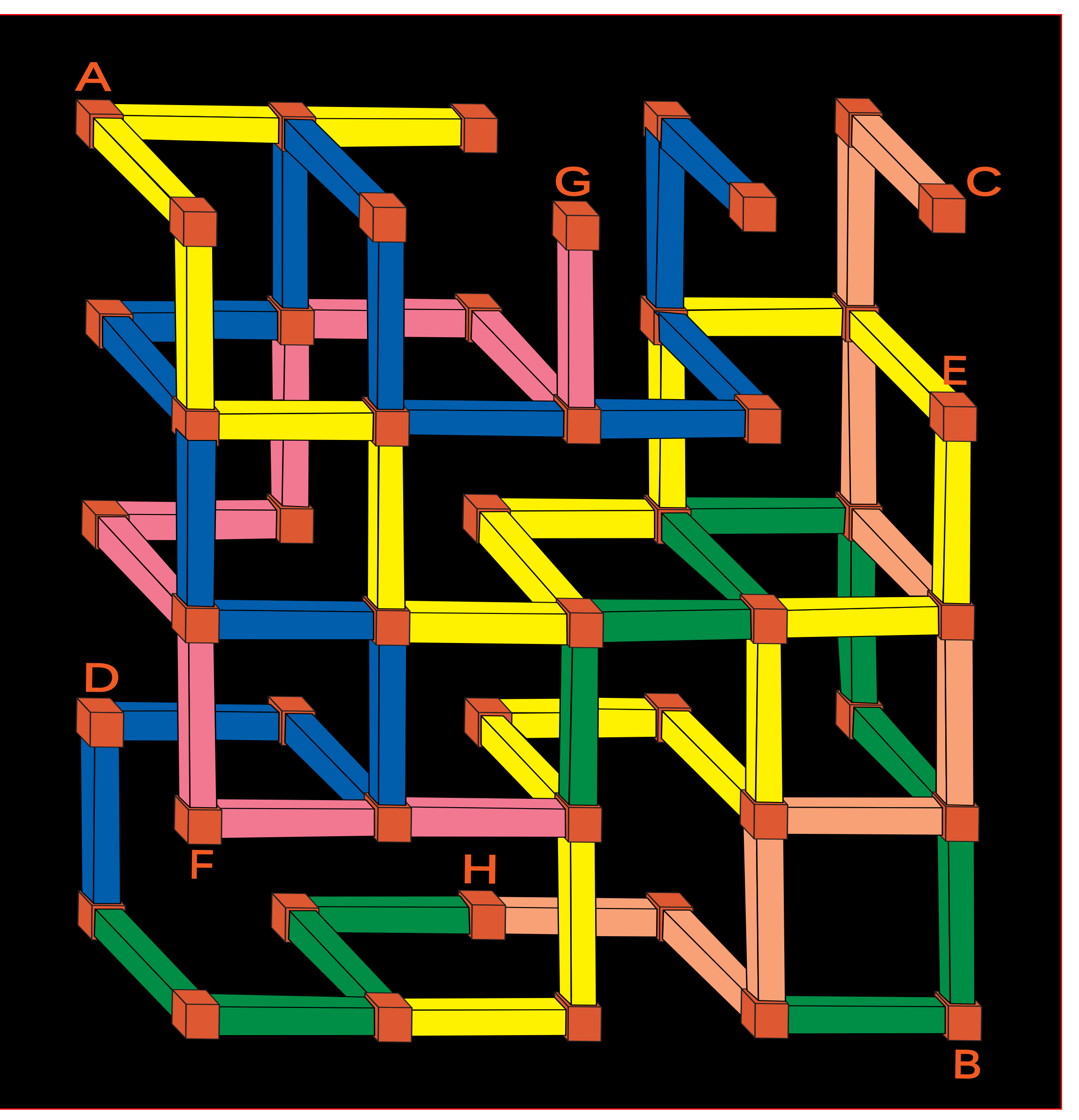
PLAYTHINK 192

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

SUBWAY

The subway is designed to be the quickest way across town—and for many trips it is. But in cities with several subway lines that have only a handful of interconnecting stations, travelers spend a lot of time waiting around. And they also have to take time simply to walk from one subway platform to another. In fact, for many subway systems the time needed to change trains is more or less equivalent to the time it takes to travel by subway from one station to another. That statistic is at the heart of this puzzle.

The object is to find the fastest route between specific stations. You have to stay on the same line, which is designated by a distinct color, unless you make a transfer at a station where two lines meet. You count each station you pass (as well as the one where you start) as one minute and any station where you change lines as two minutes. Given those rules, can you find the fastest routes from A to B, C to D, E to F and G to H?



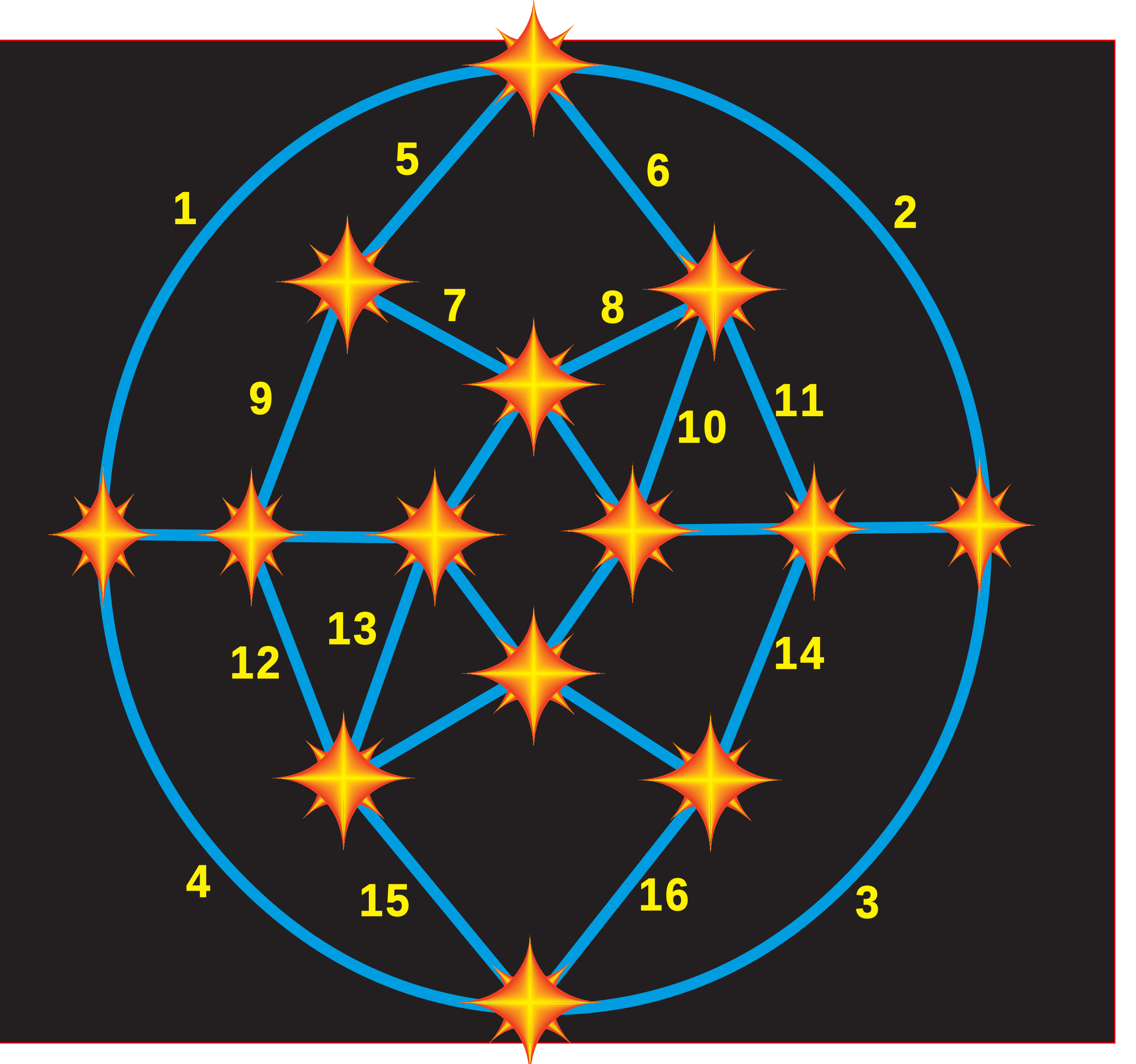
PLAYTHINK 193
DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

STAR TOURS

There are fourteen stars in the constellation shown here, each of which is connected to at least three of its neighbors by an interstellar guideway. Can you follow the guideways to visit each star once and only once?

Sixteen of the guideways have been assigned a number and will be closed for repairs, one at a time, in numerical order. Can you complete the task of visiting each star without the use of guideway 1? How about guideway 2?

Try to find routes connecting all the stars consecutively without the use of one of the sixteen numbered guideways in succession. You will find that in two of the sixteen instances, there will be no solution. Can you figure out which two?



PLAYTHINK 194
DIFFICULTY: ●●●●●●●●○○
REQUIRED:
COMPLETION: ☐ TIME: ______

MARS PUZZLE

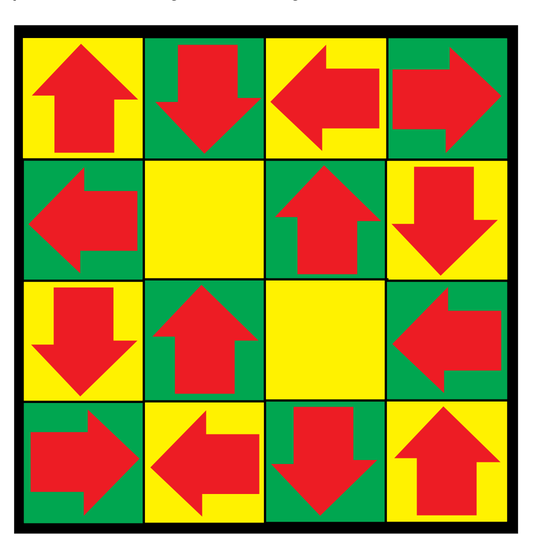
There are twenty scientific outposts scattered on the surface of Mars, each marked with a letter and each linked by a canal to at least two other stations. Starting at the outpost marked T and visiting each station just once, follow the various canals to spell out a complete English sentence.

Can you find a solution?

PLAYTHINK 195
DIFFICULTY: ●●●●○○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

MISSING ARROWS 1

Two arrows are missing from the pattern shown below. Can you add the missing arrows so that they help create a consistent pattern throughout the grid?



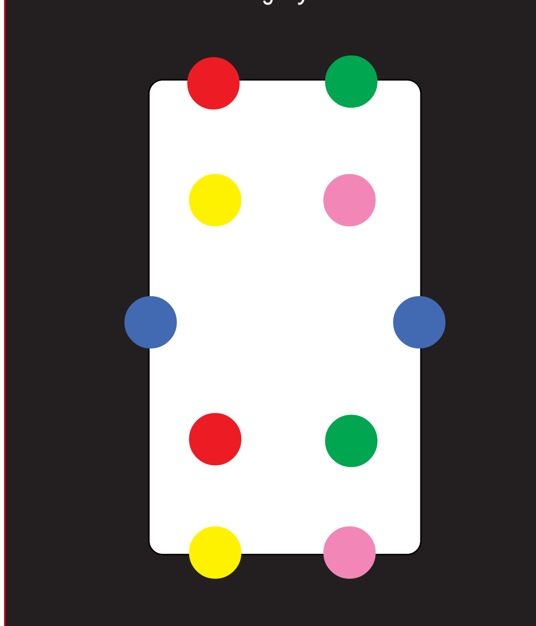
PLAYTHINK 196

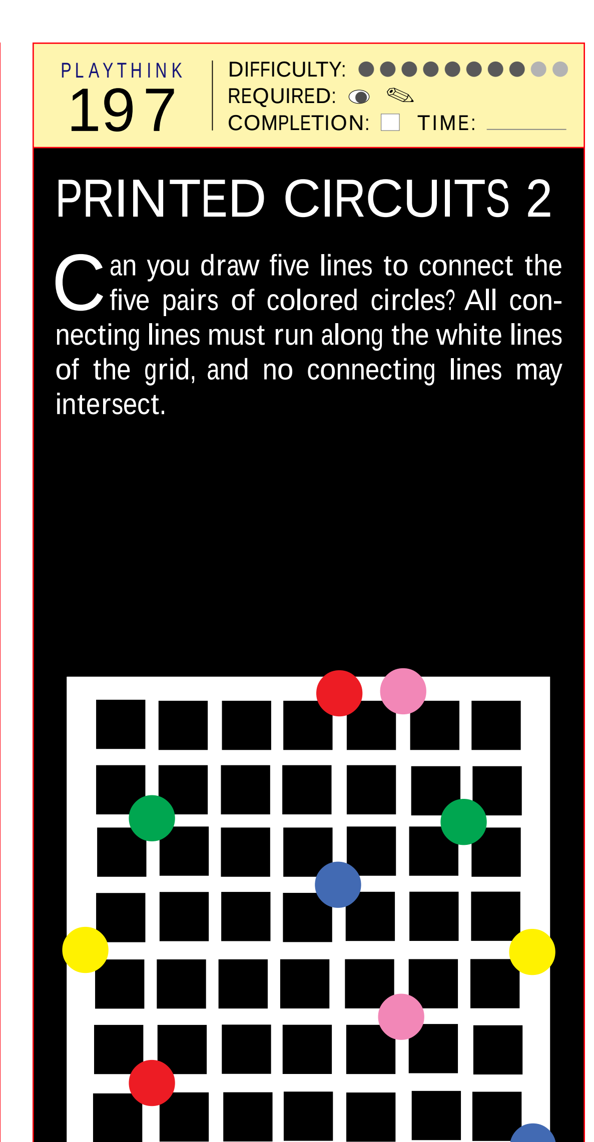
DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

PRINTED CIRCUITS 1

Printed circuits are two-dimensional graphs—the junctions carry out electronic operations, while the lines carry electrical signals from place to place. If the lines cross, there will be a short-circuit and the device will fail.

Can you connect the five pairs of colored circuits on this circuit board without crossing any lines? The connecting lines must remain in the gray area.



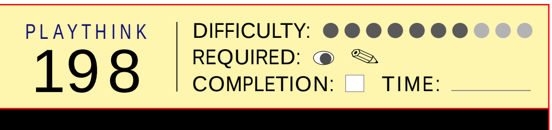


PLAYTHINK 197

DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

PRINTED CIRCUITS 2

Can you draw five lines to connect the five pairs of colored circuits? All connecting lines must run along the white lines of the grid, and no connecting lines may intersect.



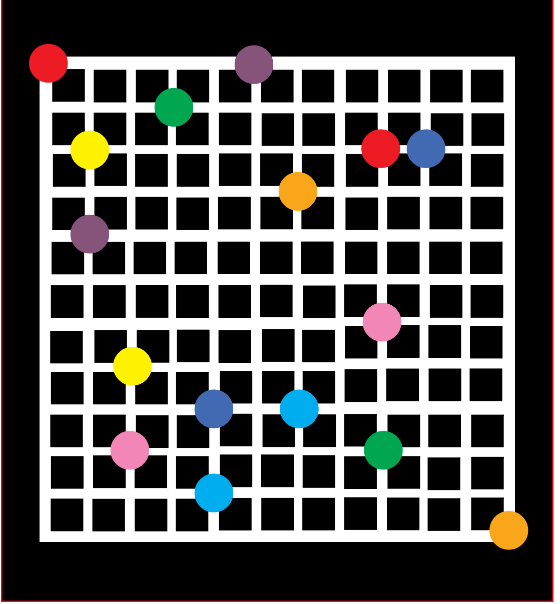
PLAYTHINK 198

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

PRINTED CIRCUITS 3

Can you draw eight lines to connect the eight pairs of colored circuits? All connecting lines must run along the white lines of the grid, and no connecting lines may intersect.

This puzzle can become a two-player game. The players take turns connecting the circles; play continues until one player is unable to make a connection.



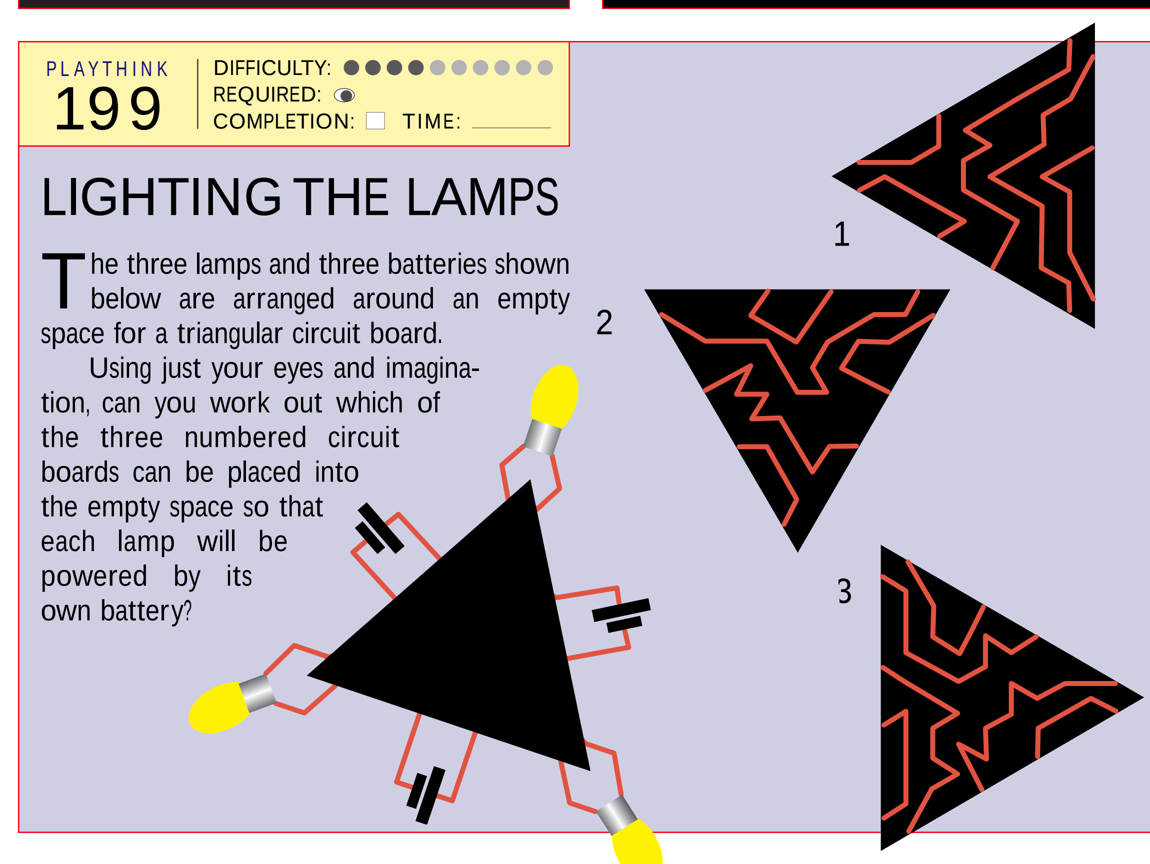
PLAYTHINK 199

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

LIGHTING THE LAMPS

The three lamps and three batteries shown below are arranged around an empty space for a triangular circuit board.

Using just your eyes and imagination, can you work out which of the three numbered circuit boards can be placed into the empty space so that each lamp will be powered by its own battery?



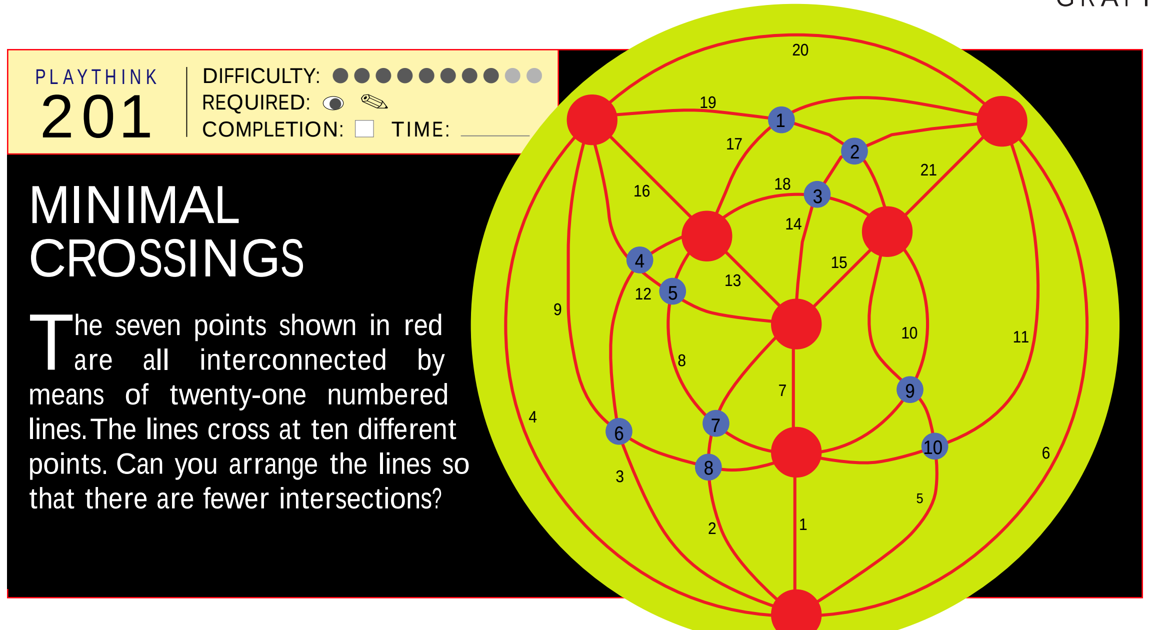
PLAYTHINK 200

DIFFICULTY: ●●●●●●●●●○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

DICE ARROWS

How many different ways can you put six arrows on the faces of a cube?

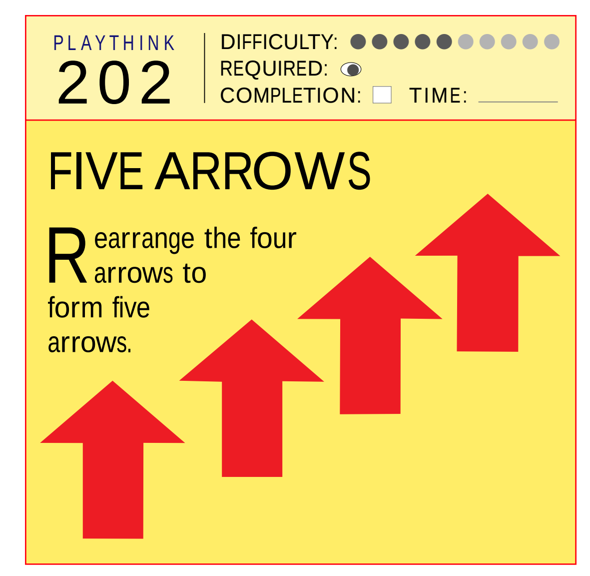




PLAYTHINK 201 | DIFFICULTY: ●●●●●●●●○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

MINIMAL CROSSINGS

The seven points shown in red are all interconnected by means of twenty-one numbered lines. The lines cross at ten different points. Can you arrange the lines so that there are fewer intersections?



PLAYTHINK 202 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

FIVE ARROWS

Rearrange the four arrows to form five arrows.

Topology and the Tree Graph

Take a shape such as a triangle and start to deform it: change the angles, lengthen the sides, add extra corners. What, from the geometric point of view, remains unchanged from the original figure? That is the kind of question answered in the field of study called topology.

Little of what is important in traditional geometry is of any use in topology. Instead, topology looks at such facts as (a) a triangle has an inside and an outside and (b) it is impossible to pass from one to the other without crossing an edge. No matter how you deform a triangle in the plane, it will still possess an inside and outside—that's a defining point in topology. In fact, to a topologist a triangle is the same as a square, a parallelogram or even a circle. The torus—the shape of an inner tube or a doughnut—has a hole in the middle and retains the hole no matter how much it is distorted; that's a property that sets it apart from triangles but gives it something in common with a coffee mug. In topology, the number *8* and the letter *B* are equivalent: each has two holes.

Many topological properties concern the way objects are connected; whether a loop of string is knotted is a topological property. The basic concepts of topology include many ideas infants learn: insideness and outsideness, right- and left-handedness, linking, knotting and disconnectedness.

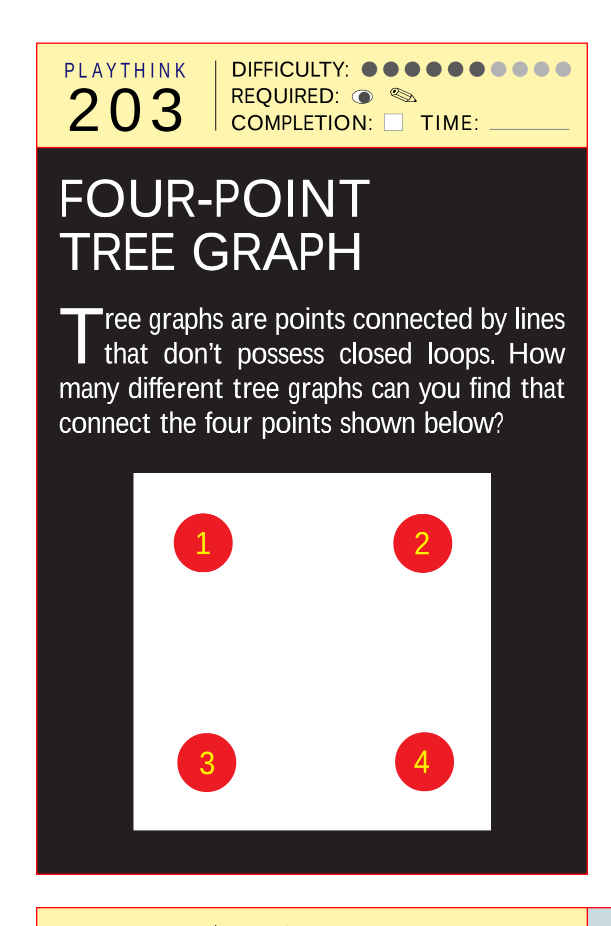
Topological ideas are crucial to the understanding of graphs. When nodes are joined by edges, what matters is not the precise position of the edges and nodes but, rather, the way they connect up. For example, a graph is connected if it is "all in one piece," that is, there's a continuous path from any node to any other node. The precise shape of the edges is irrelevant; all that matters in topology is the connectedness of the graph. Similarly, if a graph contains a circuit—a closed loop with distinct edges—it is topologically equivalent to any other graph possessing a loop.

Graphs that possess no loops are called trees (because, like real trees, these graphs have branches that never link except through a trunk). Many processes that branch may be represented as trees. For example, the positions in a game of chess form a tree whose edges are the moves of the game. The strategy in many games is generally based on viewing the game as a tree, and computer programs that play such games as chess, checkers and backgammon make essential use of this idea. Indeed, the advanced chess-playing computers that are capable of beating human grand masters work out trees of possible moves; the computer then selects the move at the present point that will ensure the best possible outcome many moves in the future.

PLAYTHINK 203 — DIFFICULTY: ●●●●●●○○○○ REQUIRED: 👁 ✎ COMPLETION: ☐ TIME: ____

FOUR-POINT TREE GRAPH

Tree graphs are points connected by lines that don't possess closed loops. How many different tree graphs can you find that connect the four points shown below?

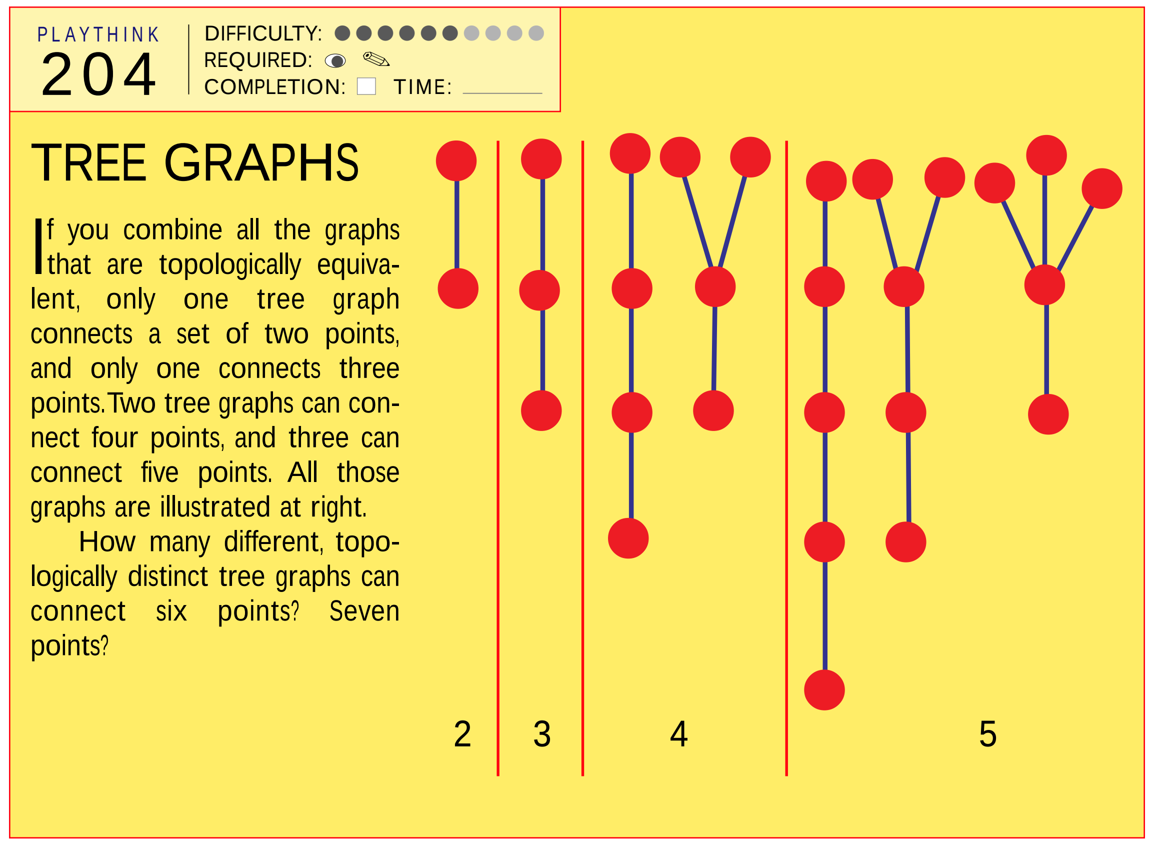


PLAYTHINK 204 — DIFFICULTY: ●●●●●●○○○○ REQUIRED: 👁 ✎ COMPLETION: ☐ TIME: ____

TREE GRAPHS

If you combine all the graphs that are topologically equivalent, only one tree graph connects a set of two points, and only one connects three points. Two tree graphs can connect four points, and three can connect five points. All those graphs are illustrated at right.

How many different, topologically distinct tree graphs can connect six points? Seven points?



PLAYTHINK 205 — DIFFICULTY: ●●●●●●●○○○ REQUIRED: 👁 ✎ 📄 ✂ COMPLETION: ☐ TIME: ____

TREE GAME

This simple stick figure game requires each player to match specific arrangements of matchsticks to patterns on playing cards.

To play the game, you need the following simple materials:

a set of cards that depicts stylized versions of tree graphs possessing three, four, five and six nodes (see PlayThink 206)

and a set of six identical sticks—matches, drinking straws or whatever is on hand—to re-create the graphs shown on the cards

The object is to reproduce the patterns on the cards in the fewest number of moves. To play, shuffle the cards and place them all face down in a pile. Place five sticks in a straight line on the table. (The sixth stick is held in a reserve pile.)

The first player takes the top three cards from the pile and places them face up. That player then has two moves to change the positions of the sticks to match the graphs shown on the exposed cards. A move consists of picking up a stick from the table and laying it in a new position, or adding a stick from the reserve pile to the graph, or removing a stick from the graph and placing it in the reserve pile. A player may also rotate as many sticks on the graph as possible, as long as the end that is attached to the graph remains fixed. (Obviously, if both ends are attached to the graph, a stick cannot be rotated.)

If a player succeeds in forming one of the graphs, he or she takes the card that depicts it and keeps it until the end of the game. Cards that the player could not duplicate remain on the table.

The second player, if necessary, draws cards to replace the cards captured by the first player and tries to duplicate one or more of the cards with just two moves of the sticks. The game continues until all the cards have been taken. The winner is the player with the most cards.

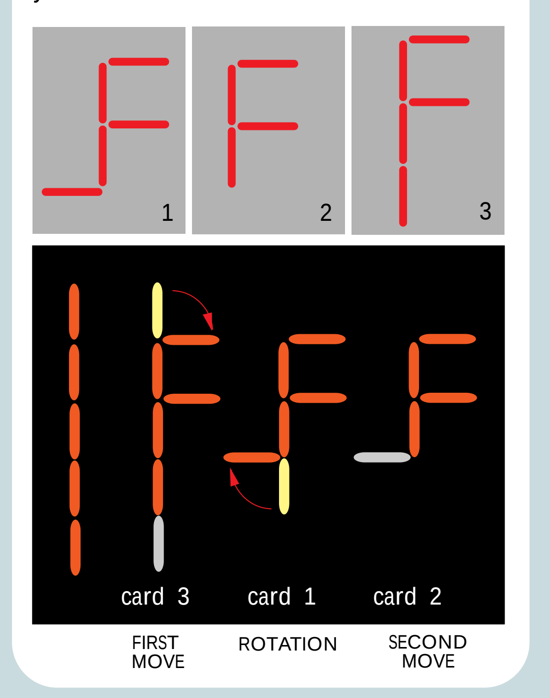
You can play two forms of the game: classical and topological. In the classical game a card may be taken only if the tree formed by the sticks is exactly the same as that depicted on the card; the topological game allows trees that are topologically equivalent. Playing both versions of the game will help drive home the difference between exact similarity and topological equivalence.

SAMPLE GAME

Three cards drawn by a player are shown at top. The cards are laid face up.

The player then makes moves according to the cards drawn. In this case the player earns a score of three points.

gray = removed stick
yellow = rotated stick



PLAYTHINK 206 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

TREE GAME CARDS AND VARIANT

These are the sixty-four cards used to play the "Tree Game": sixteen sets of topologically equivalent cards, with four variations in each set. For example, one set is made up of cards 1, 20, 35 and 61.

You can also use them to play a variant game: namely, how long will it take you to identify all sixteen sets?

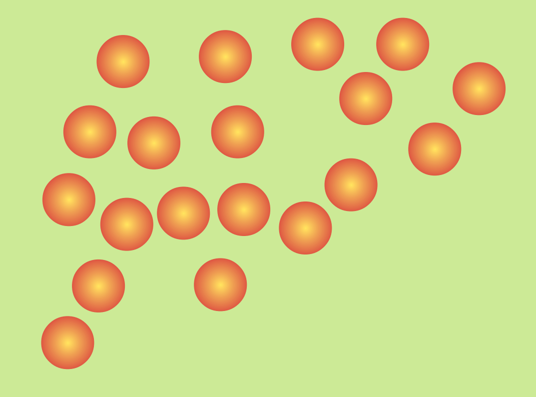
1 2 3 4 5 6 7 8
9 10 11 12 13 14 15 16
17 18 19 20 21 22 23 24
25 26 27 28 29 30 31 32
33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48
49 50 51 52 53 54 55 56
57 58 59 60 61 62 63 64

PLAYTHINK 207 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

TREE CHAIN

Nineteen beads lie on a table. Can you join them with string to create a tree graph?

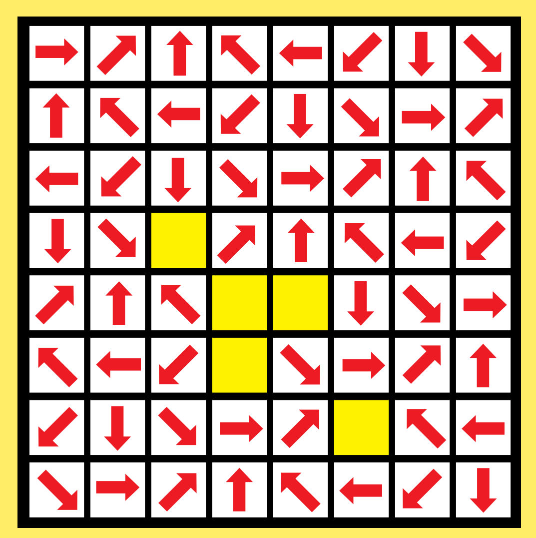
What is the smallest number of branches you can draw between nineteen beads, or nineteen points? Remember that since it is one graph, each point must be linked to every other point by some number of branches. And since it is a tree graph, there can be no closed loops. Is there a general rule for the minimum number of branches needed?



PLAYTHINK 208 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

MISSING ARROWS 2

Can you work out where the four missing arrows should point?



PLAYTHINK 209

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

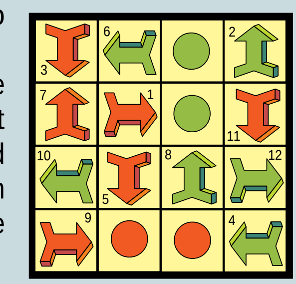
ARROWS PUZZLE AND GAME 1

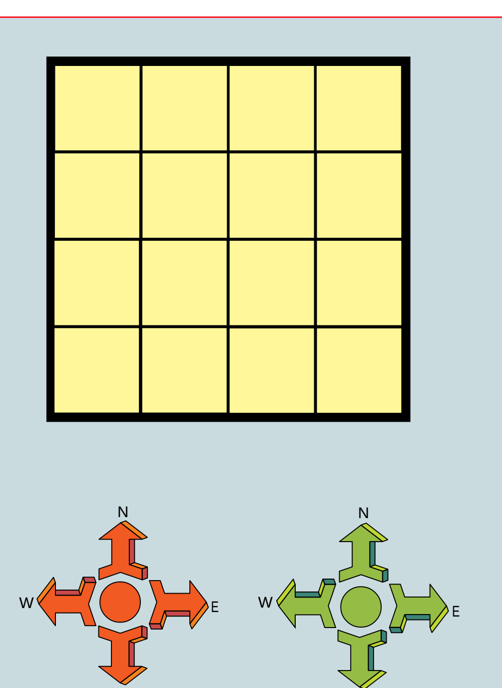
The puzzle: Place sixteen arrows on the four-by-four game board so that each row, each column and each main diagonal contains four arrows pointing in four different directions—north, south, east and west.

The game: The object of the game is to place sixteen arrows so that no row, column or main diagonal has two or more arrows pointing in the same direction. Two players take turns placing an arrow of their color on the board. The arrows must point in one of the four main directions (north, south, east or west, or perhaps up, down, right or left). After each move the board is checked to see if any squares are blocked from receiving arrows; that is, any arrows placed on such a square would violate the rules. A player whose moves create such blocked squares can mark them in her or his color before the next player's turn.

The game ends when no legal moves remain. Players receive a point for each row, column or diagonal consisting of at least three boxes of his or her color, of which at least two are arrows. A row in which a player has one arrow and two blocked squares in his or her color yields no points.

In the sample game illustrated at right, the game ended in a tie, with each player getting three points.





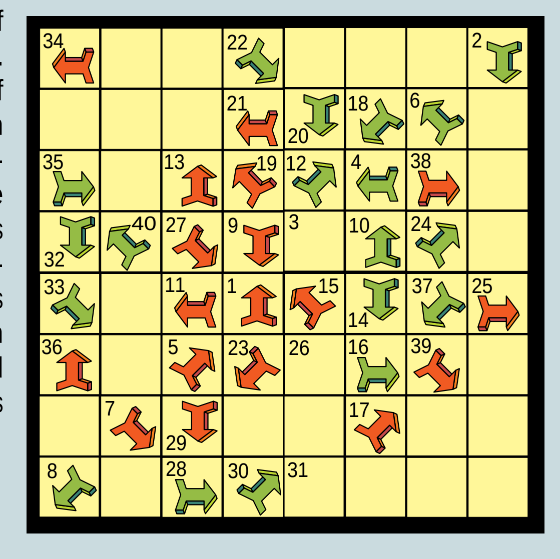
PLAYTHINK 210

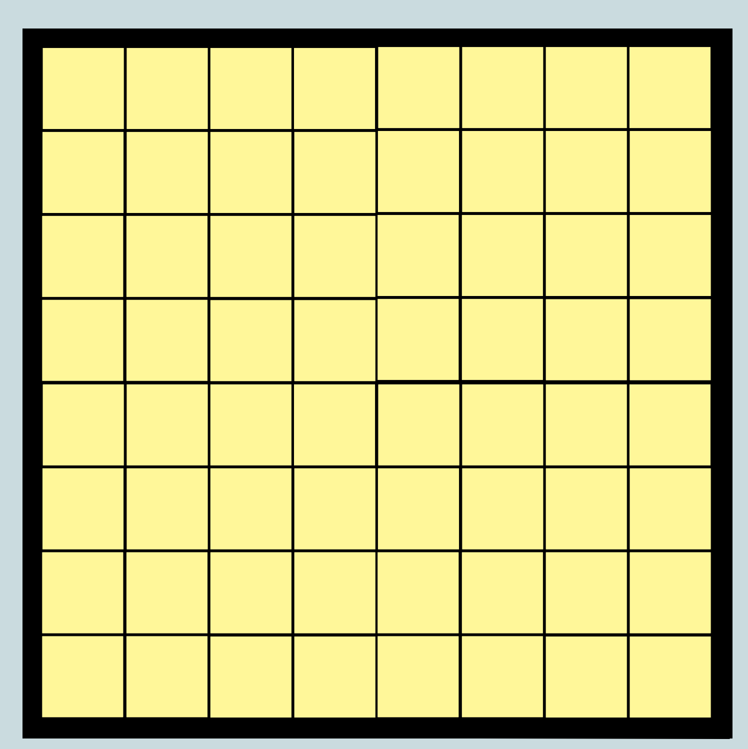
DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

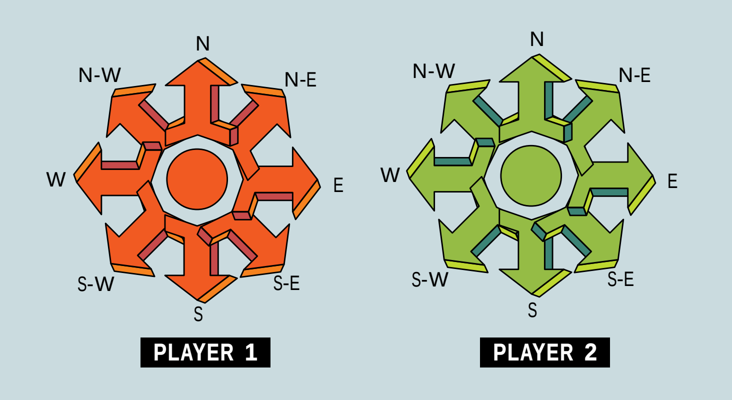
ARROWS PUZZLE AND GAME 2

The puzzle: On the eight-by-eight game board, fit sixty-four arrows so that each row and column contains eight arrows, each pointing in a different direction—north, northeast, east, southeast, south, southwest, west and northwest.

The game: The object of the game is to place arrows on the board so that no row or column has two or more arrows pointed in the same direction. Two players take turns placing arrows of their color on the game board. The arrows must be in one of the eight orientations used in the puzzle version. Play continues until no legal moves are possible. Each player receives one point for every row or column in which he or she has placed five or more arrows. In the game illustrated at right, red wins with two points versus green's one point.







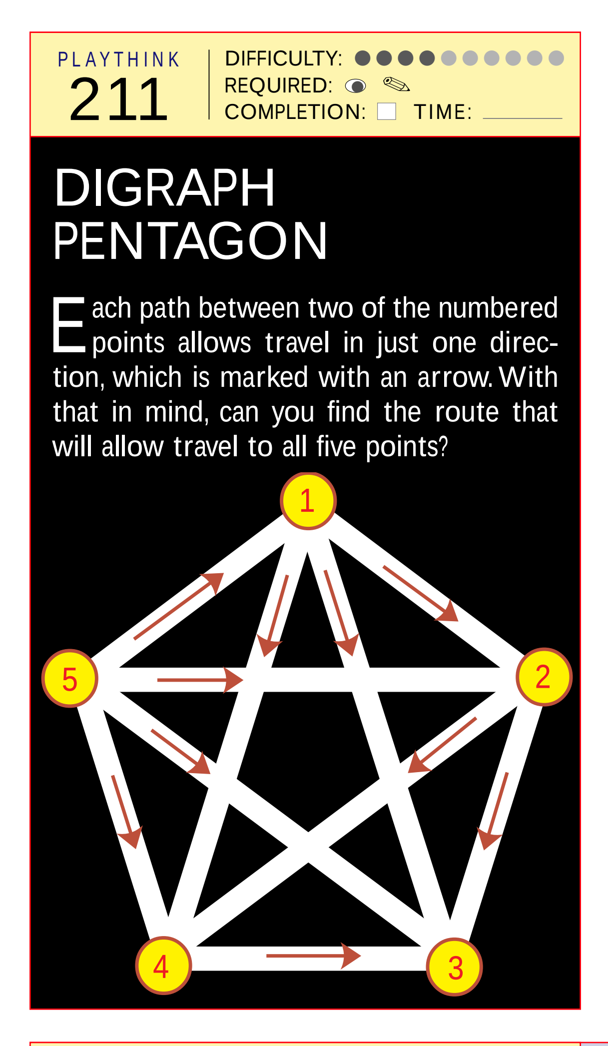
Digraphs

Add an arrowhead to each line of a graph and it becomes a directed graph, or digraph. A complete digraph, in which every edge has a direction and every pair of points is joined by a line, is called a tournament. No matter how you place the arrows, every tournament will have a Hamiltonian path—that is, a route that visits every node once. A path that returns to its starting point after visiting all the other points is called a Hamiltonian circuit; such a route is not possible for every tournament.

PLAYTHINK 211 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

DIGRAPH PENTAGON

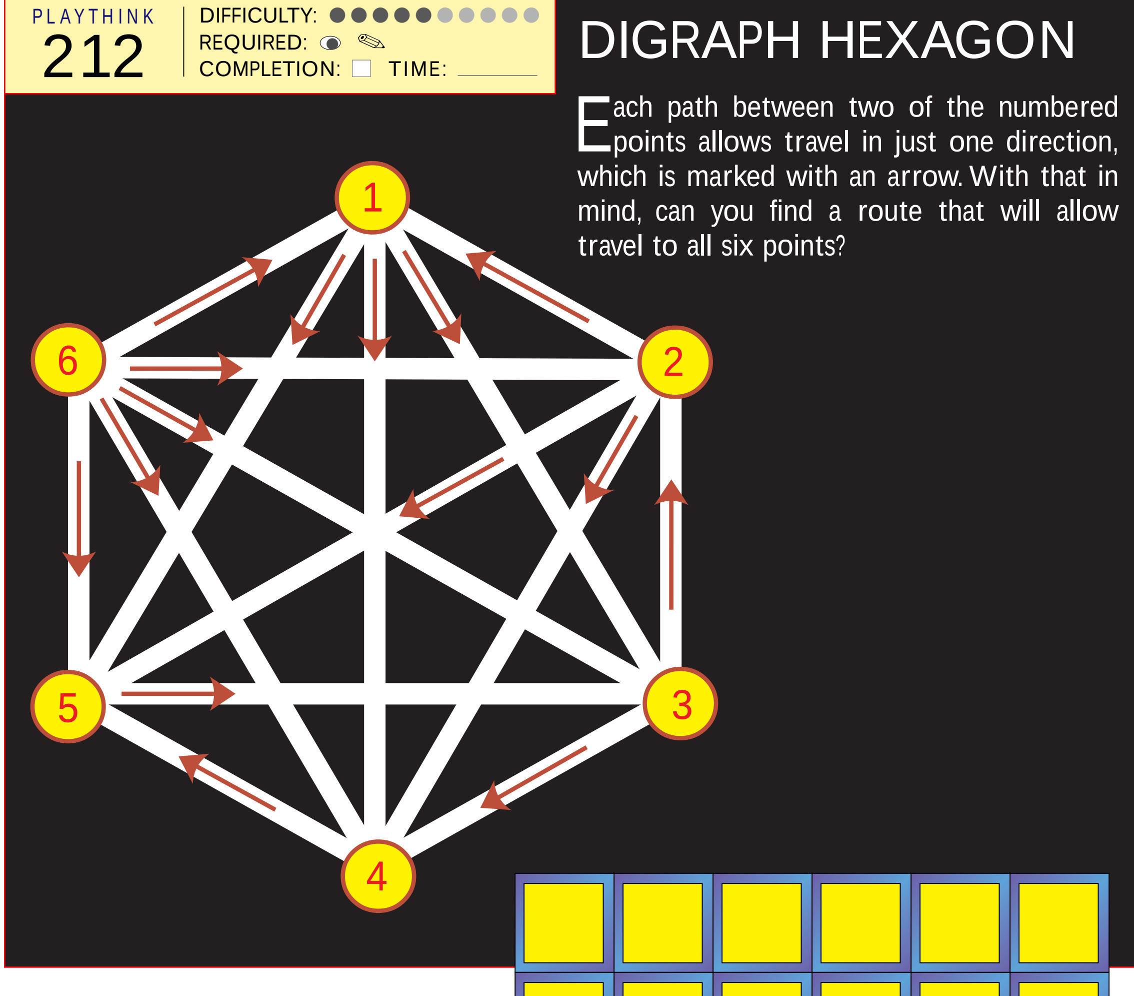
Each path between two of the numbered points allows travel in just one direction, which is marked with an arrow. With that in mind, can you find the route that will allow travel to all five points?



PLAYTHINK 212 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

DIGRAPH HEXAGON

Each path between two of the numbered points allows travel in just one direction, which is marked with an arrow. With that in mind, can you find a route that will allow travel to all six points?

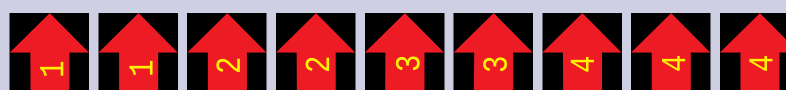
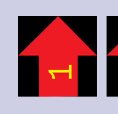


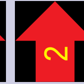
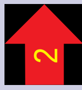
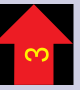
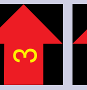
PLAYTHINK 213 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 🗎 ✂ | COMPLETION: ☐ TIME: ____

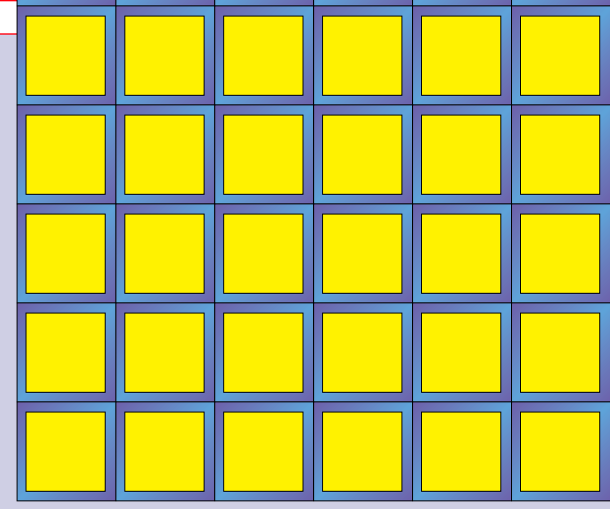
ARROWS TOUR

In this puzzle the arrows below are numbered; each number tells you how many spaces to move. For example, an arrow marked 3 means you may move three places in that direction.

Can you place the nine arrows on the board so that each arrow points at another arrow to create a continuous loop? That is, after nine jumps, you should end up where you started.

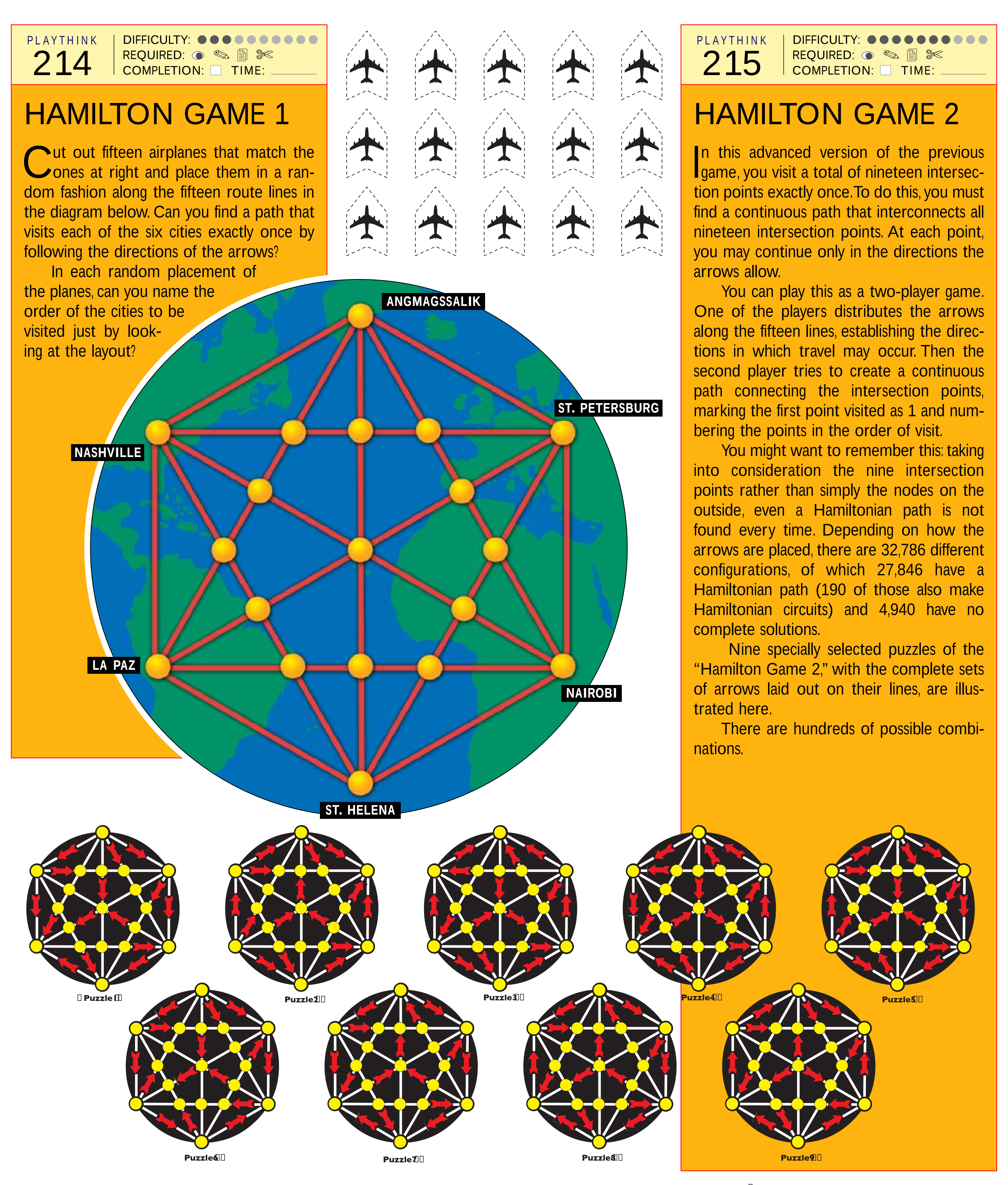
PLAYTHINK 214

DIFFICULTY: ●●●○○○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

HAMILTON GAME 1

Cut out fifteen airplanes that match the ones at right and place them in a random fashion along the fifteen route lines in the diagram below. Can you find a path that visits each of the six cities exactly once by following the directions of the arrows?

In each random placement of the planes, can you name the order of the cities to be visited just by looking at the layout?



PLAYTHINK 215

DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

HAMILTON GAME 2

In this advanced version of the previous game, you visit a total of nineteen intersection points exactly once. To do this, you must find a continuous path that interconnects all nineteen intersection points. At each point, you may continue only in the directions the arrows allow.

You can play this as a two-player game. One of the players distributes the arrows along the fifteen lines, establishing the directions in which travel may occur. Then the second player tries to create a continuous path connecting the intersection points, marking the first point visited as 1 and numbering the points in the order of visit.

You might want to remember this: taking into consideration the nine intersection points rather than simply the nodes on the outside, even a Hamiltonian path is not found every time. Depending on how the arrows are placed, there are 32,786 different configurations, of which 27,846 have a Hamiltonian path (190 of those also make Hamiltonian circuits) and 4,940 have no complete solutions.

Nine specially selected puzzles of the "Hamilton Game 2," with the complete sets of arrows laid out on their lines, are illustrated here.

There are hundreds of possible combinations.

Ramsey Theory

Although Frank Ramsey made considerable contributions to economics and philosophy, he is remembered more for his brilliance as a mathematician. The Englishman's best-known work was in set theory; indeed, a branch of that field now carries his name—quite an accomplishment for a man who died in 1930 at twenty-seven!

The appearance of disorder is really a matter of scale: a mathematical structure can be found if you look widely enough. Ramsey wanted to find the smallest set of objects that would guarantee that some of those objects would share certain properties. For example, the smallest number of people that will always include two people of the same sex is three. If there are only two, you might have a man and a woman; since the third person would be either a man *or* a woman, adding him or her guarantees at least two of one sex.

Or take this question: Can a complete graph have its edges colored using only two colors, so that no three edges of the some color form a triangle? Ramsey proved some general theorems on this question, but instances with four, five or six nodes are simple enough to analyze using pencil and paper. The famous Party Puzzle (which we present as "Love-Hate Relationships," PlayThink 216) is based on Ramsey's work.

To appreciate how elegant graphs are for solving this sort of problem, imagine listing all possible combinations of acquaintanceship among six people—a total of 32,768—and having to check if each combination included the desired relationship.

A more advanced Ramsey problem would be to imagine a party in which there must be a foursome in which everyone is a mutual friend or everyone is a mutual stranger. How large must the party be? Ramsey's work demonstrated that eighteen guests are necessary. If you draw a complete graph with eighteen nodes, no matter how you color the lines using two colors, you will inevitably create a quadrilateral formed by connecting four points (persons) in one of the colors.

The party size required to ensure at least one fivesome of mutual friends or strangers is still unknown. The answer lies between 43 and 49.

PLAYTHINK 216 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

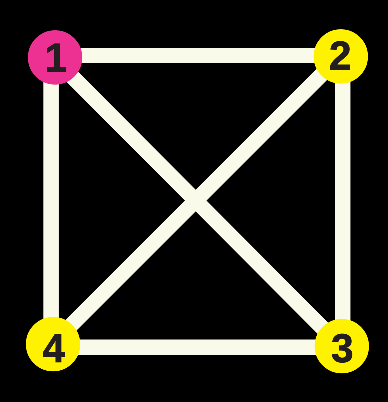
LOVE-HATE RELATIONSHIPS

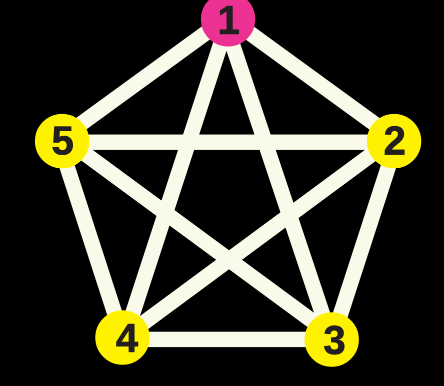
You and your friends feel your emotions very strongly—at any given time you either love someone or you hate that person. To avoid bloodshed, when you all get together, you like to arrange it so there is no group of three who all mutually hate one another and no group of three who all mutually love one another.

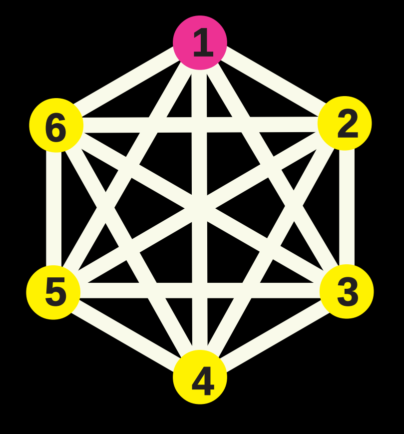
Four, five and six of you need to get together on successive evenings. Is trouble inevitable? Or is it possible to avoid both love triangles and hate triangles?

For each group, color in the lines between points in one of two colors: red for love or blue for hate. How many lines can you color before you are forced to create a love or hate triangle? Is it possible to color the lines so that, when all the relationships have been accounted for, there are no triangles?

you
your friends
red lines - love
blue lines - hate







PLAYTHINK 217

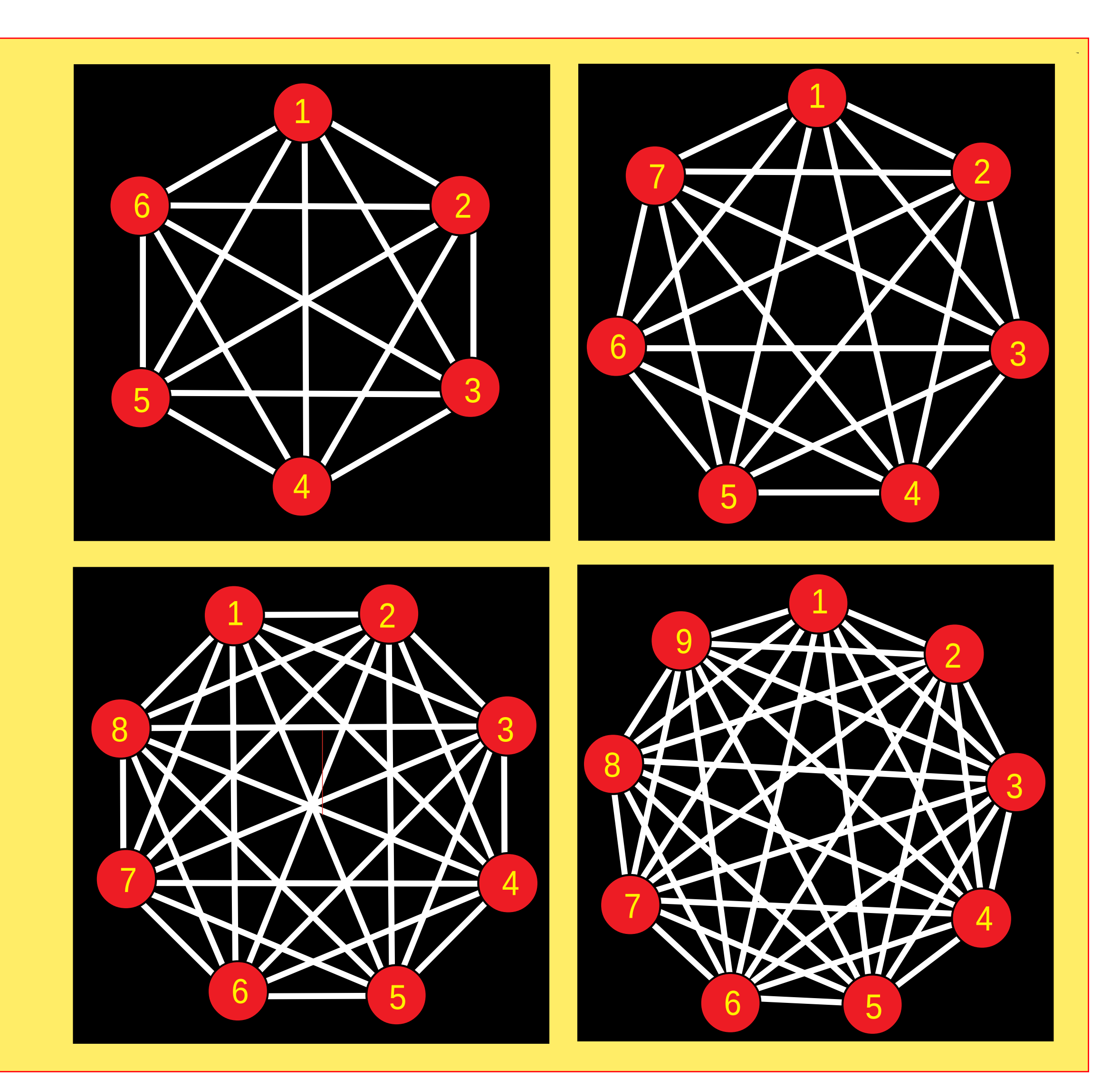
DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

SPIDER TRACK PUZZLE GAMES

This game can be played as a two-player competition or as a solitaire game.

In a two-player game the players take turns filling in the white lines between the numbered points with one of two colors—say, red and blue. Each player can use either color; the goal is to avoid creating a solid-color triangle. The game continues until one player must create a solid-color triangle. As a variant, each player can try to maneuver the other to draw a quadrilateral.

To play the game alone, fill in as many white lines as you can until you are forced to form a triangle whose vertices are the numbered points on the perimeter.



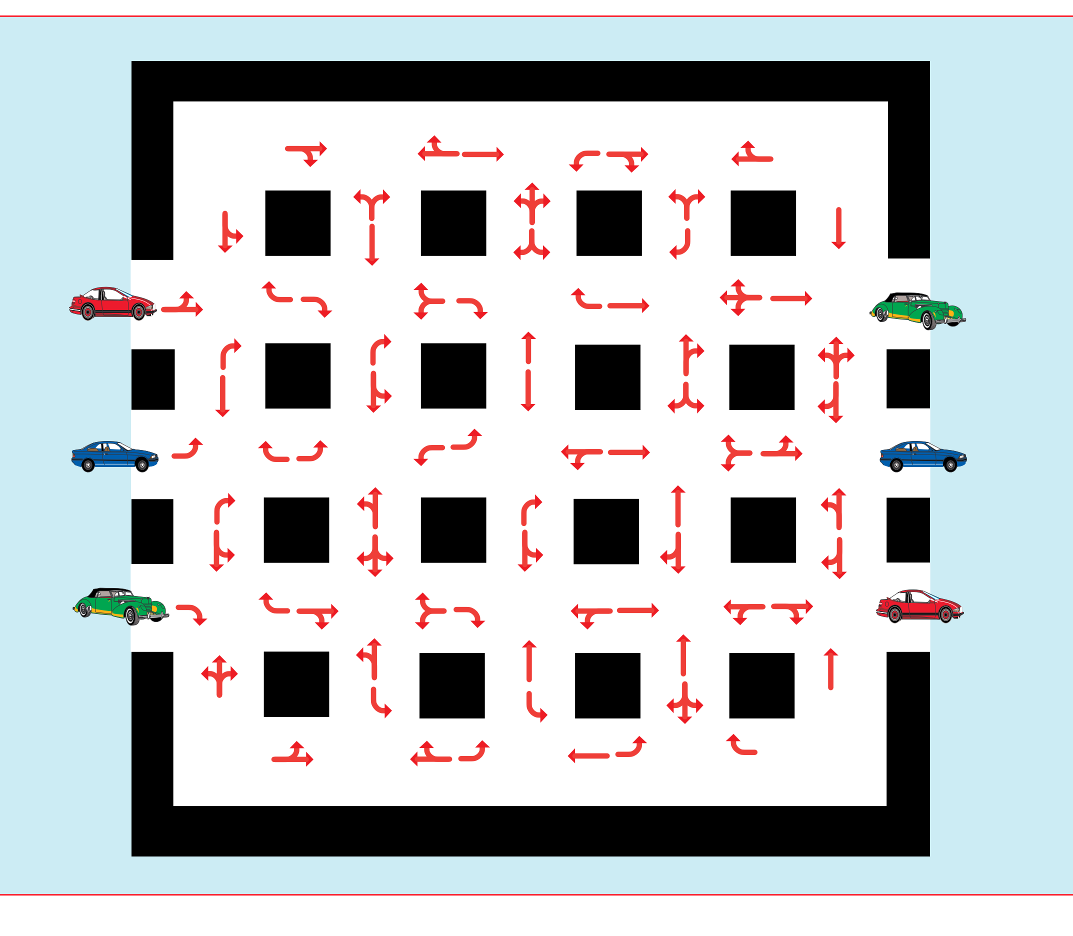
PLAYTHINK 218

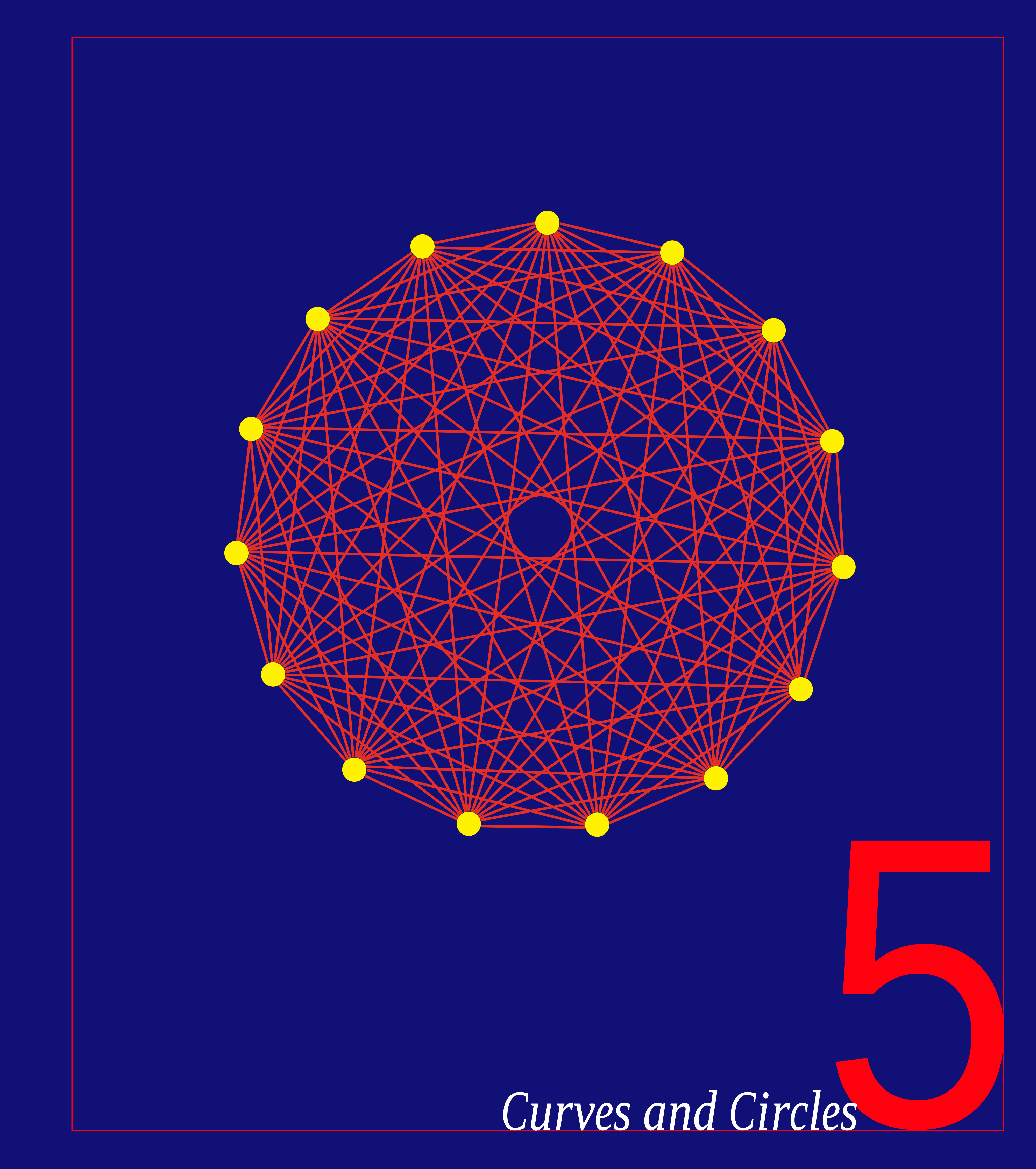
DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

TRAFFIX PUZZLE

Getting across town in Gridlock City can be a nightmare for motorists. It's not the traffic that is the problem—it's the crazy road signs that always seem to keep you from making the turn you want to make. Recently the town authorities made the problem worse by increasing the number of signposts and even inventing some new ones. The result is that at nearly every intersection there is at least one possible turn that is now forbidden. Getting from one side of town to the other now involves some surprising twists and turns.

Can you find the route across town for these three cars? For each color, enter at the left and exit at the right where indicated. And be sure to follow the road signs at each intersection.




Curves and Circles
5

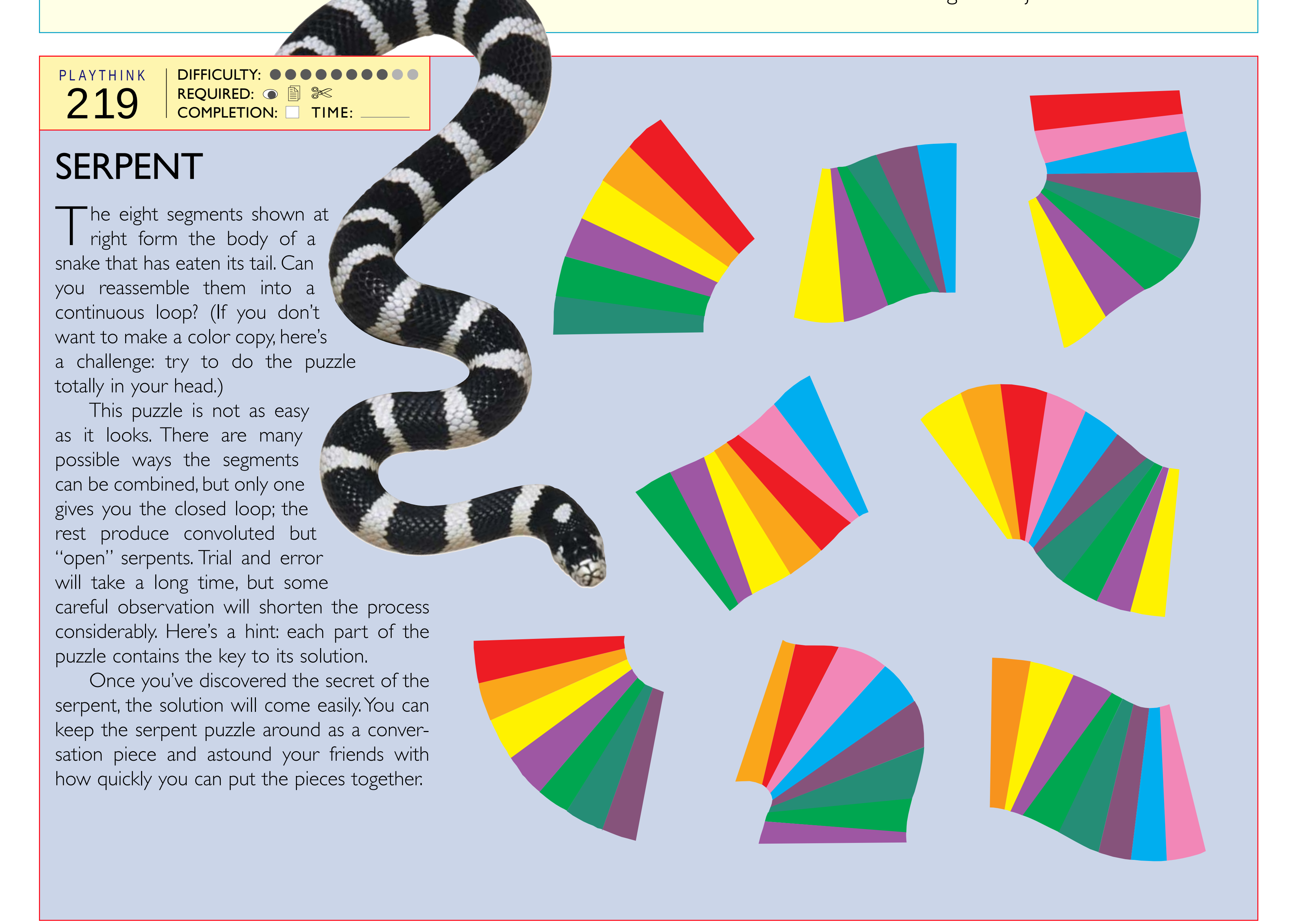
The Curves Around Us

The endless geometric pattern of a river's turns, called meanders, has a fascinating beauty. Over centuries of research, mathematicians and scientists have learned that a meander is the form a river takes in order to use the least amount of work when turning. That gives a new meaning to the phrase "lazy river"!

A thin steel strip can be bent into various configurations—all analogous to river meanders. When held firmly at two points, a strip takes on the shape in which the bend is as uniform as possible. And what is that bend? A curve—a line that continuously bends but has no angles.

Some curves, such as parabolas, are open: the line never returns to its starting point. Others join up with themselves to form closed curves, such as the circle and the ellipse. Some curves are like the helix and twist through three dimensions.

Although there are curves that are quite simple in form, others are so complex that they must be discovered experimentally. Such curves have been found through the study of soap films stretched over wire hoops. The complicated but beautiful curving glass roof over the Olympic stadium in Munich was designed in just that manner.



PLAYTHINK 219

DIFFICULTY: ●●●●●●●●○○
REQUIRED:
COMPLETION: ☐ TIME: ______

SERPENT

The eight segments shown at right form the body of a snake that has eaten its tail. Can you reassemble them into a continuous loop? (If you don't want to make a color copy, here's a challenge: try to do the puzzle totally in your head.)

This puzzle is not as easy as it looks. There are many possible ways the segments can be combined, but only one gives you the closed loop; the rest produce convoluted but "open" serpents. Trial and error will take a long time, but some careful observation will shorten the process considerably. Here's a hint: each part of the puzzle contains the key to its solution.

Once you've discovered the secret of the serpent, the solution will come easily. You can keep the serpent puzzle around as a conversation piece and astound your friends with how quickly you can put the pieces together.

Ouroboros

In ancient Egyptian and Greek mythology, the Ouroboros is a serpent with its tail in its mouth, constantly devouring itself and yet constantly reborn. A gnostic and alchemical symbol, the Ouroboros represents the unity of all things—material and spiritual—and the eternal cycle of destruction and creation.

The nineteenth-century German chemist Friedrich August Kekule von Stradonitz took inspiration from the Ouroboros when he uncovered the true nature of benzene—a molecule made of a ring of carbon atoms.

PLAYTHINK 220 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

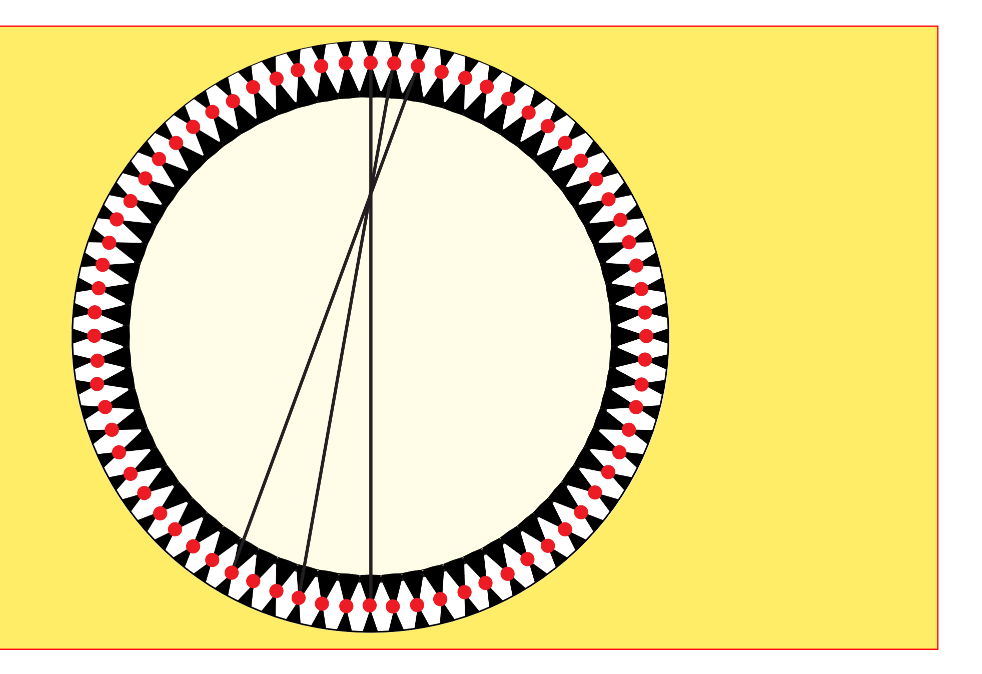
SPIDERWEB GEOMETRY 1

Three diagonals have been drawn across a circle that is divided into equal segments along its circumference. If you continue drawing lines according to the pattern set by the first three lines, what type of pattern will emerge? Will it look much like a spider's web?

PLAYTHINK 221 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

SPIDERWEB GEOMETRY 2

As before, three diagonals have been drawn across a circle that is divided into equal segments along its circumference. If you continue drawing lines according to the pattern set by the first three lines, what type of pattern will emerge?

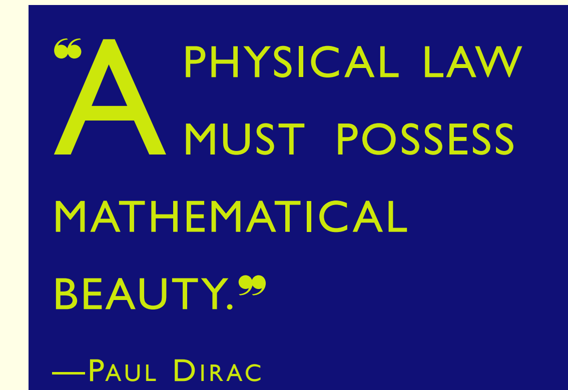


Nature's Basic Plan

Every living thing—every shell, plant or insect—embodies geometry. And little wonder: nature seems to delight in creating a multitude of geometrical shapes. Completely unrelated structures often show a surprising similarity, indicating the presence of both a basic order and basic principles in nature: the circle, the square, the triangle and the spiral.

The basic shapes of nature may be compared to the letters of an alphabet; they can be combined to establish more elaborate forms with new and unique properties. Systems that consist of a minimum number of components that can be combined to yield a great diversity of structural forms are called minimum inventory/ maximum diversity systems.

The best example of such a system is nature itself, where we can find a great number of examples. Consider the endless variety of substances formed by the combinations and permutations of a relatively small number of chemical elements. Or think of music: all the songs and symphonies ever written use a relative handful of notes. It is the way the elements are combined that is the hallmark of creativity.



PLAYTHINK 222 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ______

DRUNKEN SPIDER 1

Imagine drawing many circles, all of which have their centers on the circumference of the base circle, and all of which pass through the base point. What sort of pattern will emerge?

base point

base circle

PLAYTHINK 223 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ______

DRUNKEN SPIDER 2

Imagine drawing many circles, all of which have their centers on the circumference of the base circle and all of which touch the diameter of the base circle. What kind of pattern will emerge? You can use the illustration below to help figure this one out.

diameter

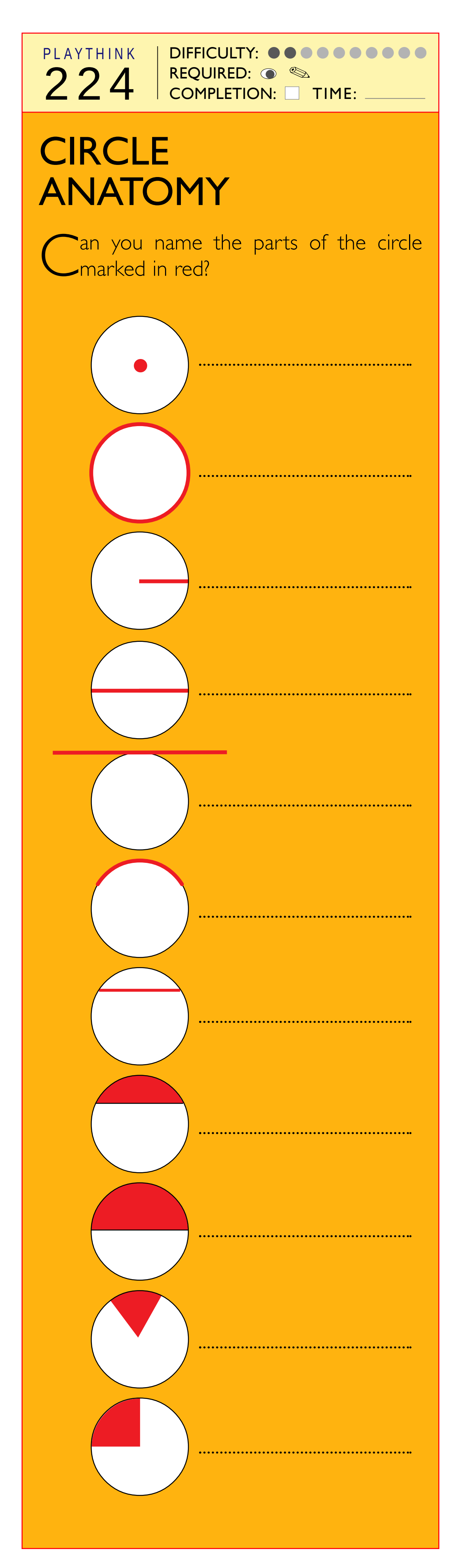
base circle

PLAYTHINK 224

DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

CIRCLE ANATOMY

Can you name the parts of the circle marked in red?



Beauty of the Spheres

Because their curvature is uniform, circles and spheres are considered the most perfect geometric shapes. With no beginning and no end, they symbolize the divine form. With that fact as his only evidence, Aristotle decreed that the paths of the planets must therefore be circular. Nearly 2,000 years later Copernicus, who understood that the sun, and not the earth, is the center of the solar system, uncritically accepted Aristotle's declaration. Even the brilliant German astronomer Johannes Kepler (1571–1630) was burdened by the "truth" of that idea until he discovered that planetary paths are actually elliptical.

Astronomers are not the only ones who have fixated on circles. Early humans certainly saw the roundness of the moon and the ripples made by a stone cast in water. Prehistoric cave paintings display a love of the form; a circle is almost always one of the first figures that a child draws.

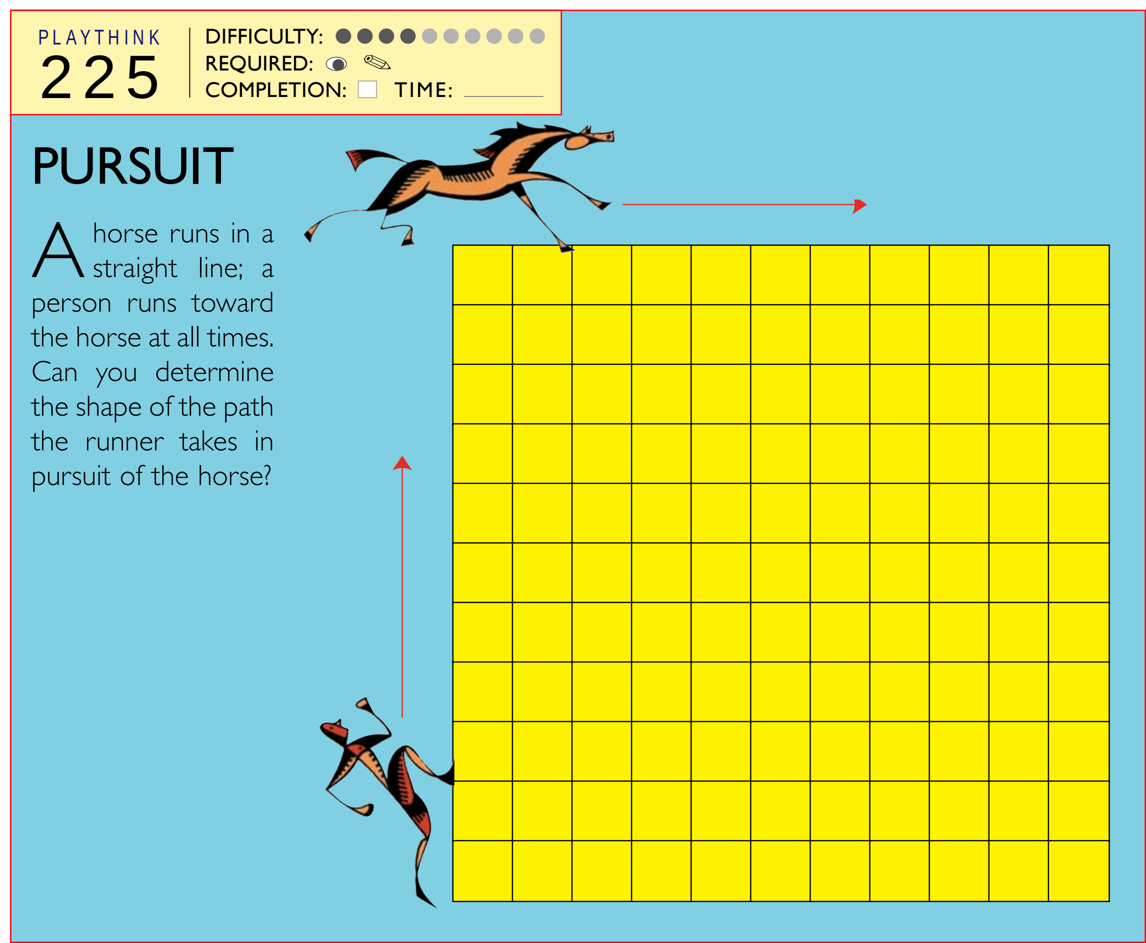
Geometrically speaking, a circle is a plane figure bounded by a curved line (called the circumference) that at every point is equally distant from a point called the center. Like many other complex curves, all circles are similar: no matter how big or how small, they are essentially the same.

PLAYTHINK 225

DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

PURSUIT

A horse runs in a straight line; a person runs toward the horse at all times. Can you determine the shape of the path the runner takes in pursuit of the horse?



The Wheel

Our civilization runs on wheels, but there is little agreement on how the technology was developed. The best available evidence indicates that, unlike the alphabet or agriculture, the wheel was invented only once in human history: in Mesopotamia about 5,000 years ago. The first vehicles probably had four wheels and were derived from platforms that originally were moved on rollers to transport heavy objects. The rollers had to be constantly picked up from the rear of the platform and moved to the front end. Notching the underside of such a platform to keep the rollers in place eliminated the need to cycle the rollers from back to front. Eventually rollers held in place evolved into the wheel and axle. The invention of proper wheels had to wait for the discovery of metals, with which more useful tools could be made. (Copper came into use about 4000 B.C. and bronze some time before 2500 B.C.)

The introduction of the wheel represented an event of enormous importance in technical history. It took thousands of years for humans to conceive the idea of a form of motion not evident in their immediate surroundings. After all, no animal uses wheels for moving about. The discovery of the wheel required a capacity for abstract thinking and the ability to pass from the object itself to the idea of it—from phenomenon to theory.

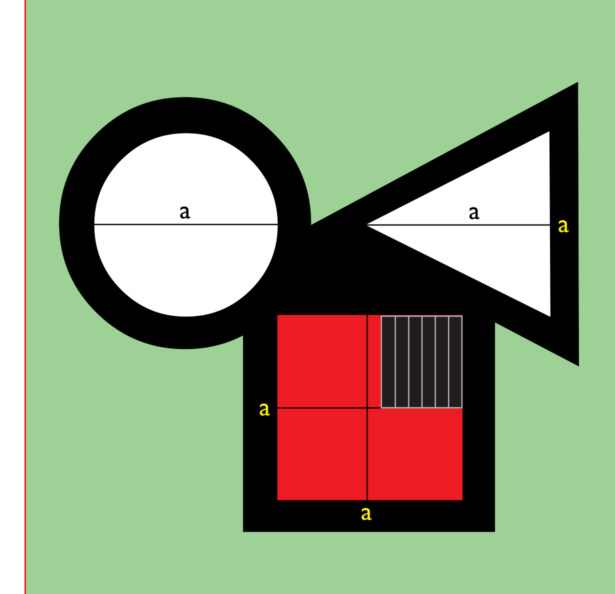
Once this problem was solved, the wheel remained fairly static. The only essential difference between the first wheel of Mesopotamia and the contemporary wheel is the widespread use of pneumatic tires.

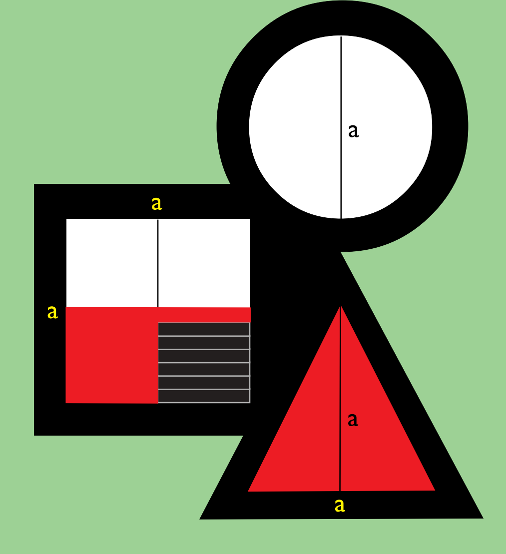
PLAYTHINK 226 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

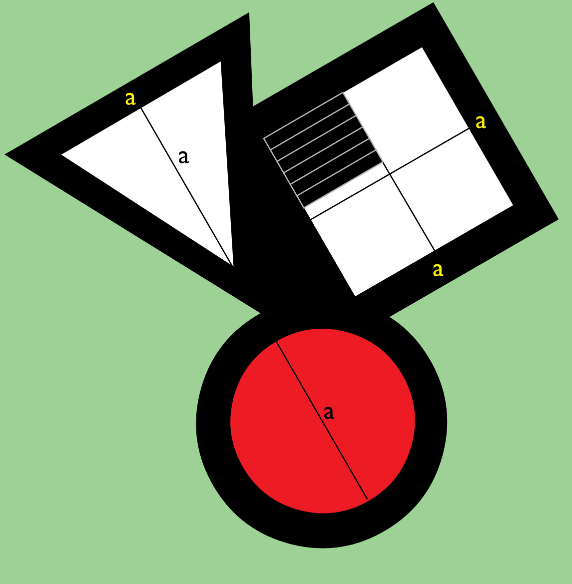
CIRCLE-SQUARE-TRIANGLE AREA

A three-chambered vessel for holding liquids is illustrated here. As the vessel rotates, the red fluid moves from chamber to chamber, filling one of them completely at each turn.

Based on this illustration, can you work out the relationship between a circle, a square and a triangle all possessing the same diameter, height and sides? (Don't forget: the area of a circle is r2π.) Also, can this demonstration give you a way to evaluate the number π? (See page 96.)







PLAYTHINK 227 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ____

AROUND

There are many classic circle dissection puzzles, such as the old circular tangram, parts of which are combined to make many different patterns and figures.

Our circle dissection puzzle is much more subtle. It consists of ten parts that when combined will form a perfect circle. The subtlety lies in the fact that the circle was dissected using a compass set at the radius of the circle itself—so that every curve is identical.

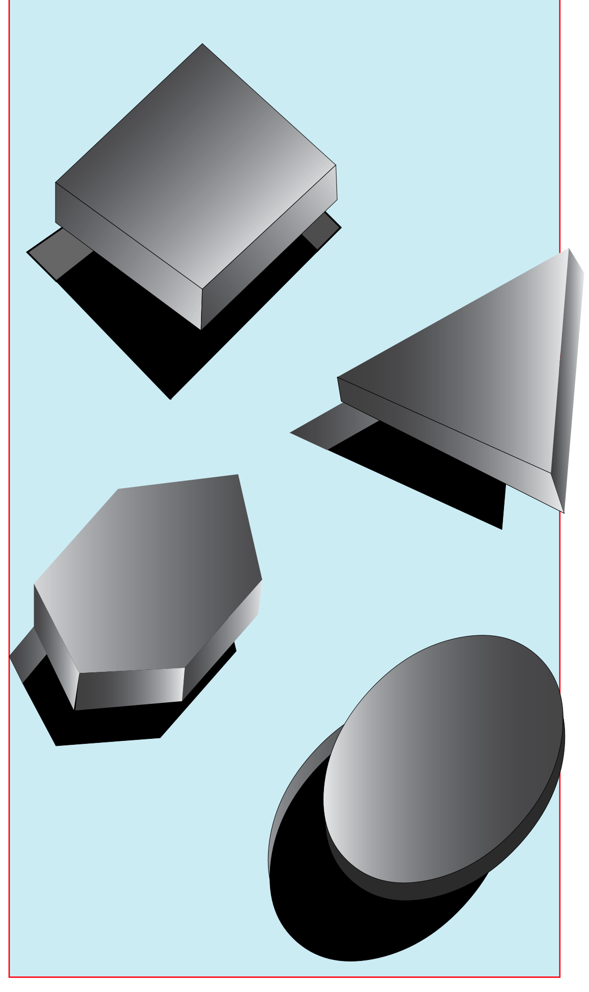
How long will it take you to reassemble the circle?



PLAYTHINK 228 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ____

WHY ROUND?

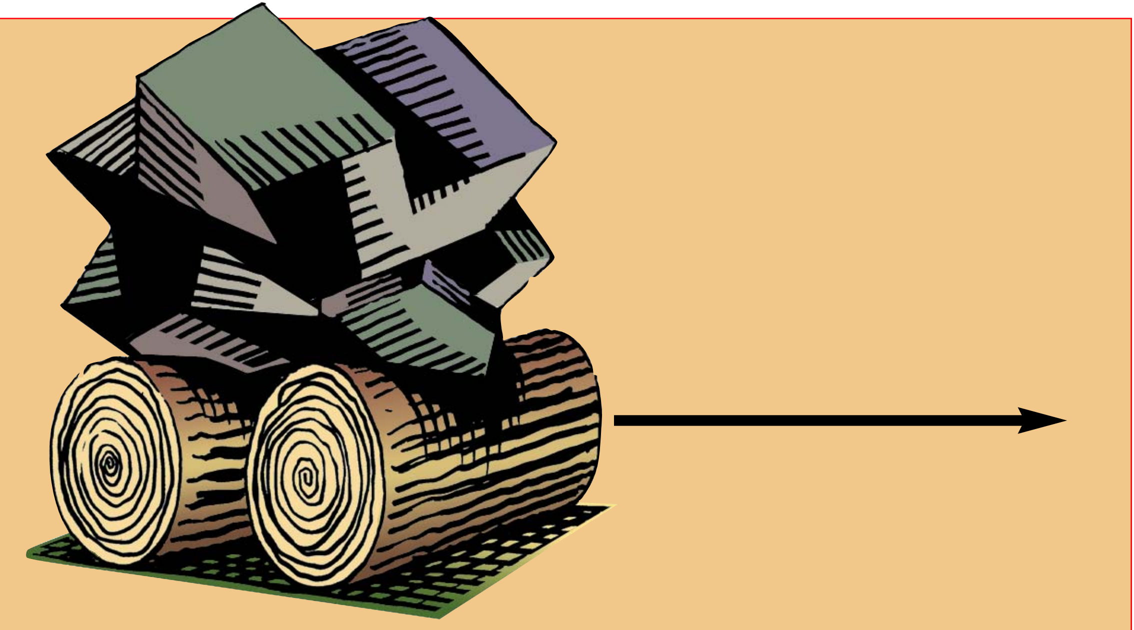
Why are manhole covers round? Can you find three reasons why round is the best possible shape? And the answer "Because manholes are round" doesn't count!



PLAYTHINK 229 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ____

ROLLING STONE

People once moved heavy weights by means of rollers made of logs. The circumference of the two identical logs shown here is exactly 1 meter. If the logs roll one whole turn, how far will the weight be carried forward?



The Number π: 3.14159265358979323846264338327950288...

The ratio between the circumference of a circle and its diameter is one of the most fascinating numbers in mathematics. The Babylonians gave the ratio as simply 3, as does the Bible, though other ancient mathematicians strove for greater precision. The Egyptians, for instance, arrived at a ratio of 3.16 (which has an accuracy of 1 percent) as early as 1500 B.C. In 225 B.C., the Greek mathematician Archimedes inscribed and circumscribed a circle with a ninety-six-sided polygon and found that the ratio lies between $3\frac{1}{7}$ and $3\frac{10}{71}$. Ptolemy in A.D. 150 found a value of 3.1416, which is sufficiently accurate for most practical purposes.

These days π (the Greek letter pi), as that ratio is known, has been calculated to millions of decimal places. Why should anyone bother to carry π to such fantastic lengths through the ages, let alone today? There are three good reasons:

- π is there. Its mere existence, not to mention its great fame, is cause enough for mathematicians to tackle the problem.
- Such calculations often have useful spin-offs. Today the calculation of π provides a way to test new computers and train programmers.
- The more digits of π are known, the more mathematicians hope to answer a major unsolved problem of number theory: Is the sequence of digits behind the decimal place completely random? Thus far there seems to be no hidden pattern, but π does contain an endless variety of remarkable patterns that are the result of pure chance. For example, starting with the 710,000th decimal place, π begins to stutter 3333333. Similar runs occur of every digit except 2 and 4.

The ratio was named π in 1737 by none other than Leonhard Euler (see page 71). In 1882 the German mathematician Ferdinand von Lindenmann proved that π is a transcendental number; that is, neither π itself nor any of its whole powers can be expressed as a simple fraction. No fraction, with integers above and below the line, can exactly equal π, and no straight line of length π can be constructed with compass and ruler alone.

The importance of π lies not simply in its role as a geometric ratio; π appears in the formulas engineers use to calculate the force of magnetic fields and physicists use to describe the structure of space and time.

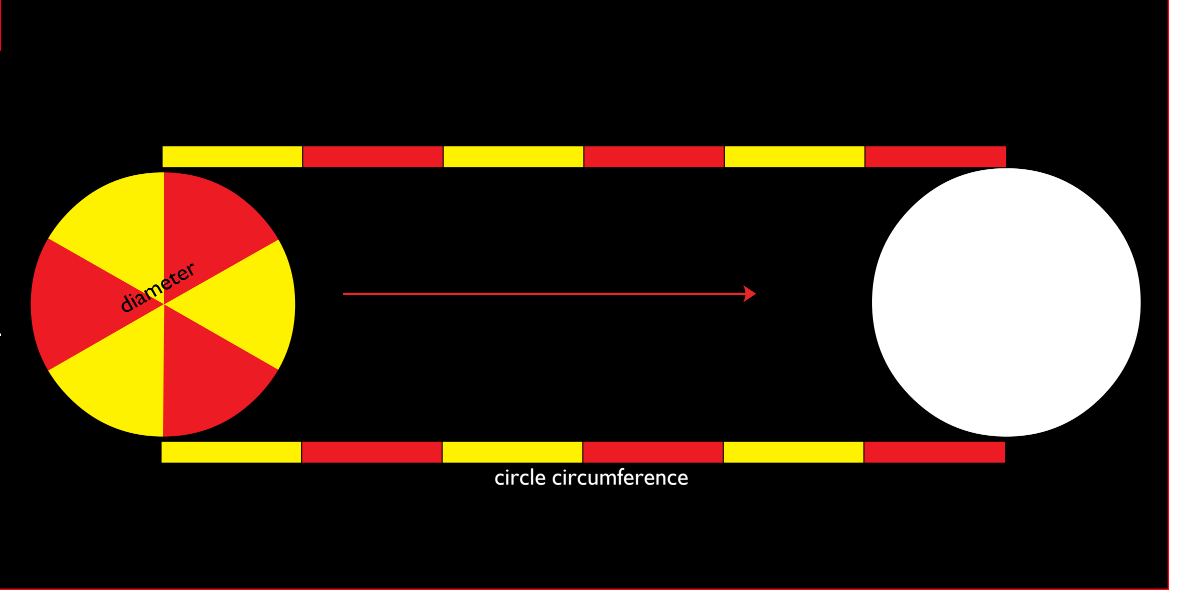
PLAYTHINK 230

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

CIRCLE CIRCUMFERENCE AND THE NUMBER π

Roll a circle one full turn along a line. The line is equal to the circumference of the circle. Then imagine rolling more circles of various sizes along a line always equal to the circumference of the circle.

What can you tell about the relationship between the circumference and the diameter of a given circle? Is it the same for all circles?



Squaring the Circle

One of the most famous geometric problems of antiquity was that of squaring the circle. The problem was to construct a square with an area equal to that of a given circle, using only a straightedge and a compass. The ancient Greek mathematicians, with their great geometrical skill, tried hard but were unable to solve such an apparently simple problem. Ironically, in their trials in squaring the circle, the ancient Greeks succeeded in squaring many more complex curves, which made for exciting discoveries and theorems.

For more than 2,000 years mathematicians and amateurs devoted untold hours to solving this problem. Ferdinand von Lindenmann's proof that π is a transcendental number—and thus cannot be constructed with a compass and ruler—finally made official what all mathematicians who have tackled the problem have declared in frustration: Squaring the circle is impossible!

PLAYTHINK 231

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

CRESCENTS OF HIPPOCRATES

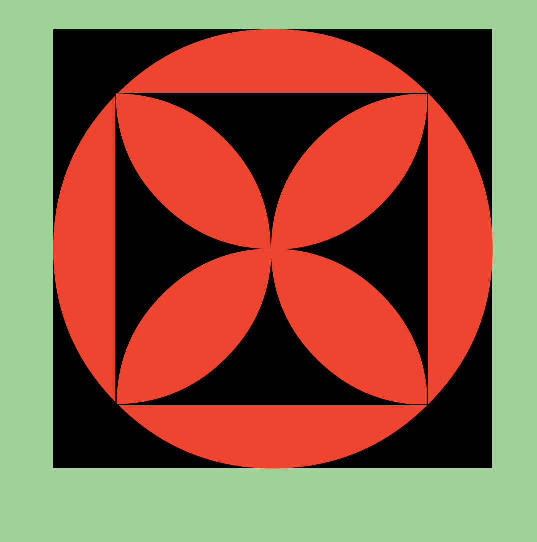
The ancient Greek geometer Hippocrates of Chios discovered this problem while trying to square the circle. He constructed overlapping semicircles on the sides of a right triangle, as shown below. Can you determine the total area of the two red crescents?

PLAYTHINK 232

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

CIRCLE IN THE SQUARE

Which is greater, the sum of the black areas or the sum of the red areas?

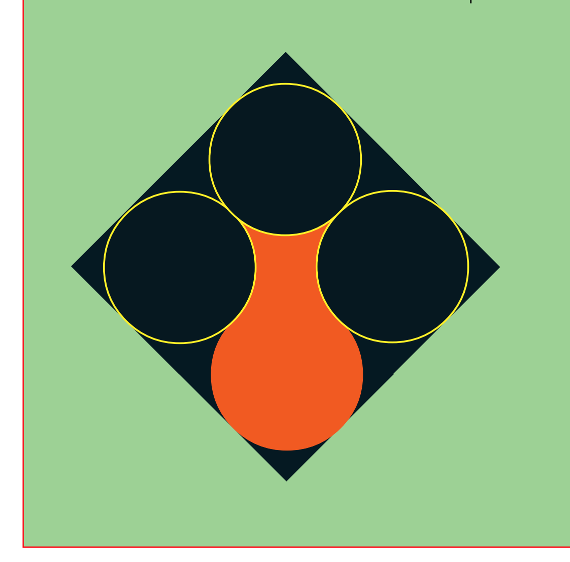


PLAYTHINK 233

DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

SQUARE VASE

Can you divide the red vase and reassemble the parts to form a perfect square? This is possible in two different ways, one that divides the vase into three parts and another that divides it into four parts.



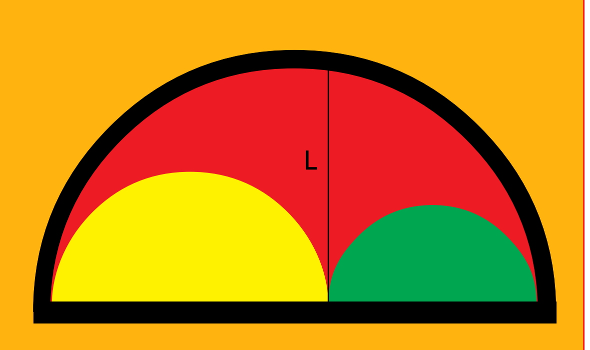
PLAYTHINK 234

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

SICKLE OF ARCHIMEDES

A circle is divided in half along its diameter, and two additional semicircles are constructed along that diameter, as shown here. A line (L) is drawn from the point on the diameter where the two circles meet and extended perpendicularly from the diameter to the circumference of the large semicircle.

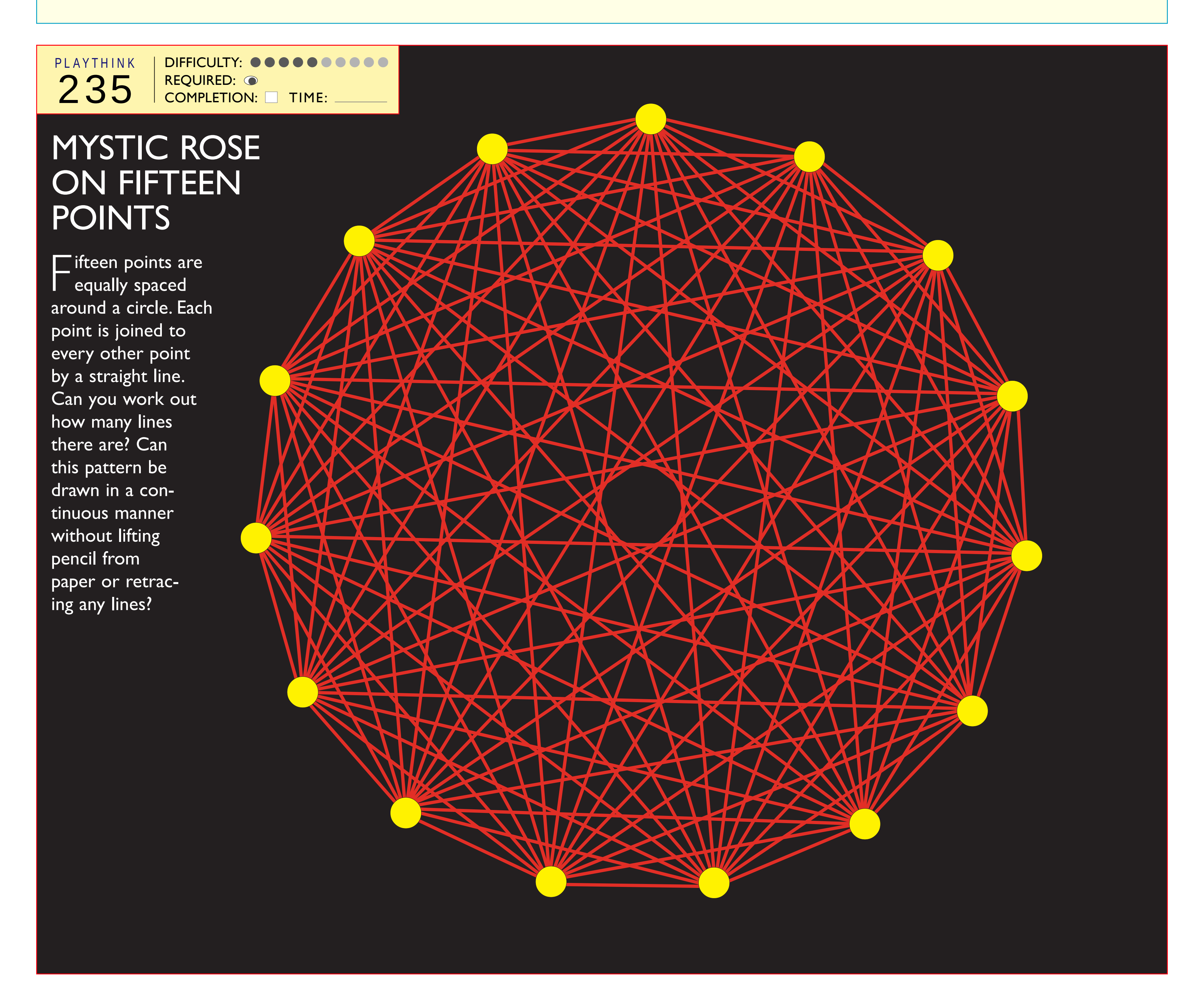
The area of the large semicircle that is not covered by the smaller semicircles has the form of a sickle, an ancient tool used for harvesting grain. Can you guess what the area of the sickle might be?



Mystic Roses

To create a mystic rose, a set of points is evenly spaced along the circumference of a circle, and each point is connected to every other point by a straight line. A small number of points leads to a relatively simple rose. As the number of points increases, the complexity rises substantially.

In 1809 the French mathematician Louis Poinsot asked what was the minimum number of continuous lines needed to draw mystic roses of various sizes. (A continuous line is drawn without lifting the pen from the paper or retracing any of the lines.) A three-point mystic rose can be drawn with one continuous line, but it is impossible to draw a four-point rose in one continuous line. Two continuous lines will always be needed.



PLAYTHINK 235

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

MYSTIC ROSE ON FIFTEEN POINTS

Fifteen points are equally spaced around a circle. Each point is joined to every other point by a straight line. Can you work out how many lines there are? Can this pattern be drawn in a continuous manner without lifting pencil from paper or retracing any lines?

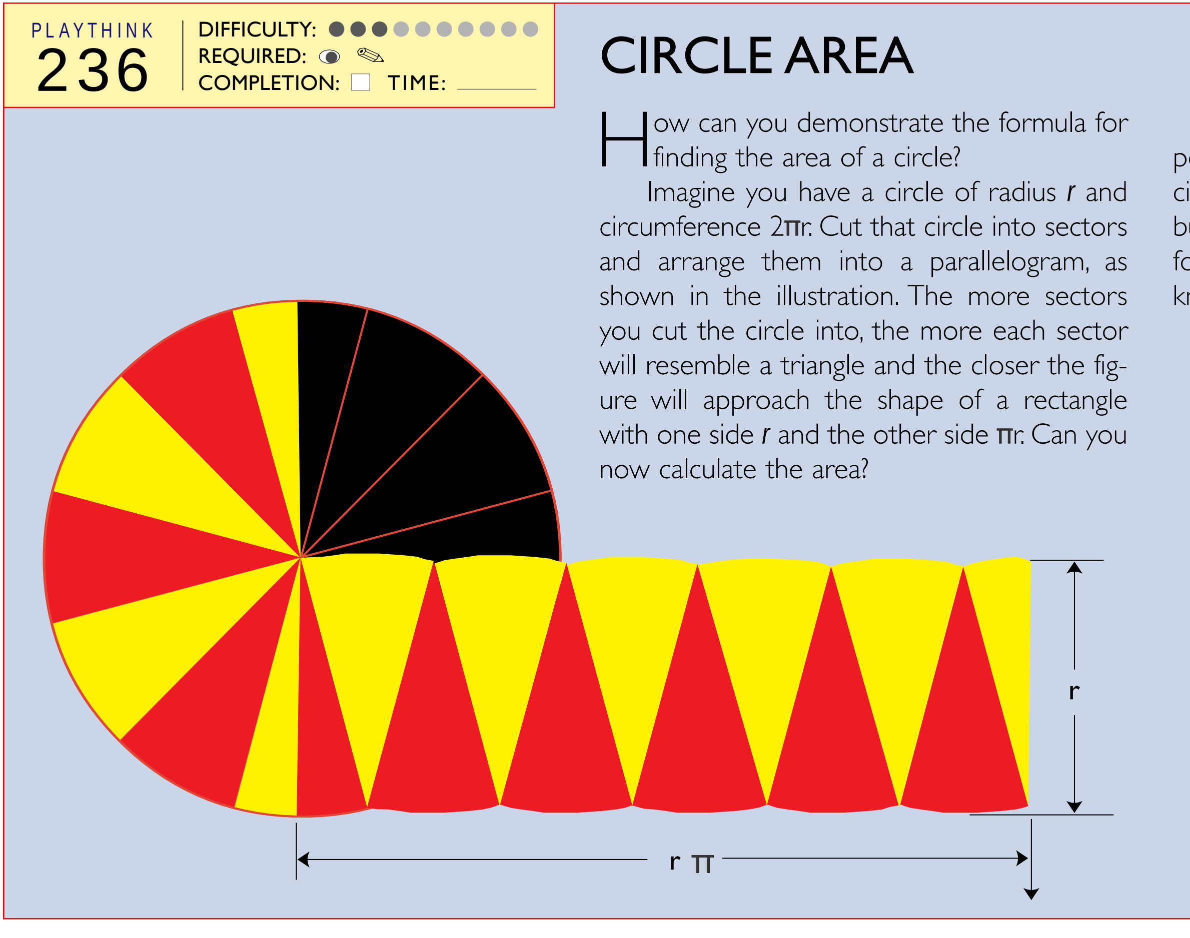
PLAYTHINK 236 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ______

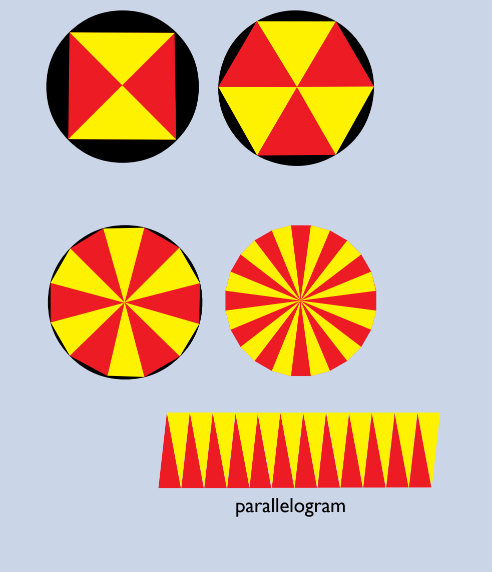
CIRCLE AREA

How can you demonstrate the formula for finding the area of a circle?

Imagine you have a circle of radius *r* and circumference 2πr. Cut that circle into sectors and arrange them into a parallelogram, as shown in the illustration. The more sectors you cut the circle into, the more each sector will resemble a triangle and the closer the figure will approach the shape of a rectangle with one side *r* and the other side πr. Can you now calculate the area?

"In the limit," as mathematicians say, the polygons inscribed in the circle become the circle itself. That limit can never be reached, but approaching it as nearly as we like is the foundation of the mathematical discipline known as calculus.



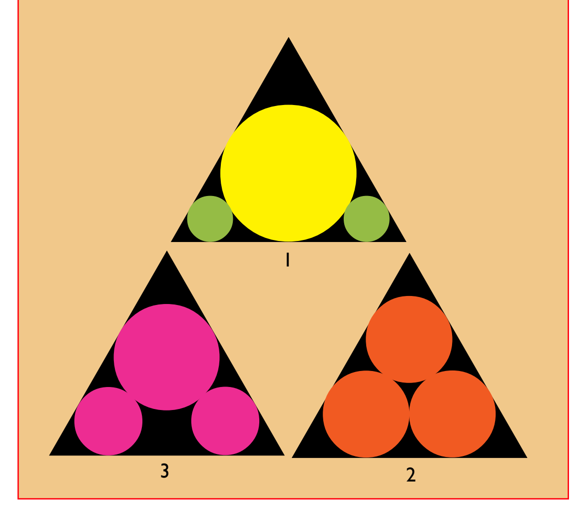


PLAYTHINK 237 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ______

THREE CIRCLES

Three identical equilateral triangles are inscribed with circles, as shown. Which case provides the circles with the largest total area?

1. An incircle (the largest circle that can be inscribed on a triangle) and two smaller circles **2.** Three identical circles of the largest size possible **3.** One big circle and two smaller ones

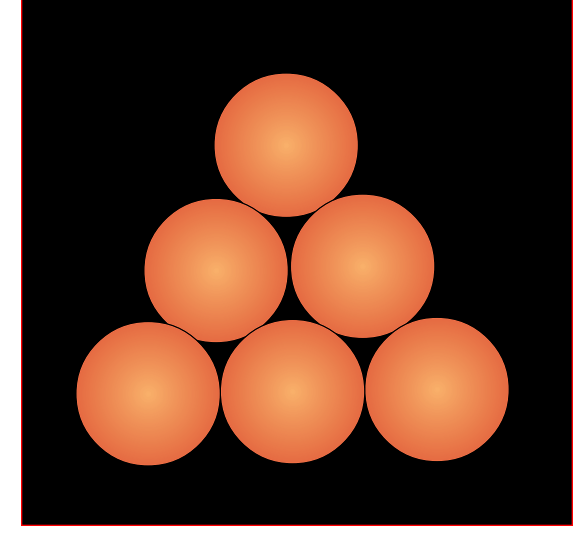


PLAYTHINK 238 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ______

COIN MATTERS

You must rearrange the pyramid of six coins into a hexagon that possesses a hole large enough for a seventh coin. Can you do this in just five moves?

A move consists of sliding a single coin along a flat surface to a new position so that it is in contact with at least two other coins. When moving a coin, you cannot move or jostle any other coin.

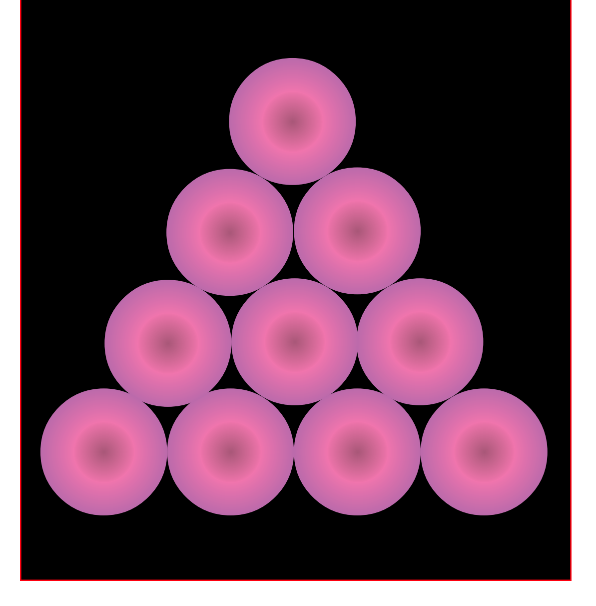


PLAYTHINK 239 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ______

UPSIDE-DOWN COINS

The object is to turn the pyramid of ten coins upside-down, moving one coin at a time to a new position in which it touches two or more coins.

It is easy to do this in six moves. Can you do it in three?



PLAYTHINK 240

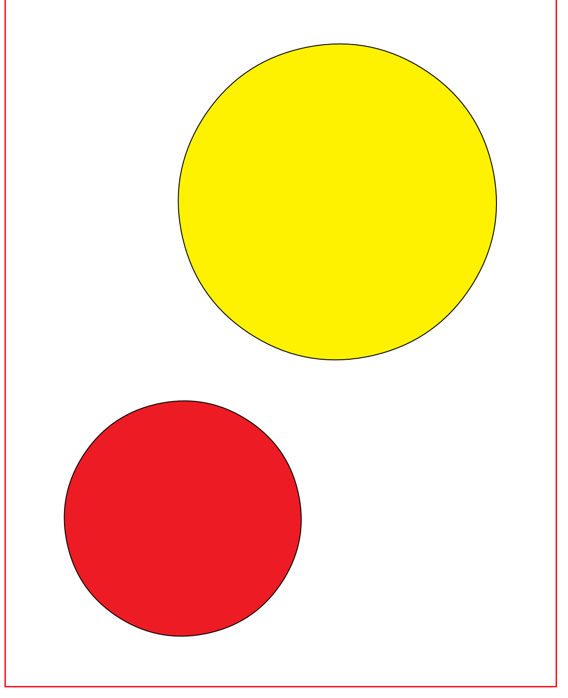
DIFFICULTY: ●●●●●○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

CIRCLES AND TANGENTS

How many essentially different ways can you find to arrange two circles of unequal size on a plane?

If a tangent is a straight line touching a curve at a single point, and a common tangent is a straight line tangent to two circles, can you find the total number of common tangents to the two circles for all the arrangements of two circles?

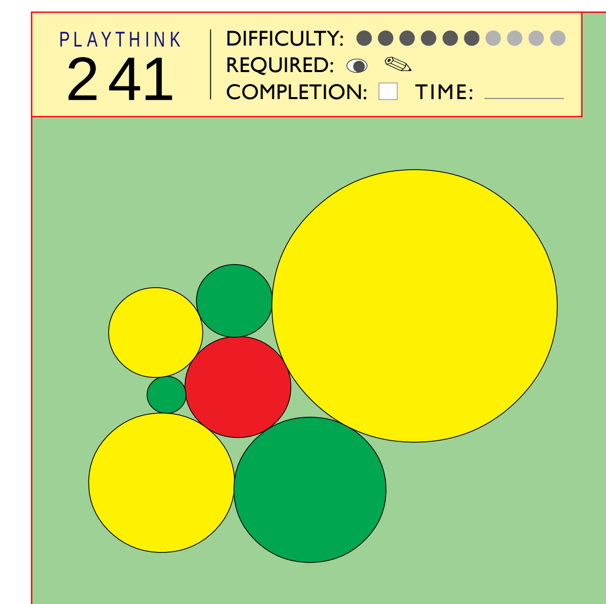
Would it make any difference if the circles were the same size?



PLAYTHINK 241

DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

SEVEN CIRCLES PROBLEM



Start with any circle. (Use the red one in the diagram as a reference.) Add six circles around the circumference of the circle so that each of the new circles touches two other new circles and the red circle. Imagine that three of the circles (yellow in the diagram) become larger and larger and the green circles become smaller and smaller, though the green and yellow still remain in contact. Imagine that the yellow circles become so large that they even intersect. What will be the ultimate outcome?

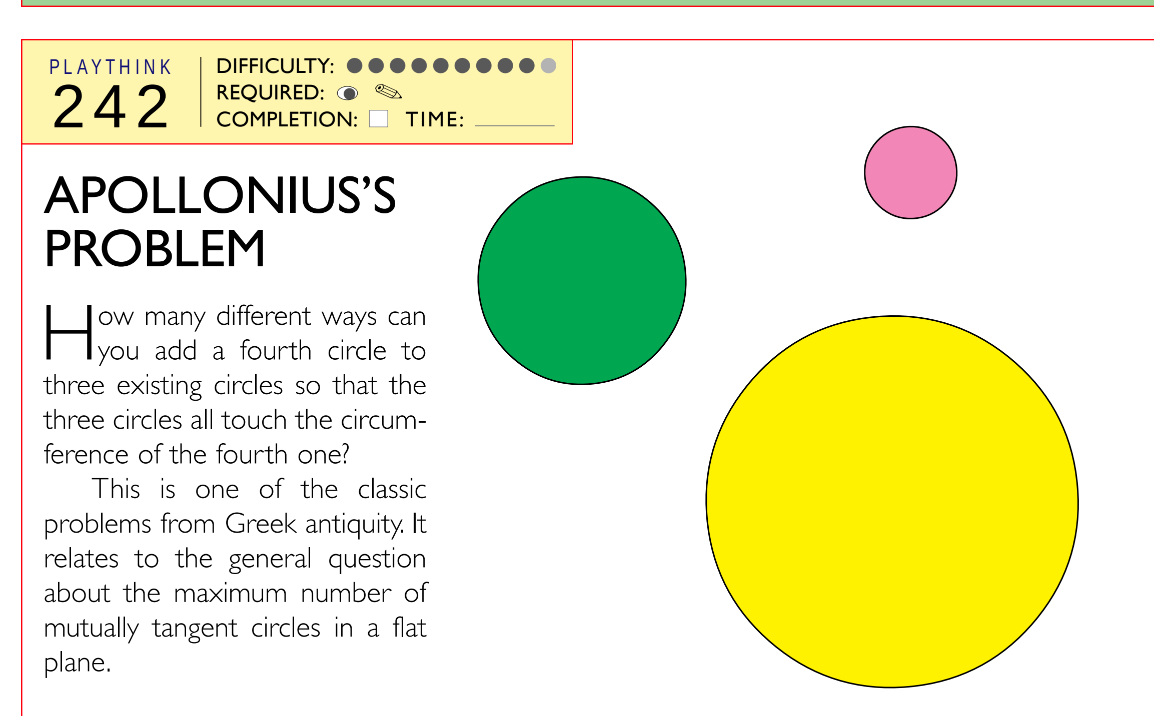
PLAYTHINK 242

DIFFICULTY: ●●●●●●●●●○
REQUIRED:
COMPLETION: ☐ TIME: ______

APOLLONIUS'S PROBLEM

How many different ways can you add a fourth circle to three existing circles so that the three circles all touch the circumference of the fourth one?

This is one of the classic problems from Greek antiquity. It relates to the general question about the maximum number of mutually tangent circles in a flat plane.



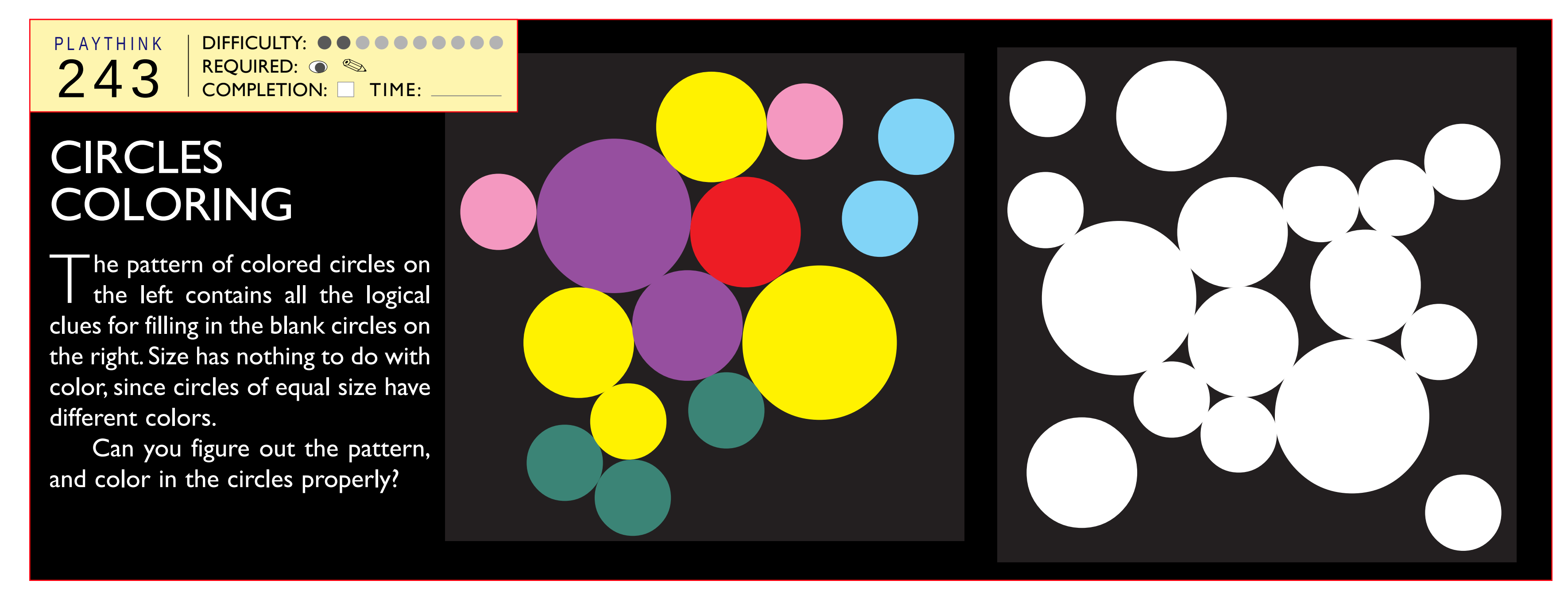
PLAYTHINK 243

DIFFICULTY: ●●○○○○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

CIRCLES COLORING

The pattern of colored circles on the left contains all the logical clues for filling in the blank circles on the right. Size has nothing to do with color, since circles of equal size have different colors.

Can you figure out the pattern, and color in the circles properly?

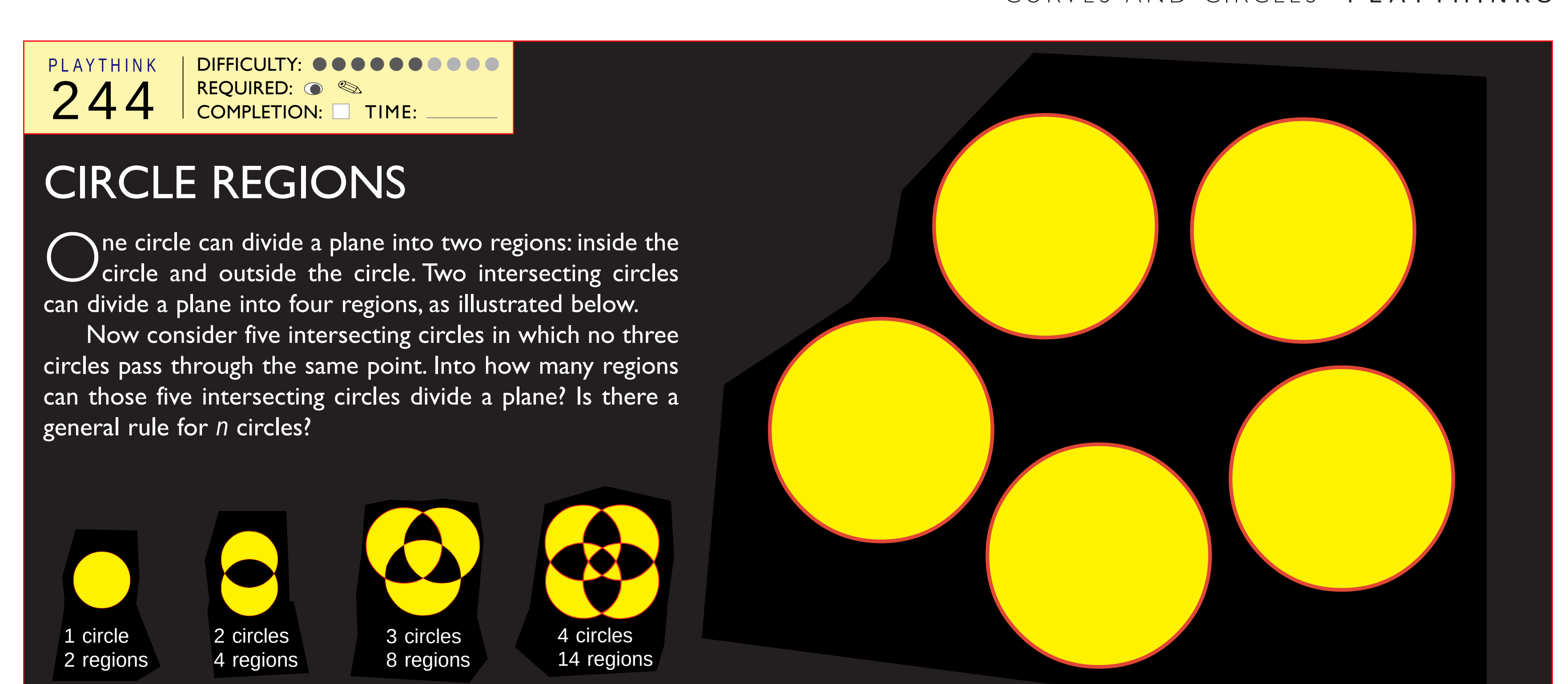


PLAYTHINK 244 | DIFFICULTY: ●●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

CIRCLE REGIONS

One circle can divide a plane into two regions: inside the circle and outside the circle. Two intersecting circles can divide a plane into four regions, as illustrated below.

Now consider five intersecting circles in which no three circles pass through the same point. Into how many regions can those five intersecting circles divide a plane? Is there a general rule for *n* circles?



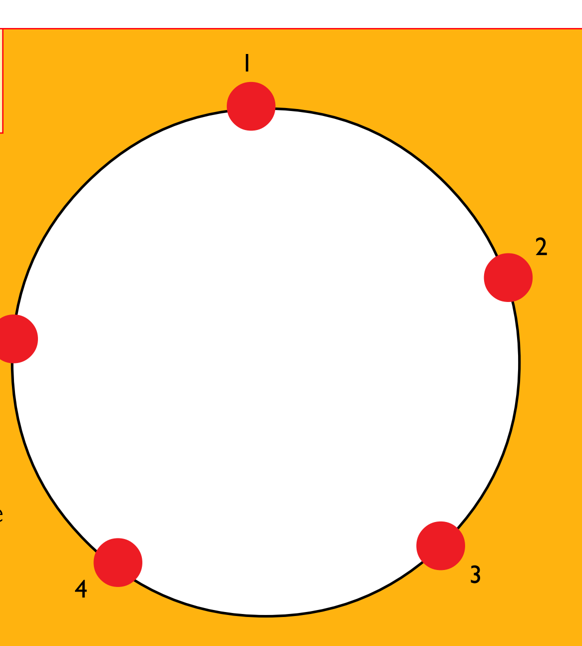
PLAYTHINK 245 | DIFFICULTY: ●●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

POLYGONS IN A CIRCLE

Five points are randomly distributed on the circumference of a circle. From any of those points, a continuous line may be drawn that connects the other points in the polygon before returning to the original point.

How many different polygons can be drawn with these five points?

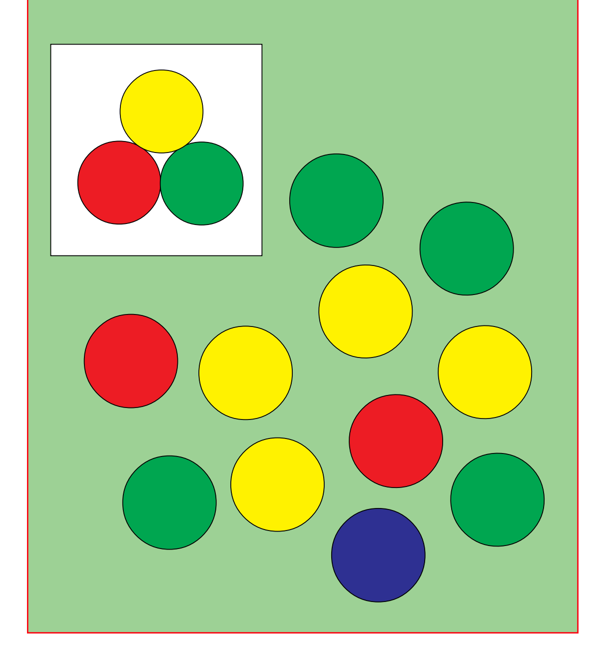




PLAYTHINK 246 | DIFFICULTY: ●●●●●●●●●●○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

TOUCHING CIRCLES 2

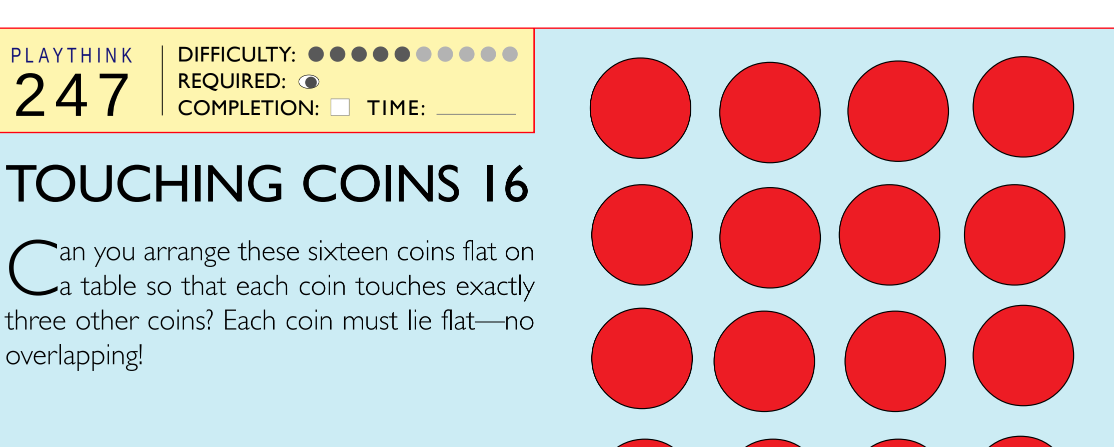
Three differently colored circles of identical size can be arranged in a way so that all three are in contact but no two circles of the same color touch. (See the inset for an example of this.) Can you arrange identical circles in a way so that four colors are needed to avoid contact between two circles of the same color? What is the smallest number of circles needed for this to occur?



PLAYTHINK 247 | DIFFICULTY: ●●●●●○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

TOUCHING COINS 16

Can you arrange these sixteen coins flat on a table so that each coin touches exactly three other coins? Each coin must lie flat—no overlapping!



PLAYTHINK 248

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

CIRCLE RELATIONSHIP

One circle is circumscribed around a square; another circle is inscribed within the same square. How are the areas of the two circles related?

PLAYTHINK 249

DIFFICULTY: ●●○○○○○○○○
REQUIRED: 👁 ✂
COMPLETION: ☐ TIME: ______

TUBE ILLUSION

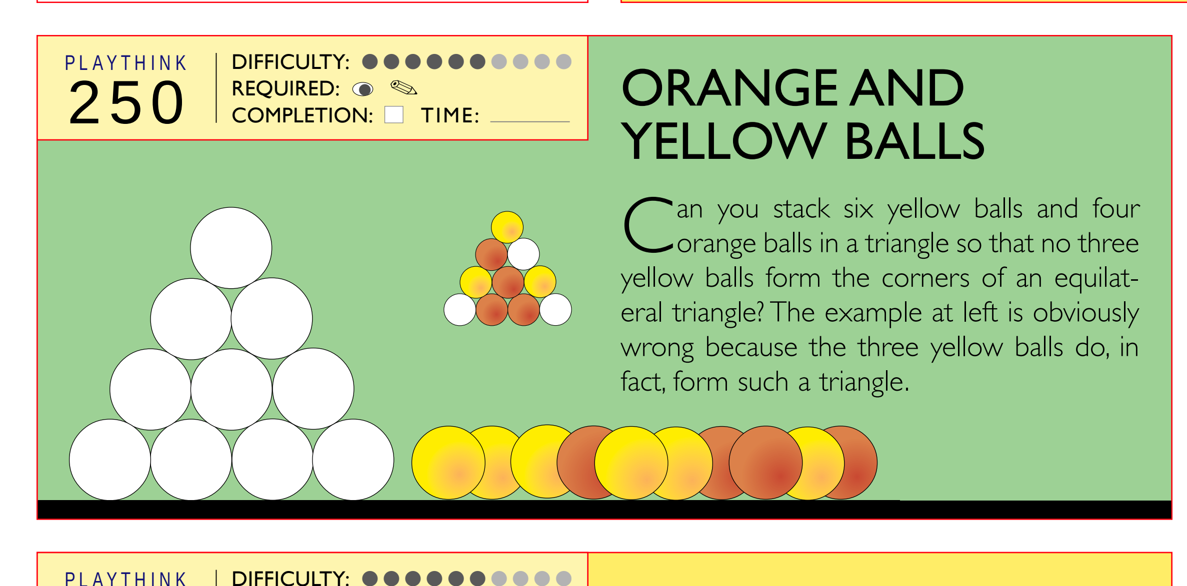
What will you see if you look through the hole of a cardboard tube such as the one in the illustration?

PLAYTHINK 250

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

ORANGE AND YELLOW BALLS

Can you stack six yellow balls and four orange balls in a triangle so that no three yellow balls form the corners of an equilateral triangle? The example at left is obviously wrong because the three yellow balls do, in fact, form such a triangle.



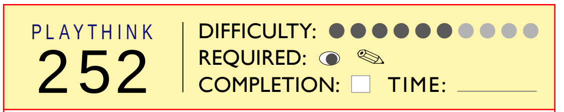
PLAYTHINK 252

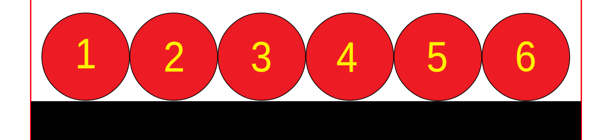
DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

JUMPING COINS

You must stack the six numbered coins into two piles of three coins each. But in order to do so, you must move each coin by jumping over exactly three other coins. As an example of an allowable first move, coin 2 can jump over 3, 4 and 5 to stack on coin 6.

Can you stack the coins in five moves or less?

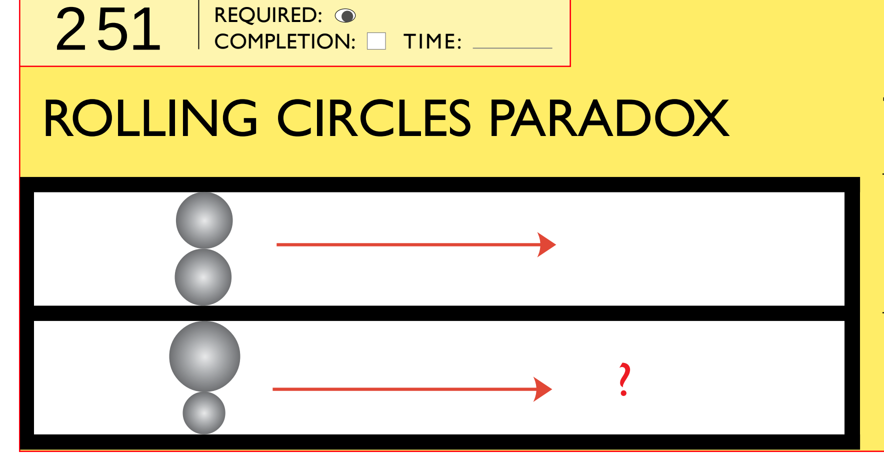




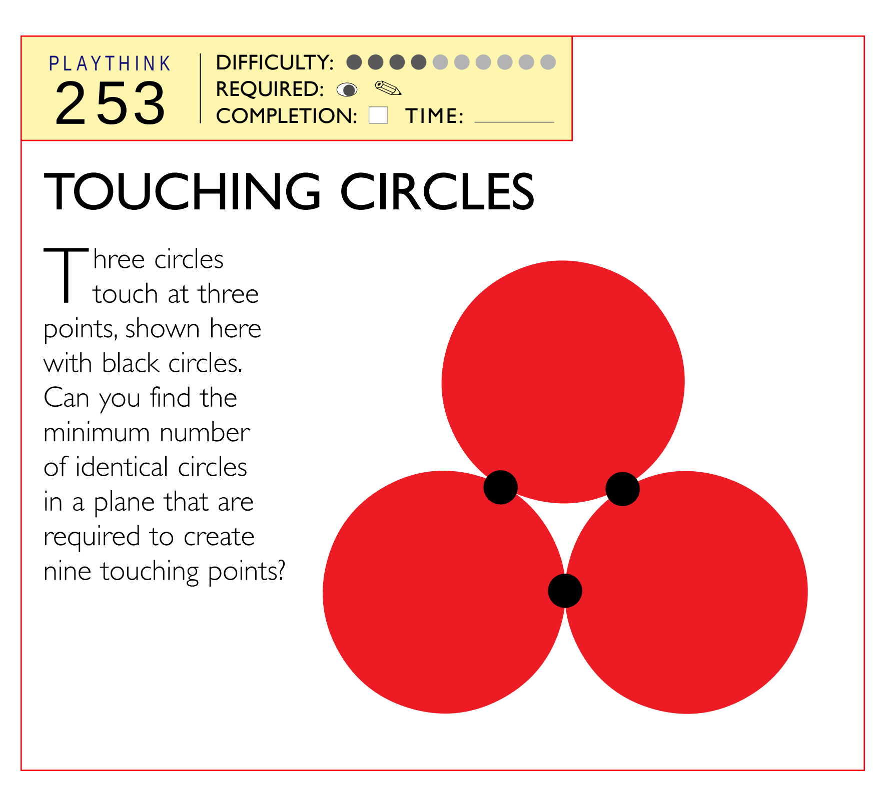
PLAYTHINK 251

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

ROLLING CIRCLES PARADOX



Two identical rollers between two parallel rails can roll and retain their relative positions, one over the other. Would that be possible if one roller were twice as big as the other?

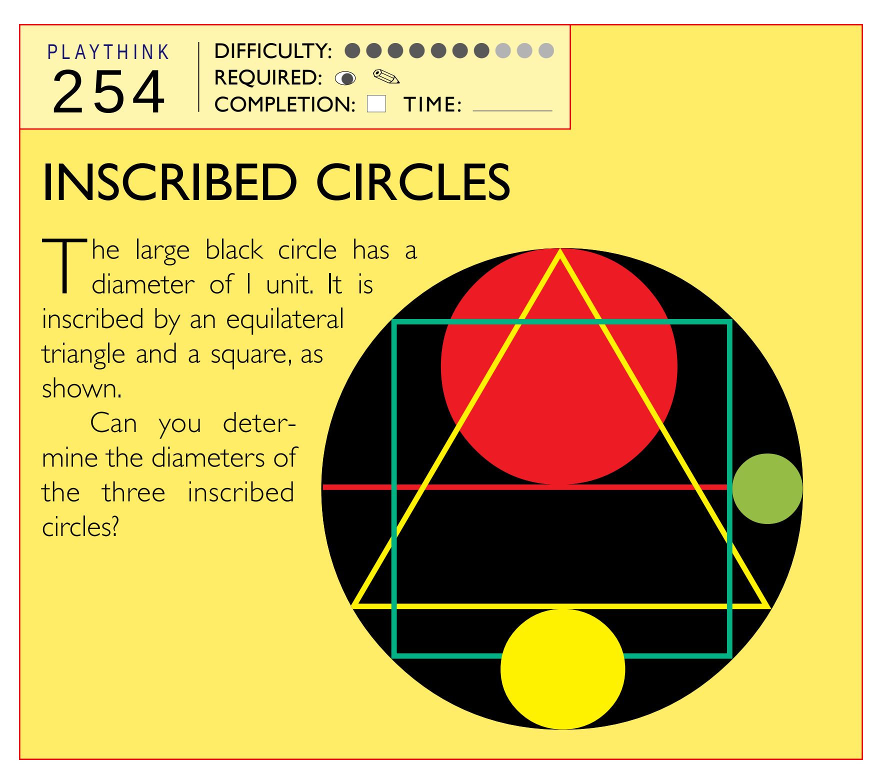


PLAYTHINK 253

DIFFICULTY: ●●●●○○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

TOUCHING CIRCLES

Three circles touch at three points, shown here with black circles. Can you find the minimum number of identical circles in a plane that are required to create nine touching points?



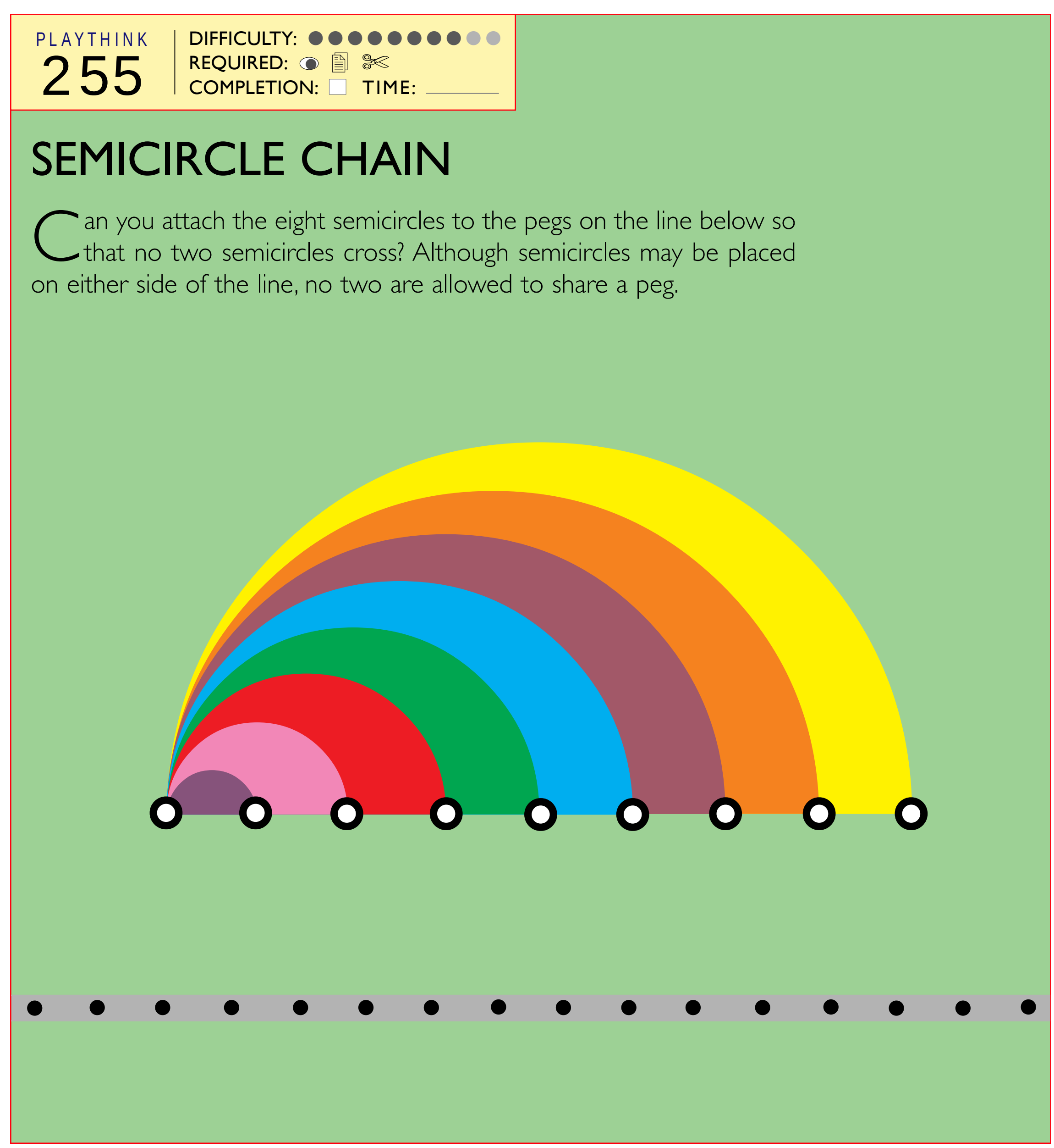
PLAYTHINK 254

DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

INSCRIBED CIRCLES

The large black circle has a diameter of 1 unit. It is inscribed by an equilateral triangle and a square, as shown.

Can you determine the diameters of the three inscribed circles?

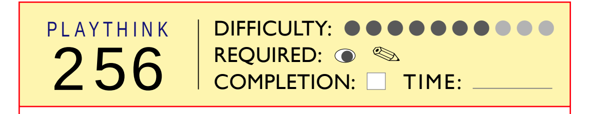


PLAYTHINK 255

DIFFICULTY: ●●●●●●●●○○
REQUIRED:
COMPLETION: ☐ TIME: ______

SEMICIRCLE CHAIN

Can you attach the eight semicircles to the pegs on the line below so that no two semicircles cross? Although semicircles may be placed on either side of the line, no two are allowed to share a peg.

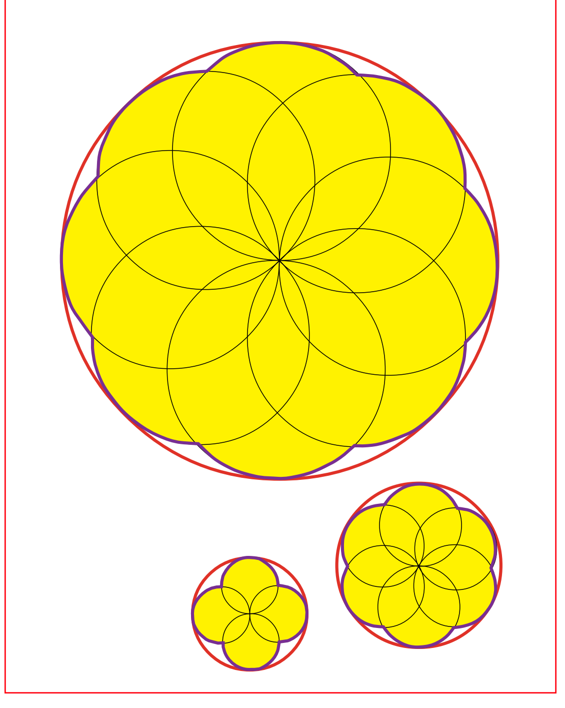


PLAYTHINK 256

DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

ROSETTE CIRCUMFERENCE

When a number of circles of the same radius are drawn through a point, the result is a shape called a rosette. Can you tell which is greater, the perimeter of a rosette formed by circles of radius equal to 1 unit, or the circumference of a larger circle with a radius equal to 2? The illustration below may be helpful.

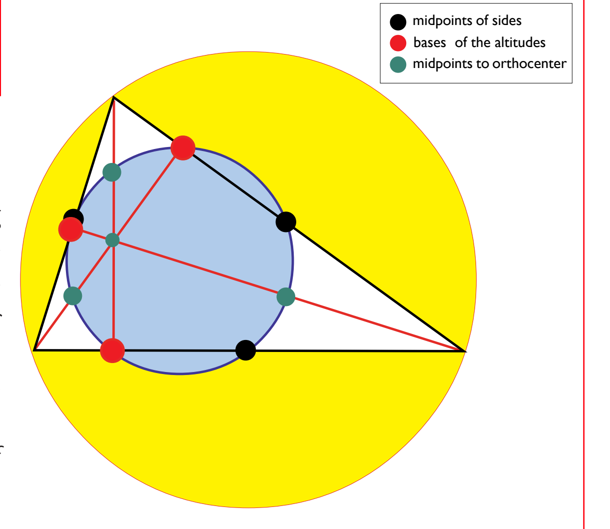


PLAYTHINK 257 | DIFFICULTY: ●●●●●●●●●○ | REQUIRED: | COMPLETION: ☐ TIME: ______

NINE-POINT CIRCLE

The white triangle has some interesting properties: the midpoints of the sides, the bases of the altitudes and the midpoints of the line joining the vertices to the orthocenter (the common intersection of all three altitudes of the triangle) all line up on the circumference of a circle.

Does every triangle form that sort of nine-point circle?



PLAYTHINK 258 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: | COMPLETION: ☐ TIME: ______

INDIANA ESCAPE

Jones is running down a square tunnel, desperately trying to avoid being crushed by a giant round stone that is rolling toward him. The width of the tunnel is just about the same as the diameter of the sphere; both are 20 meters.

The end of the tunnel is too far for Jones to reach in time. Is he doomed?

PLAYTHINK 259 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ______

JAPANESE TEMPLE TABLET

The same polygon is inscribed in two identical circles, but it is triangulated in different ways. The biggest circle that can be inscribed is placed in each resultant triangle.

Can you compare the sum of the diameters of the two sets of circles? Is one set larger than the other?

PLAYTHINK 260 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ______

THREE INTERSECTING CIRCLES

We interconnected three intersecting circles of random size by their common chords, as shown. The result should surprise: the common chords passed through a single point.

Will this happen regardless of the size and position of the three circles?

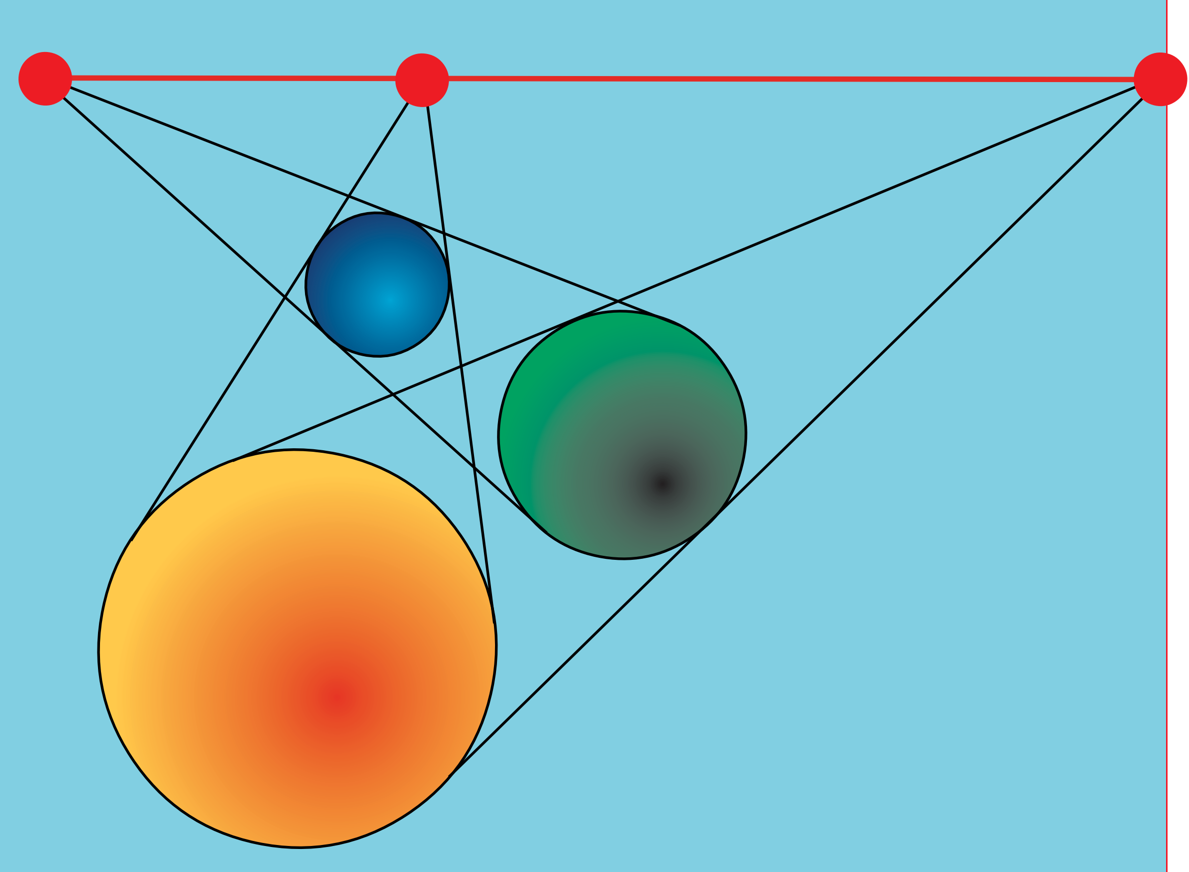
PLAYTHINK 261

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

TANGENTS TO THE CIRCLE

Three circles of different sizes are distributed randomly, as shown. Pairs of tangents are drawn around the circles, with a surprising result: the three intersection points for the tangents lie along a straight line.

Is this just a coincidence, or will it always happen?

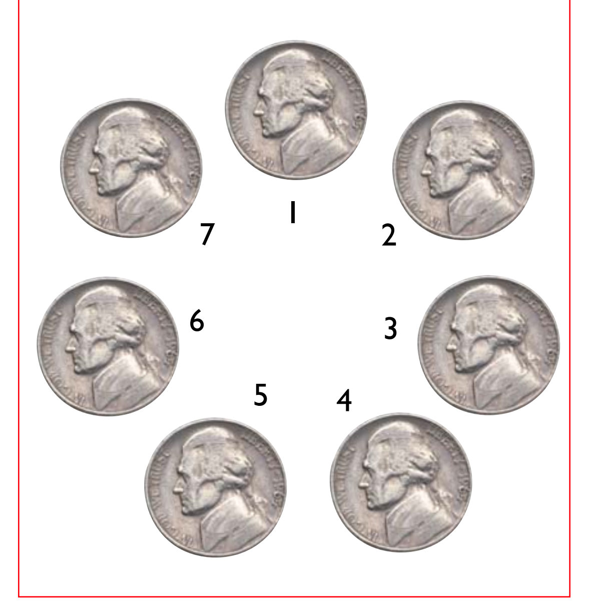


PLAYTHINK 262

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

COINS REVERSE

Seven coins are placed heads up in a circle. You would like them all to be tails up, but you are allowed to move them only if you turn five over at a time. Can you follow that rule repeatedly to eventually wind up with all seven coins tails up? How many moves will it take?



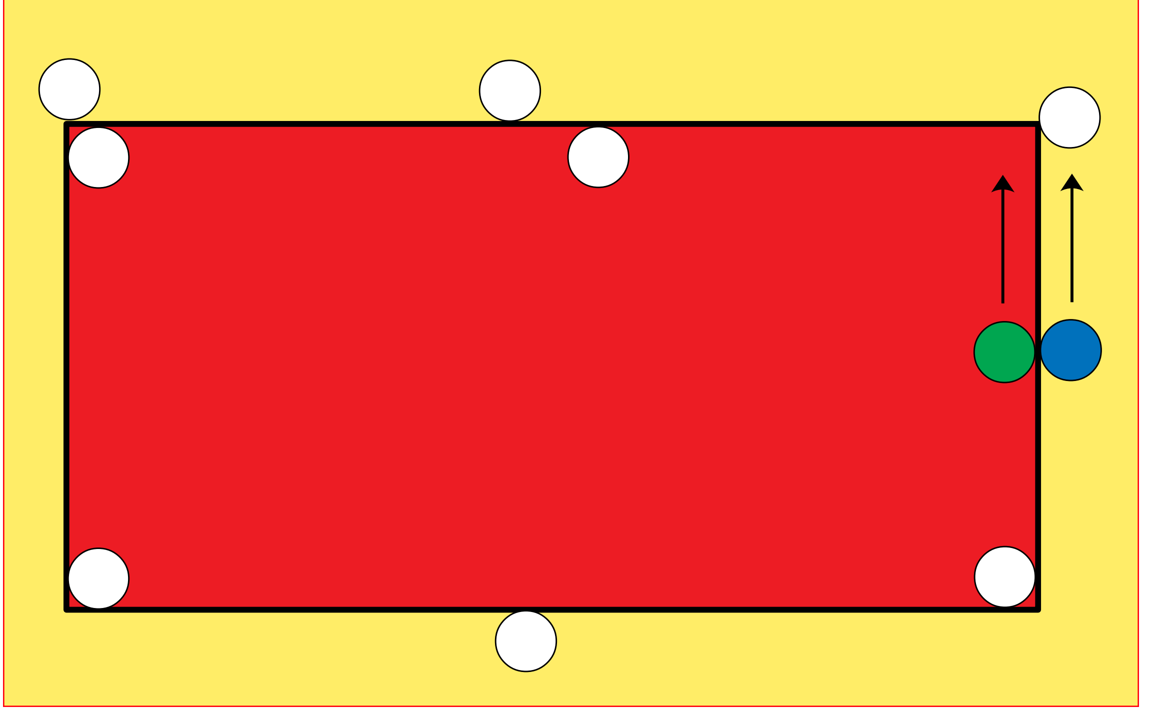
PLAYTHINK 263

DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

ROLLING INSIDE OUT

Two identical circles touch the same point of a rectangle—one from the inside, one from the outside. Both circles begin rolling in the plane along the perimeter of the rectangle until they return to the starting point.

If the height of the rectangle is twice the circumference of the circles, and if the width is twice the height, how many revolutions will each circle make?



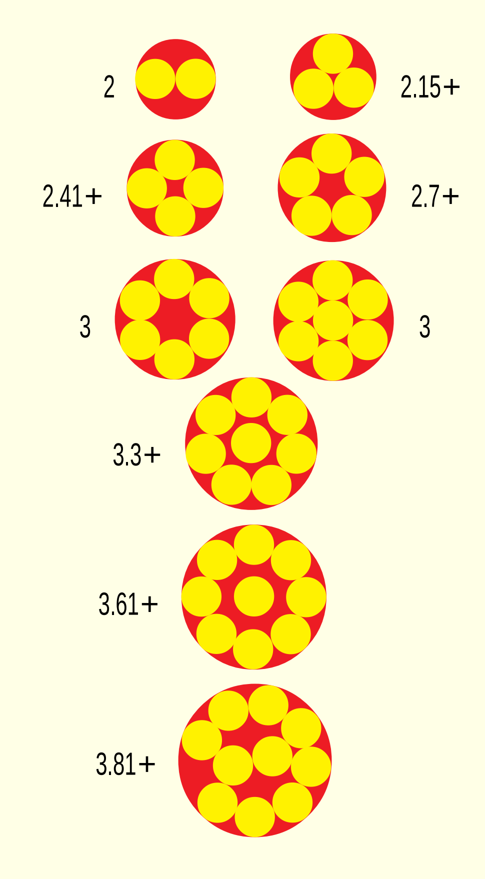
Packing Circles

Walk down the halls of some prestigious universities, and you will find grown men and women trying to figure out how to pack steel balls into boxes. This isn't a case of adults getting in touch with their inner children: What they are trying to do has a direct impact on such cutting-edge fields as information theory and solid state physics. Packing regular objects—circles on a plane or spheres in a space—is one of the most important problems in mathematics.

Balls of equal size do not fill a space completely, nor do circles in a plane. It is fairly easy to show that the densest possible configuration—a packing similar to a honeycomb, called a hexagonal lattice—is the most efficient regular packing of circles. It is enormously more difficult—though it has been done nonetheless—to show that no irregular packing can be denser.

The analogous problem of spheres packed into a volume has proven to be even more difficult. The densest regular packing is known, but whether any irregular packing can do better is still a mystery. The best guess is no, but there is no proof.

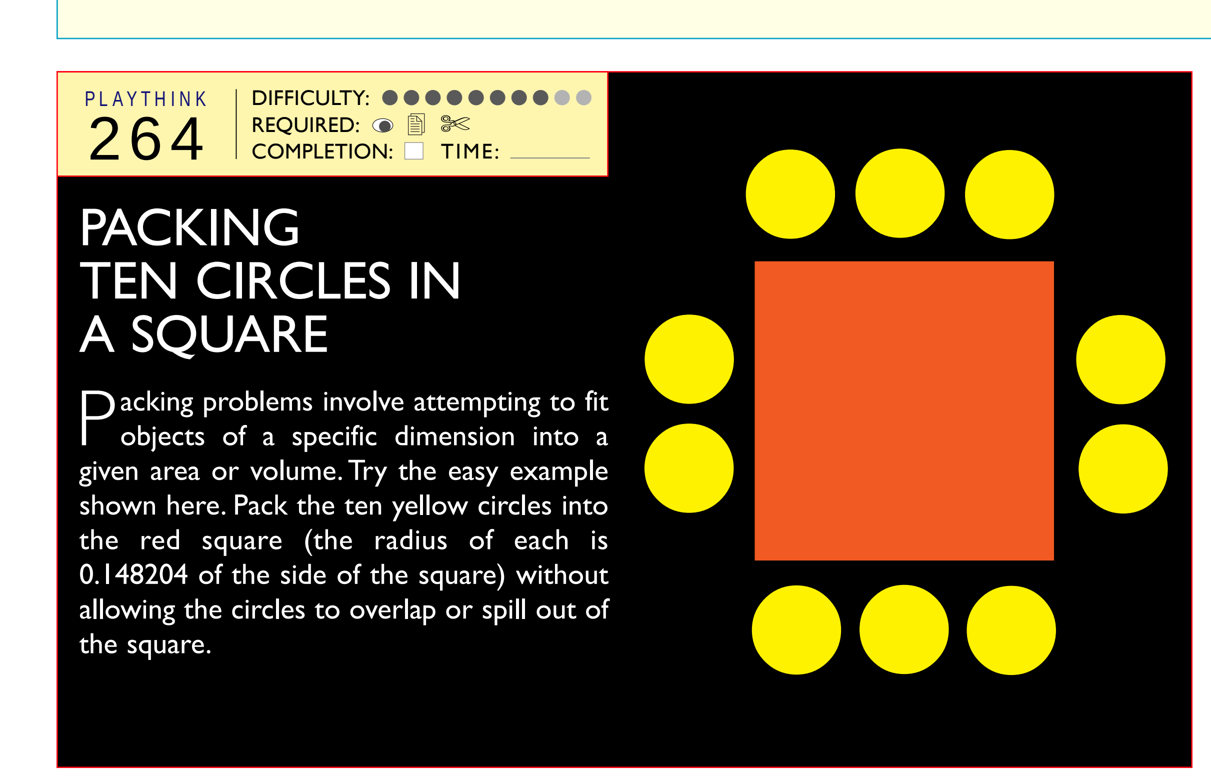
A more recent problem involves packing a given number of circles into a specific boundary of the smallest area—a square, say, or a circle. No general solution is yet known, even when the boundary of the region is very simple; the best solutions that have been found apply to only a very few circles packed in a very regular space. For example, the solution for packing circles within a larger circle has been proved up to only ten circles. The densest packings for instances up to ten circles are illustrated here. The numbers next to each example are the diameters of the outer circles, in terms of the unit circles they contain.



PLAYTHINK 264 DIFFICULTY: ●●●●●●●●○○ REQUIRED: COMPLETION: TIME:

PACKING TEN CIRCLES IN A SQUARE

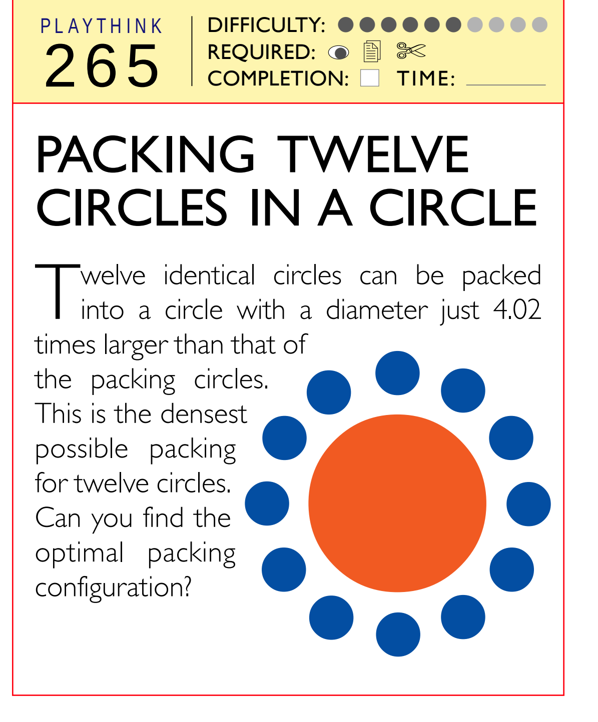
Packing problems involve attempting to fit objects of a specific dimension into a given area or volume. Try the easy example shown here. Pack the ten yellow circles into the red square (the radius of each is 0.148204 of the side of the square) without allowing the circles to overlap or spill out of the square.



PLAYTHINK 265 DIFFICULTY: ●●●●●●○○○○ REQUIRED: COMPLETION: TIME:

PACKING TWELVE CIRCLES IN A CIRCLE

Twelve identical circles can be packed into a circle with a diameter just 4.02 times larger than that of the packing circles. This is the densest possible packing for twelve circles. Can you find the optimal packing configuration?



Spheres

To get their marbles exceedingly smooth and round, glassmakers have devised a simple and yet ingenious process. They melt the glass at the top of a tower and allow small amounts to dribble off into a shaft. As the globs of glass fall, they contract to form nearly perfect spheres. By the time the globs reach the bottom of the shaft, they have cooled to become hard and round.

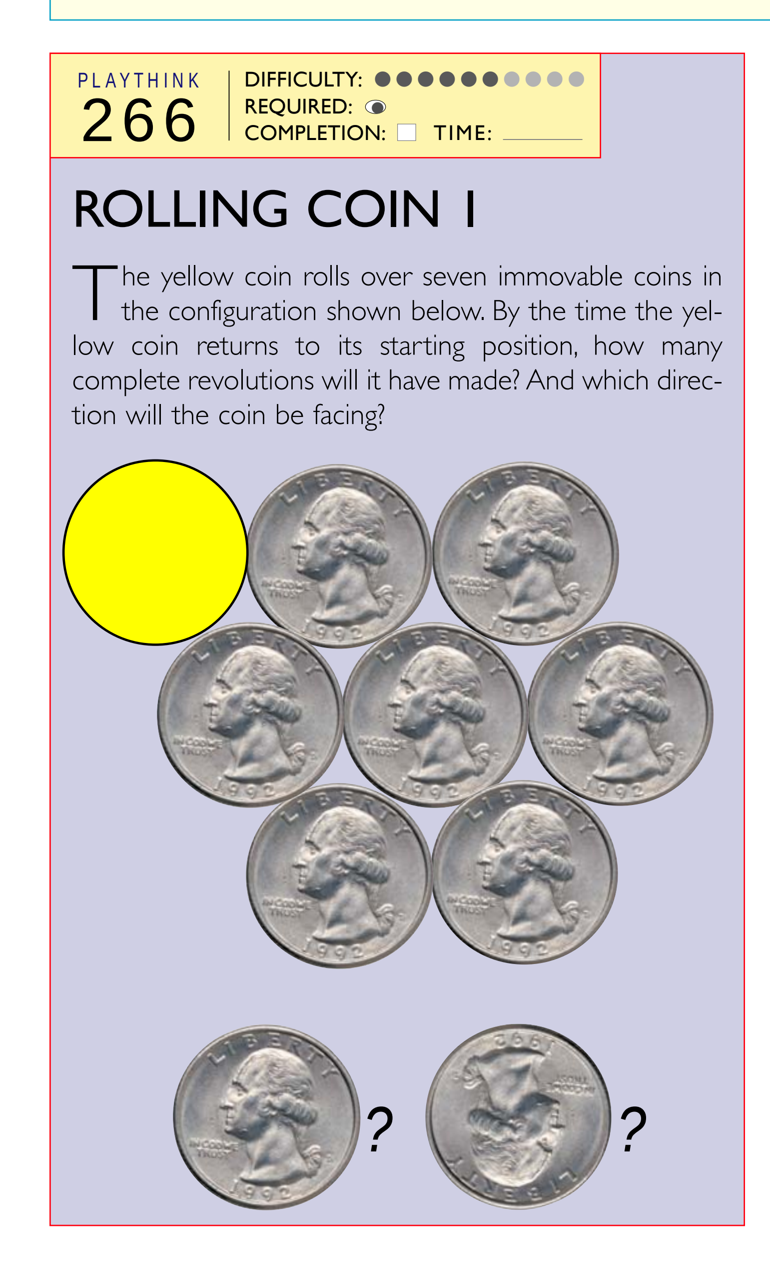
Although the traditional icon for a drop is the "teardrop" shape, photography using a strobe flash has shown that most drops in midfall are spherical. Drops of liquid are spherical in shape because electrical forces pull the loose materials toward the middle. Molecules moving in from the outer parts of the drop fill in any open space close to the center of mass. Once the drop has reached its most compact form, it has taken on the shape of a sphere.

A sphere, or ball, is perhaps the simplest solid shape that one can imagine. It has no corners or edges. Every spot on the outside of a ball is exactly the same distance from the center as every other spot. A sphere is also one of the most common shapes in the universe. Stars and planets are subject to the constant pull of their own gravity and take on nearly spherical shapes; indeed, astronauts in orbit find that any spilled liquids quickly form little quivering balls.

PLAYTHINK 266 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ______

ROLLING COIN 1

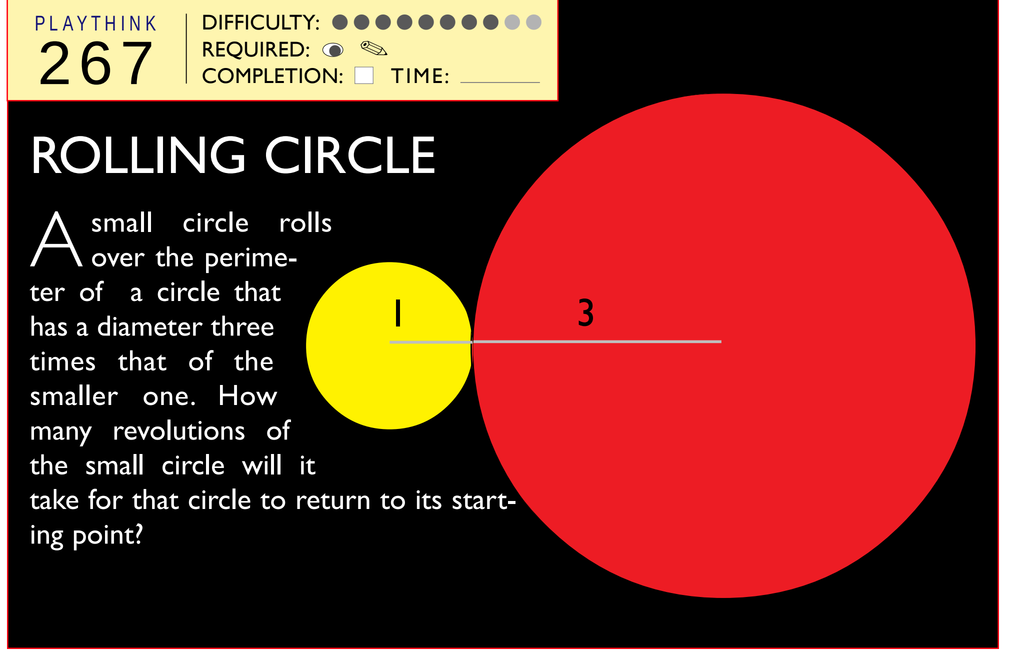
The yellow coin rolls over seven immovable coins in the configuration shown below. By the time the yellow coin returns to its starting position, how many complete revolutions will it have made? And which direction will the coin be facing?



PLAYTHINK 267 | DIFFICULTY: ●●●●●●●●○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ______

ROLLING CIRCLE

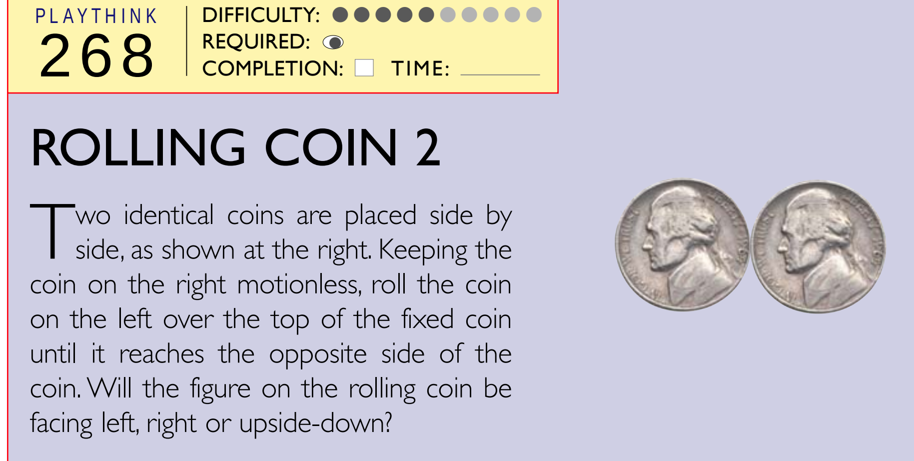
A small circle rolls over the perimeter of a circle that has a diameter three times that of the smaller one. How many revolutions of the small circle will it take for that circle to return to its starting point?



PLAYTHINK 268 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ______

ROLLING COIN 2

Two identical coins are placed side by side, as shown at the right. Keeping the coin on the right motionless, roll the coin on the left over the top of the fixed coin until it reaches the opposite side of the coin. Will the figure on the rolling coin be facing left, right or upside-down?



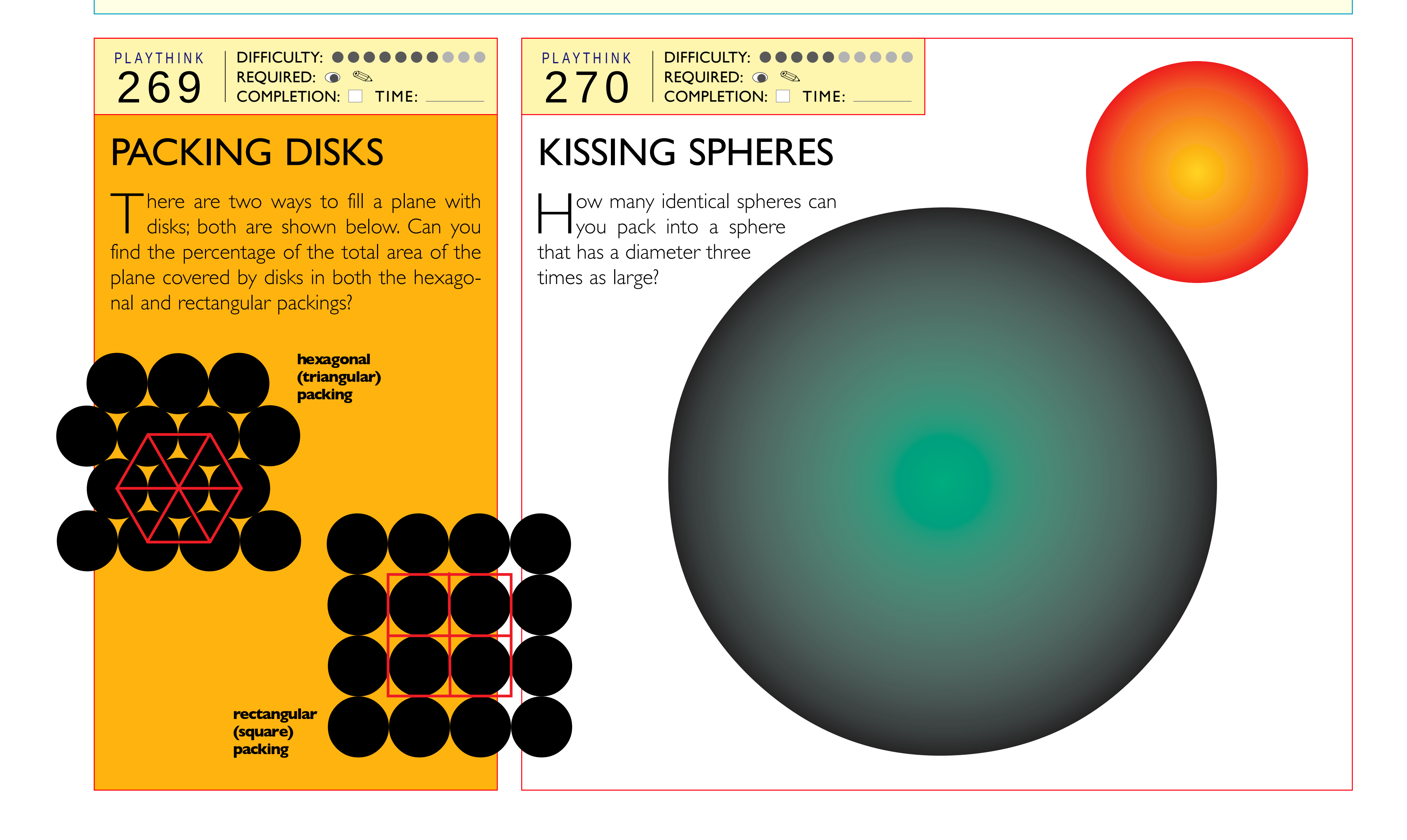
Packing Spheres

The astronomer Johannes Kepler revolutionized the study of the orbits of planets. He also researched the problem of packing spheres. Kepler found that there are two ways to arrange spheres in a plane: the square lattice and the hexagonal (or honeycomb) lattice. Those two arrangements can then be stacked to fill a volume in several ways.

Square layers, for example, can be stacked so that the spheres are vertically above each other, or the spheres in one layer can nestle into the gaps between the four spheres in the layer below—the so-called face-centered cubic lattice. Hexagonal layers also have two possibilities, either aligned or staggered, although this last instance is essentially no different than the face-centered cubic lattice.

One way to tell which arrangement is the most compact is by imagining that the spheres were allowed to expand to fill in the available space. What shape would the spheres then have? Spheres in a cubic lattice would simply form cubes, while spheres in a hexagonal lattice would form hexagonal prisms. But spheres packed in a face-centered cubic lattice, as Kepler found, would form a rhombic dodecahedron, which leads to the tightest possible packing.

The efficiency of a packing lattice is measured in the proportion of space that is filled with spheres. For spheres in a plane, the efficiency for a square lattice is 78.54 percent; for a hexagonal lattice, 90.69 percent. For spheres in a three-dimensional volume, the efficiency for a cubic lattice is 52.36 percent; for a hexagonal lattice, 60.46 percent; for a face-centered cubic lattice, 74.04 percent.



Cycloids

No point in the universe is truly fixed. A point that remains stationary within a car may be tracing a linear path as the car speeds down the highway. A point on a mountain follows the earth around the sun. And even the sun and the Milky Way galaxy have their own paths through an ever-expanding universe.

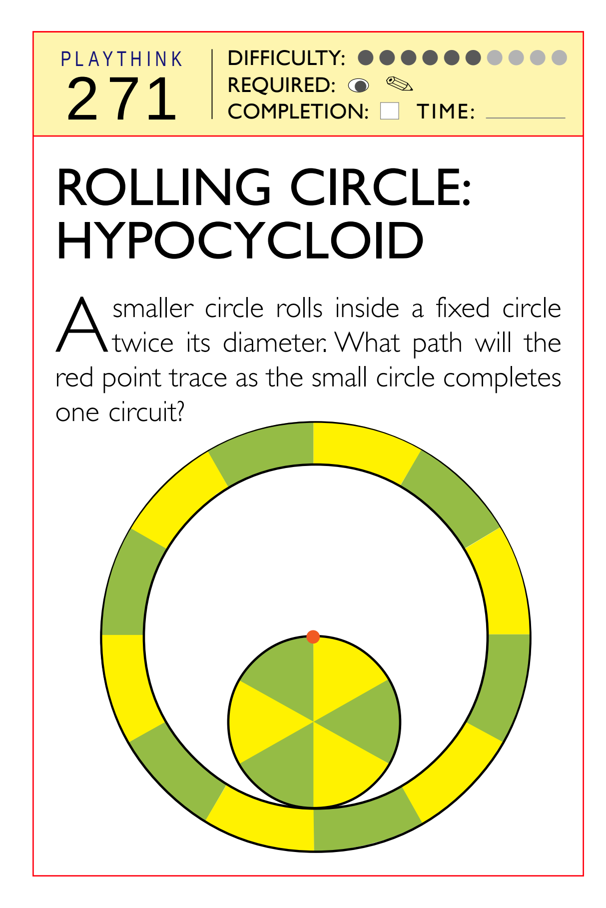
The motion of a fixed point on a moving body traces a curve that can have very unusual properties. For example, the curve traced by a point on a rotating circle is called a cycloid. The cycloidal curve appears in many places in modern society: mechanical gears have teeth whose sides possess a cycloidal curve; a machine engraves an elaborate cycloid on the plates used for printing bank notes; a popular science toy known as the Spirograph produces an endless variety of cycloidal shapes with just a few round parts.

Other similar curves include the spiral and the involute—the line traced by the end of a taut thread as it is unwound off a spool.

PLAYTHINK 271 | DIFFICULTY: ●●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

ROLLING CIRCLE: HYPOCYCLOID

A smaller circle rolls inside a fixed circle twice its diameter. What path will the red point trace as the small circle completes one circuit?

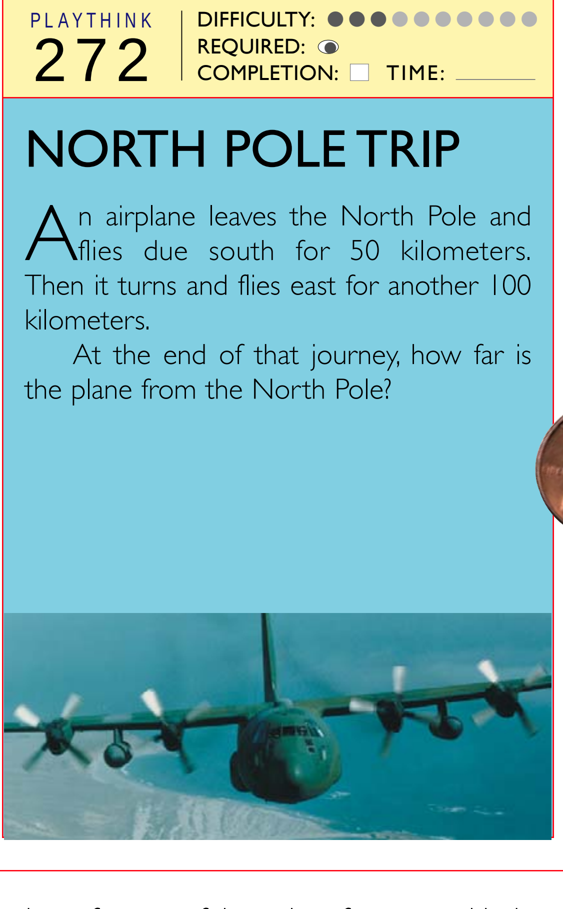


PLAYTHINK 272 | DIFFICULTY: ●●●○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

NORTH POLE TRIP

An airplane leaves the North Pole and flies due south for 50 kilometers. Then it turns and flies east for another 100 kilometers.

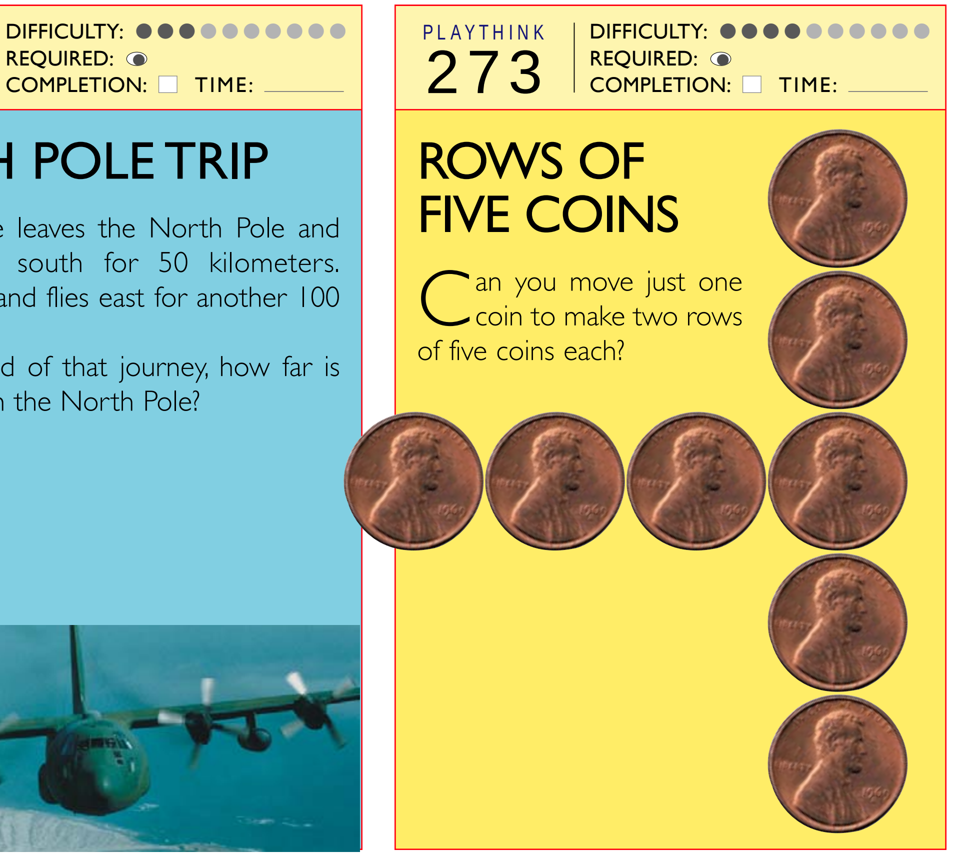
At the end of that journey, how far is the plane from the North Pole?



PLAYTHINK 273 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

ROWS OF FIVE COINS

Can you move just one coin to make two rows of five coins each?

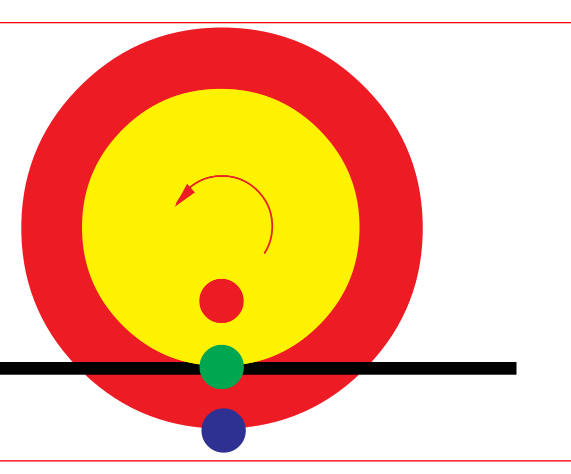


PLAYTHINK 274 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

ROLLING WHEEL

The wheel of a train rolls along a rail. To keep the train on the tracks, each wheel has a flange that extends below the circumference of the point of contact with the train. Can you envision the path traced by these three points?

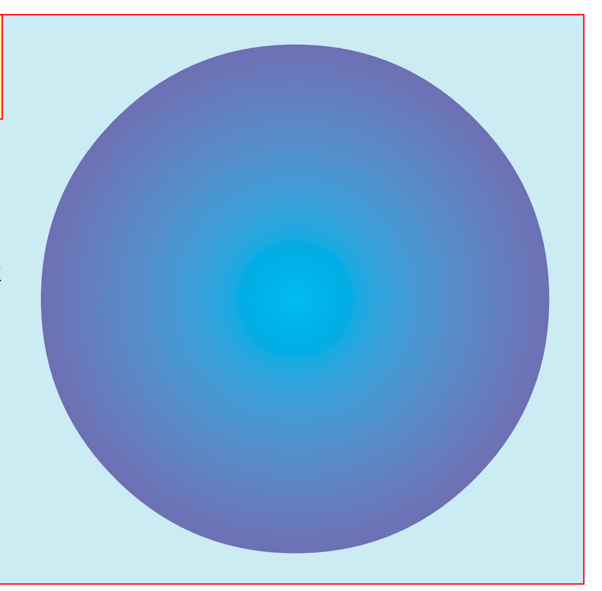
- A point on the inside of the rolling wheel
- A point on the circumference of the rolling wheel
- A point on the outer flange of the rolling wheel



PLAYTHINK 275 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

CUTTING A SPHERE

Imagine that this sphere has been divided with four straight cuts, all of which go right through the sphere. Can you determine the maximum number of pieces into which the sphere has been divided?

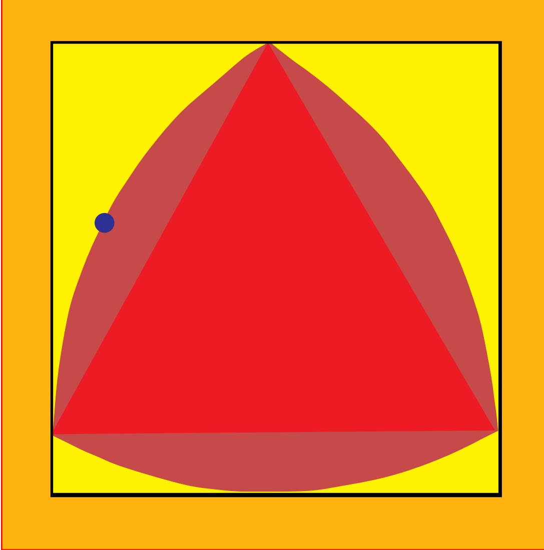


PLAYTHINK 276 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ ✂ | COMPLETION: ☐ TIME: ____

REULEUX'S TRIANGLE

A curiously shaped triangle revolves inside a fixed square frame. To construct such a shape, begin with an equilateral triangle; with each of the three corners as centers, draw a circular arc passing through the other two corners. What you will have is called Reuleux's triangle, named after the man who discovered it in 1875. The width of the curve in every direction is equal to the side of an equilateral triangle.

Imagine the triangle rolling along the inside of the square, as shown here. Can you imagine the path traced by the blue dot through several complete rotations?

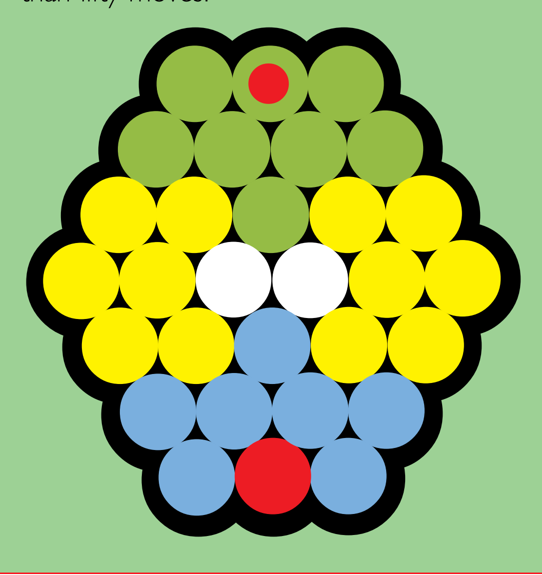


PLAYTHINK 277 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 📄 ✂ | COMPLETION: ☐ TIME: ____

HEXSTEP SOLITAIRE: A SLIDING DISK PUZZLE

In this game the object is to transfer the red disk at the bottom to the space marked with the red dot at the top. To do so, you must slide disks one at a time into one of the two empty spaces (shown in the illustration as white circles). For example, the two possible first moves would be to slide either the green disk down or the blue disk up into one of the empty spaces. The yellow disks cannot reach the white spaces on the first move because the gap for them to move through is too narrow. As a rule, only two moves are possible at any given time.

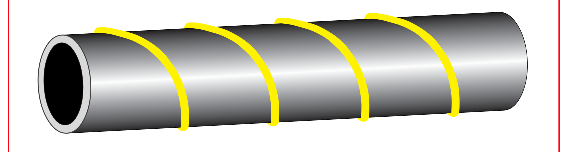
Can you accomplish the goal in fewer than fifty moves?



PLAYTHINK 278 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

HELIX

A rope winds around a large cylindrical pipe, making four complete turns, as shown. The circumference of the pipe is 4 meters and its length is 12 meters. Can you figure out how long the rope is?

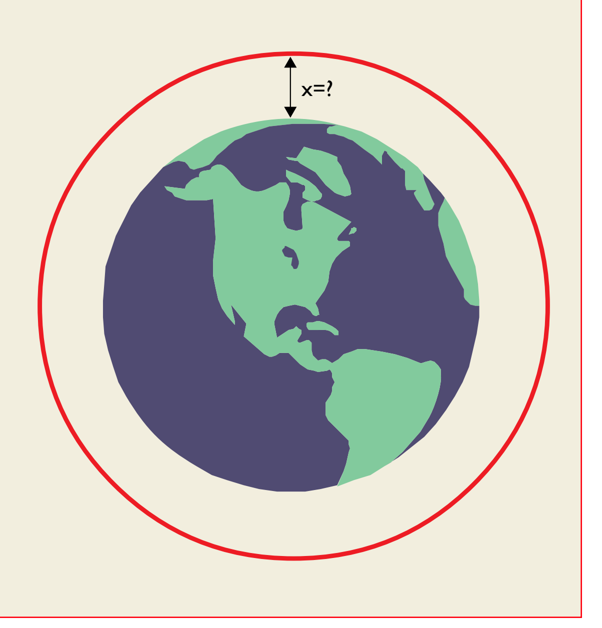


PLAYTHINK 279 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

LOOPED EARTH

Imagine the earth as a perfect sphere. (It is not, but picture it this way for the sake of this puzzle.) Then imagine the equator is a long belt that has been looped around the earth and fastened snugly.

If you loosened that belt by 2 meters and pulled the belt away from the surface, how much slack would there be? In other words, how high could you pull the belt? The answer is either .03 meters, .33 meters or 3.3 meters—but which?

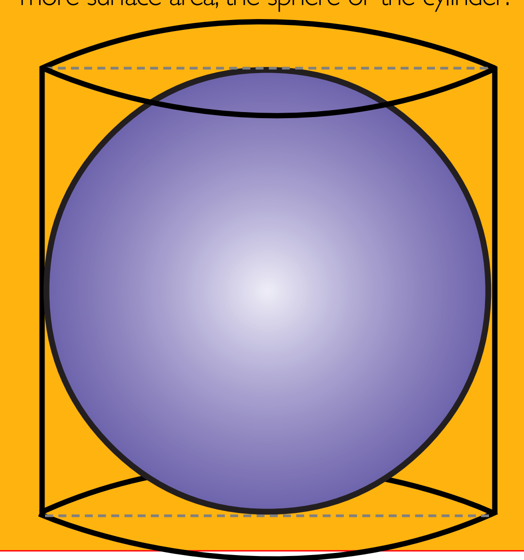


PLAYTHINK 280

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

SPHERE SURFACE AREA

A sphere fits exactly inside a thin-walled cylinder that has a height and diameter equal to the diameter of the sphere. Which object has more surface area, the sphere or the cylinder?

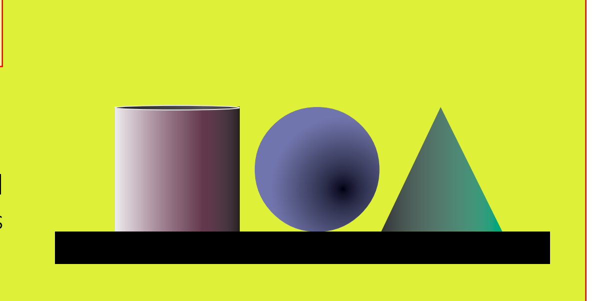


PLAYTHINK 281

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

SPHERE VOLUME

A cylinder, a sphere and a cone are identical in height and width. Do their volumes have any sort of special relationship?

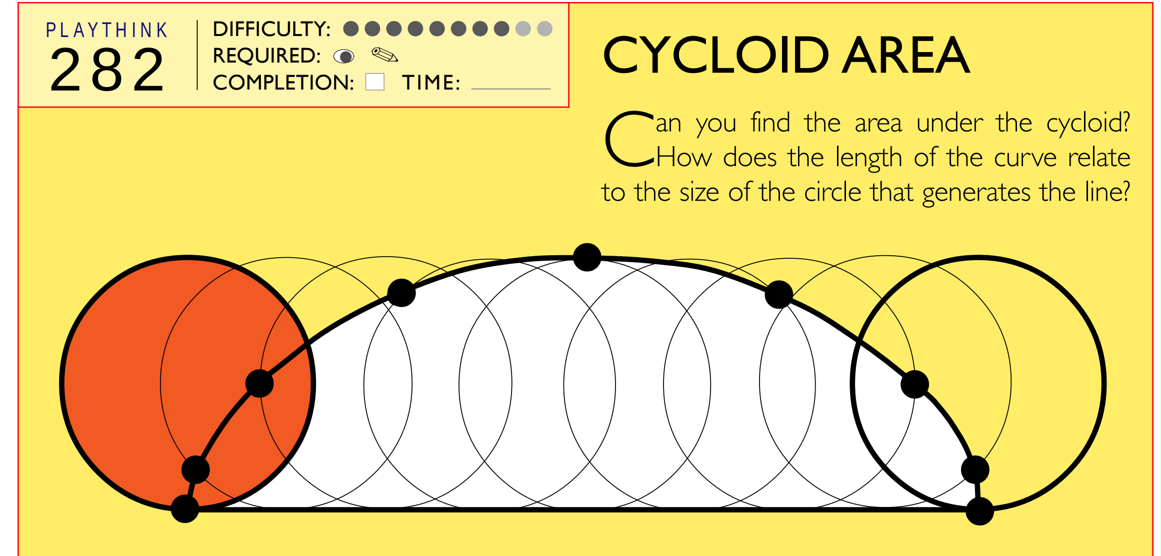


PLAYTHINK 282

DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

CYCLOID AREA

Can you find the area under the cycloid? How does the length of the curve relate to the size of the circle that generates the line?



PLAYTHINK 283

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

PACKING BOX

Is this story possible?

The king has a rectangular chest filled tightly with twenty golden spheres. Each sphere is held securely by the spheres touching it, so that when picked up, the spheres do not move about in the box.

How many spheres can be removed without disrupting the tightness of the fit?

. . . Once upon a time the king had all his money made into identical gold spheres. He packed the money into a big chest tightly. He knew that the chest was full, because it didn't rattle.
Soon the queen took out some money, repacked the chest, and still the chest didn't rattle. Then the treasurer took out some more money, repacked the chest, and still the chest didn't rattle. Then the prime minister took out some more money and still it didn't rattle. . . .



Curves of Constant Width

Curves that have the same width in every direction are called curves of constant width. Any curve of constant width can turn between two fixed parallel lines or within a square. Although some curves of constant width, such as the circle, are smooth, others have corners; and while some are highly symmetrical, others are quite irregular. As a matter of fact, any regular polygon with an odd number of sides can be rounded up to create a curve of constant width.

But every curve of constant width has one thing in common: the length of the curve is equal to π times that constant width. This is known as Minkowski's theorem and is seen most obviously in the formula for the circumference of a circle, which is π times the diameter.

PLAYTHINK 284

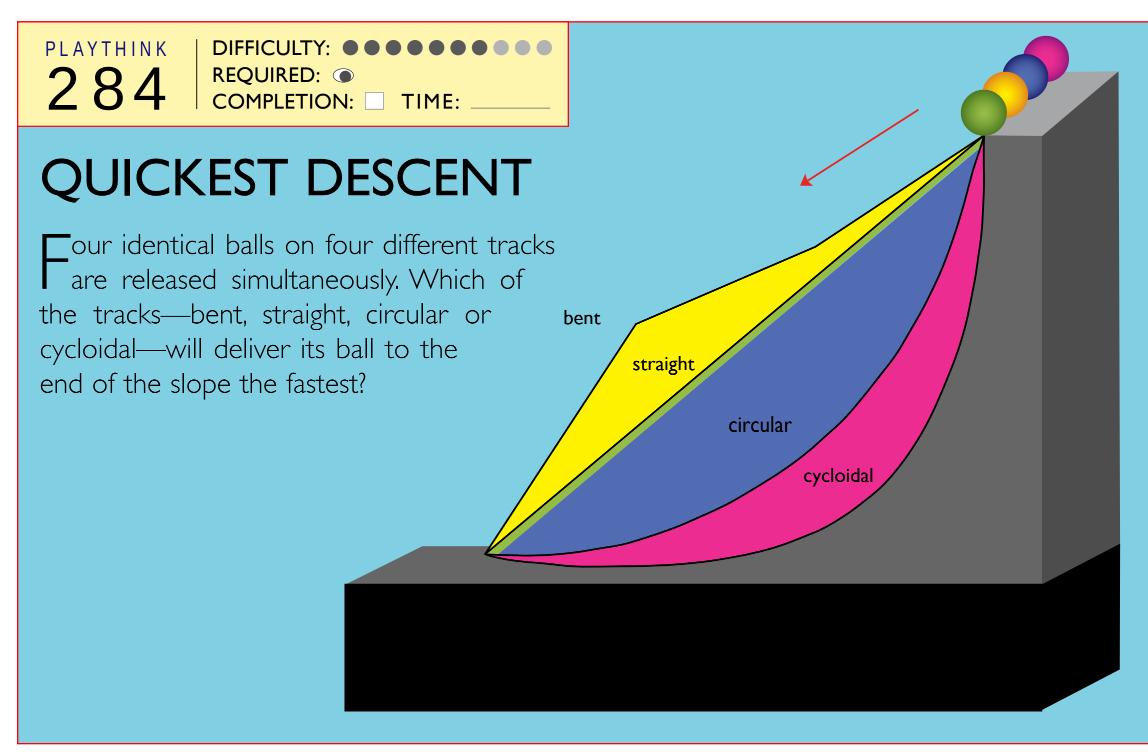
DIFFICULTY: ●●●●●●●○○○

REQUIRED: 👁

COMPLETION: ☐ TIME: ______

QUICKEST DESCENT

Four identical balls on four different tracks are released simultaneously. Which of the tracks—bent, straight, circular or cycloidal—will deliver its ball to the end of the slope the fastest?



PLAYTHINK 285

DIFFICULTY: ●●●●●●○○○○

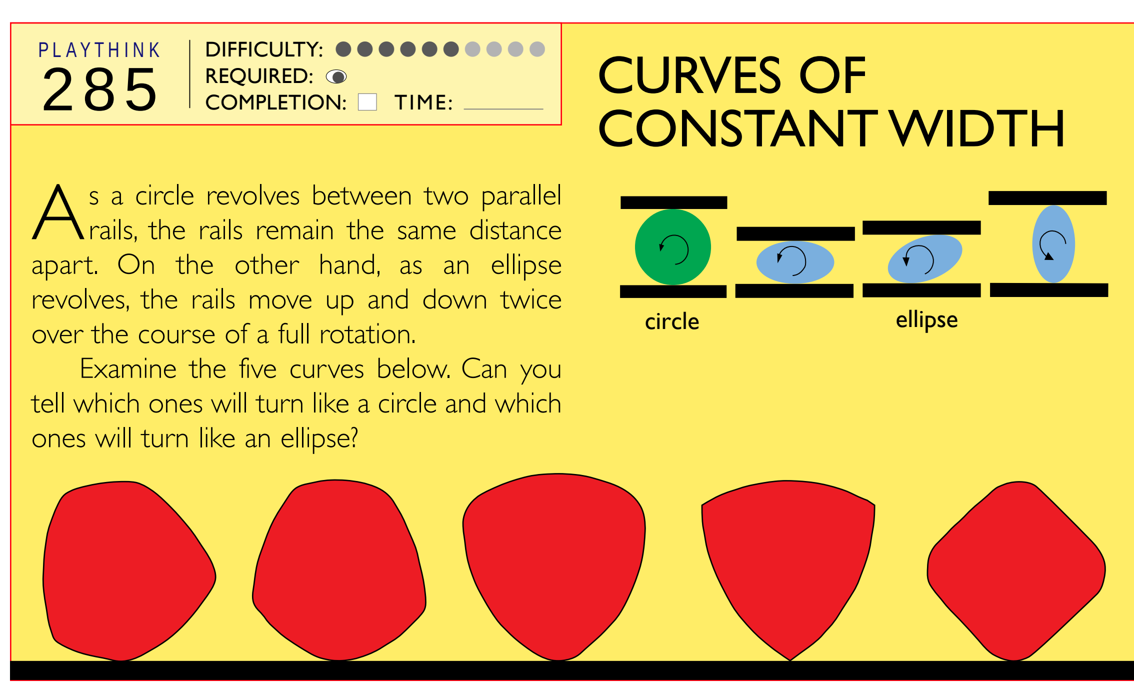
REQUIRED: 👁

COMPLETION: ☐ TIME: ______

CURVES OF CONSTANT WIDTH

As a circle revolves between two parallel rails, the rails remain the same distance apart. On the other hand, as an ellipse revolves, the rails move up and down twice over the course of a full rotation.

Examine the five curves below. Can you tell which ones will turn like a circle and which ones will turn like an ellipse?



PLAYTHINK 286

DIFFICULTY: ●●●●●●●○○○

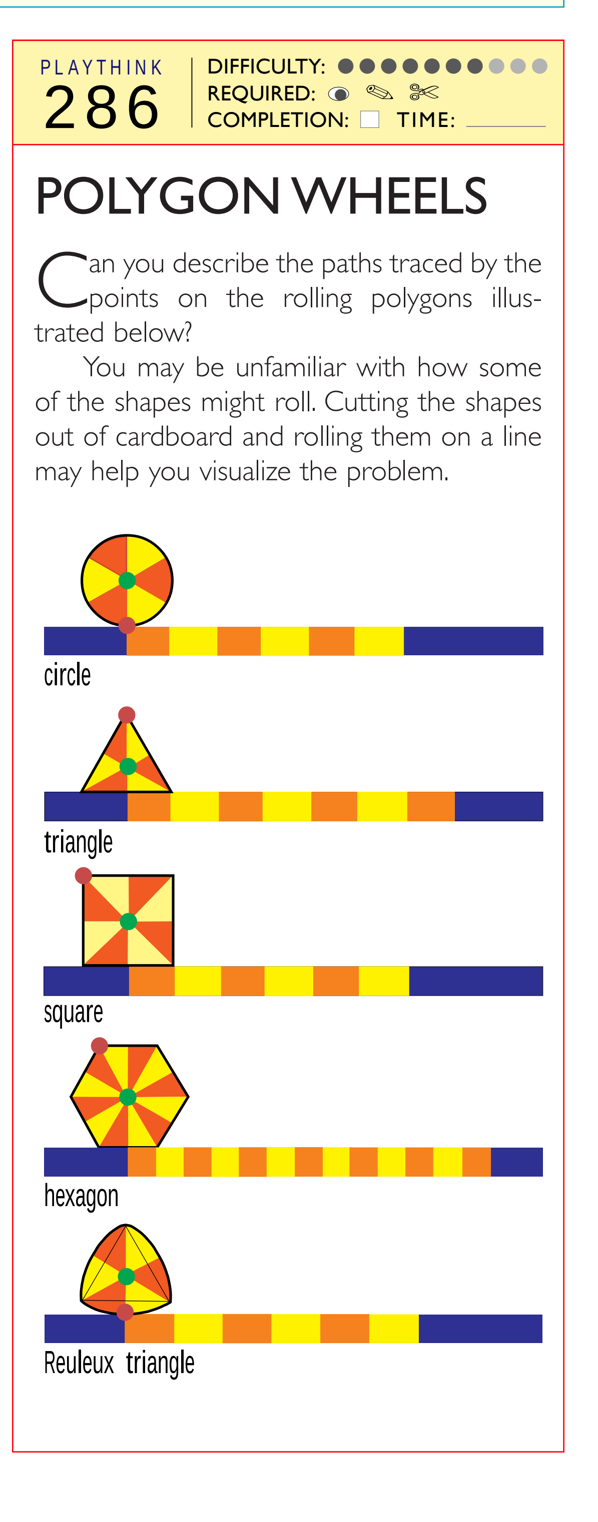
REQUIRED: 👁 ✎ ✂

COMPLETION: ☐ TIME: ______

POLYGON WHEELS

Can you describe the paths traced by the points on the rolling polygons illustrated below?

You may be unfamiliar with how some of the shapes might roll. Cutting the shapes out of cardboard and rolling them on a line may help you visualize the problem.



Conic Sections and Spirals

The curves formed by passing a plane through a cone—known as conic sections—were a subject of intense study in ancient Greece. The aesthetic properties of ellipses, hyperbolas and parabolas fascinated Euclid and other geometers of that era. But they could find no uses for them and regarded conic sections as merely interesting geometrical recreations.

Mathematicians have a habit of studying utterly useless objects just for fun. But often those studies become enormously important to scientists in later centuries. That is what happened with conic sections. The work of Johannes Kepler and Isaac Newton relied on the study of conic sections to describe the paths traced by celestial bodies moving through space. Planets, comets and even galaxies move exclusively in ellipses, hyperbolas and parabolas.

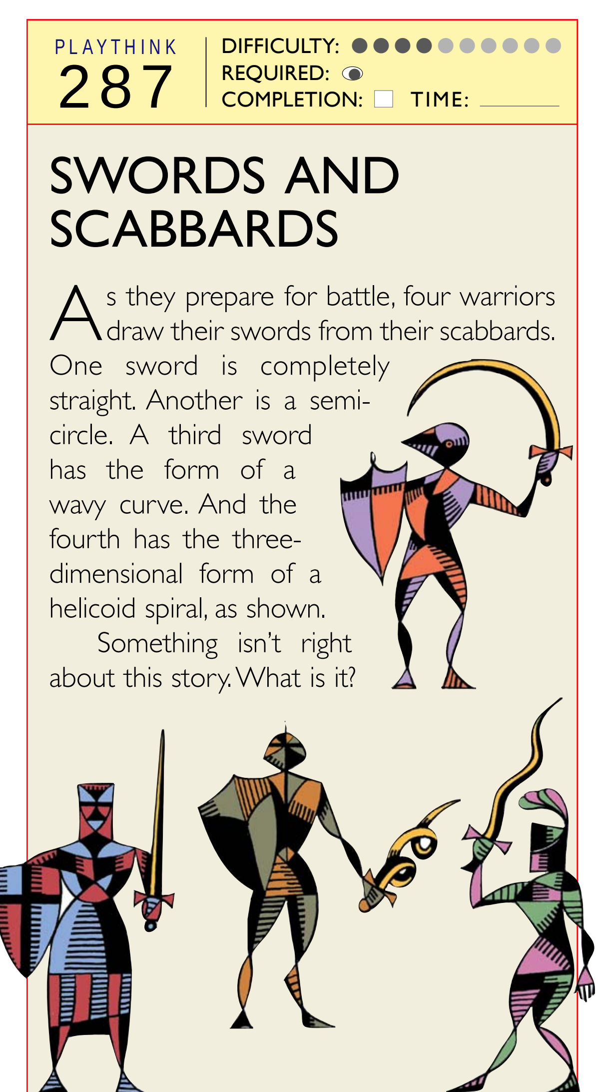
The same is true of objects in flight on earth. The path of a ball through the air is a parabola. In fact, every projectile—every bullet, arrow, rocket or stream of water emerging from a nozzle—follows a parabolic path. The reason for this, Newton discovered, is that gravity's pull affects the path of an object at every point in its flight. Rather than being straight, the line traced by an object in flight is constantly curved, approaching but never reaching a perfectly vertical path over time. If the object is thrown fast enough, however—as in the case of a satellite launched by a rocket—the path will curve in such a way that the object (satellite) doesn't fall; instead, it orbits the earth.

PLAYTHINK 287 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ______

SWORDS AND SCABBARDS

As they prepare for battle, four warriors draw their swords from their scabbards. One sword is completely straight. Another is a semicircle. A third sword has the form of a wavy curve. And the fourth has the three-dimensional form of a helicoid spiral, as shown.

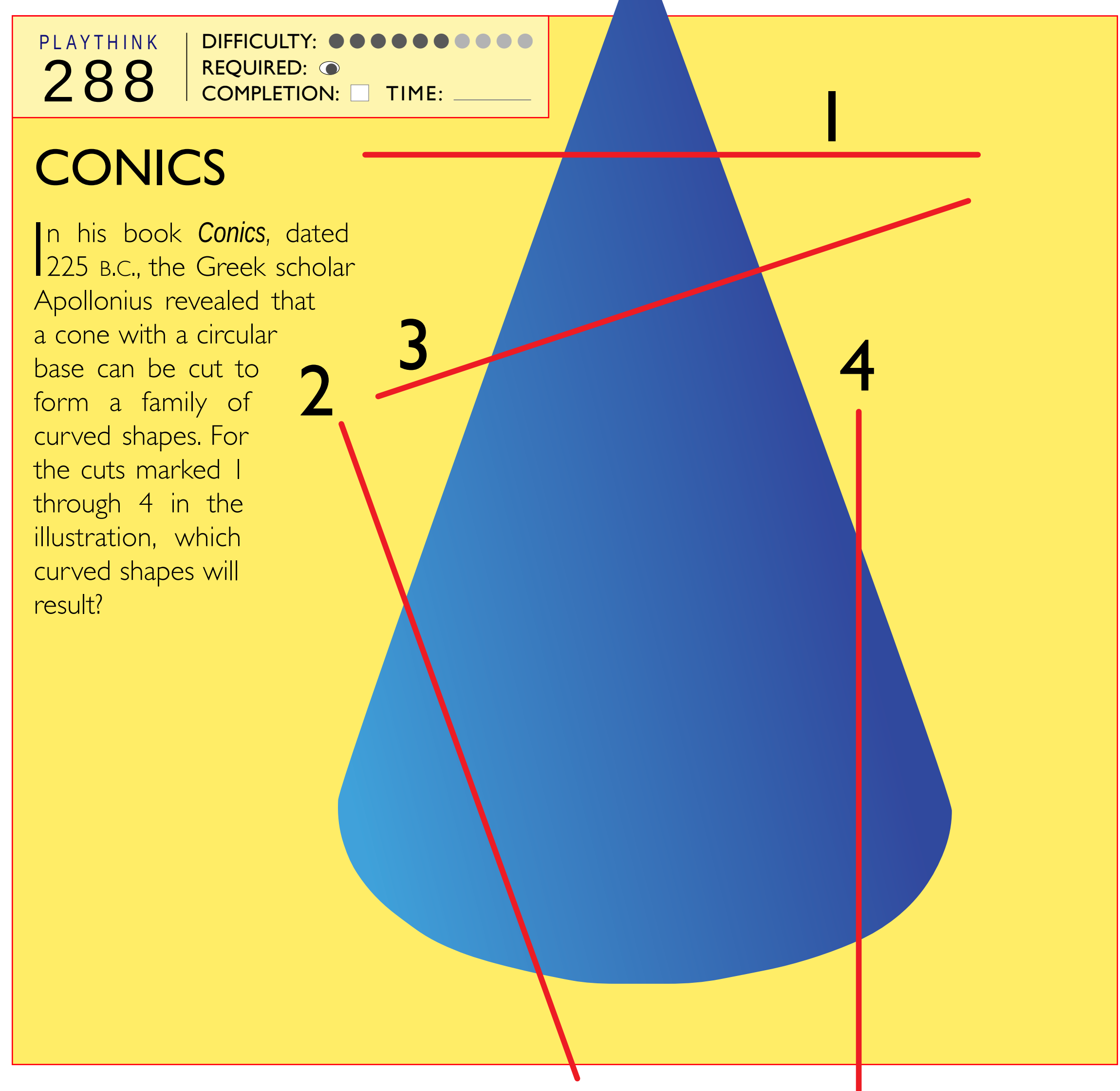
Something isn't right about this story. What is it?



PLAYTHINK 288 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ______

CONICS

In his book *Conics*, dated 225 B.C., the Greek scholar Apollonius revealed that a cone with a circular base can be cut to form a family of curved shapes. For the cuts marked 1 through 4 in the illustration, which curved shapes will result?



PLAYTHINK 289 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

ELLIPSE WHERE?

The man in the illustration desperately needs to see an ellipse. How can he make one while sitting at the table without touching his pen, compass, ruler or computer?

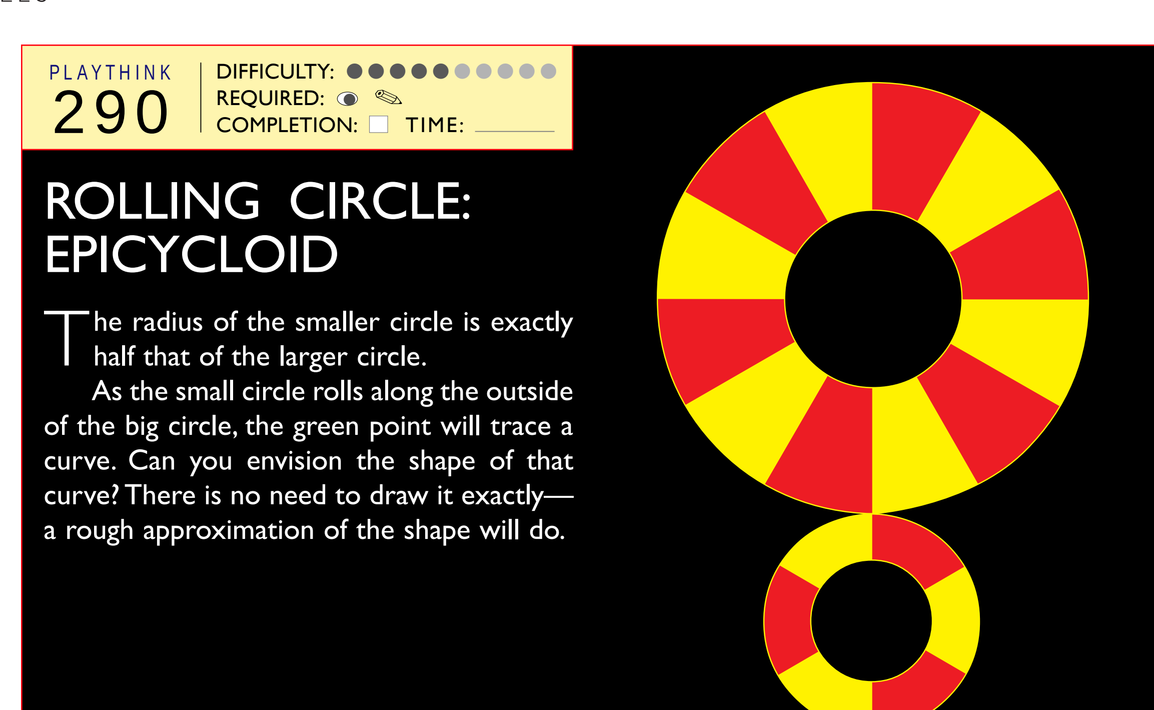


PLAYTHINK 290 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 ✎ | COMPLETION: ☐ | TIME: ____

ROLLING CIRCLE: EPICYCLOID

The radius of the smaller circle is exactly half that of the larger circle.

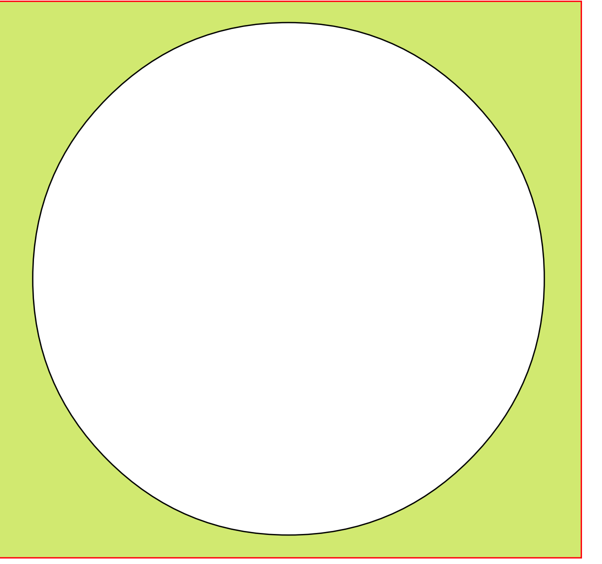
As the small circle rolls along the outside of the big circle, the green point will trace a curve. Can you envision the shape of that curve? There is no need to draw it exactly—a rough approximation of the shape will do.



PLAYTHINK 291 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: ✂ | COMPLETION: ☐ | TIME: ____

ELLIPSE BY PAPER FOLDING

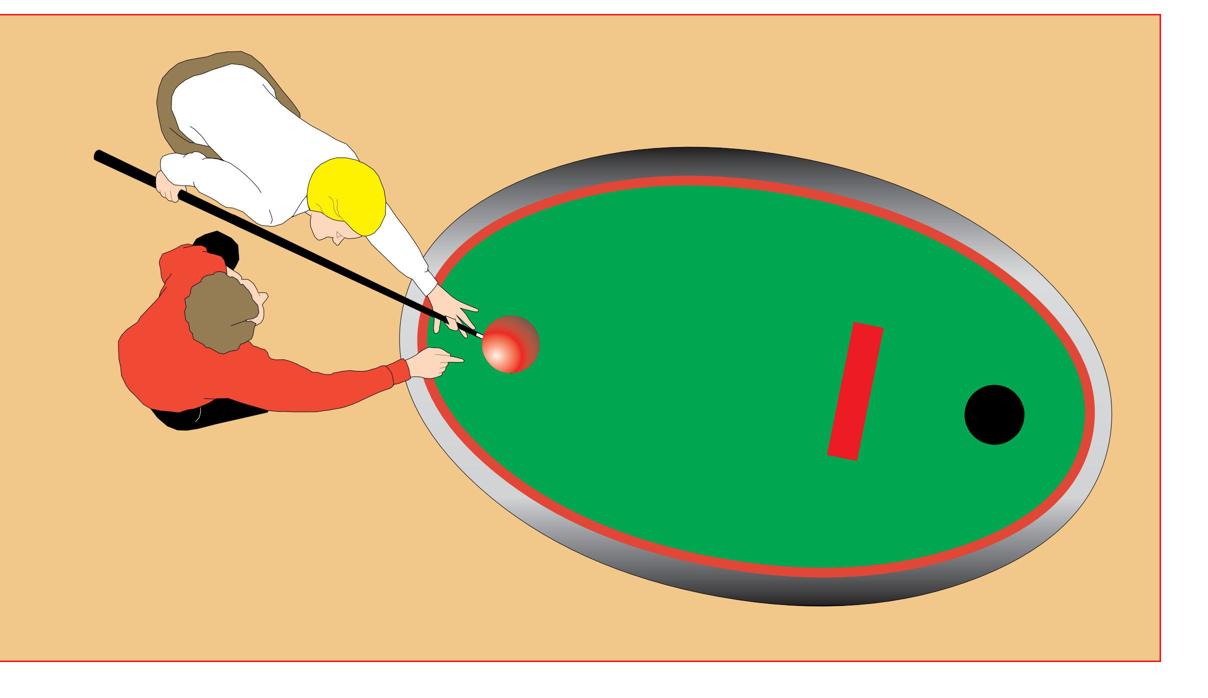
How can you create an ellipse from a circular piece of paper without using a pen or any other object?

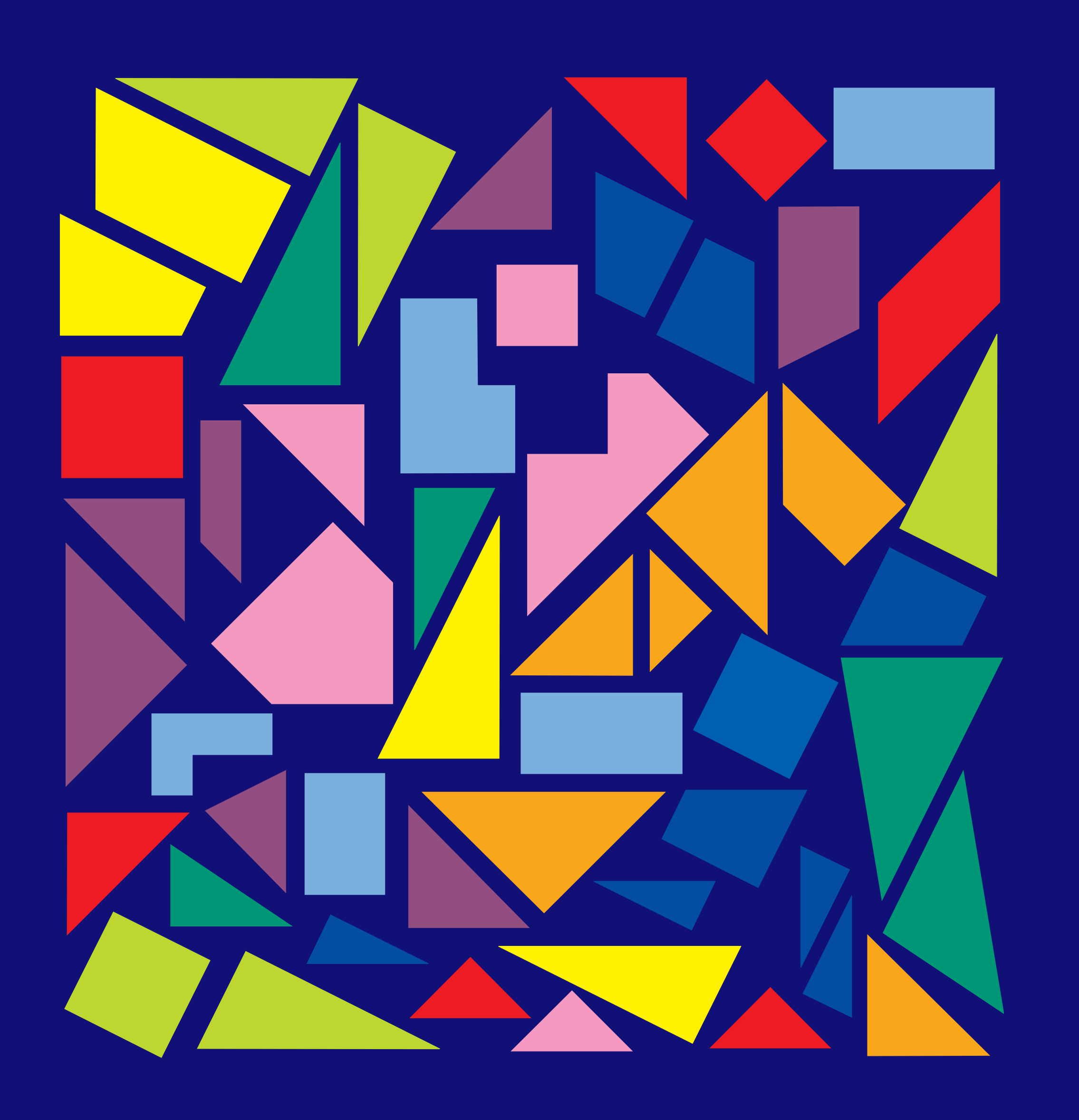


PLAYTHINK 292 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

ELLIPTICAL POOL TABLE

This elliptical pool table has a pocket at one focal point and a ball at the other focal point. Is it possible to strike the ball so that it lands in the pocket despite the obstacle in between?





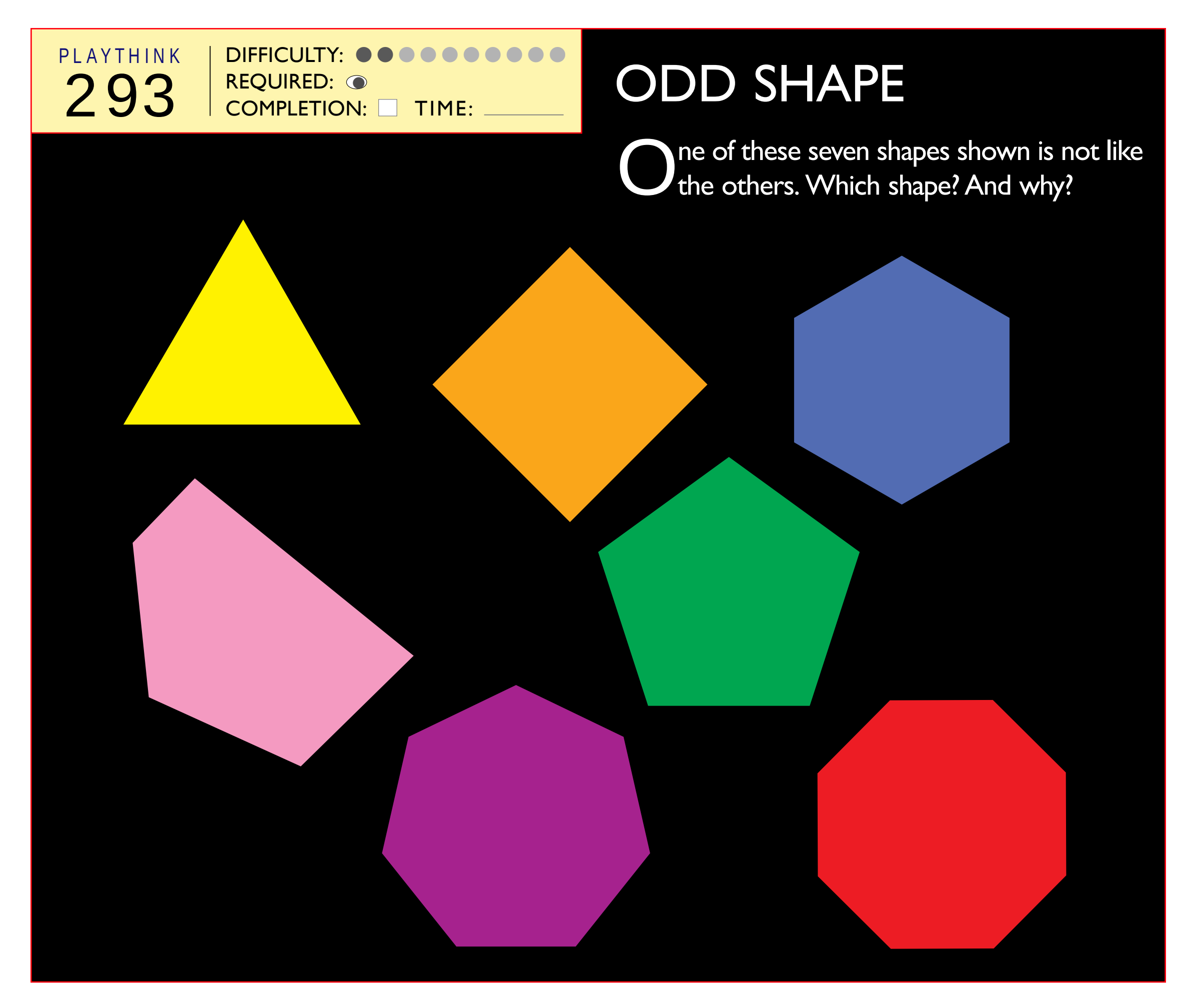
6

Shapes and Polygons

PLAYTHINK 293
DIFFICULTY: ●●○○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

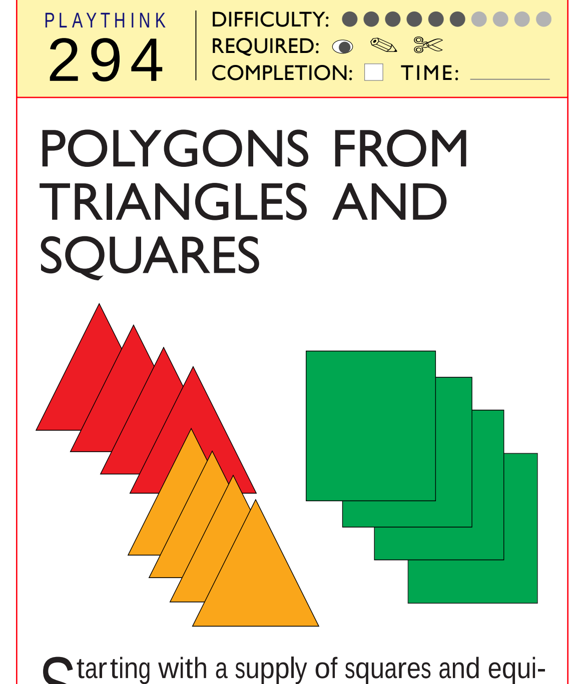
ODD SHAPE

One of these seven shapes shown is not like the others. Which shape? And why?



PLAYTHINK 294
DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎ ✂
COMPLETION: ☐ TIME: ____

POLYGONS FROM TRIANGLES AND SQUARES

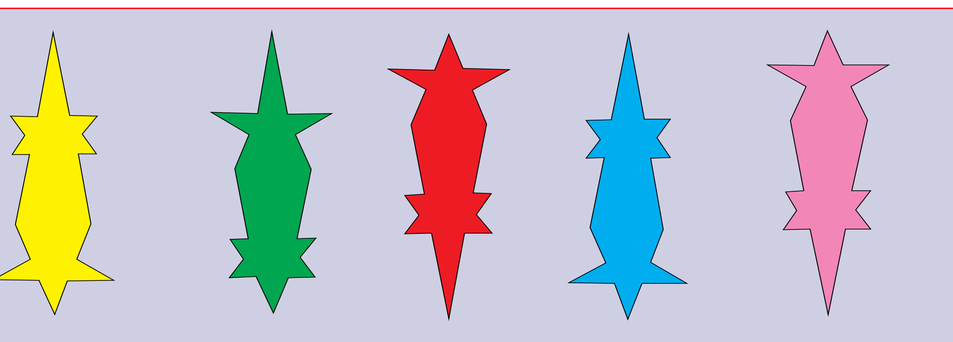


Starting with a supply of squares and equilateral triangles that all have sides of identical length, put the pieces together to form convex polygons. Can you form polygons that have a number of sides ranging from five to ten? How many triangles and squares do you need to form each polygon?

PLAYTHINK 295
DIFFICULTY: ●●●○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

ODD ONE OUT

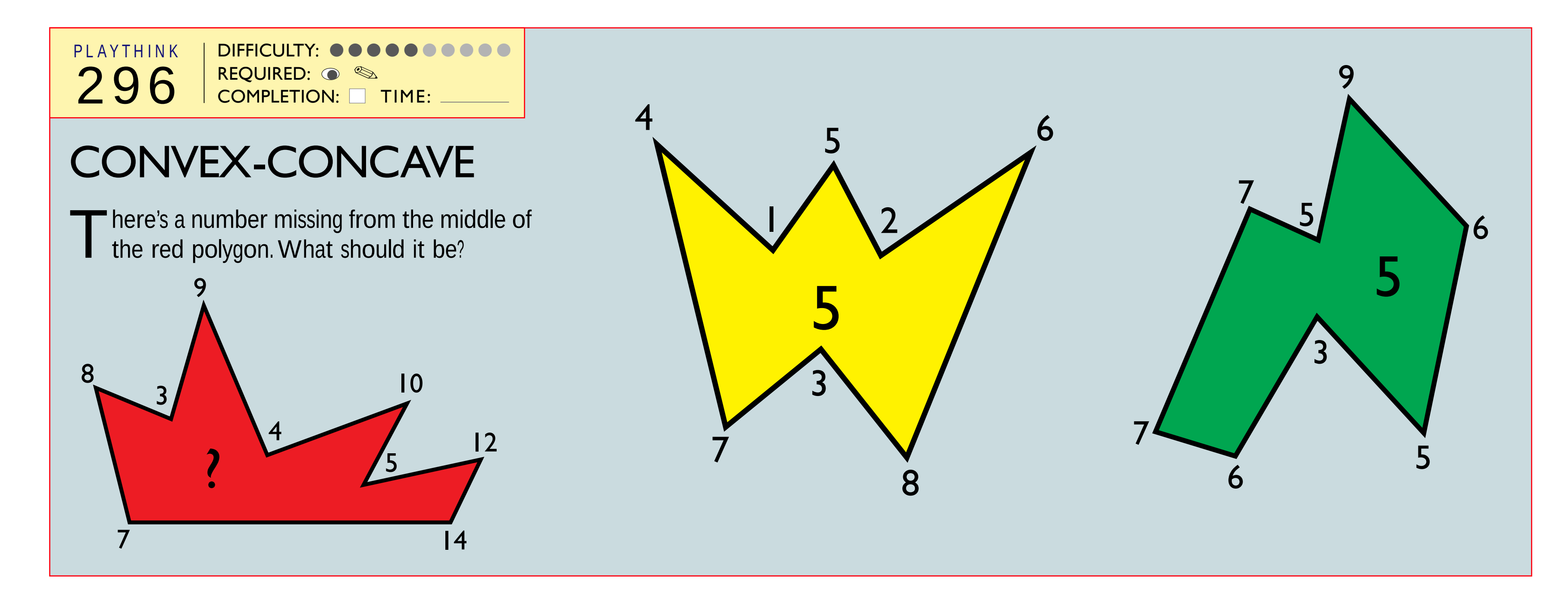
Which of these shapes is different from the other four?



PLAYTHINK 296
DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

CONVEX-CONCAVE

There's a number missing from the middle of the red polygon. What should it be?



PLAYTHINK 297 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

SHAPES AND HOLES

A mechanism for making various extruded profiles is shown in the schematic below. Each of the four cam wheels can be set in one of four positions to create a cross-section through which the profile may be extruded. The resultant profile is shown in blue in the center of the circle.

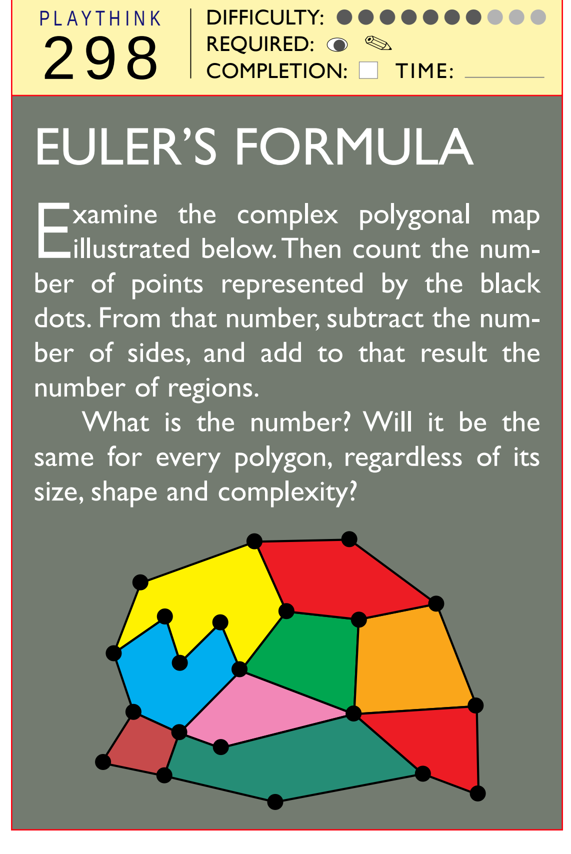
Six desired profiles are shown at left. Which of them can be made by this mechanism? Which are impossible to make?

PLAYTHINK 298 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

EULER'S FORMULA

Examine the complex polygonal map illustrated below. Then count the number of points represented by the black dots. From that number, subtract the number of sides, and add to that result the number of regions.

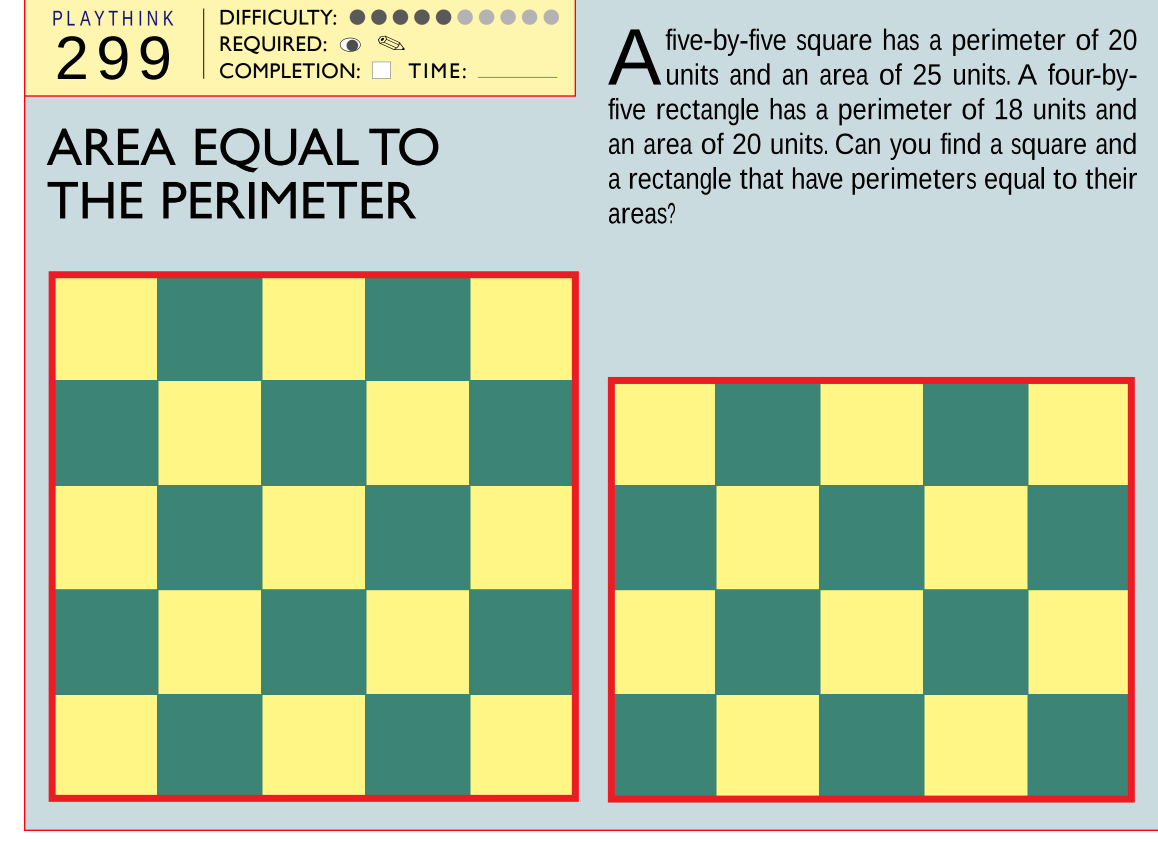
What is the number? Will it be the same for every polygon, regardless of its size, shape and complexity?



PLAYTHINK 299 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

AREA EQUAL TO THE PERIMETER

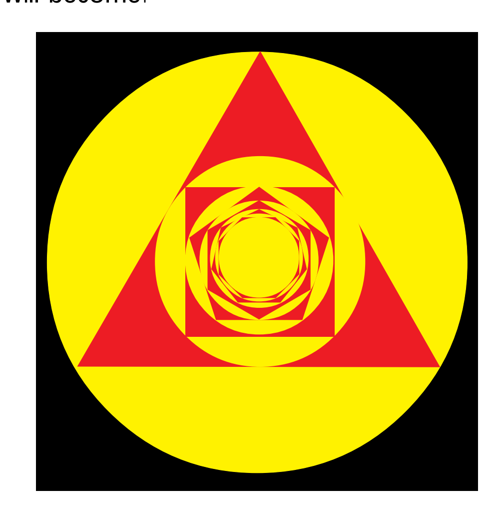
A five-by-five square has a perimeter of 20 units and an area of 25 units. A four-by-five rectangle has a perimeter of 18 units and an area of 20 units. Can you find a square and a rectangle that have perimeters equal to their areas?



PLAYTHINK 300 | DIFFICULTY: ●●●●●●●●●○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

INSCRIBED POLYGONS

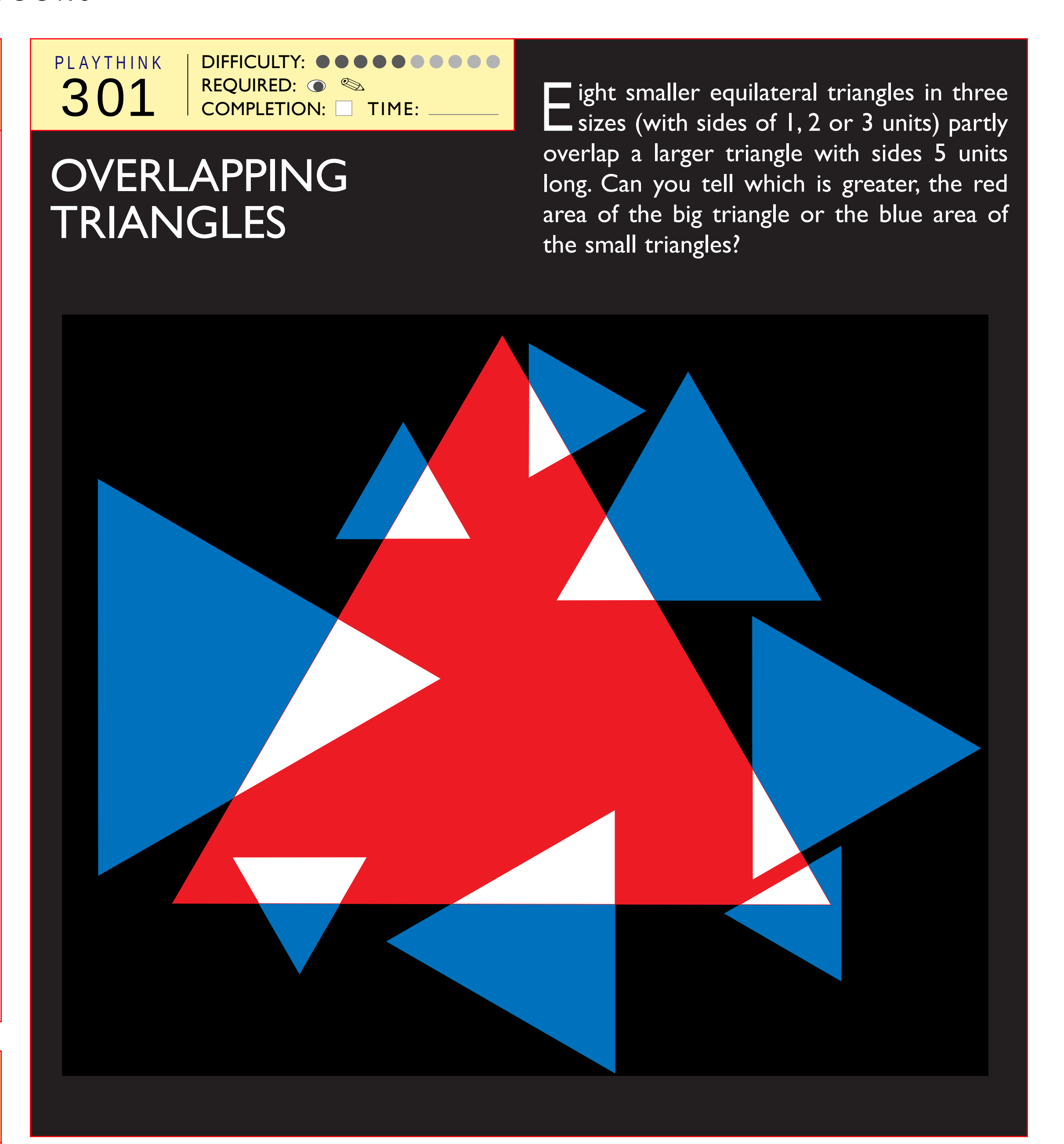
The outermost circle has a radius of 1 unit. In that circle, inscribe an equilateral triangle. In the triangle, inscribe a circle. Inside the circle, a square, then another circle, then a regular pentagon and so on. At each step the number of sides of the regular polygon will become larger, and the size of the circle will become smaller. Can you make a rough guess as to how small the circle eventually will become?



PLAYTHINK 301 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

OVERLAPPING TRIANGLES

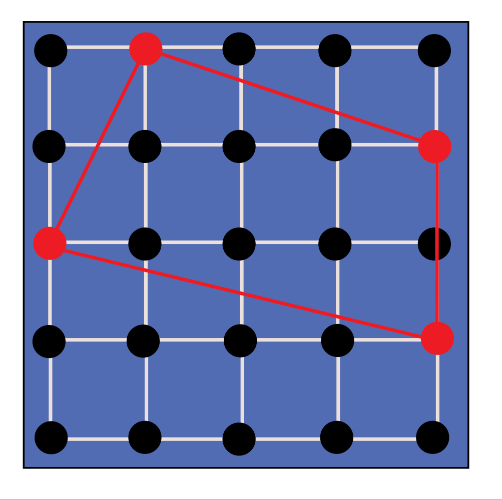
Eight smaller equilateral triangles in three sizes (with sides of 1, 2 or 3 units) partly overlap a larger triangle with sides 5 units long. Can you tell which is greater, the red area of the big triangle or the blue area of the small triangles?



PLAYTHINK 302 | DIFFICULTY: ●○○○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

PEG-BOARD AREA

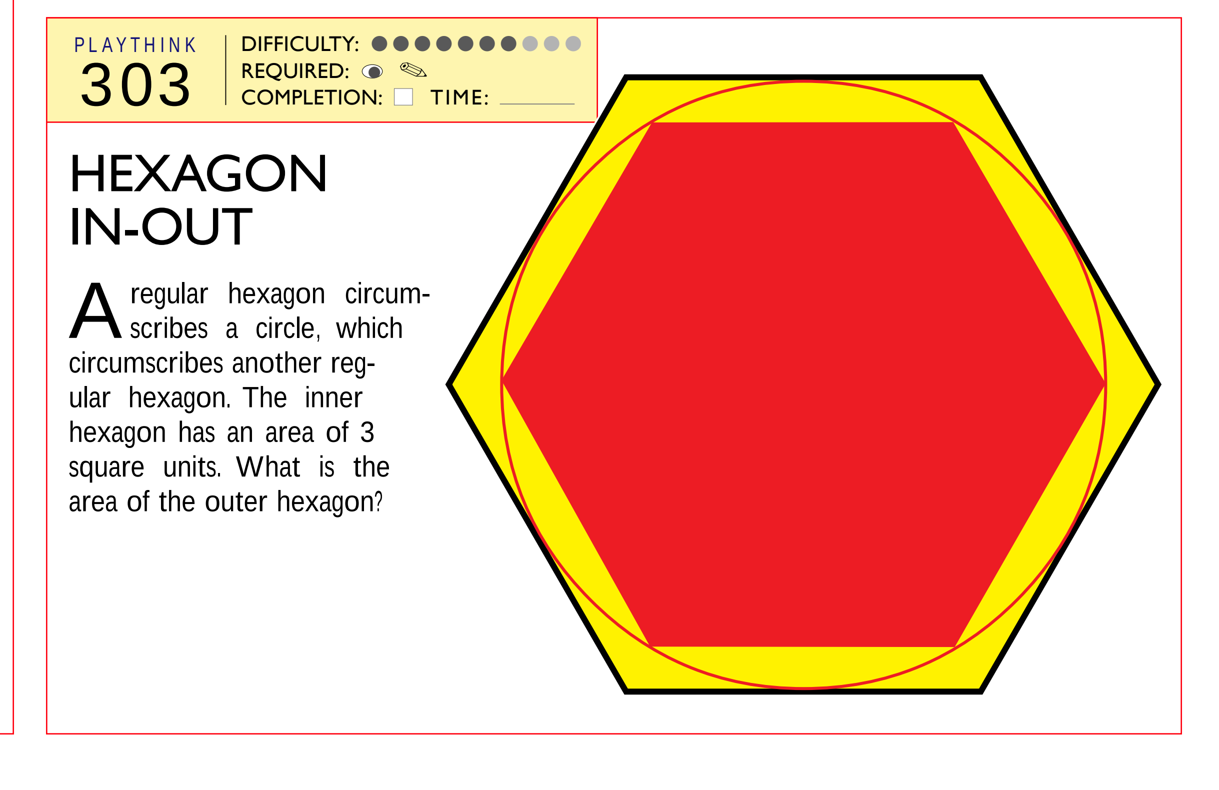
The Peg-Board shown below has a rubber band stretched around the four red pegs. Can you calculate the area enclosed by the rubber band without measuring anything?



PLAYTHINK 303 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

HEXAGON IN-OUT

A regular hexagon circumscribes a circle, which circumscribes another regular hexagon. The inner hexagon has an area of 3 square units. What is the area of the outer hexagon?

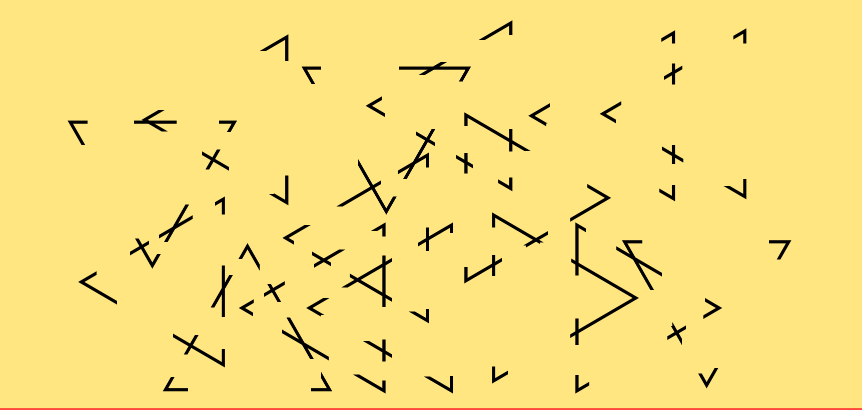


PLAYTHINK 304

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

TRIANGLE COUNT

A mask of unknown shape has been placed over this collection of triangles. Based on what you can see, how many triangles were there to begin with?



PLAYTHINK 305

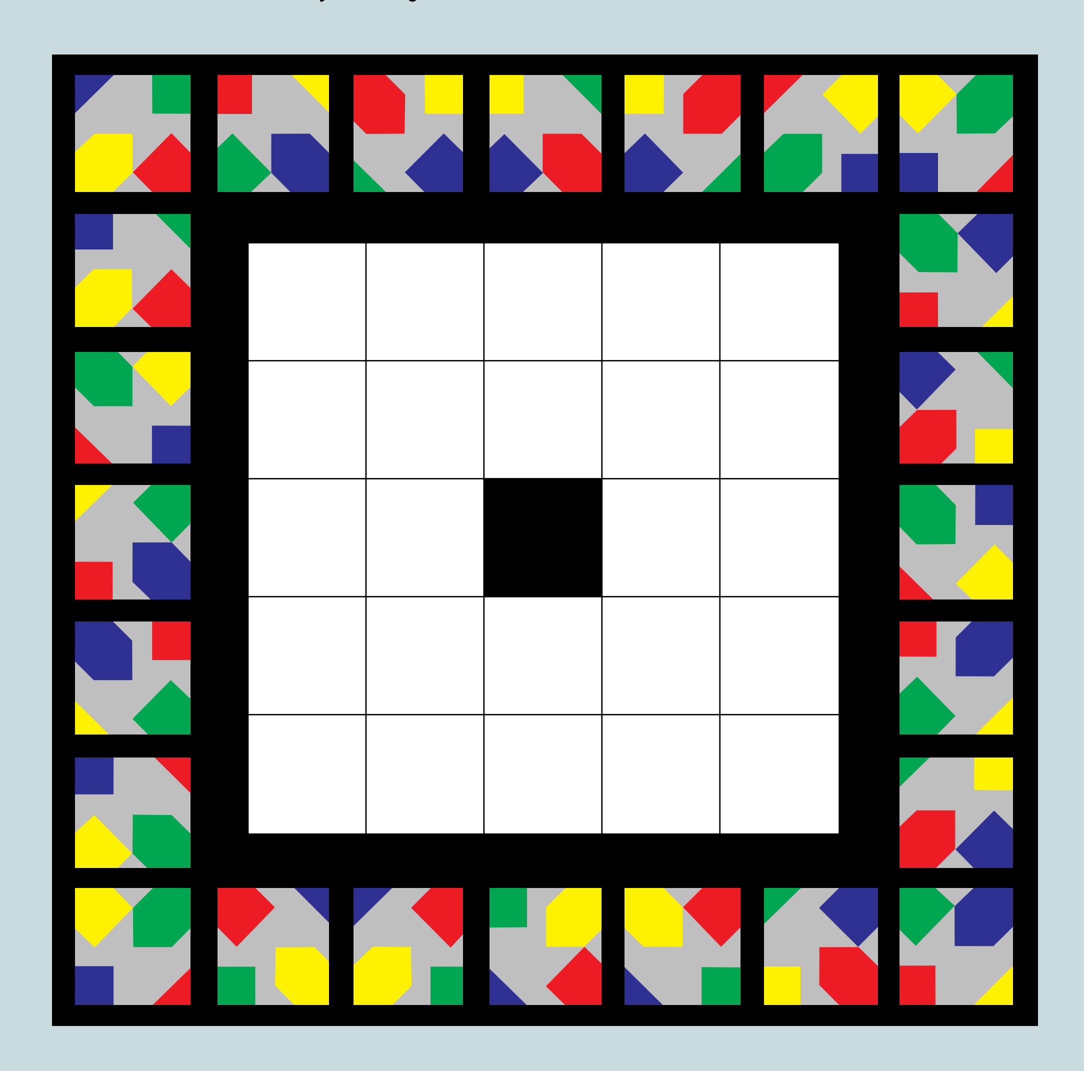
DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎ 📄 ✂
COMPLETION: ☐ TIME: ____

POLYGO

Polygo is a game based on the creation and recognition of complex shapes built up from four simple polygons: triangles, squares, pentagons and hexagons. By assembling these basic shapes, it is possible to construct a great variety of new polygons.

Each tile possesses polygons filled in with four different colors—red, yellow, green and blue. In a two-person game, each player chooses to be represented by two of those colors. The object is to create complex shapes of solid colors by joining four tiles side by side. Each polygon so created has a value, which is the sum of the four tiles from which it was built: one point for each triangle, two for each square, three for each pentagon and four for each hexagon. The player with the most points at the end wins.

To play a game of solitaire, fit the twenty-four square tiles into the grid, matching the colors at each corner.



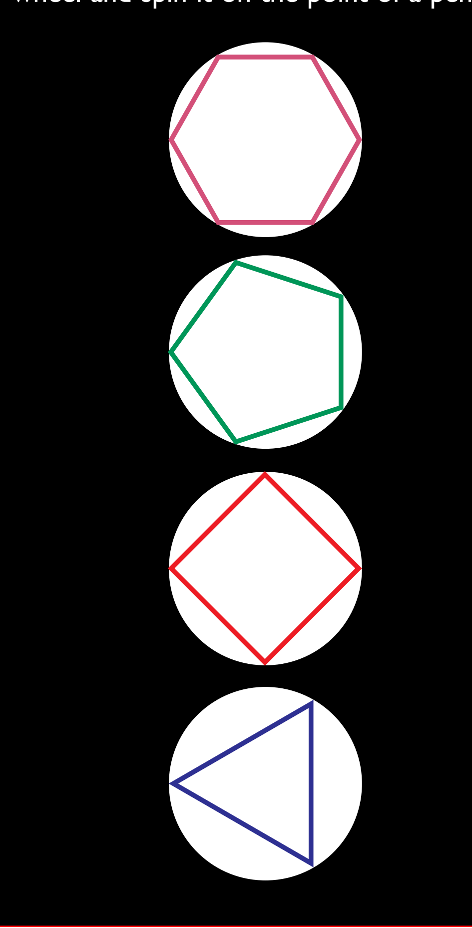
PLAYTHINK 306

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎ ✂
COMPLETION: ☐ TIME: ____

WHIRLING POLYGONS

A blue triangle, a red square, a green pentagon and a pink hexagon are inscribed in the same circle and are revolving within it. (Imagine that the four circles shown below are actually one circle.) Can you envision what you would see as the circle turns?

To double-check your answer, make a paper wheel inscribed with the four shapes. Make a small hole in the center of the wheel and spin it on the point of a pencil.



The Island Problem

As any primary school-teacher knows, area and volume are difficult ideas to grasp. Water will be spilled all over the classroom floor long before most children begin to understand the basic concept of conservation: that the amount of liquid inside a container does not depend on the container's shape.

Children are not the only ones who are confused by area and volume. Clever packaging fools many adults into thinking that they are buying more than they really are. Areas and volumes are easy to estimate for rectangular plots and boxes; estimating is more difficult for other shapes, especially ones with curved sides.

The ancient Greeks knew all about the significance of perimeter in terms of area enclosed—indeed, the word *meter* is derived from the Greek word for "measure around." Since many Greeks lived on islands, they had good reason to be aware of the pitfalls of measurement. After all, it is easy to see that the area of an island cannot be assessed using the time it takes to walk around it; a long coastline might simply mean that the shape of the island is irregular rather than that the island is large. Nevertheless, the custom was for landowners to base real estate values on the perimeter of their holdings, not the area.

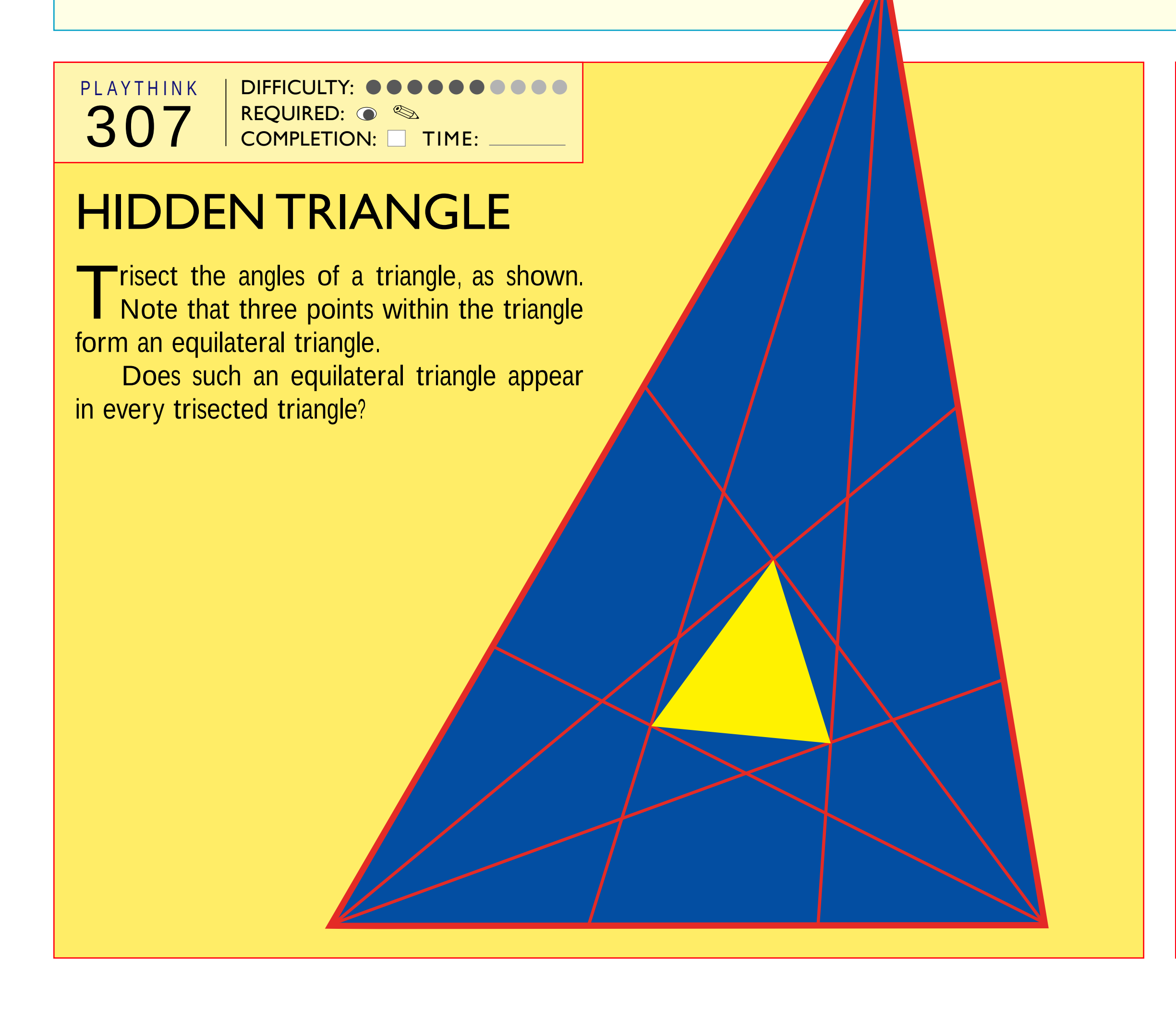
One ancient story tells of Dido, the princess of Tyre, who fled to a spot on the North African coast. There she was given a grant of land that was terribly small—equal to what could be covered by the hide of an ox. Undaunted, Dido had the hide cut into strips and sewn together to make one ribbon about a mile in length. Then, using the shoreline as one boundary, she had her supporters stretch the ribbon of hide in as big a semicircle as was possible. In this way one ordinary ox hide encompassed about 25 acres of land. On that spot Dido founded the famous and powerful city of Carthage.

PLAYTHINK 307 — DIFFICULTY: ●●●●●●○○○○ — REQUIRED: 👁 ✎ — COMPLETION: ☐ TIME: ____

HIDDEN TRIANGLE

Trisect the angles of a triangle, as shown. Note that three points within the triangle form an equilateral triangle.

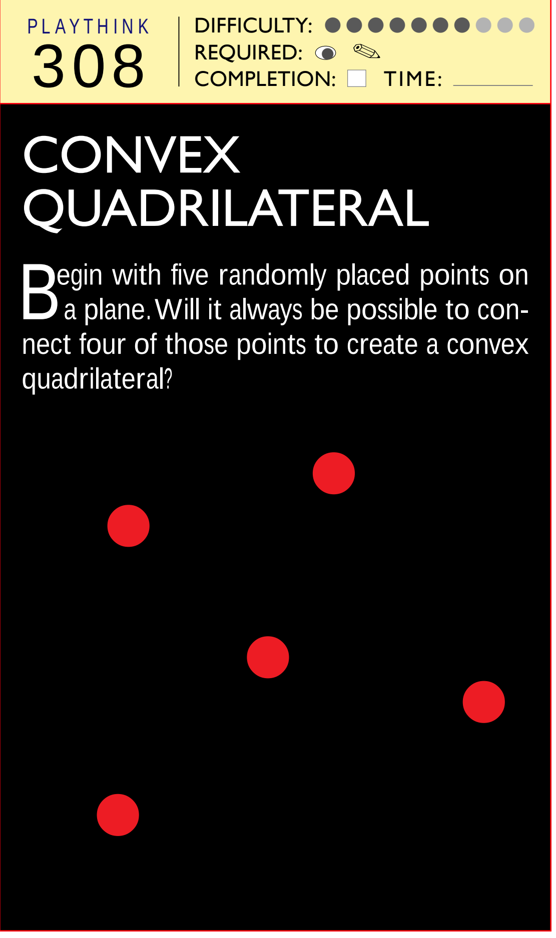
Does such an equilateral triangle appear in every trisected triangle?



PLAYTHINK 308 — DIFFICULTY: ●●●●●●●○○○ — REQUIRED: 👁 ✎ — COMPLETION: ☐ TIME: ____

CONVEX QUADRILATERAL

Begin with five randomly placed points on a plane. Will it always be possible to connect four of those points to create a convex quadrilateral?

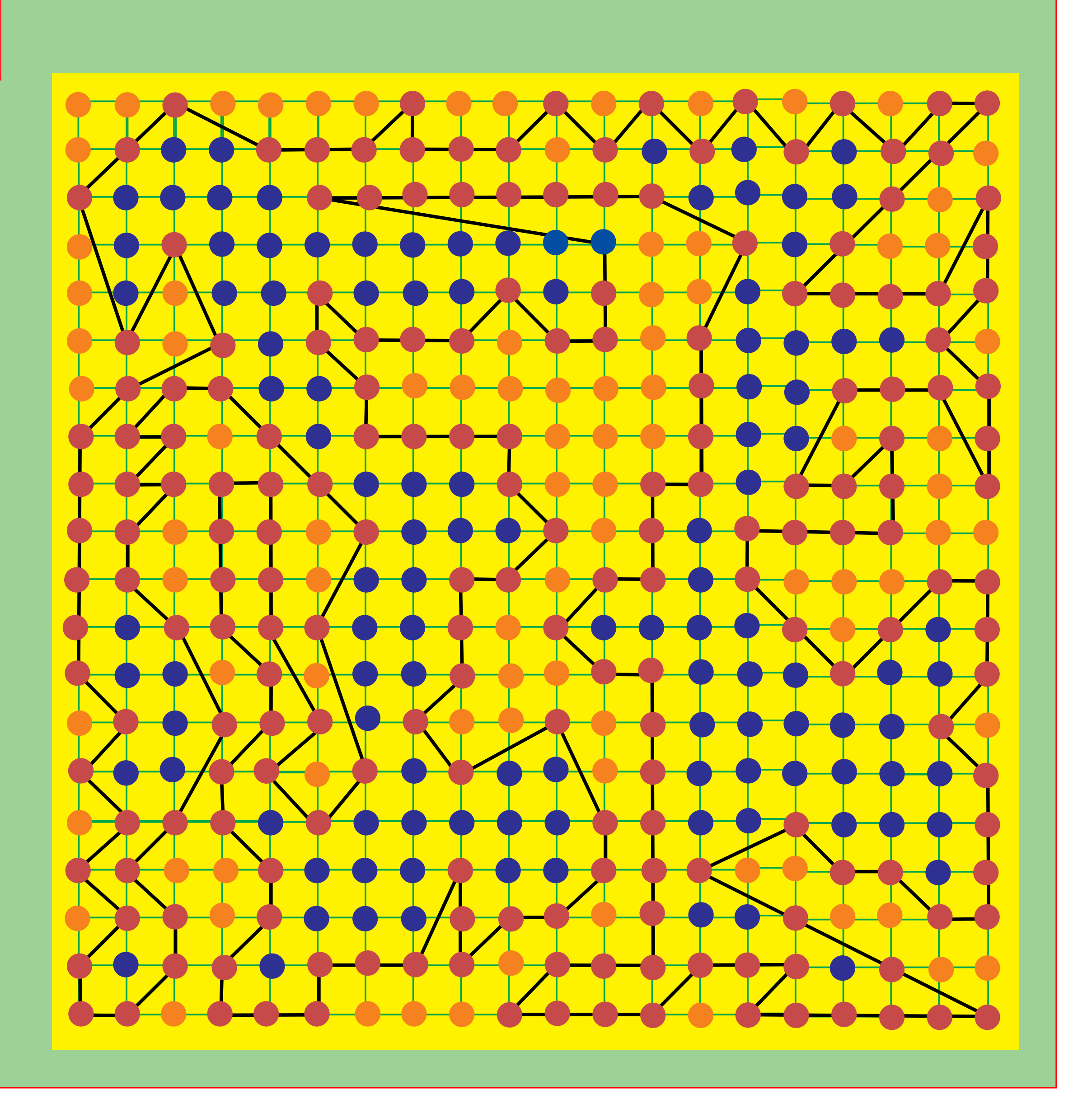


PLAYTHINK 309

DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

GOATS AND PEG-BOARDS

The blue pegs in the Peg-Board represent goats grazing in the fenced-off part of an orchard. Each goat needs an area equal to 1 square unit of the grid to graze on. Within the fenced-in areas (the fences are the black lines), how many goats can graze?

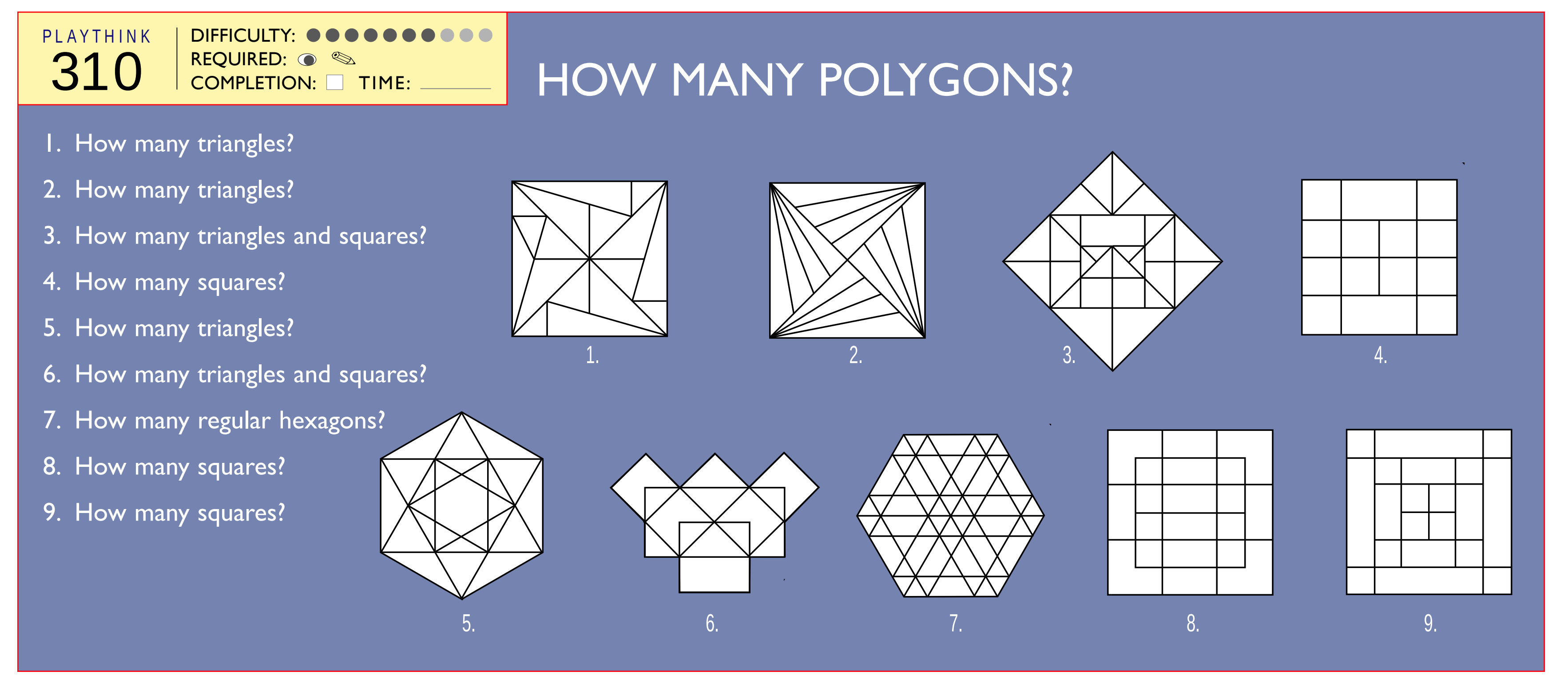


PLAYTHINK 310

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

HOW MANY POLYGONS?

1. How many triangles?
2. How many triangles?
3. How many triangles and squares?
4. How many squares?
5. How many triangles?
6. How many triangles and squares?
7. How many regular hexagons?
8. How many squares?
9. How many squares?

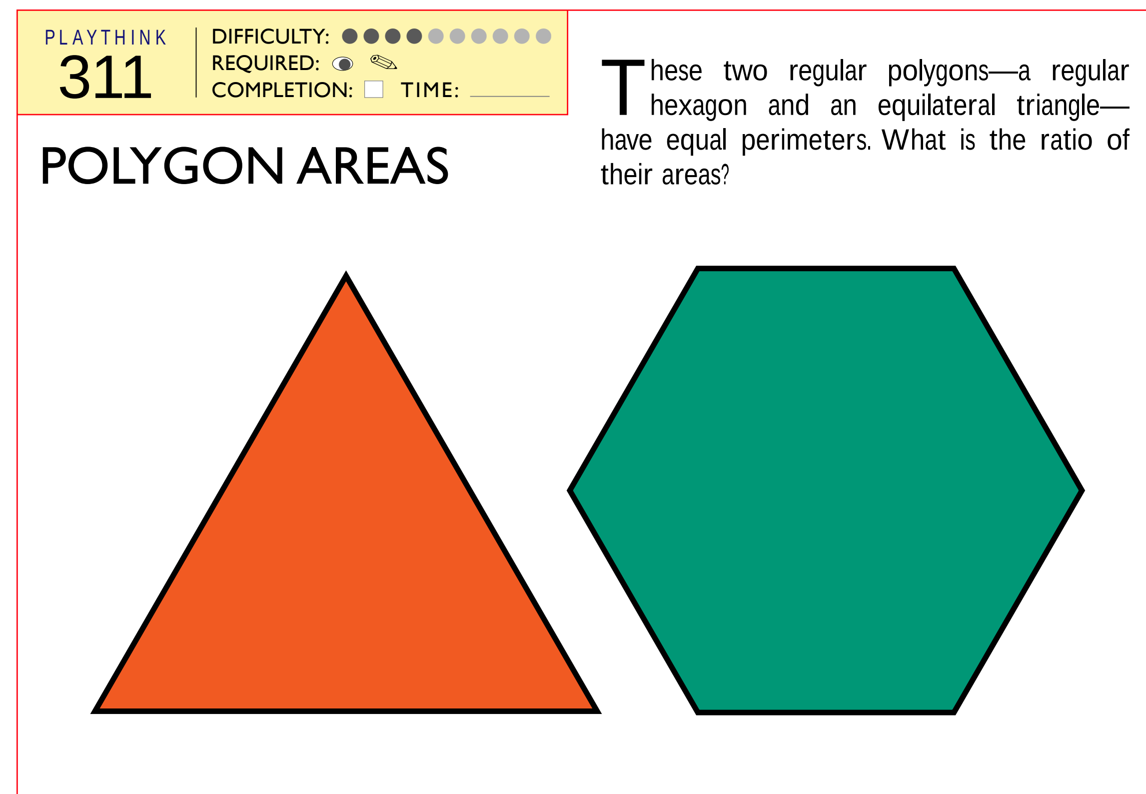


PLAYTHINK 311

DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

POLYGON AREAS

These two regular polygons—a regular hexagon and an equilateral triangle—have equal perimeters. What is the ratio of their areas?

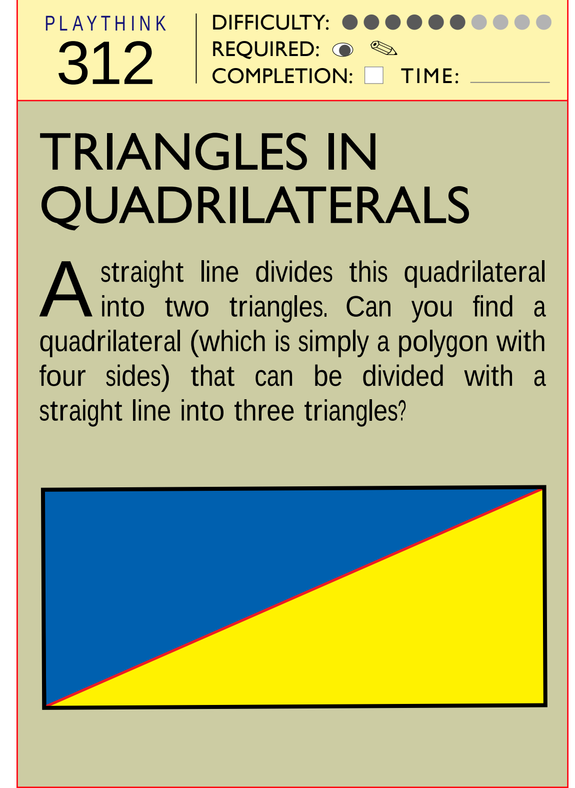


PLAYTHINK 312

DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

TRIANGLES IN QUADRILATERALS

A straight line divides this quadrilateral into two triangles. Can you find a quadrilateral (which is simply a polygon with four sides) that can be divided with a straight line into three triangles?

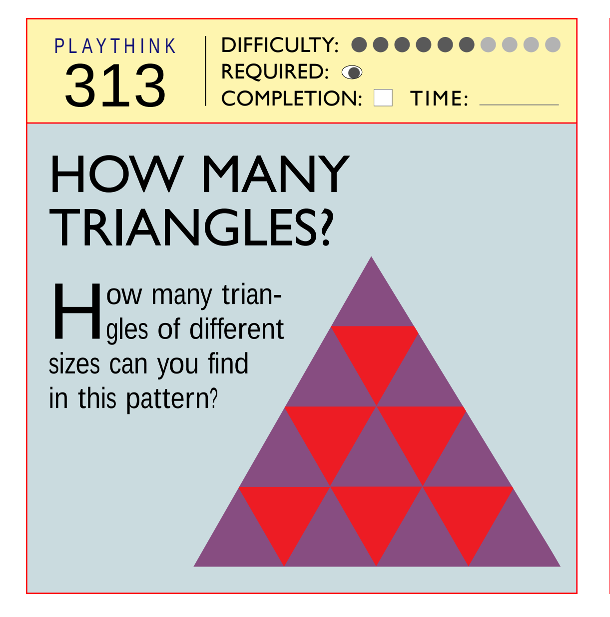


PLAYTHINK 313

DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

HOW MANY TRIANGLES?

How many triangles of different sizes can you find in this pattern?

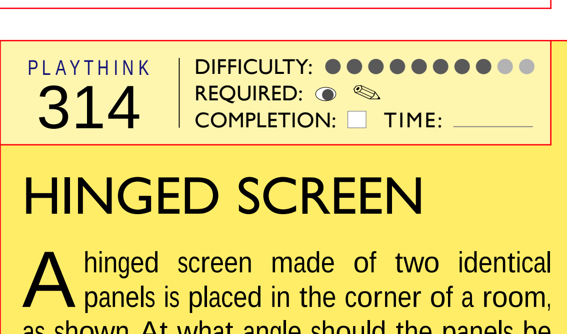


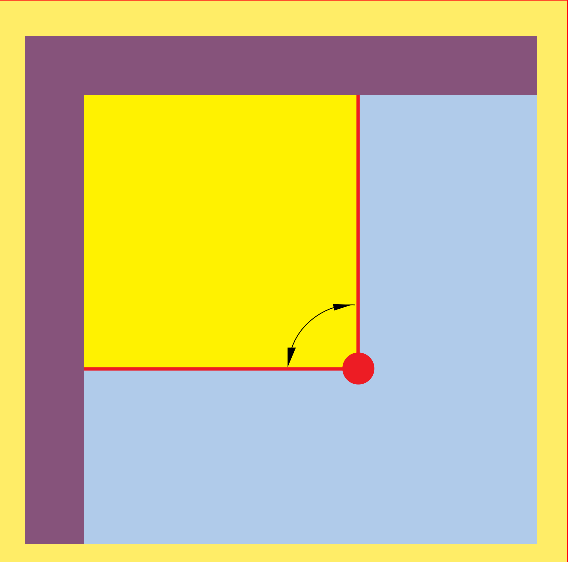
PLAYTHINK 314

DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

HINGED SCREEN

A hinged screen made of two identical panels is placed in the corner of a room, as shown. At what angle should the panels be opened to enclose—with the walls—the largest possible area?



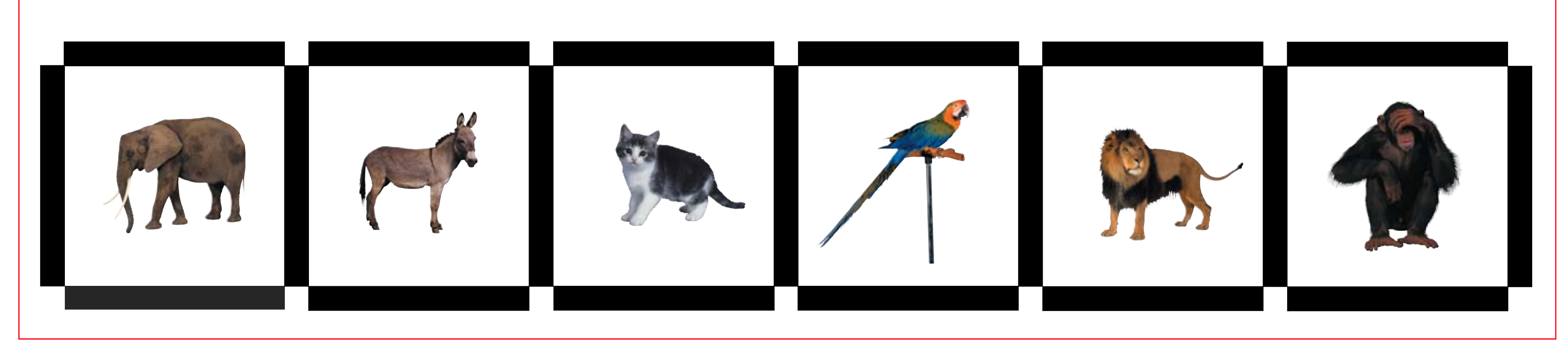


PLAYTHINK 315

DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

BUILDING CAGES

Six cages, built of nineteen panels of equal lengths, hold six different animals. Disaster strikes, and seven of those panels become unusable. With just the twelve remaining panels, can you construct six new cages to hold the animals? Each animal must be completely surrounded by panels and must not share its cage.



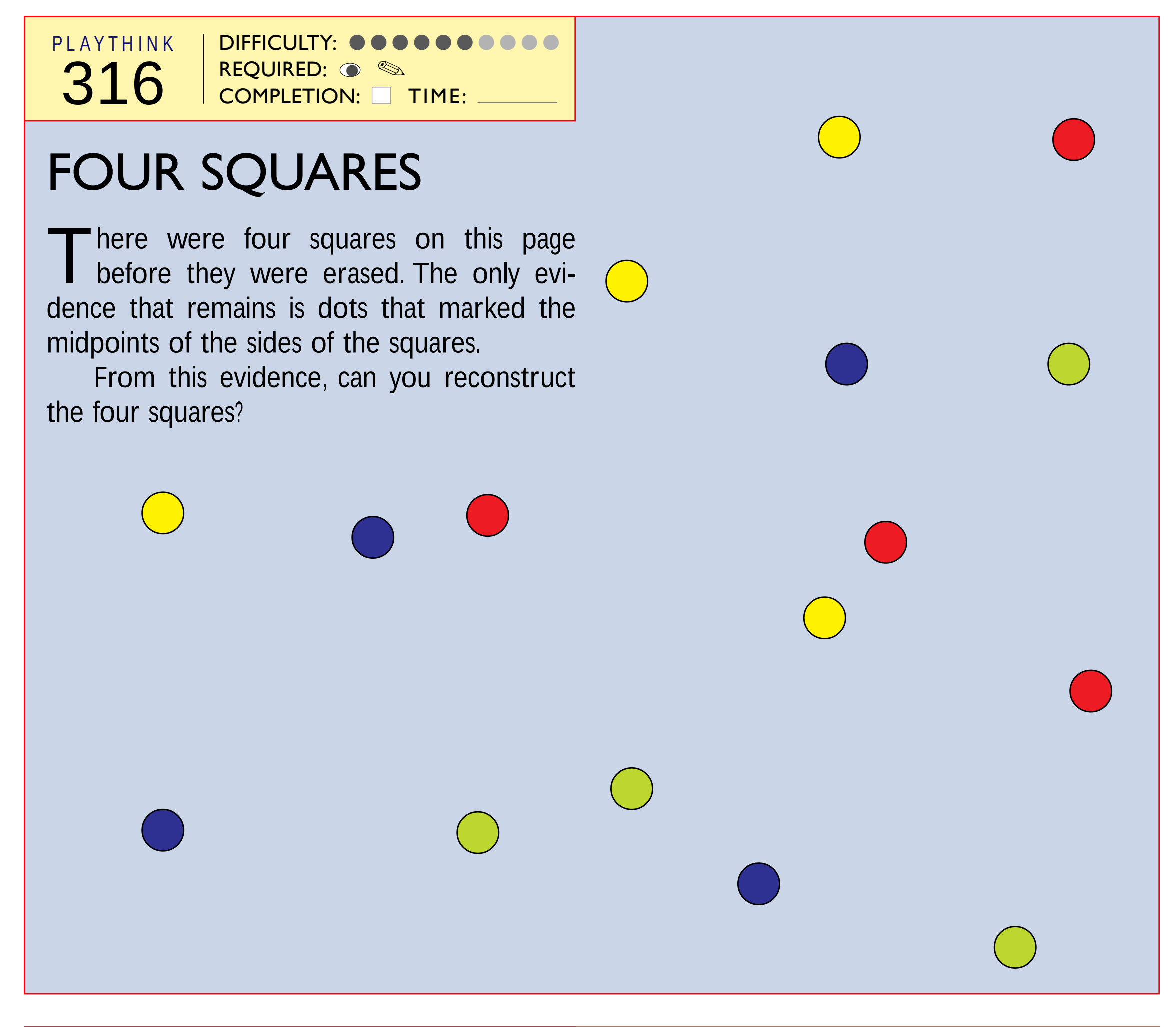
PLAYTHINK 316

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

FOUR SQUARES

There were four squares on this page before they were erased. The only evidence that remains is dots that marked the midpoints of the sides of the squares.

From this evidence, can you reconstruct the four squares?

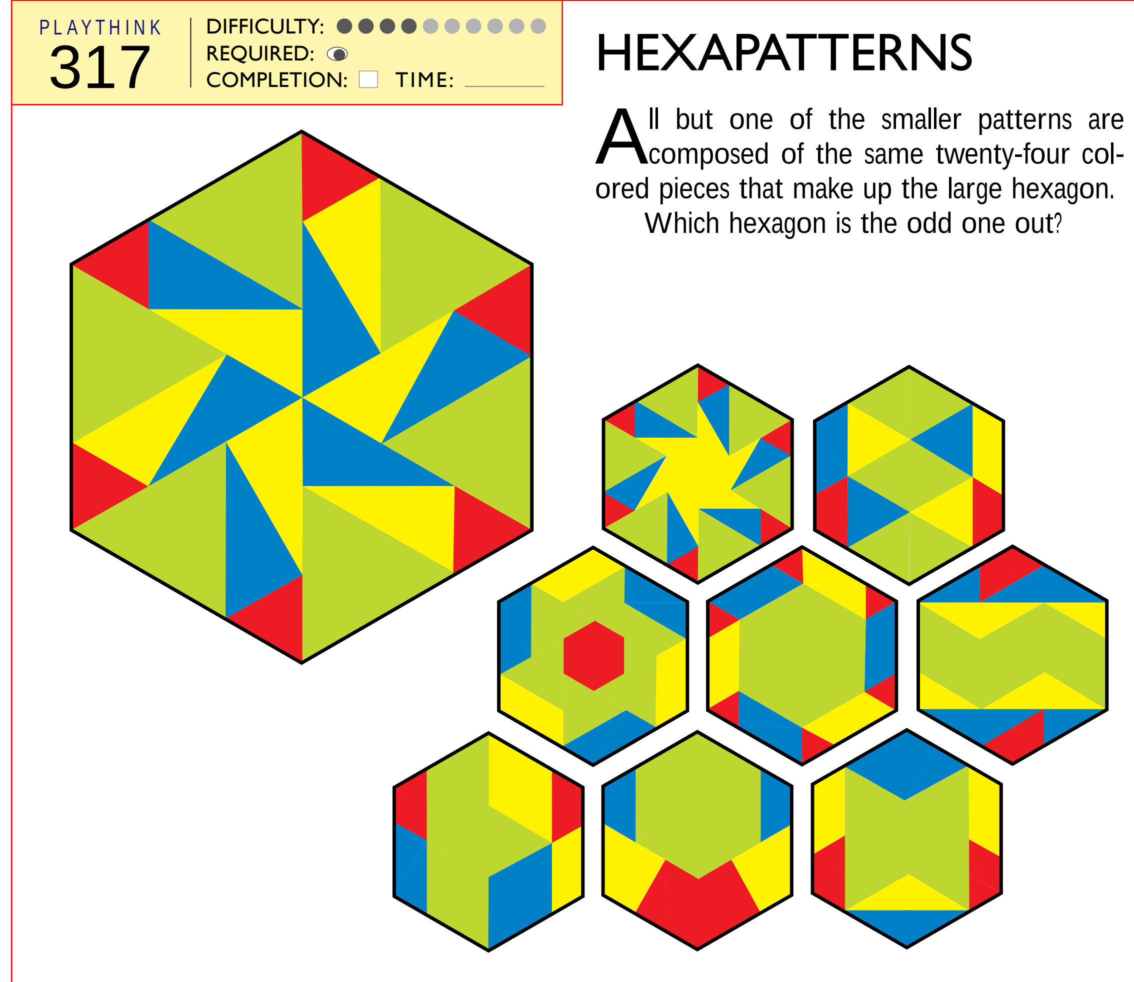


PLAYTHINK 317

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

HEXAPATTERNS

All but one of the smaller patterns are composed of the same twenty-four colored pieces that make up the large hexagon. Which hexagon is the odd one out?



PLAYTHINK 318

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

TRIANGLES INSCRIBED 1

How many triangles can you draw from the vertices of a heptagon that don't possess sides that are also sides of the heptagon?

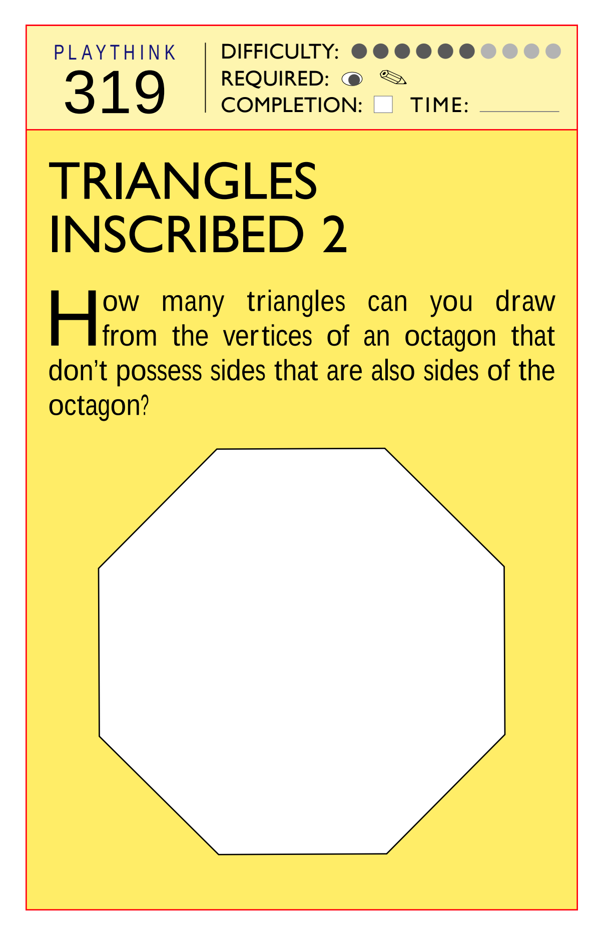
In a square and a pentagon, for instance, you can't draw such a triangle; in a regular hexagon you can draw two, as shown below.

PLAYTHINK 319

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

TRIANGLES INSCRIBED 2

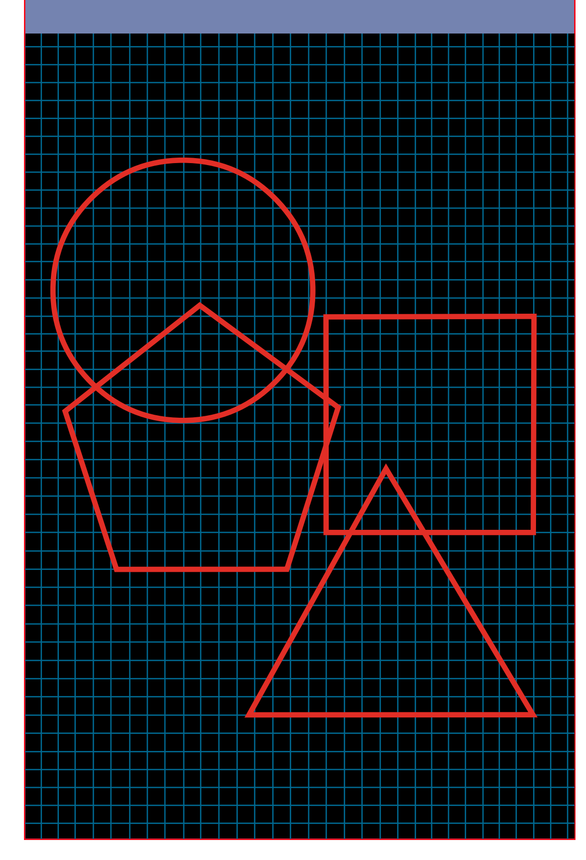
How many triangles can you draw from the vertices of an octagon that don't possess sides that are also sides of the octagon?



PLAYTHINK 320 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: | COMPLETION: ☐ TIME: ______

EQUAL PERIMETERS

All four shapes—circle, square, triangle and pentagon—are equal in perimeter. Rank the shapes in order of area, from largest to smallest. You may use logic, calculations or the superimposed grid to find your answer.

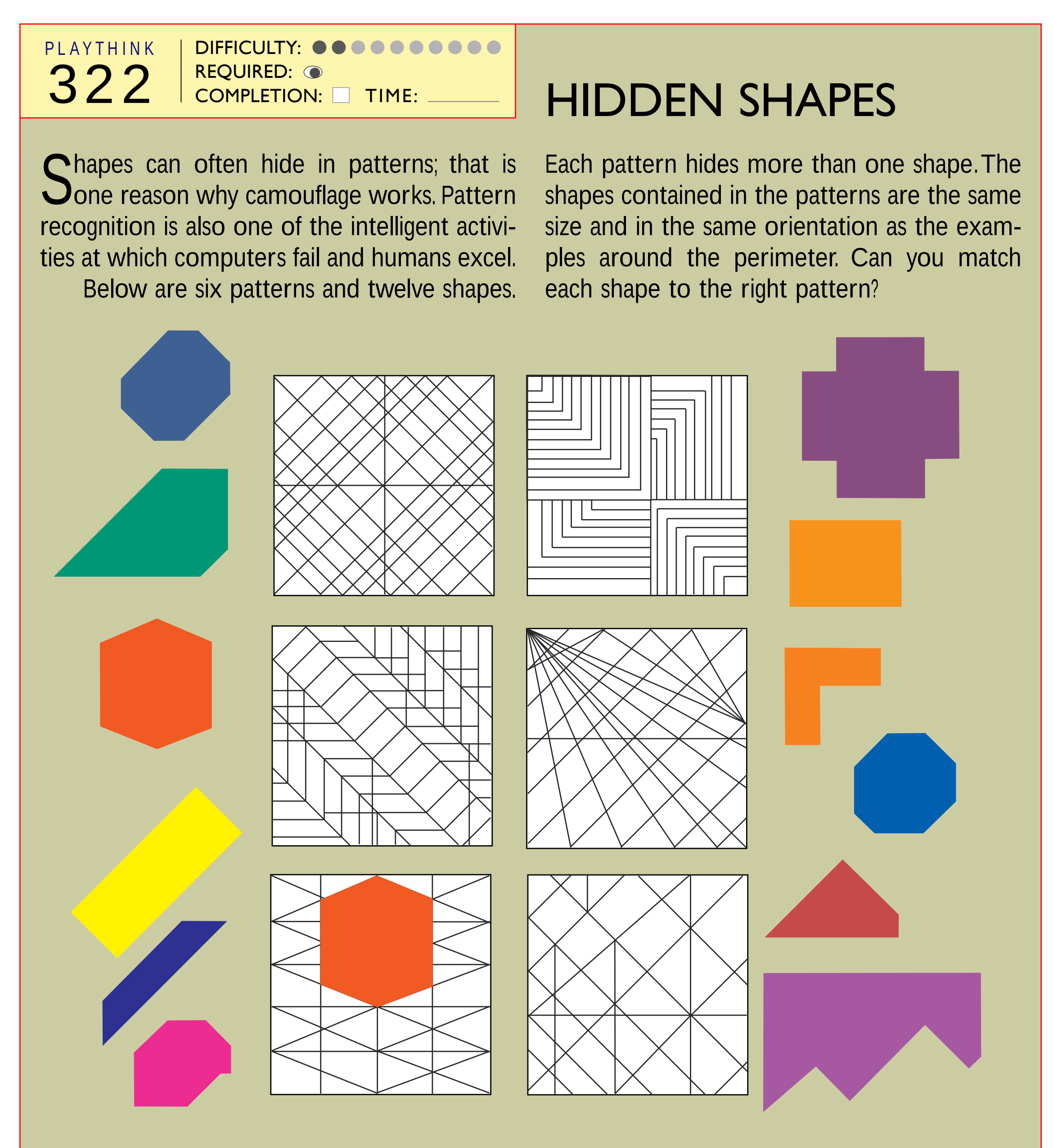


PLAYTHINK 322 | DIFFICULTY: ●●○○○○○○○○ | REQUIRED: | COMPLETION: ☐ TIME: ______

HIDDEN SHAPES

Shapes can often hide in patterns; that is one reason why camouflage works. Pattern recognition is also one of the intelligent activities at which computers fail and humans excel. Below are six patterns and twelve shapes.

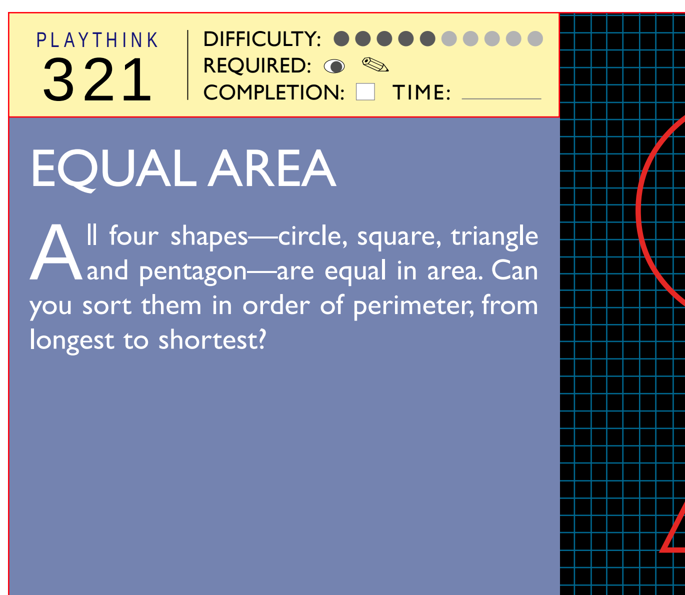
Each pattern hides more than one shape. The shapes contained in the patterns are the same size and in the same orientation as the examples around the perimeter. Can you match each shape to the right pattern?

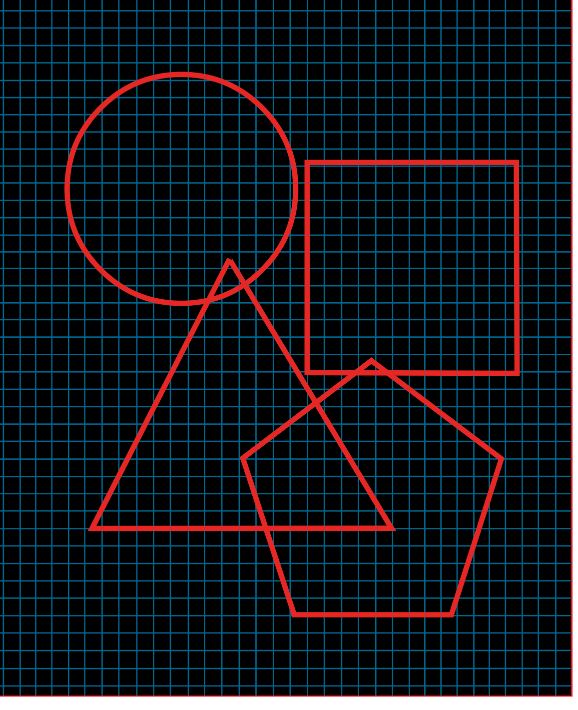


PLAYTHINK 321 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: | COMPLETION: ☐ TIME: ______

EQUAL AREA

All four shapes—circle, square, triangle and pentagon—are equal in area. Can you sort them in order of perimeter, from longest to shortest?

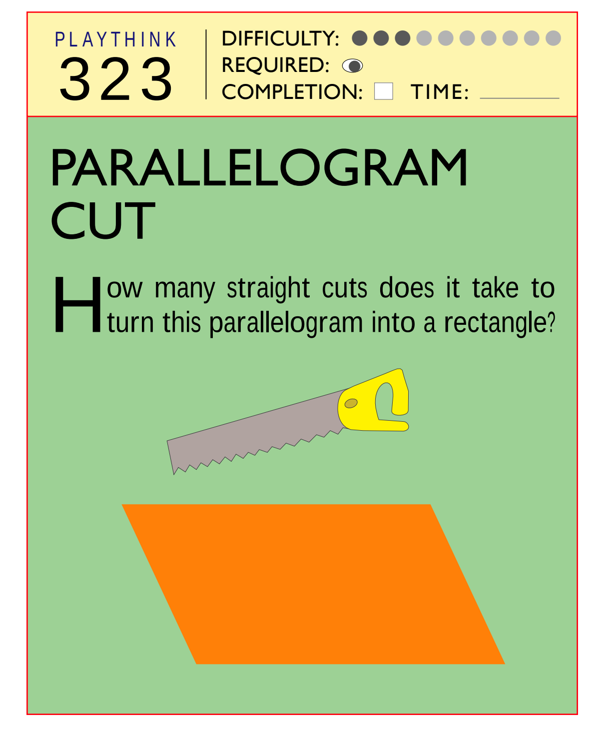




PLAYTHINK 323 | DIFFICULTY: ●●●○○○○○○○ | REQUIRED: | COMPLETION: ☐ TIME: ______

PARALLELOGRAM CUT

How many straight cuts does it take to turn this parallelogram into a rectangle?



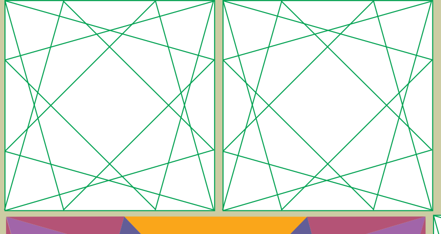
PLAYTHINK 324 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

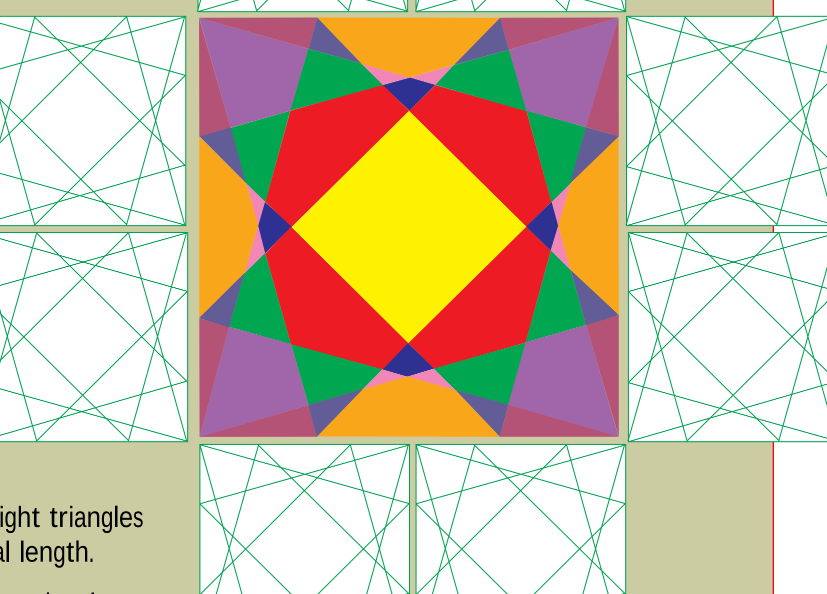
FIND THE POLYGONS

At first glance the designs shown at right may seem like just squares crisscrossed with lines. But look again: you will spot regularities and symmetries—squares, triangles, rhombuses, kites and so on. In fact, there is an even more remarkable property in this pattern. It is composed simply of four equilateral triangles of the largest size that will fit within the square. A vertex of one of the triangles is at each corner of the square.

The object of this puzzle is to find the items listed below. To make things easier for you, I have provided a design that shows the shapes you are looking for. A pencil or pen may come in handy for marking the shapes you find.

1. First, find the four large equilateral triangles that create the pattern in each square.
2. Then find four squares. They are not all the same size.
3. Find four medium-sized equilateral triangles.
4. Find eight small equilateral triangles.
5. Find four halves of regular hexagons. (A regular hexagon has six sides of equal length.)
6. Find two large identical but irregular six-sided polygons.
7. Find two medium-sized identical but irregular six-sided polygons.
8. Find two small identical but irregular six-sided polygons.
9. Find an irregular eight-sided polygon.
10. Find four large right-angled isosceles triangles (that is, a right triangle with two equal legs).
11. Find four medium-sized right-angled isosceles triangles.
12. Find the eight largest right triangles that do not have sides of equal length.
13. Find eight medium-sized right triangles that do not have sides of equal length.
14. Find the eight smallest right triangles that do not have sides of equal length.
15. Find two large rhombuses of equal area.
16. Find four large parallelograms.
17. Find four medium-sized parallelograms.

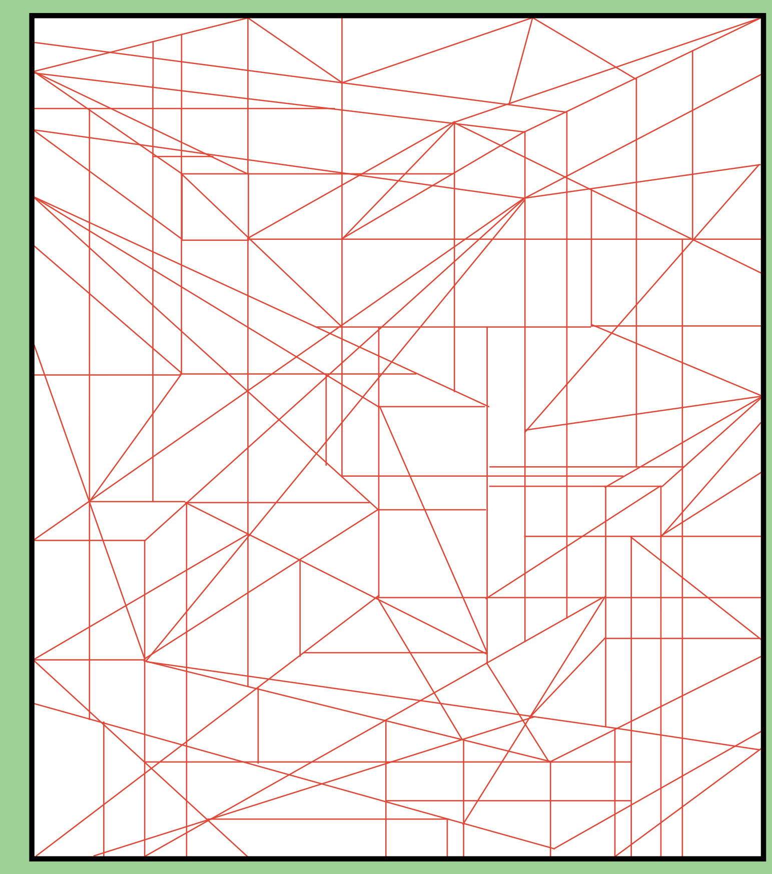




PLAYTHINK 325 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

HOW MANY CUBES?

Can you find six cubes portrayed in perspective in the pattern at right?



The Square

The square is the simplest, most symmetrical and most perfect quadrilateral. Its sides are all equal, and its angles are all right angles. But its simplicity is deceptive: the square conceals within its austere geometry untold intellectual depths. From the Pythagorean theorem to Einstein's theory of general relativity, from the flat geometry of Euclid to the curvature of space, there are only three or four short steps, and the square is their common thread.

The square is found in the crystals of many minerals, including common salt. It played a role in the structure of the Hebrew alphabet and gave birth to the ancient games of chess, go, solitaire and dominoes. The square has provided the proportions of famous ancient structures, as well as daring modern buildings. The endless tracts of the American Midwest are laid out in squares one mile on a side.

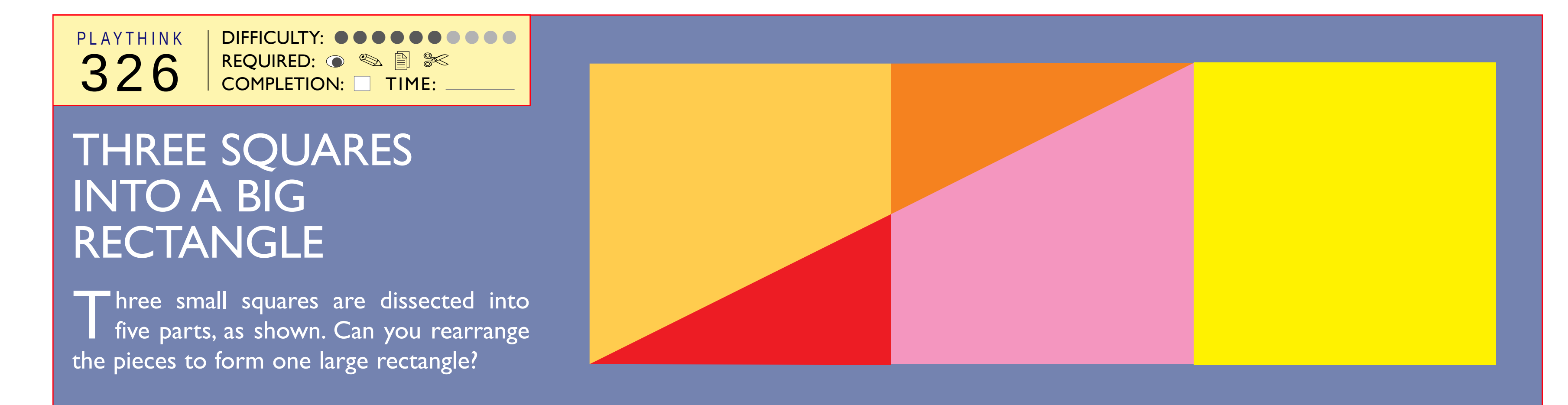
The square is everywhere.

> "IT'S A SQUARE: BEAUTIFUL, EQUILATERAL AND RECTANGULAR."
>
> —LEWIS CARROLL

PLAYTHINK 326 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: eye, pencil, paper, scissors | COMPLETION: ☐ TIME: ____

THREE SQUARES INTO A BIG RECTANGLE

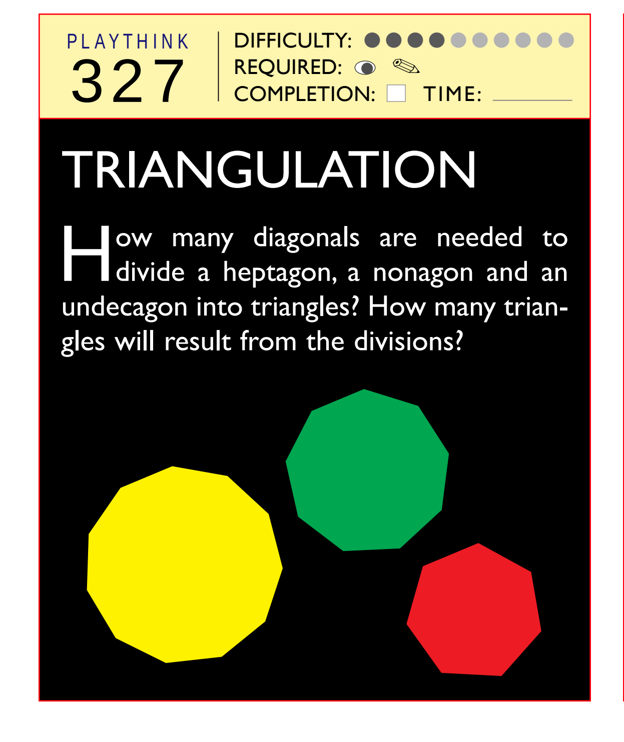
Three small squares are dissected into five parts, as shown. Can you rearrange the pieces to form one large rectangle?



PLAYTHINK 327 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: eye, pencil | COMPLETION: ☐ TIME: ____

TRIANGULATION

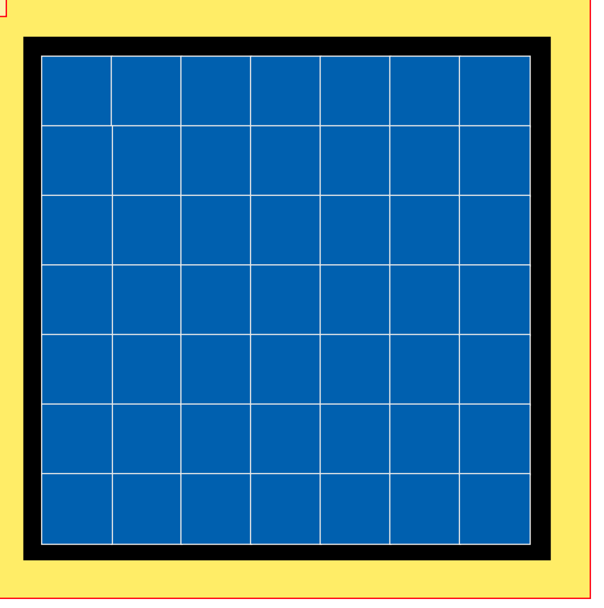
How many diagonals are needed to divide a heptagon, a nonagon and an undecagon into triangles? How many triangles will result from the divisions?



PLAYTHINK 328 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: eye, pencil | COMPLETION: ☐ TIME: ____

INSCRIBED SQUARE

Can you inscribe a square on this seven-by-seven square matrix so that the sides of the inscribed square are of an integer length, in units of the grid? The vertices of the new square must lie on grid-line intersections.



PLAYTHINK 329

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

CONDITION TRIANGLE

Four sets of strips of different lengths are shown. The sets are of lengths 3, 4 and 6; 3, 5 and 7; 4, 5 and 9 and 3, 5 and 9. Are there any sets of strips that cannot form a triangle when joined together?

3 4 6 3 5 7 4 5 9 3 5 9

PLAYTHINK 330

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

ART GALLERY

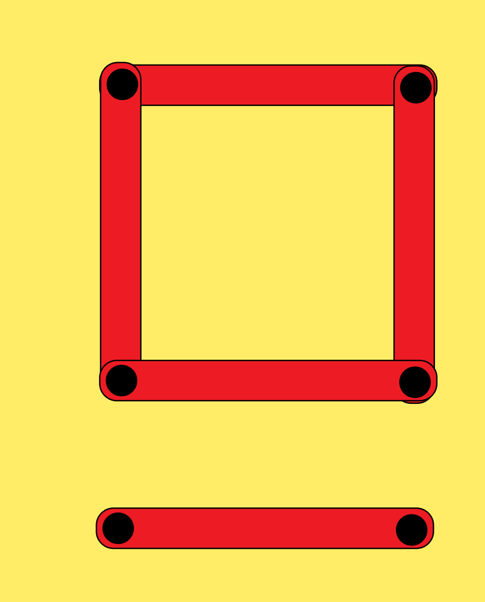
An art gallery has fourteen walls of identical length. Several revolving security cameras keep a watch on the walls. The gallery owner would like to redesign the space so that the total number and length of walls stay the same and yet every square inch of wall can be watched with just one revolving camera. What design accomplishes this goal?



PLAYTHINK 331 | DIFFICULTY: ●●●●●●●●●○ | REQUIRED: | COMPLETION: ☐ TIME: ____

RIGID SQUARE

Construct a square from four identical linkages hinged at the corners, as shown. Such a figure is capable of moving on its hinges to become a rhombus. How many linkages of the same length must be added in the same plane to make the square rigid? The linkages must be in the same plane as the square, and each one can be connected only at the hinges.

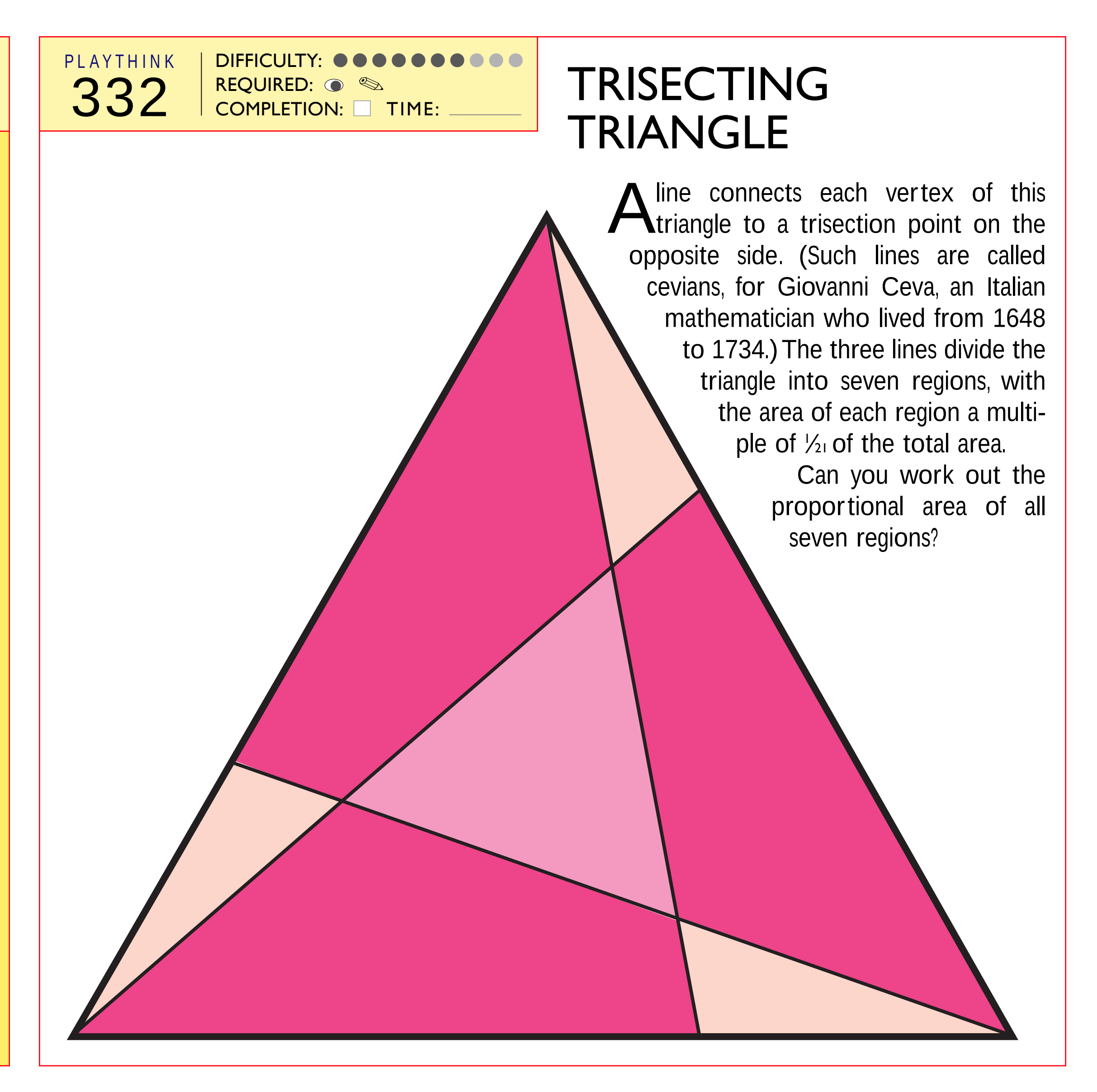


PLAYTHINK 332 | DIFFICULTY: ●●●●●●●●○○○ | REQUIRED: | COMPLETION: ☐ TIME: ____

TRISECTING TRIANGLE

A line connects each vertex of this triangle to a trisection point on the opposite side. (Such lines are called cevians, for Giovanni Ceva, an Italian mathematician who lived from 1648 to 1734.) The three lines divide the triangle into seven regions, with the area of each region a multiple of $\frac{1}{21}$ of the total area.

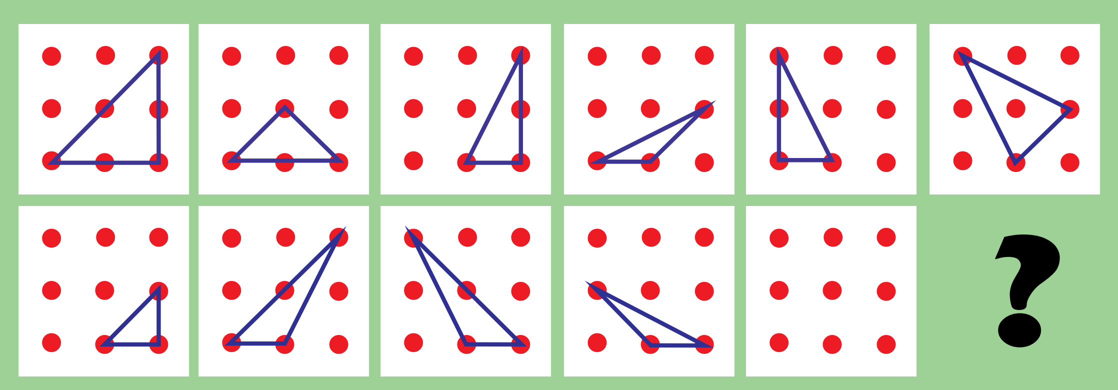
Can you work out the proportional area of all seven regions?



PLAYTHINK 333 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: | COMPLETION: ☐ TIME: ____

PEG-BOARD TRIANGLES

Not counting rotationally symmetrical variations and translations, there are exactly eleven different triangles that can be formed by connecting three points on a three-by-three Peg-Board. Ten are shown here. Can you find the eleventh?

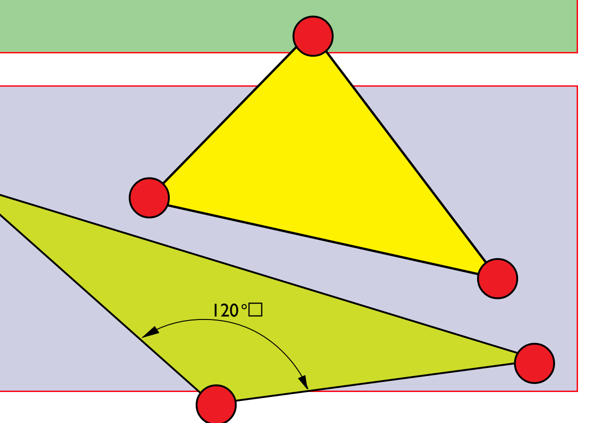


PLAYTHINK 334 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: | COMPLETION: ☐ TIME: ____

MINIMAL TRIANGLES

Three villages would like to be connected by a set of paths but want to do this in the most economical way. Can you find a general way of determining how this can be done?

To make it easier to solve this problem, examine the two triangles at right. Can you find the point in each triangle that is at a minimum total distance from the three vertices?



PLAYTHINK 335

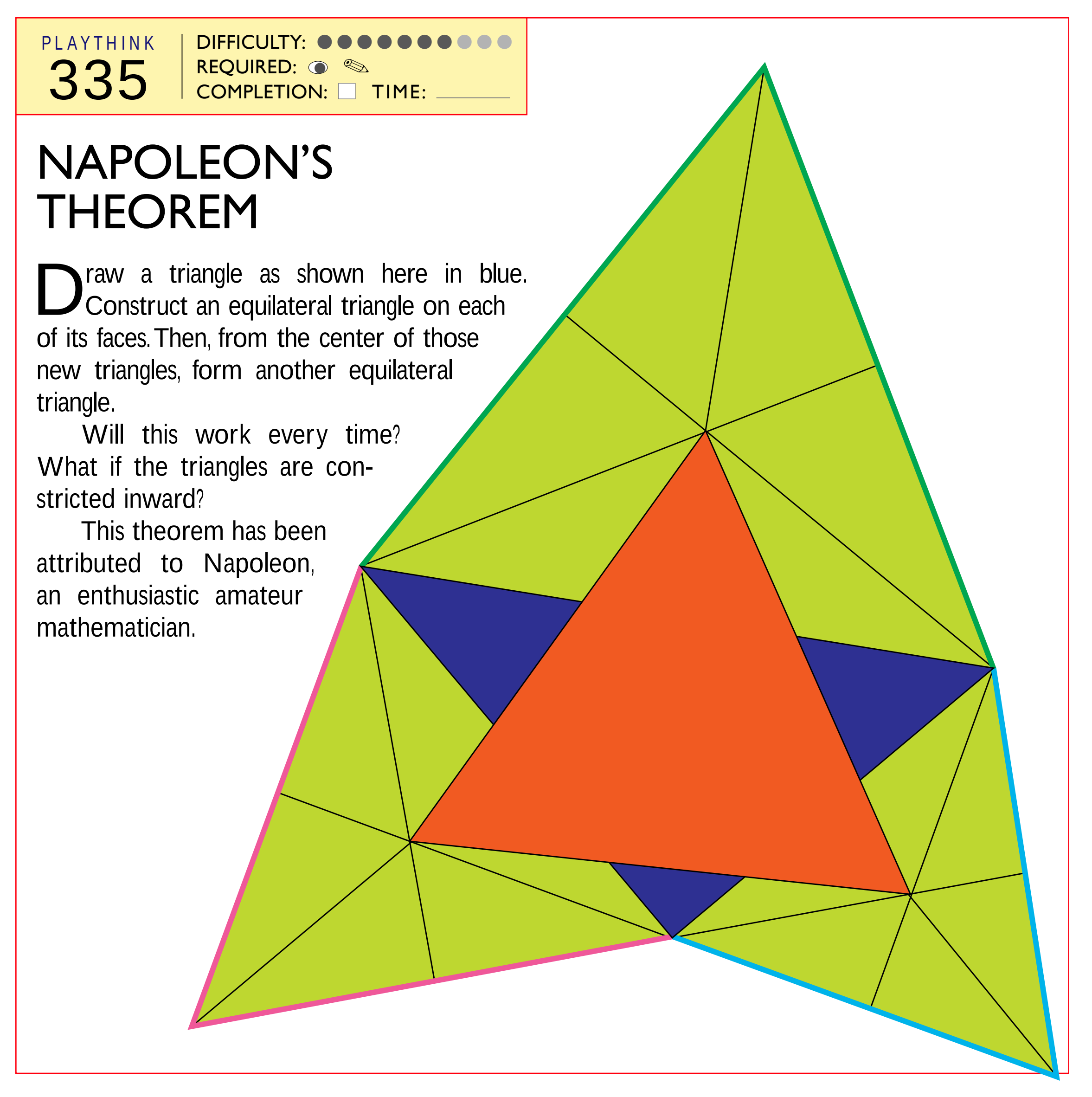
DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

NAPOLEON'S THEOREM

Draw a triangle as shown here in blue. Construct an equilateral triangle on each of its faces. Then, from the center of those new triangles, form another equilateral triangle.

Will this work every time? What if the triangles are constricted inward?

This theorem has been attributed to Napoleon, an enthusiastic amateur mathematician.

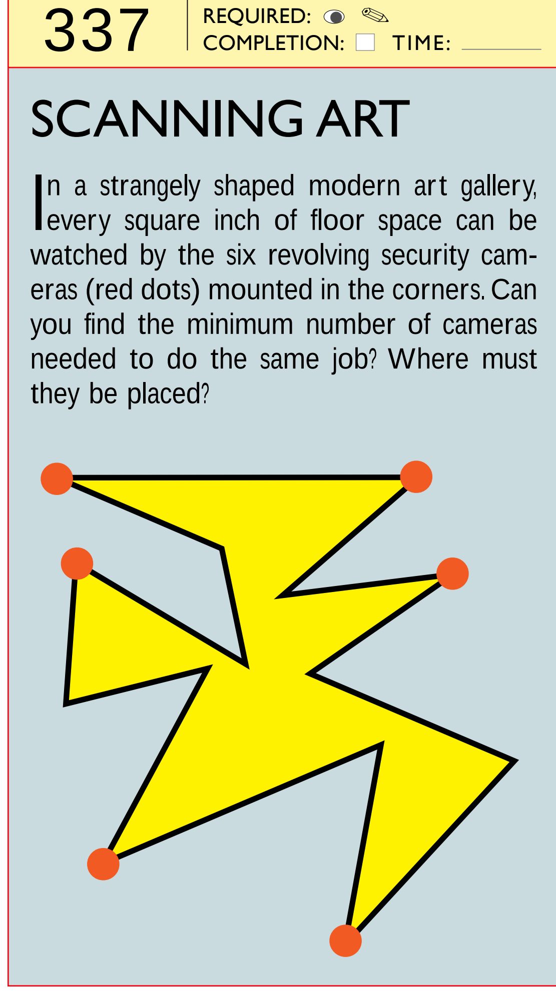


PLAYTHINK 337

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

SCANNING ART

In a strangely shaped modern art gallery, every square inch of floor space can be watched by the six revolving security cameras (red dots) mounted in the corners. Can you find the minimum number of cameras needed to do the same job? Where must they be placed?



PLAYTHINK 336

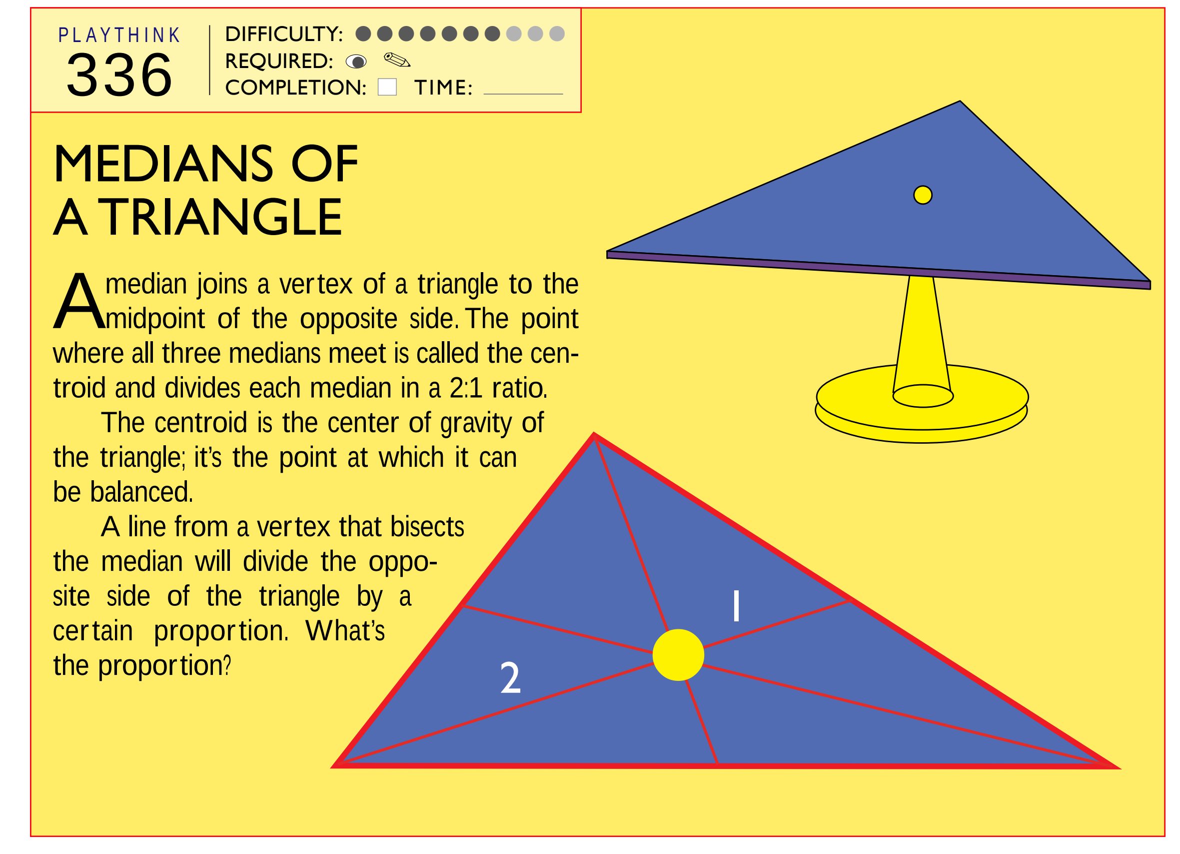
DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

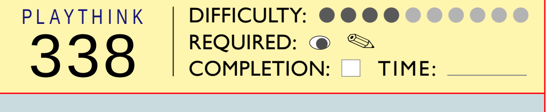
MEDIANS OF A TRIANGLE

A median joins a vertex of a triangle to the midpoint of the opposite side. The point where all three medians meet is called the centroid and divides each median in a 2:1 ratio.

The centroid is the center of gravity of the triangle; it's the point at which it can be balanced.

A line from a vertex that bisects the median will divide the opposite side of the triangle by a certain proportion. What's the proportion?

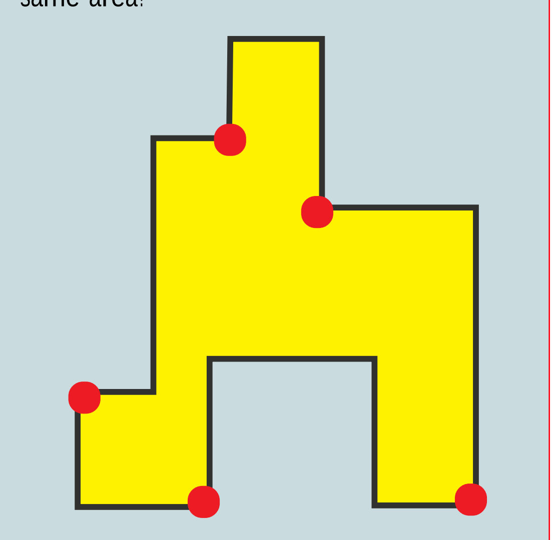



PLAYTHINK 338

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

SCANNING BANK

Five revolving security cameras (red dots) are installed in the corners of a bank. The cameras can cover every square inch of floor area. Where would you mount just three cameras so that they can cover the same area?



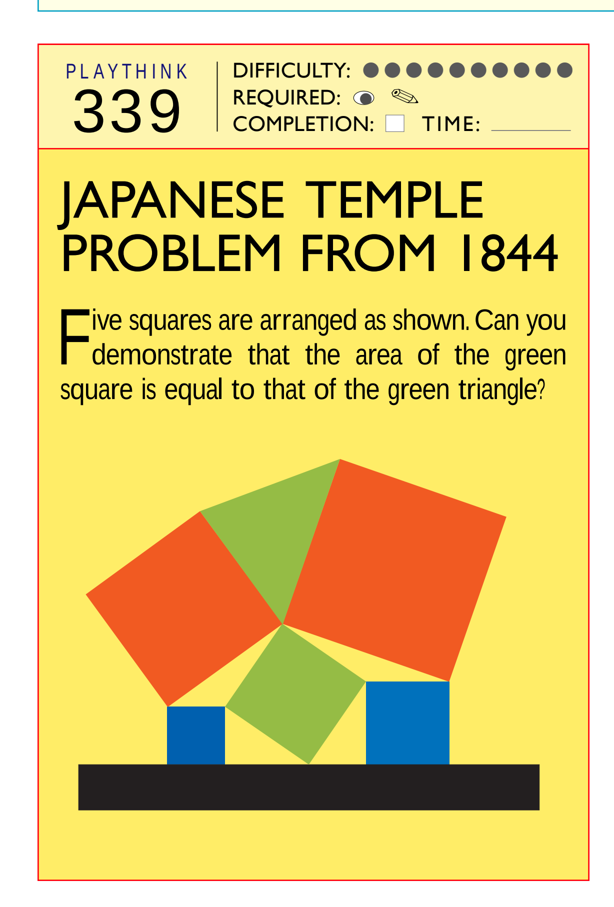
Polygons

Of all the possible polygons—closed figures bordered by straight lines—the triangle is the simplest. (If you don't believe it, try enclosing a figure with just two straight lines.) And as the engineer and architect Buckminster Fuller so aptly explained it, the triangle is also the only inherently stable form. What Fuller meant was that triangles are hard to deform. (And if you don't believe that, just try to push a triangular cardboard tube flat.)

Engineers take advantage of the stiffness of triangles by inserting them within structural forms; even rectangular girders are generally composed of triangular parts. But actually every polygon can be divided into triangles, or triangulated. The number of possible triangles is two less than the number of sides: a square can be divided into two triangles, and a heptagon into five. The number of diagonals from any vertex is three less than the number of sides. And the sum of the internal angles of a polygon is similarly related—two less than the number of sides, times 180.

Regular polygons are a special subset of polygons: all their sides are equal and all their angles are equal. (These qualities are not necessarily related; both the rhombus and the rectangle are instances in which only one of the equalities is present.) Regular polygons are an essential building block of regular solids, called polyhedrons. Indeed, the faces of regular polyhedrons—as well as other, irregular solids—are made up of just three basic shapes: the regular pentagon, the square and the equilateral triangle.

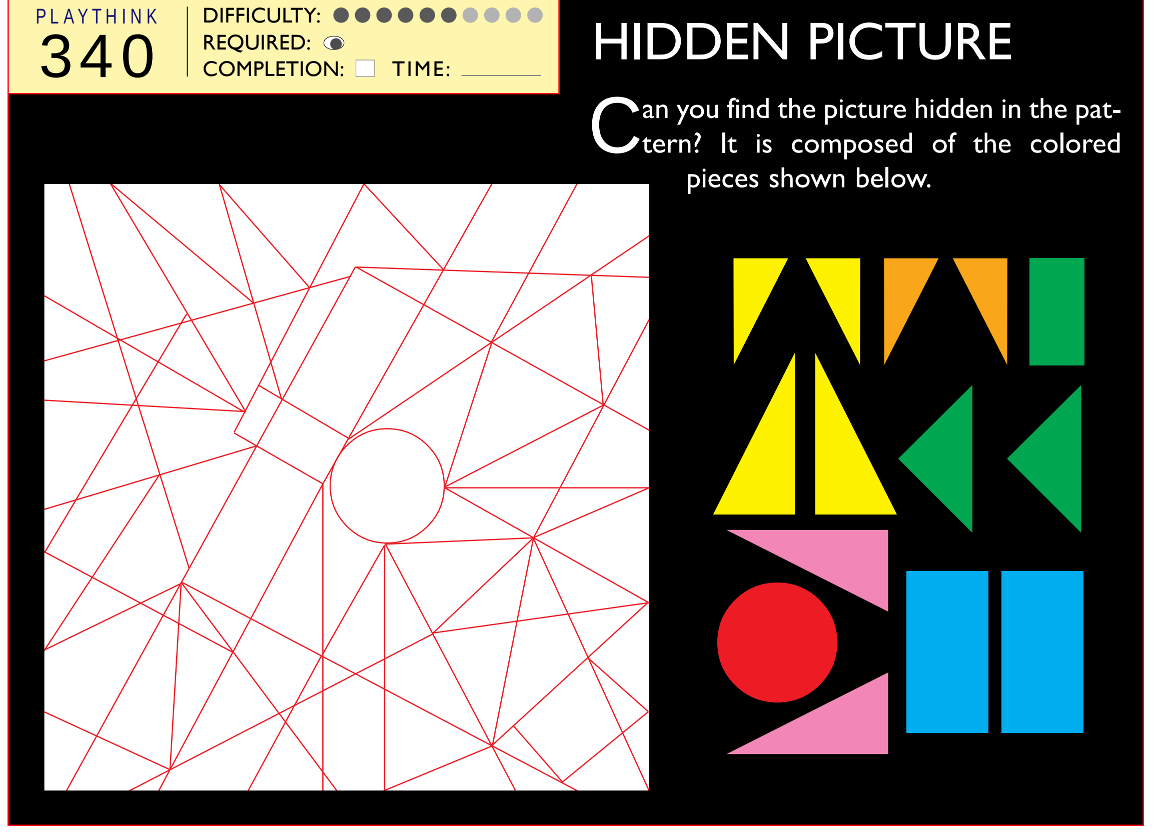
When ancient stargazers looked at the night sky, they combined the points of polygons—squares, triangles, rectangles, other shapes—into more elaborate figures, such as monsters, warriors and gods. The ancients assumed that such figures must have been placed there by some guiding hand. Modern mathematicians now know that whenever a collection of random points is great enough, they will inevitably begin to show signs of shapes and patterns.



PLAYTHINK 339 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: TIME:

JAPANESE TEMPLE PROBLEM FROM 1844

Five squares are arranged as shown. Can you demonstrate that the area of the green square is equal to that of the green triangle?



PLAYTHINK 340 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: TIME:

HIDDEN PICTURE

Can you find the picture hidden in the pattern? It is composed of the colored pieces shown below.

PLAYTHINK 341

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

STAINED GLASS WINDOW

There are four regular stars in this window: a three-pointed, a four-pointed, a five-pointed and a six-pointed star. Can you find all four?



PLAYTHINK 342

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

SHARING CAKES

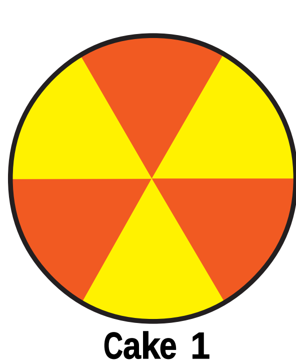
At a birthday party three cakes are cut as shown and divided between two groups. One group gets the red pieces, while the other gets the yellow.

Cake 1 is cut through the center three times, making six 60° angles.

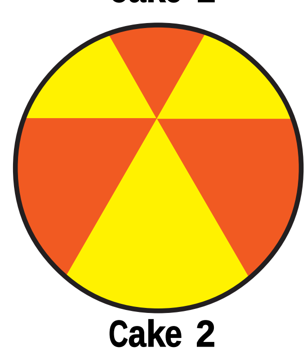
Cake 2 is also cut three times, but through an off-center point. Again, the cuts make six 60° angles.

Cake 3 is cut through the same off-center point, but now four times, making eight 45° angles.

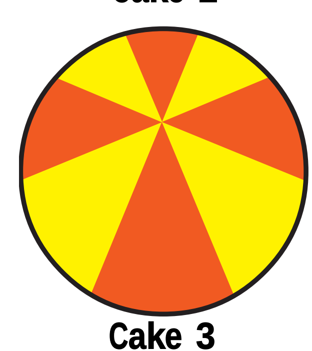
Did each group get identical shares of the three cakes?



Cake 1



Cake 2



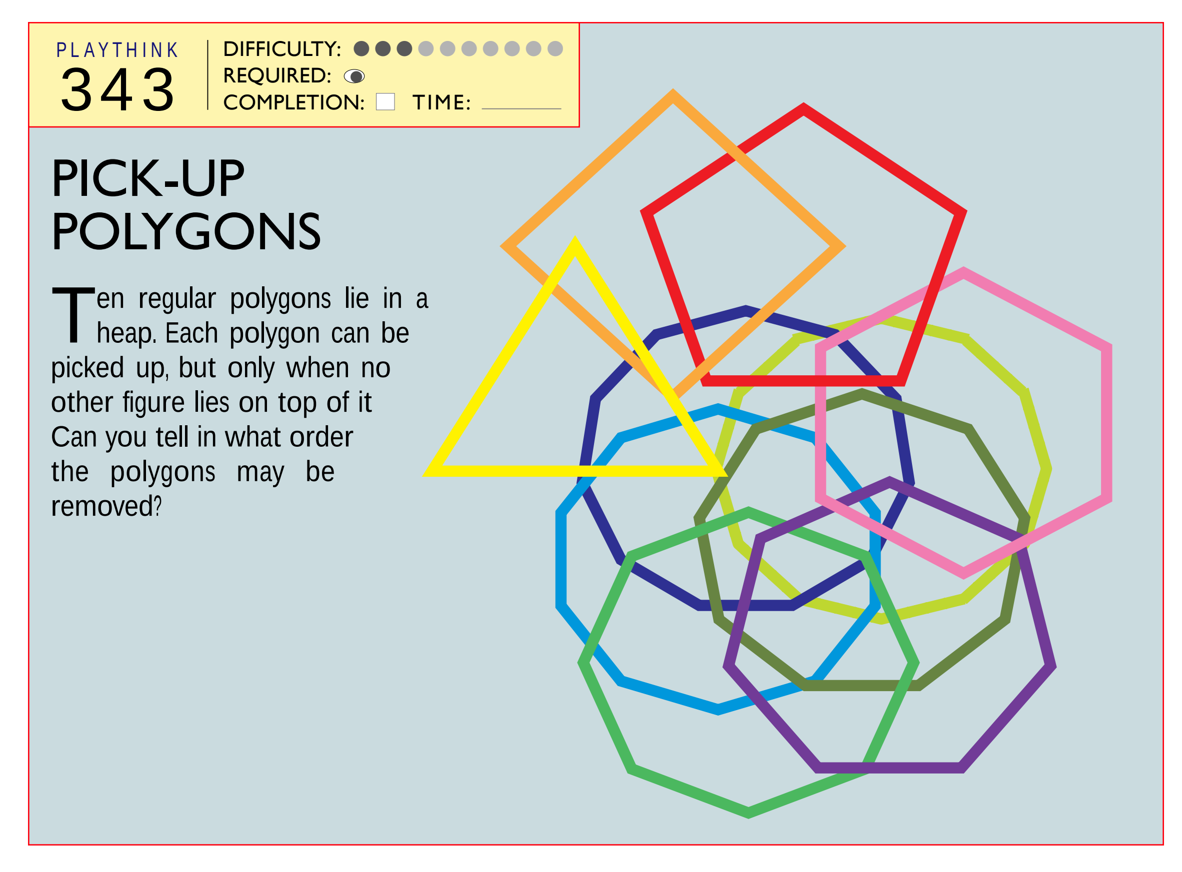
Cake 3

PLAYTHINK 343

DIFFICULTY: ●●●○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

PICK-UP POLYGONS

Ten regular polygons lie in a heap. Each polygon can be picked up, but only when no other figure lies on top of it Can you tell in what order the polygons may be removed?



PLAYTHINK 344

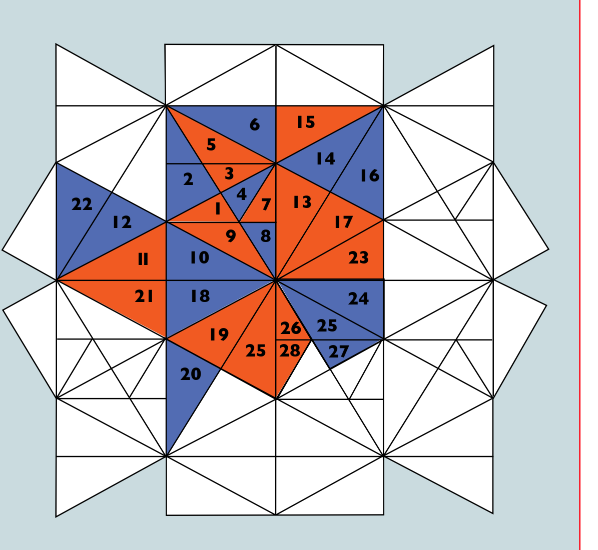
DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

QUADRILATERALS GAME

The object of this game is to form quadrilaterals. Players choose a color and take turns filling in the triangular fields along the grid lines with their colors. Each newly colored field must touch a field that has already been filled in. Points are scored when a quadrilateral is formed, one point per triangle within its boundaries, regardless of color. Certain conditions, however, must first be met:

- More than half of the triangles forming the quadrilateral must be of the color of the player who formed it.
- The perimeter of the quadrilateral cannot pass between two triangles of the same color.
- The quadrilateral cannot have any empty spaces.
- The quadrilateral must be symmetrical.
- Each triangle can be part of only one quadrilateral.

The game ends when all the triangles have been filled. The unfinished sample game demonstrates the above rules.

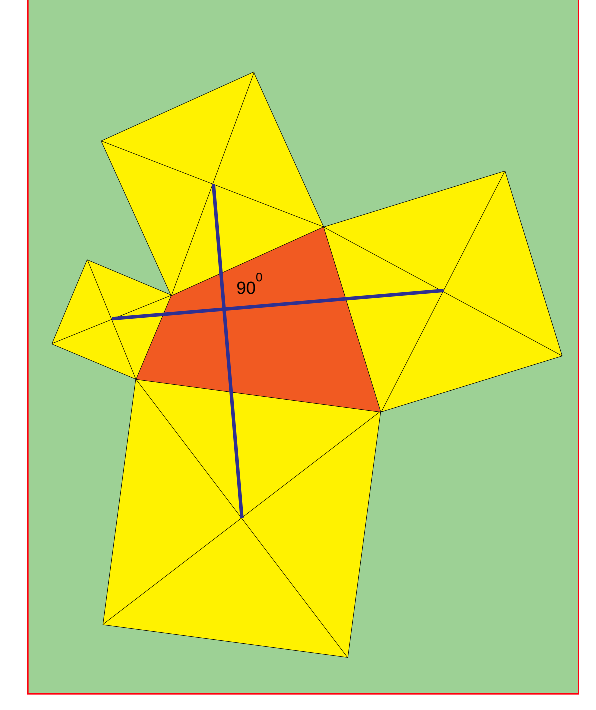


PLAYTHINK 345
DIFFICULTY: ●●●○○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

SQUARES ON A QUADRILATERAL

In the drawing below, squares are drawn on the sides of the red quadrilateral. The centers of the squares on opposite sides are joined, and not only do the two lines intersect at 90 degrees, but they are of equal length as well.

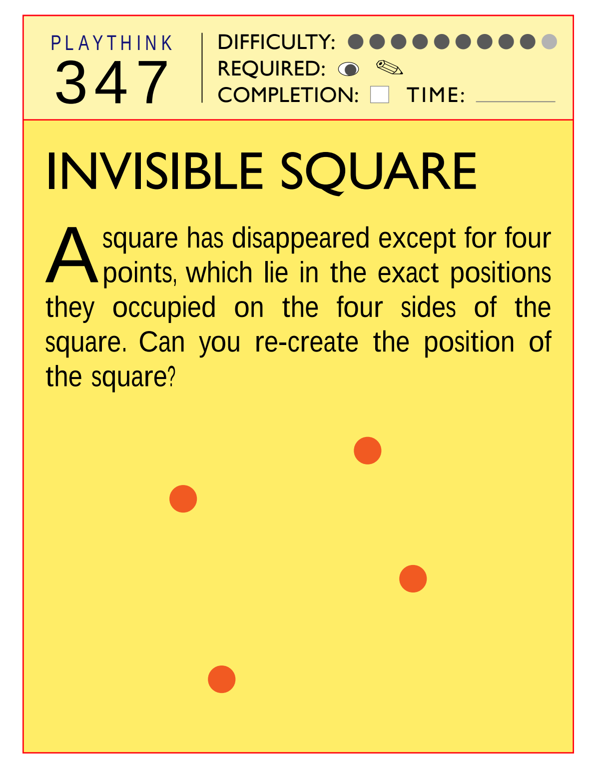
Will every quadrilateral—no matter its shape—lead to the same result?



PLAYTHINK 347
DIFFICULTY: ●●●●●●●●●○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

INVISIBLE SQUARE

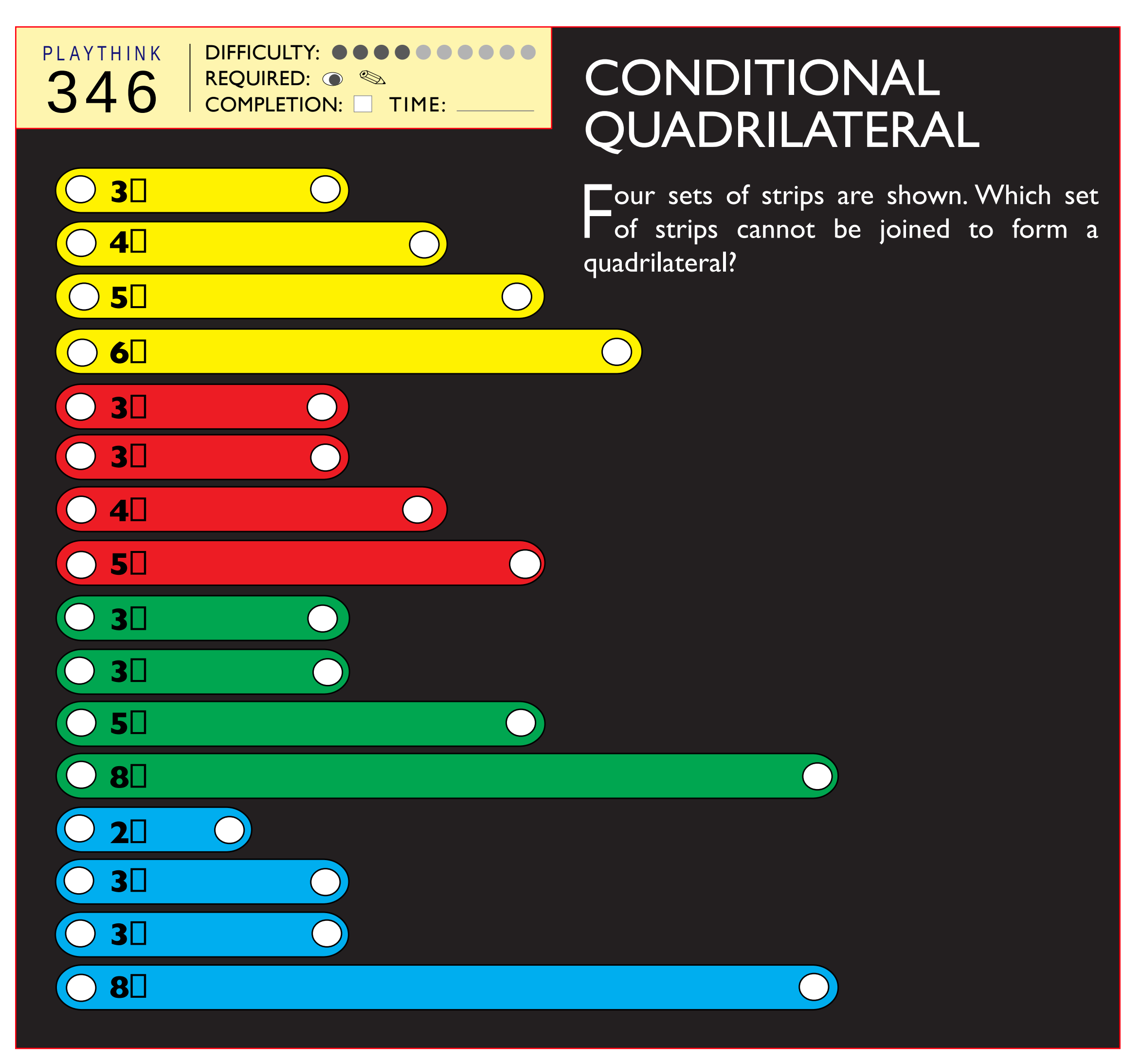
A square has disappeared except for four points, which lie in the exact positions they occupied on the four sides of the square. Can you re-create the position of the square?



PLAYTHINK 346
DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

CONDITIONAL QUADRILATERAL

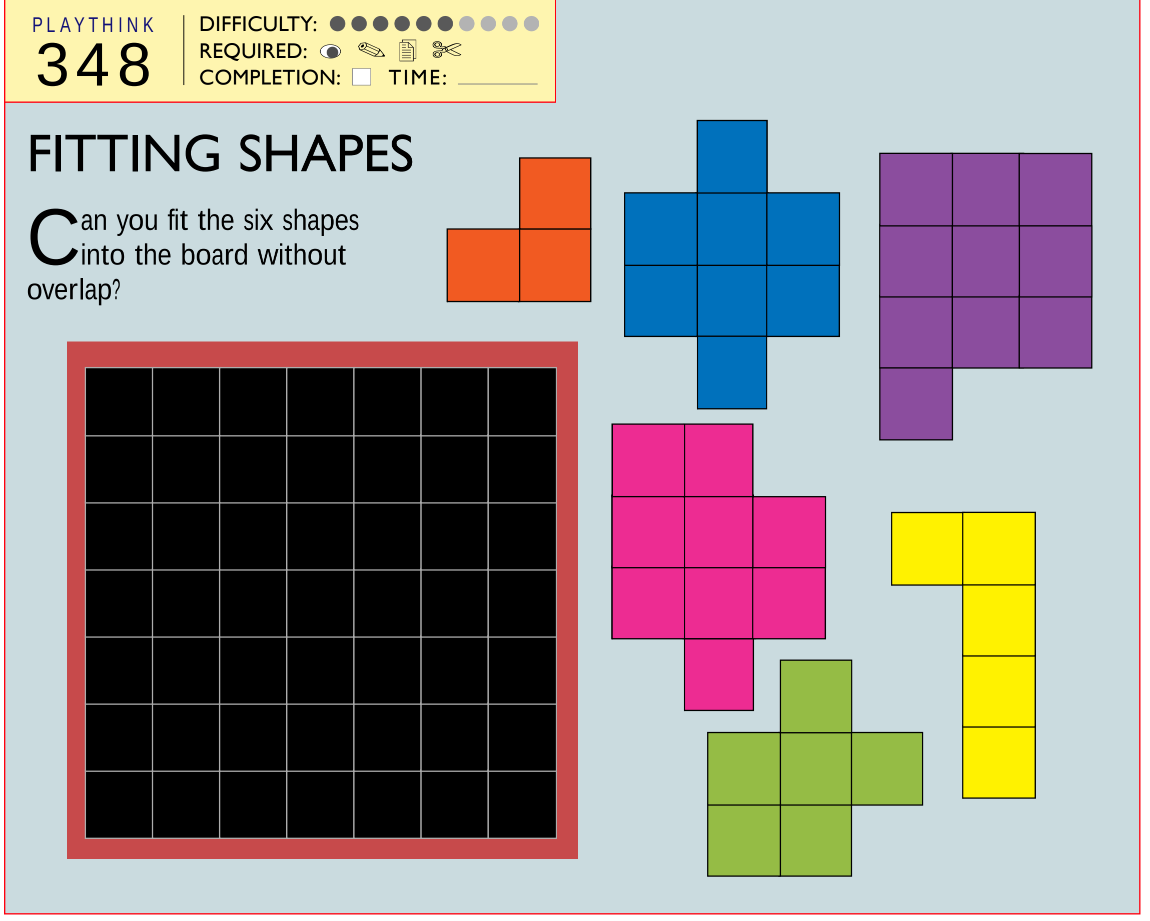
Four sets of strips are shown. Which set of strips cannot be joined to form a quadrilateral?



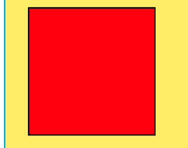
PLAYTHINK 348
DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎ 🗐 ✂
COMPLETION: ☐ TIME: ______

FITTING SHAPES

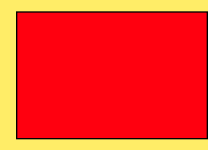
Can you fit the six shapes into the board without overlap?



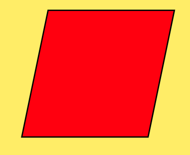
Quadrilateral Definitions



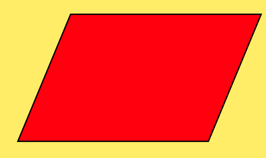
Square—
a quadrilateral with four equal sides and four right angles



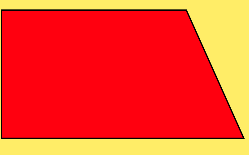
Rectangle—
a quadrilateral with opposite sides parallel and four right angles



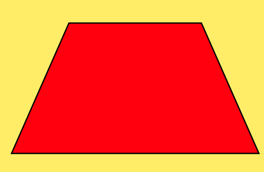
Rhombus—
a quadrilateral with opposite sides equal and parallel



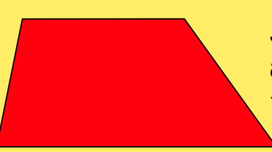
Parallelogram—
a quadrilateral with opposite sides parallel



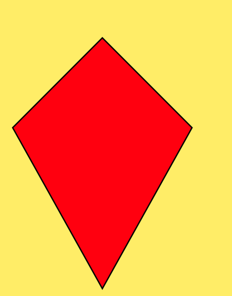
Right-angle trapezoid
—a quadrilateral with two parallel sides and a right angle



Isosceles trapezoid—
a quadrilateral with two parallel sides and two sloping sides equal



Scalene trapezoid—
a quadrilateral with two parallel sides

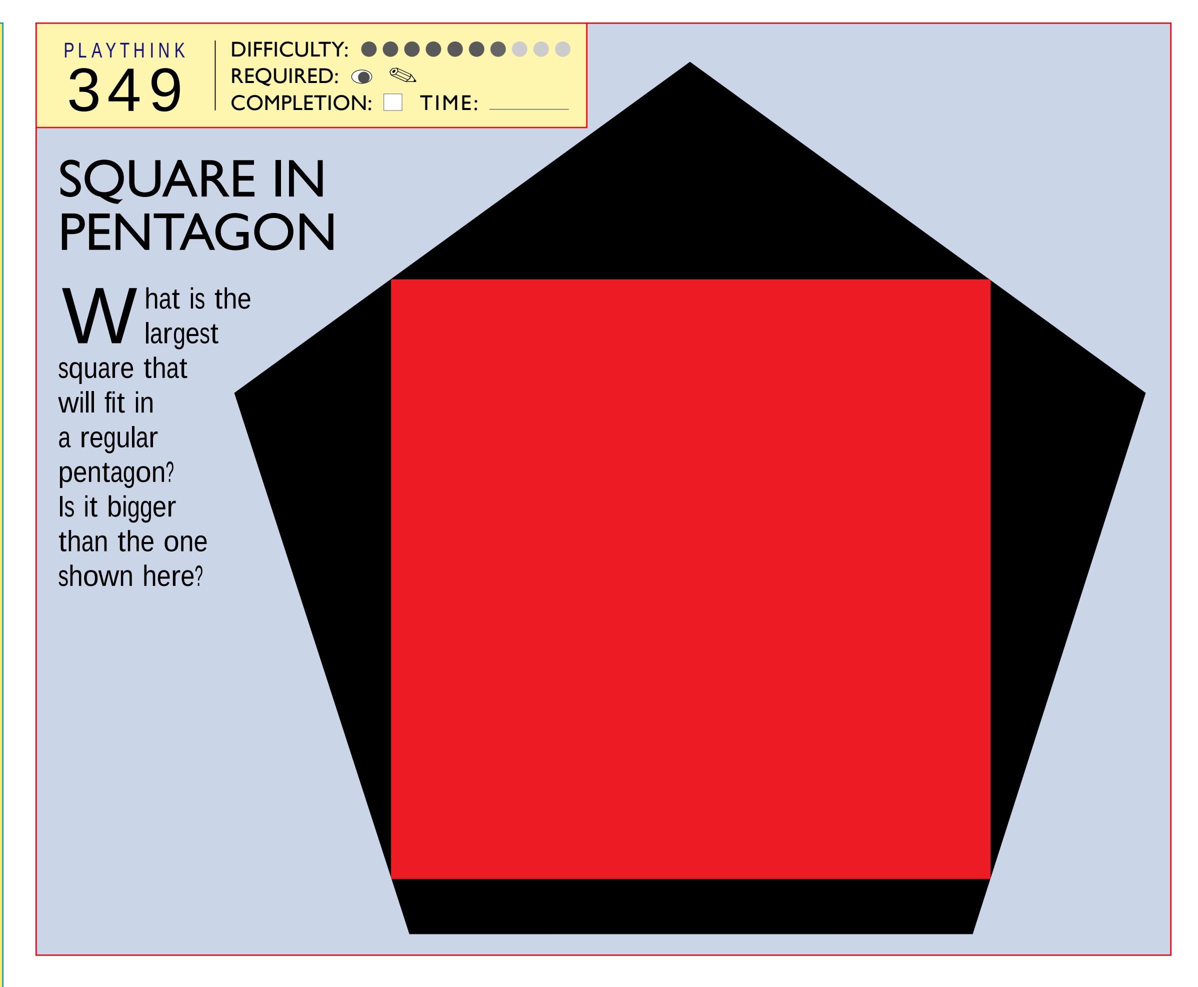


Deltoid—
a quadrilateral with two pairs of adjacent sides of equal length

PLAYTHINK 349 DIFFICULTY: ●●●●●●●○○○ REQUIRED: COMPLETION: ☐ TIME: ____

SQUARE IN PENTAGON

What is the largest square that will fit in a regular pentagon? Is it bigger than the one shown here?



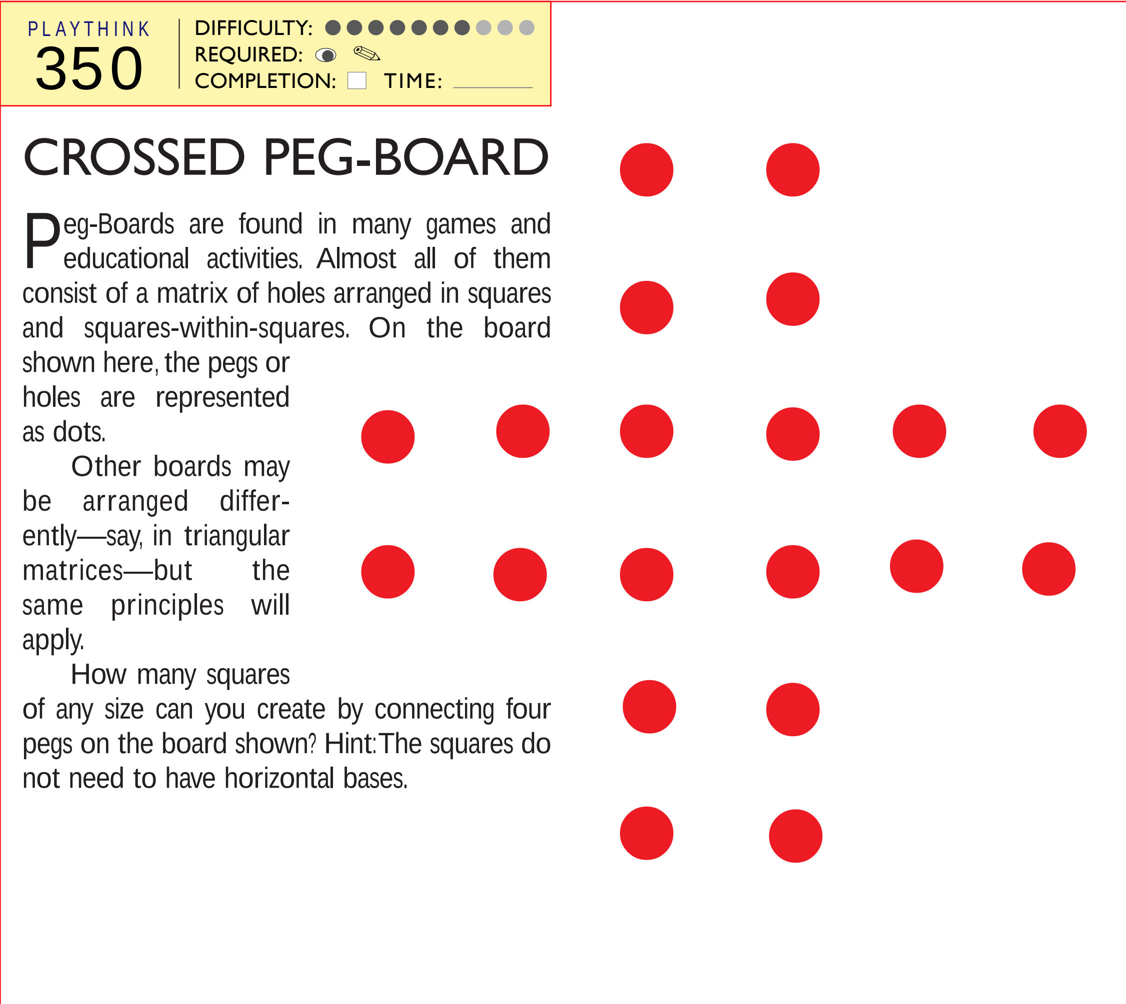
PLAYTHINK 350 DIFFICULTY: ●●●●●●●○○○ REQUIRED: COMPLETION: ☐ TIME: ____

CROSSED PEG-BOARD

Peg-Boards are found in many games and educational activities. Almost all of them consist of a matrix of holes arranged in squares and squares-within-squares. On the board shown here, the pegs or holes are represented as dots.

Other boards may be arranged differently—say, in triangular matrices—but the same principles will apply.

How many squares of any size can you create by connecting four pegs on the board shown? Hint: The squares do not need to have horizontal bases.

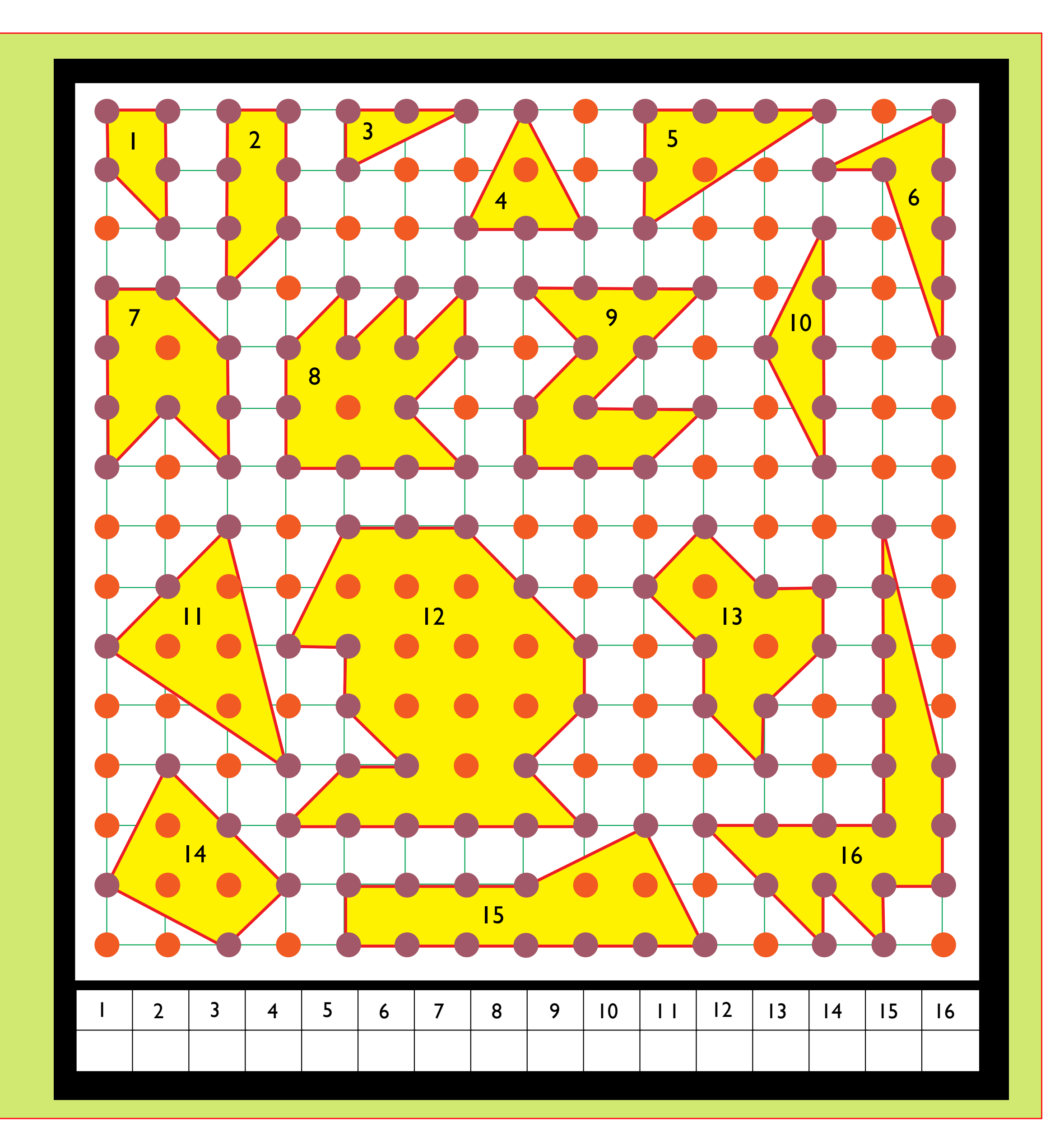


PLAYTHINK 351

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

PEG-BOARD POLYGONS

If the small square between four pegs represents 1 square unit, how much area is enclosed by each of the Peg-Board polygons, numbered 1 through 16?



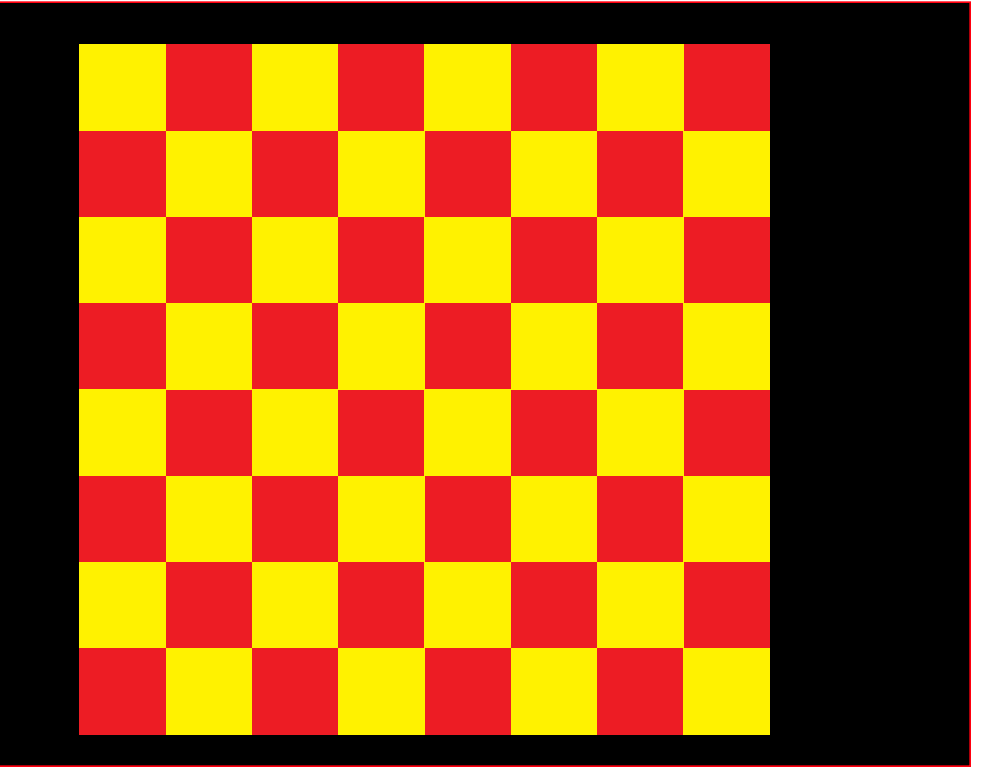
PLAYTHINK 352

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

CHESSBOARD SQUARES

How many squares of different sizes can you find along the grid of a chessboard?

One place to start on this problem is with the sixty-four individual squares that make up the chessboard. But there are other squares that are composites, made up of several square units. Can you find them all?

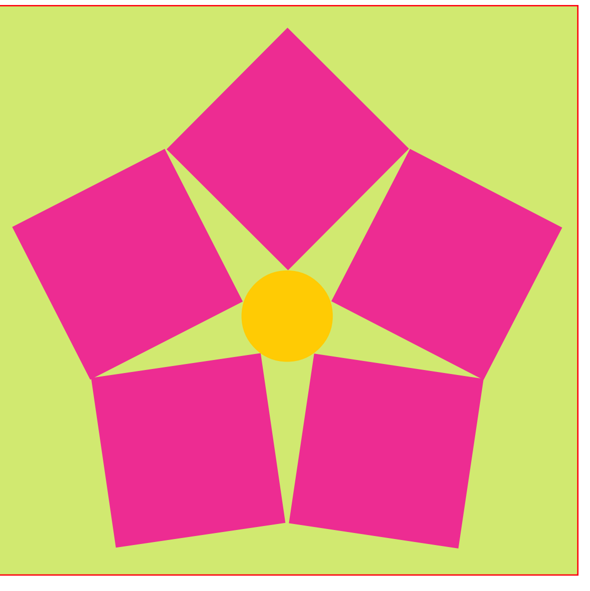


PLAYTHINK 353 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

SQUARES AROUND

In the illustration here, five identical squares are arranged symmetrically around a circle so that their corners touch one another and each square touches the circle.

Given a circle with a radius equal to the sides of the squares, how many squares would it take to be similarly arranged?



PLAYTHINK 354 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: ✎ 🗎 ✂ | COMPLETION: ☐ TIME: ____

SQUARE DISSECTIONS

Trace the colored shapes and join them to form nine identical squares.

Hint: The nine dissection puzzles here have all been created by bisection and trisection of the sides of the squares.



PLAYTHINK 355 | DIFFICULTY: ●●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

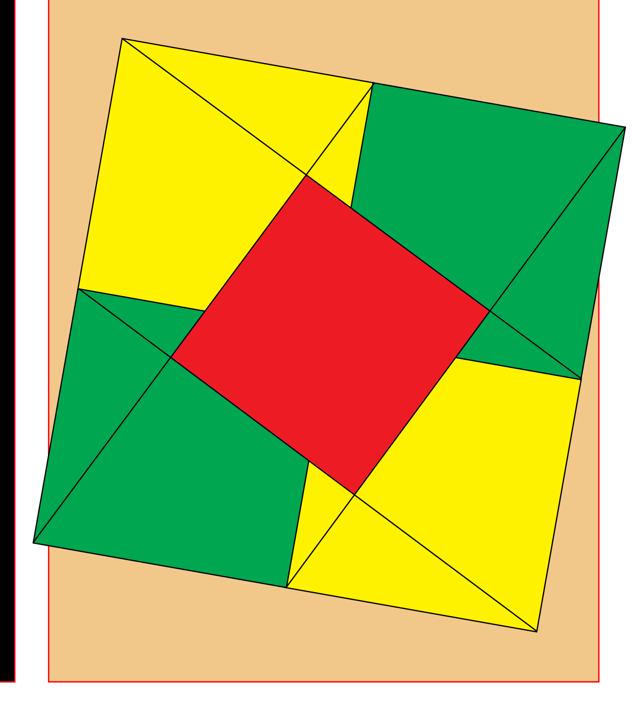
TRIANGLE–CIRCUMCENTER–INCENTER

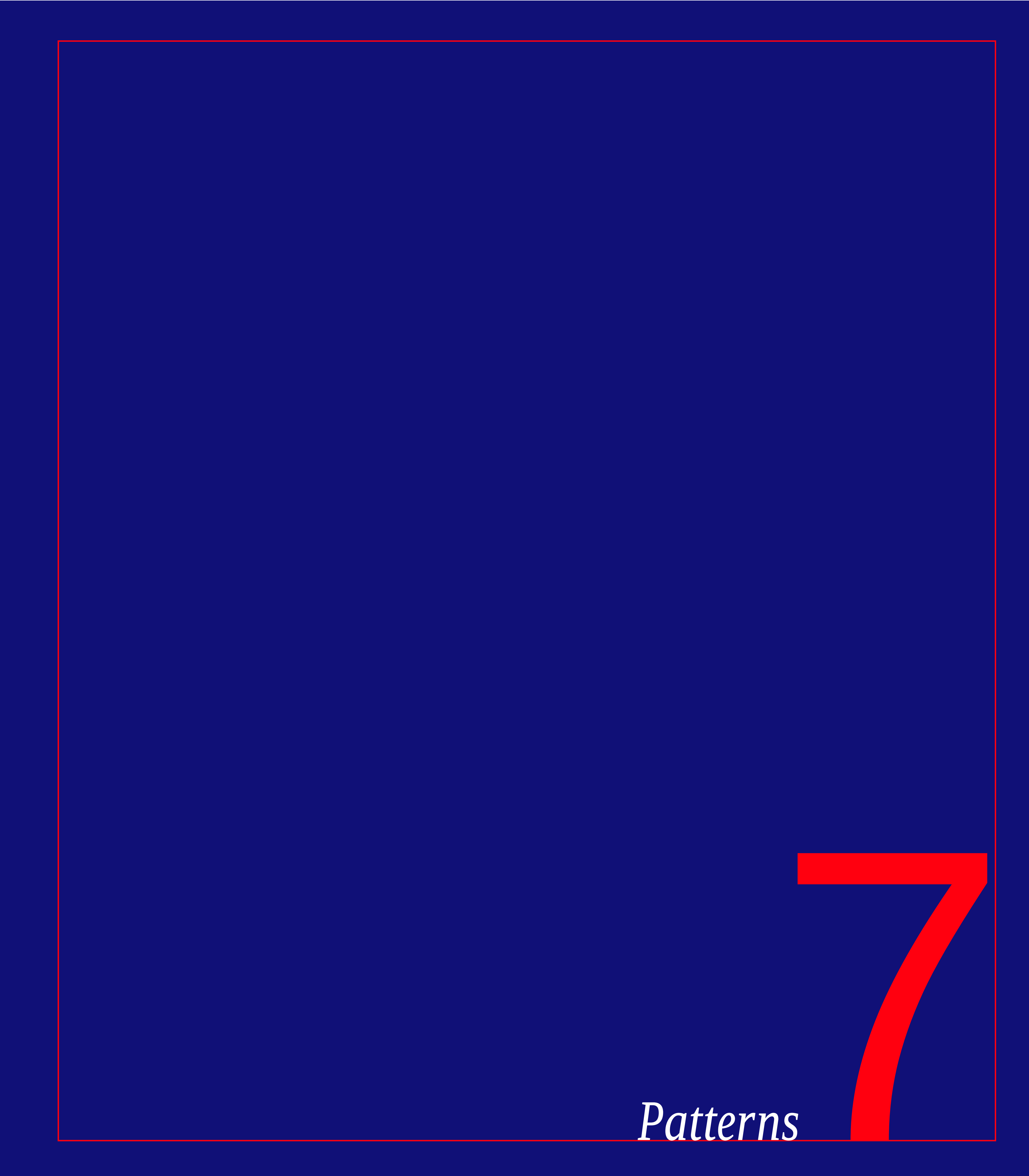
Can you discover how to find both the center of a circle inscribing the triangle—touching the three sides—and the center of a circle circumscribing the triangle—that is, passing through the three vertices?

PLAYTHINK 356 | DIFFICULTY: ●●●●●●●●●○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

SQUARE CUT

Just by looking at the figure below, can you figure out the area of the red square?

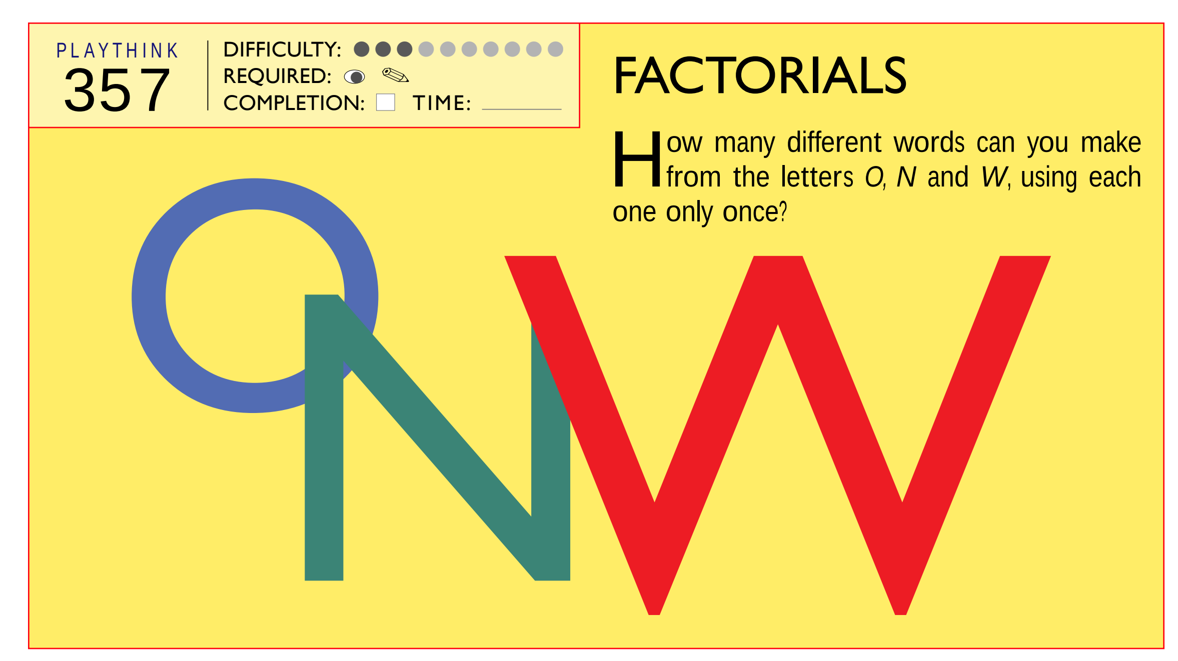



Patterns
7

PLAYTHINK 357
DIFFICULTY: ●●●○○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

FACTORIALS

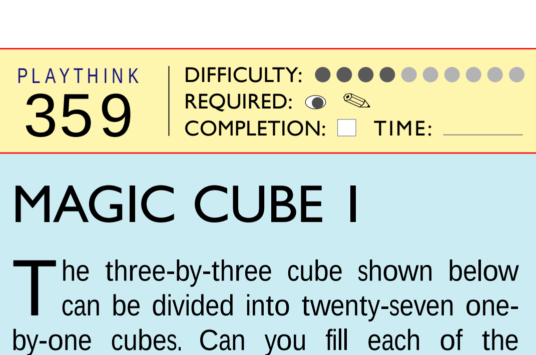
How many different words can you make from the letters *O*, *N* and *W*, using each one only once?



PLAYTHINK 358
DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

COLOR DIADEM

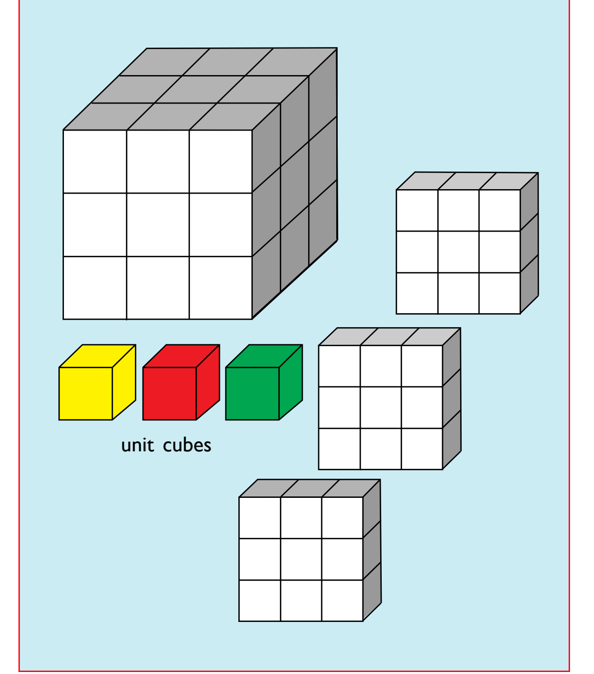
How many different diadems can you make using seven different precious stones? Configurations that differ only through rotation are not considered different.

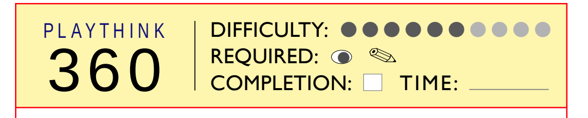


PLAYTHINK 359
DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

MAGIC CUBE I

The three-by-three cube shown below can be divided into twenty-seven one-by-one cubes. Can you fill each of the smaller cubes with one of three colors (red, green or yellow) in such a way that each vertical column and each horizontal row contains all three colors? Each color will appear exactly nine times.





PLAYTHINK 360
DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

BOYS AND GIRLS

Elementary schoolchildren on a field trip sit in groups of four, so that every girl sits next to at least one other girl. How many permutations are possible?



Pattern Recognition

Patterns are inescapable. Found in fantastic variety in the natural world, they show up in everything from atomic structures to snowflakes to spiral galaxies. Patterns are the basis of art as varied as Egyptian tomb painting and contemporary minimalism. And because patterns are everywhere—and so exquisitely beautiful—they make us curious. Children call their curiosity play; mathematicians name theirs research.

And what have we learned through all this research and play? That lines drawn on a flat surface divide the area into smaller bits. That if the lines are drawn so that the groups of smaller areas look the same, or at least similar, and those areas are aligned in a ordered manner, a pattern is formed. And that an area that is divided according to precise measurements to make a pattern that can then be applied to measuring or further drawing makes a grid.

The human talent for pattern recognition is simply the understanding that there is a systematic relationship between the elements in a group. These patterns, like the ones found in nature, indicate an underlying system of order. When this order is sought out, found and expressed, we are speaking the language of mathematics.

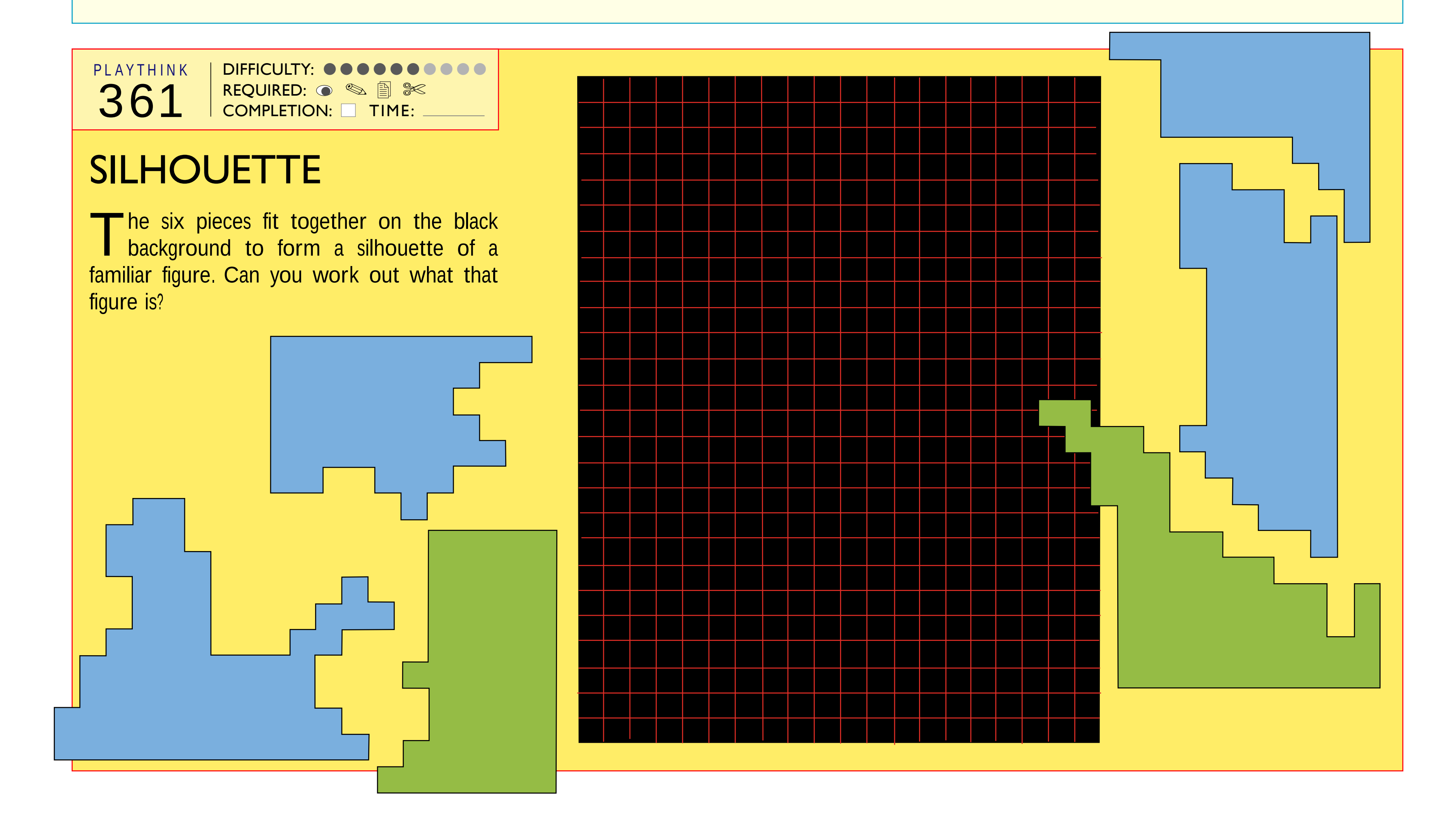
> "THE MATHEMATICIAN'S PATTERN, LIKE THE PAINTER'S OR THE POET'S, MUST BE BEAUTIFUL. . . . THERE IS NO PERMANENT PLACE IN THE WORLD FOR UGLY MATHEMATICS."
>
> —GODFREY H. HARDY

PLAYTHINK 361

DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

SILHOUETTE

The six pieces fit together on the black background to form a silhouette of a familiar figure. Can you work out what that figure is?



Combinations and Permutations

Probability, computer theory and many everyday situations depend on the principles of combination and permutations The number of possible arrangements in a system may seem small at first, but possibilities rise quickly with the number of elements and soon become impossibly large.

The basic instance is simplicity itself: one object by itself can be arranged in just one way and come in just one order.

Two objects, call them *a* and *b*, can be arranged as *ab* or *ba* for a total of two permutations. Three objects—*a*, *b* and *c*—can be arranged in six ways: *abc*, *acb*, *bac*, *bca*, *cab*, *cba*.

For the general case with *n* number of objects, the way to work out the permutations is to take the objects one at a time. The first object can fall at any of the *n* possible positions; for each of those possibilities, the second object can fall at one of n – 1 possible places (since it can't occupy the place the first object takes up); for every one of those n(n – 1) permutations, the third object can fall in one of n – 2 places; and so on.

In general, for *n* objects there are *n* times as many more permutations as there are in systems with only n – 1 objects. For example, there are four times as many possible permutations in a system with four objects than there are in a system with three — in other words, 24 permutations. There are 5 x 24, or 120, different ways to arrange five things and 6 x 120, or 720, ways to arrange six things. These numbers are called factorials and are designated with a *!*, as in 6!, or six factorial, to stand in for 720.

Therefore, the general formula is

$$n! = n \times (n - 1) \times (n - 2) \times (n - 3) \times \ldots 3 \times 2 \times 1$$

What about cases that deal not simply with ordering one group but with finding the permutations of *n* things taken *k* at a time? The mathematics here is only a bit trickier. Say you wanted to know how many ordered groups of three can be made from five different elements (such as colors or letters). You would calculate:

$$n!/(n - k)! = 5!/(5 - 3)! = 120/2 = 60$$

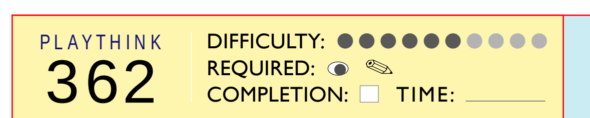
That means there are ten different possible groups of three elements (out of five), and each group has six possible permutations, for a total of 60. In the general formula, as you can see, *n* stands for the number of elements, and *k* stands for the size of the group.

Of course, the order of the elements does not always matter. Often it is only the raw constitution of the group that matters, such as the selection of a team from a group of athletes. A combination is a set of things chosen from a given group when no significance is attached to the order of the thing within the set. The number of combinations can be found by:

$$n!/k!(n - k)!$$

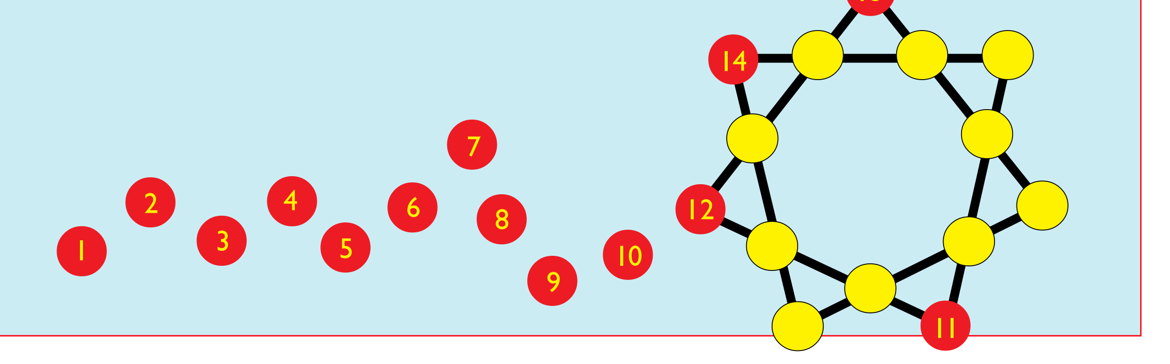
And, of course, sometimes there are objects within the group that are identical, so that picking one or the other doesn't change the set at all. In a case where the group is made up of *a* number of one thing, *b* number of another and *c* of a third, the number of different combinations can be found by:

$$n!/(a! \times b! \times c!)$$



MAGIC STAR I

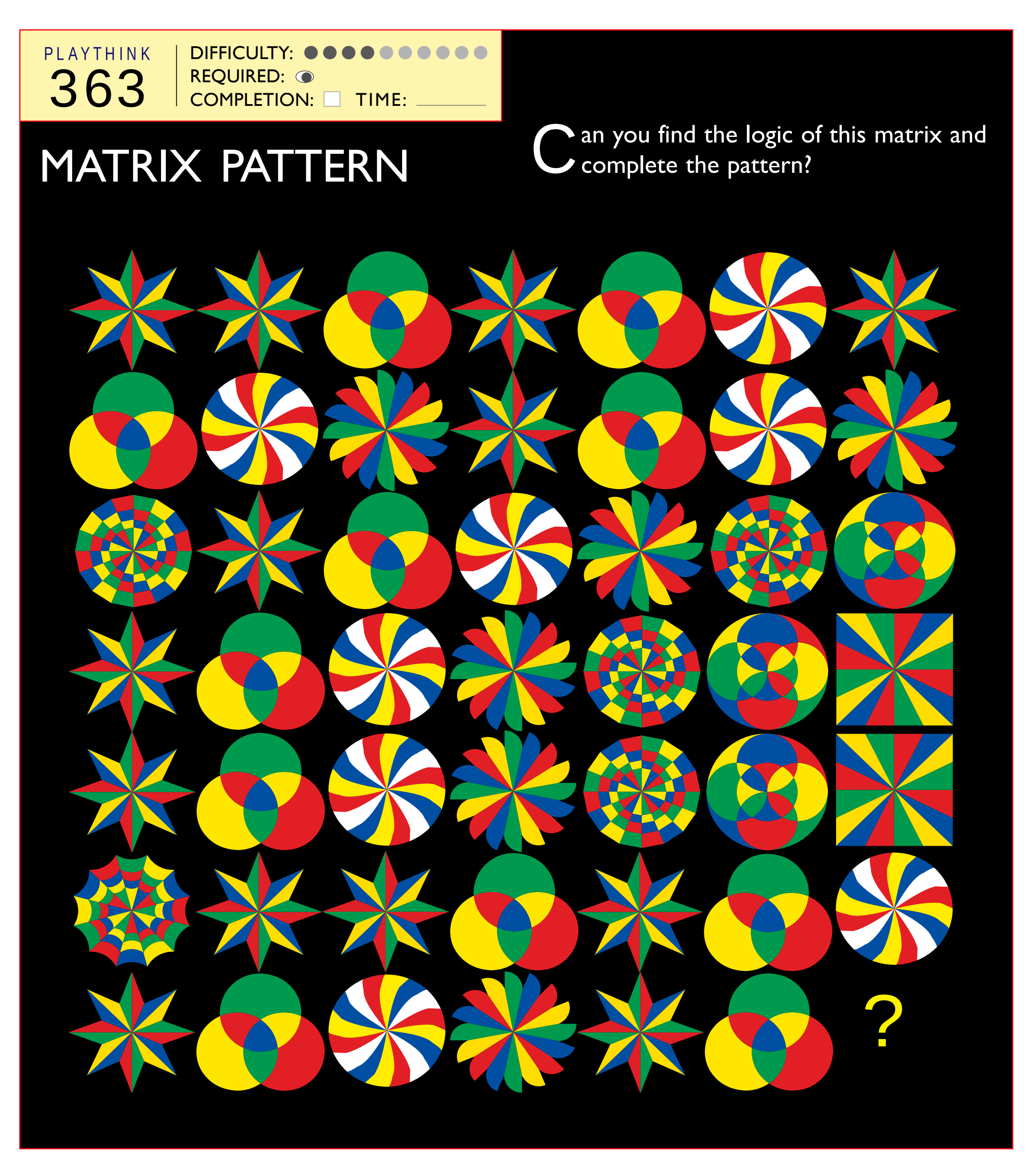
Can you place the numbers from 1 to 10 on the blank circles so that the sum along any straight line equals 30?



PLAYTHINK 363 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ____

MATRIX PATTERN

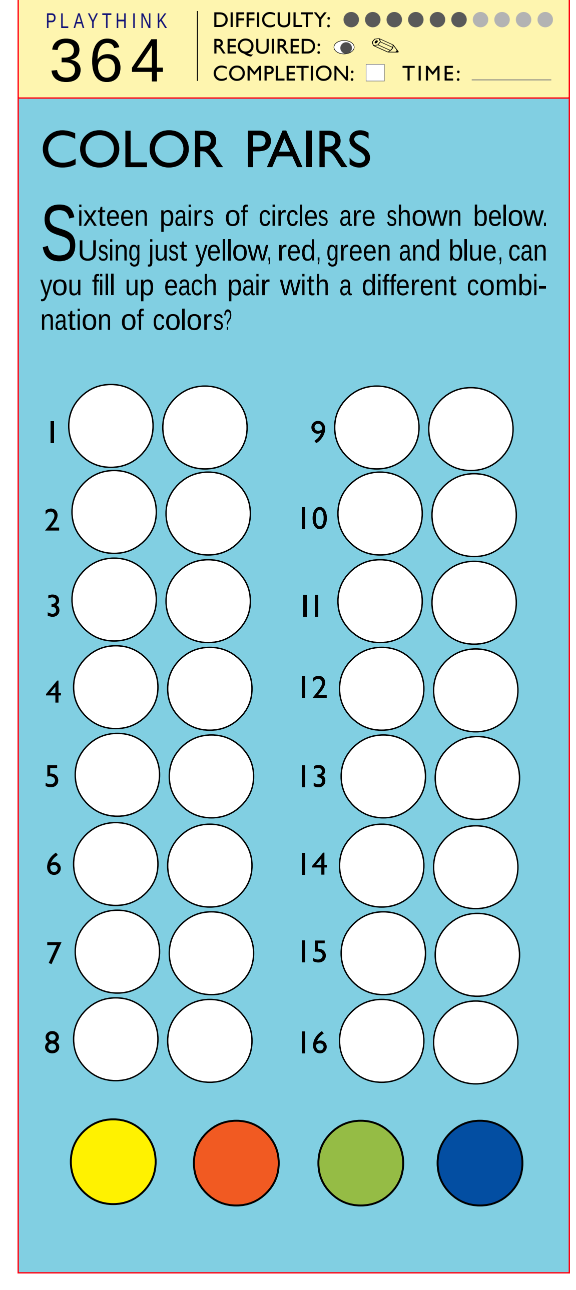
Can you find the logic of this matrix and complete the pattern?



PLAYTHINK 364 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ____

COLOR PAIRS

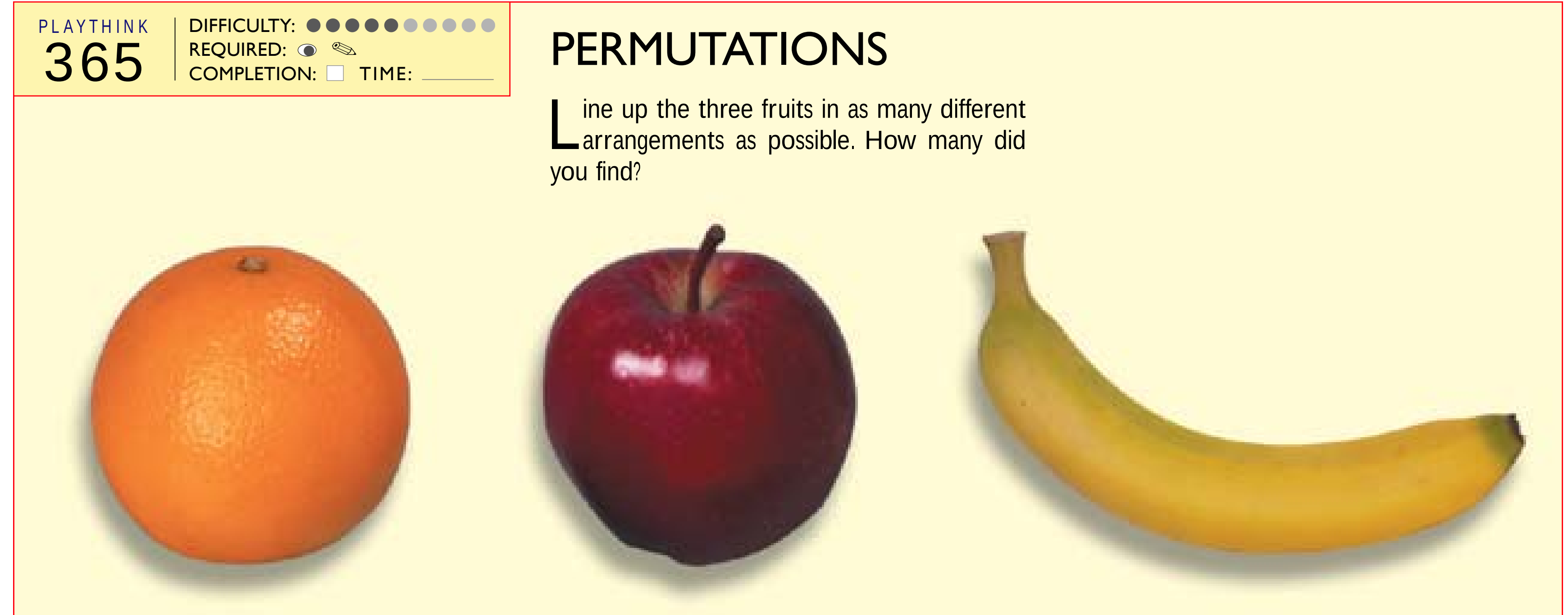
Sixteen pairs of circles are shown below. Using just yellow, red, green and blue, can you fill up each pair with a different combination of colors?



PLAYTHINK 365 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ____

PERMUTATIONS

Line up the three fruits in as many different arrangements as possible. How many did you find?

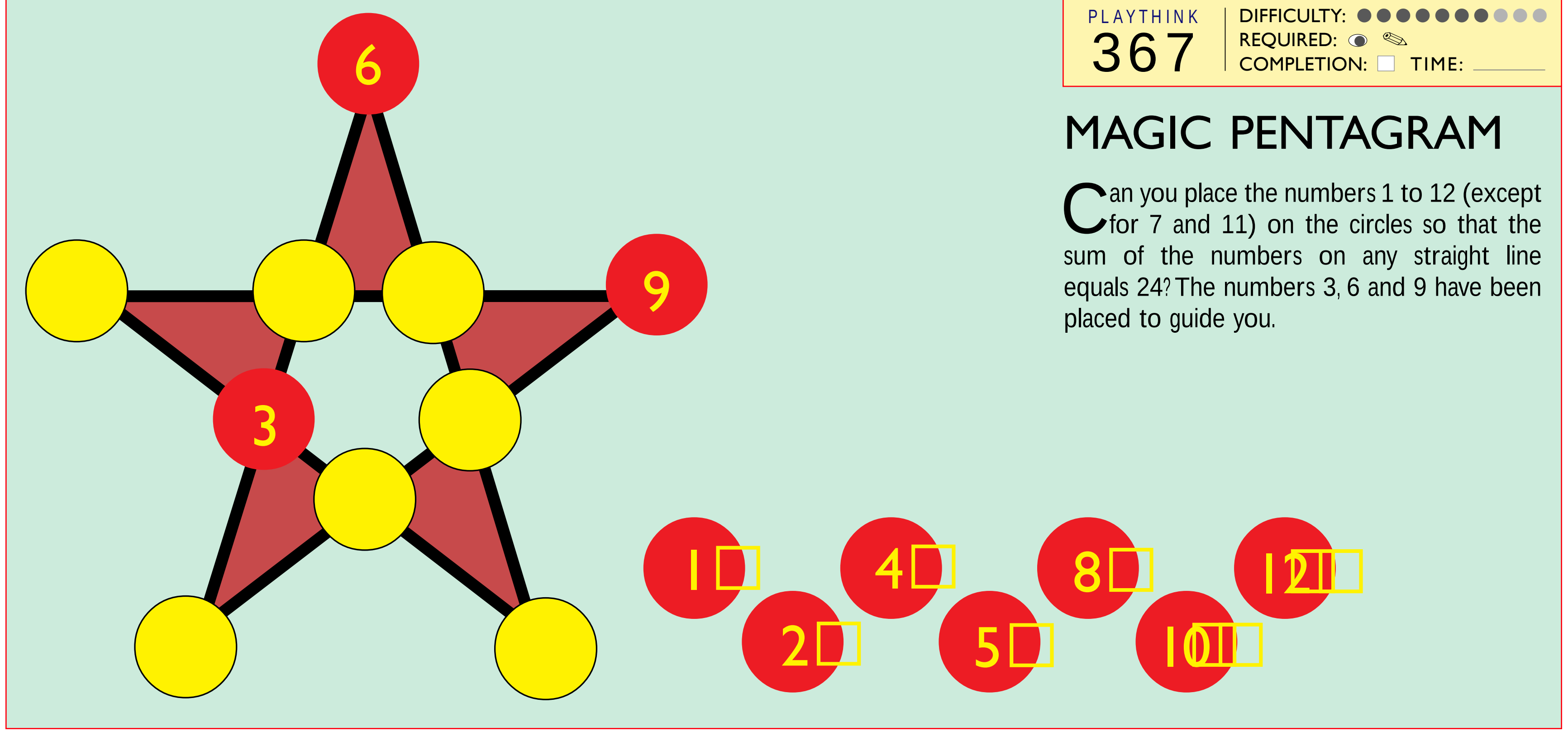


PLAYTHINK 366

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

SEATING PROBLEM

In how many different ways (ignoring rotations) can eight family members seat themselves around an octagonal dinner table?



PLAYTHINK 367

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

MAGIC PENTAGRAM

Can you place the numbers 1 to 12 (except for 7 and 11) on the circles so that the sum of the numbers on any straight line equals 24? The numbers 3, 6 and 9 have been placed to guide you.

PLAYTHINK 368

DIFFICULTY: ●●●●●●●○○○

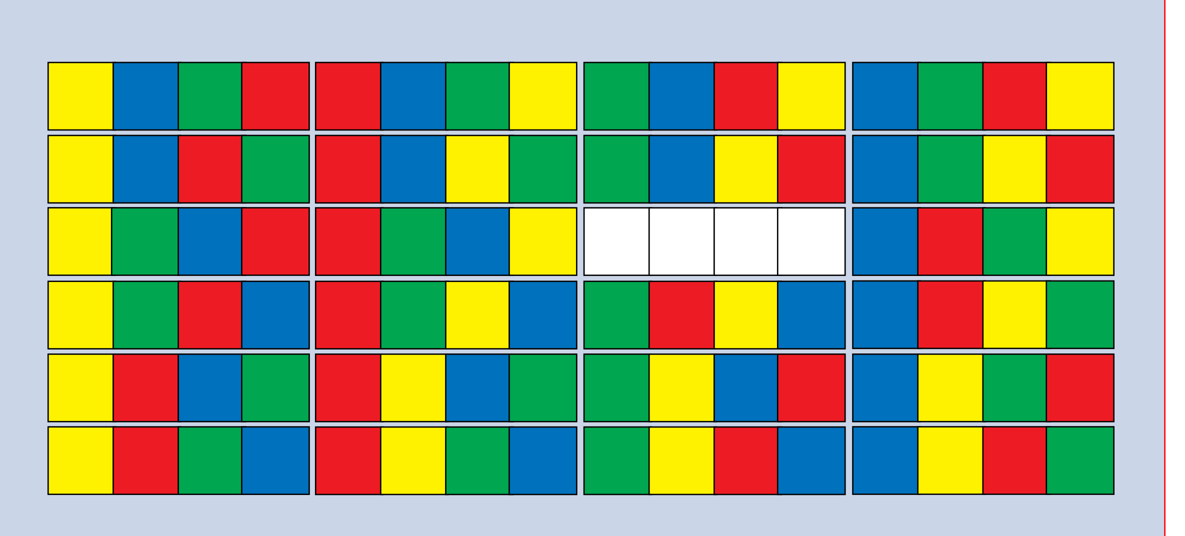
REQUIRED: 👁 ✎ 📄 ✂

COMPLETION: ☐ TIME: ______

PERMUTINO

The strips here are made from the possible permutations of four blocks of color. One of the strips is missing. Can you figure out what its sequence should be?

Copying and cutting out the set of strips offers the possibility of playing many puzzles and games, including the "Permutino Game" (PlayThink 370).



PLAYTHINK 369

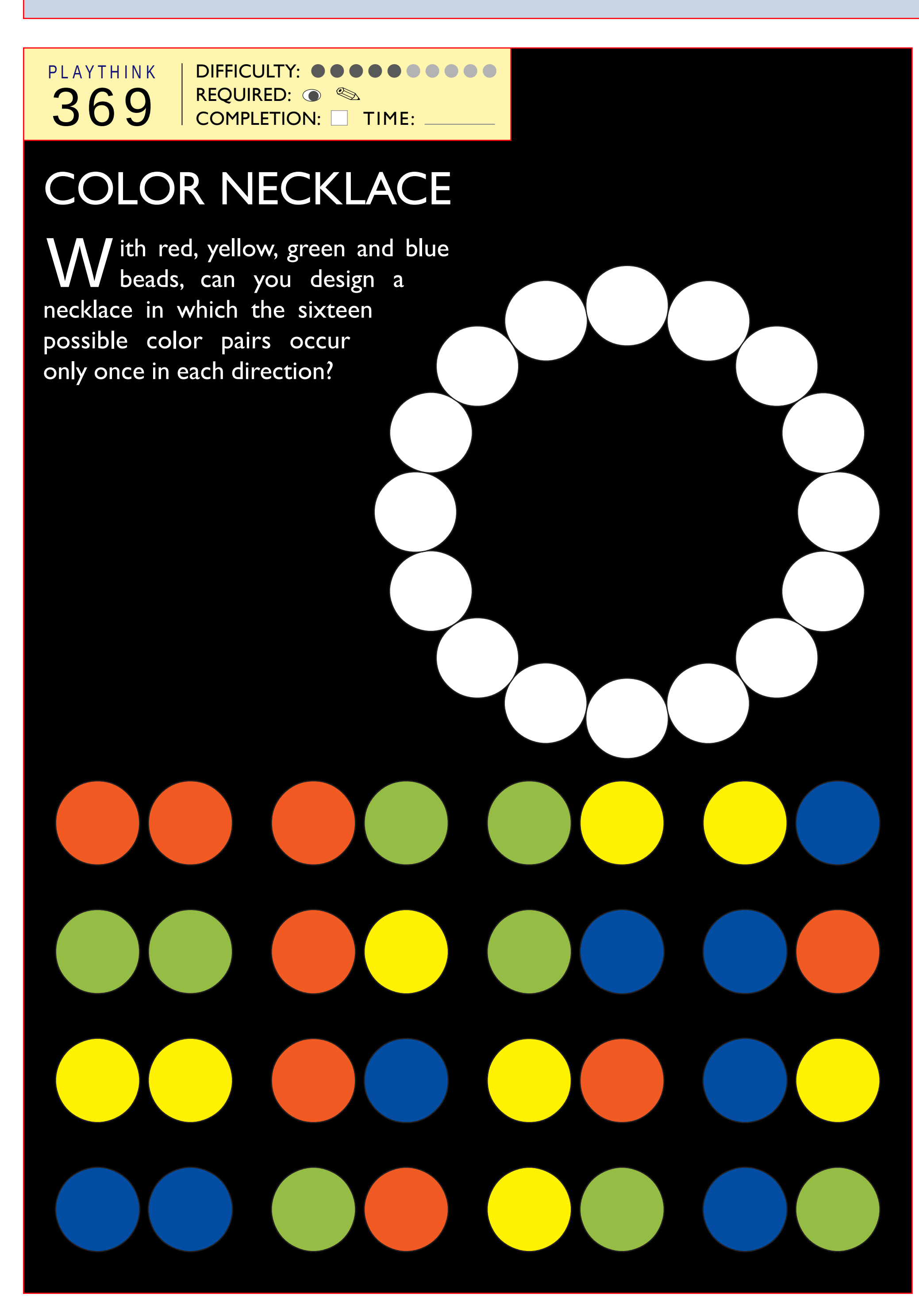
DIFFICULTY: ●●●●●○○○○○

REQUIRED: 👁 ✎

COMPLETION: ☐ TIME: ______

COLOR NECKLACE

With red, yellow, green and blue beads, can you design a necklace in which the sixteen possible color pairs occur only once in each direction?



PLAYTHINK 370

DIFFICULTY: ●●●●●●○○○○

REQUIRED: 👁 ✎

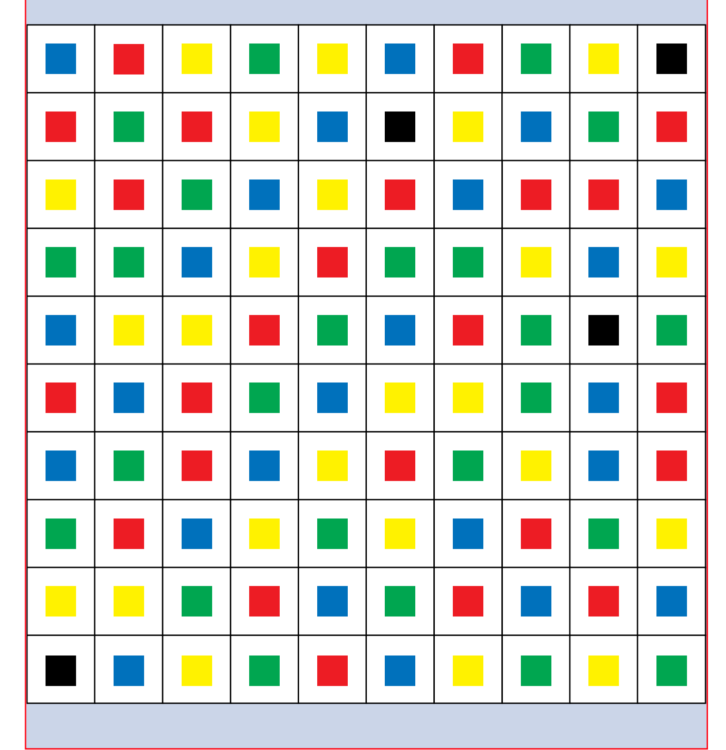
COMPLETION: ☐ TIME: ______

PERMUTINO GAME

The twenty-four four-color bands representing the twenty-four possible permutations of red, yellow, blue and green were placed on this ten-by-ten grid. The color for each block was recorded; the four empty spaces were marked black.

How long will it take you to fill in the outlines with the twenty-four strips from "Permutino" (PlayThink 368)?

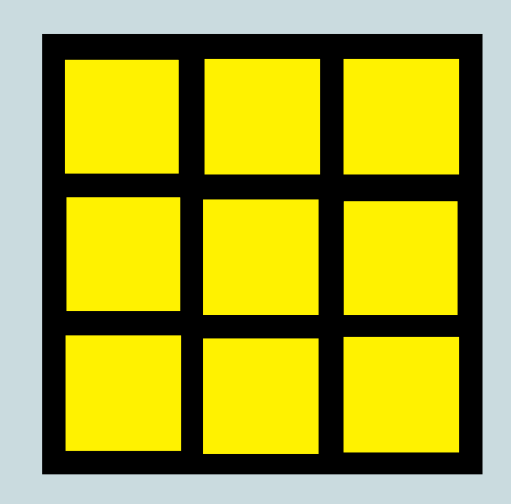
This puzzle can be played as a two-person game, with players taking turns putting down strips that match the pattern on the game board. The last player to successfully place a strip on the game board wins.

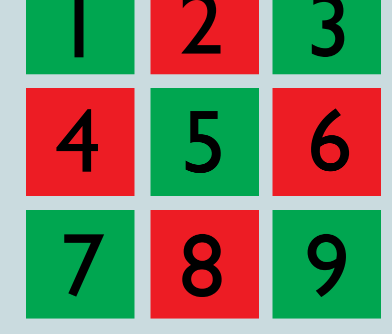


PLAYTHINK 371 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

MAGIC SQUARE 1

Can you distribute the numbers 1 through 9 in such a way that when the central number in any horizontal, vertical or diagonal line of three is subtracted from the outer two, the sum is always the same?

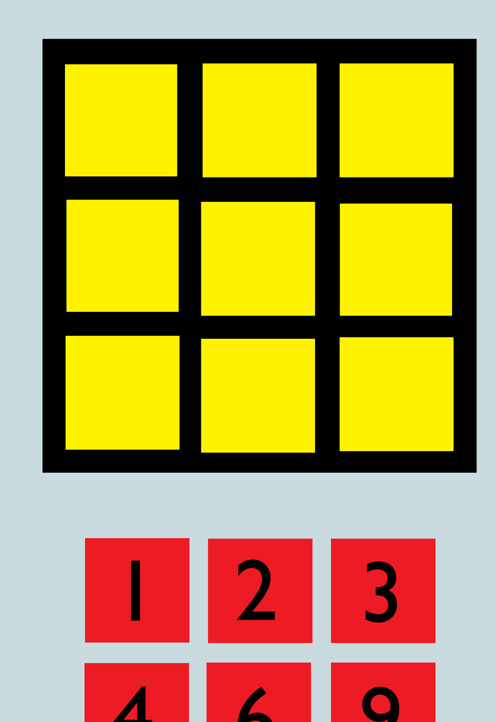




PLAYTHINK 372 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

MAGIC SQUARE 2

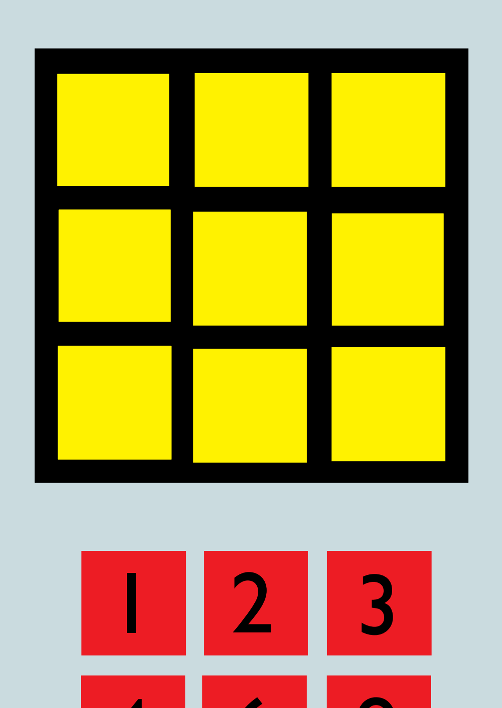
Can you distribute the numbers 1, 2, 3, 4, 6, 9, 12, 18 and 36 in such a way that when multiplied, each horizontal, vertical and diagonal line has the same result?



PLAYTHINK 373 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

MAGIC SQUARE 3

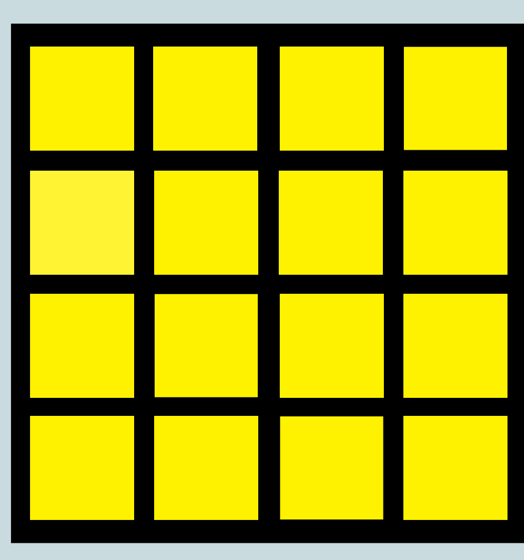
Can you distribute the numbers 1, 2, 3, 4, 6, 9, 12, 18 and 36 in such a way that when the central number of any horizontal, vertical or diagonal line is divided into the product of the outer two numbers of the line, the result is always the same?

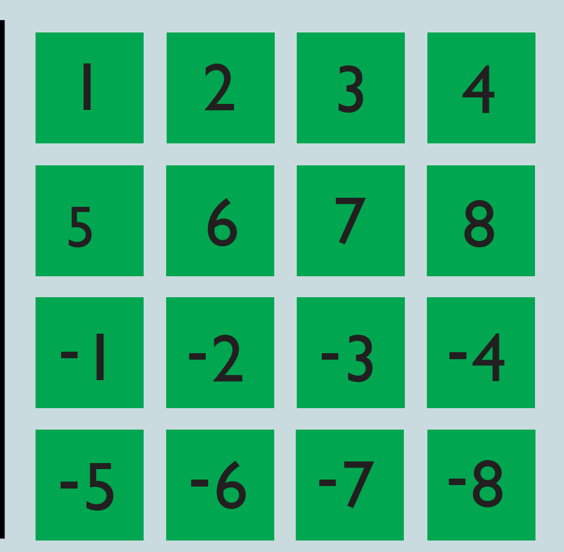


PLAYTHINK 374 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

MAGIC SQUARE 4

Can you distribute the numbers 1 through 8 and –1 through –8 so that the sum of every horizontal, vertical and main diagonal line equals zero?





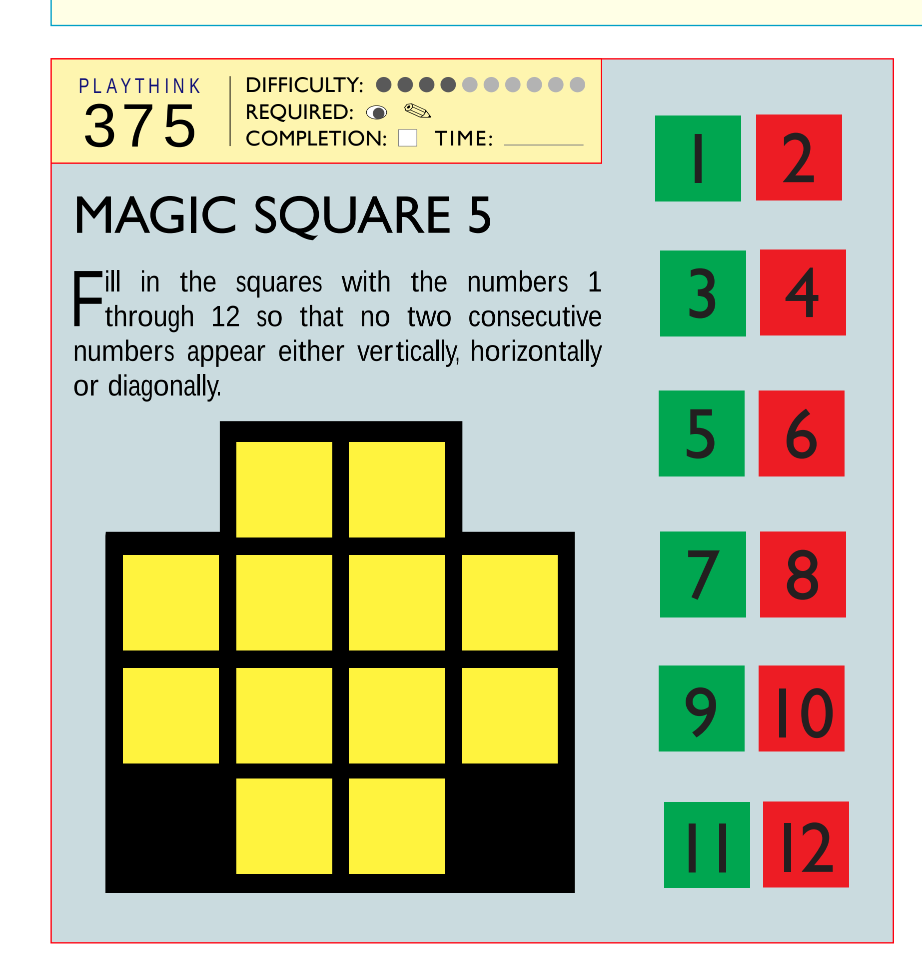
Magic Squares

The Rubik's Cube wasn't the first popular pastime that involved squares. As early as 4,500 years ago, people spent hours putting numbers in little boxes in hopes that the results would lead to mathematical beauty. What they were playing with was an ancient form of a puzzle called the magic square.

The writing of numbers in patterns started in ancient China, where numbers were often represented by circles or dots in a regular pattern, such as a triangle or a square. Since they were already thinking about numbers as forms unto themselves, Chinese mathematicians needed just a short step to create the Lo-Shu (PlayThink 378), which was the first magic square. A magic square is a group of cells, each filled with one of a set of natural numbers, generally a series that runs in order from 1 to a number equaling the number of cells. A five-by-five magic square, for instance, would contain the numbers from 1 to 25. What's more, the numbers must be entered into the cells in a very specific way—the sum of any row or column (and often either diagonal) of numbers must be the same. That total is called the magic constant.

Magic squares are described by their order—that is, the number of cells on one side. It turns out that there are no order 2 magic squares and only one of order 3: the Lo-Shu. Past order 3, the number of possible magic squares grows quickly. There are exactly 880 different types of order 4 magic squares, many of which are more "magic" than required by the definition of the magic square (see "Magic Square of Dürer," PlayThink 377), and there are millions of order 5 magic squares.

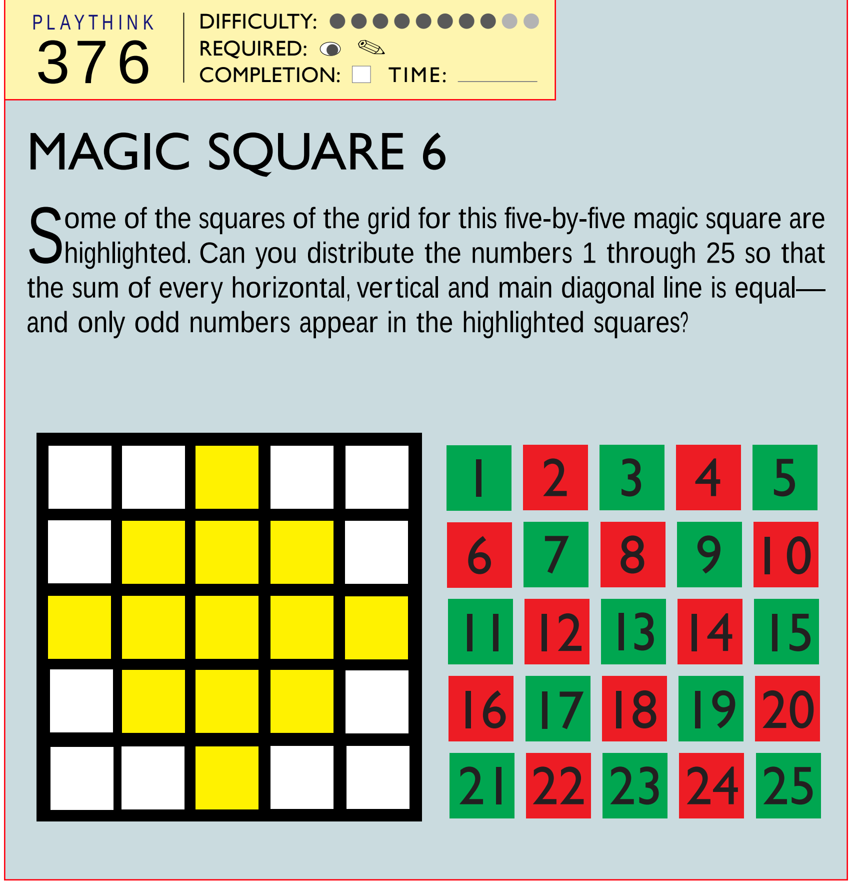
Magic squares have been widely popular throughout the ages, and some people have ascribed a different kind of magic to them. By A.D. 900, for instance, one Arab treatise recommended that pregnant women wear a charm marked with a magic square for a favorable birth.



PLAYTHINK 375 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

MAGIC SQUARE 5

Fill in the squares with the numbers 1 through 12 so that no two consecutive numbers appear either vertically, horizontally or diagonally.



PLAYTHINK 376 | DIFFICULTY: ●●●●●●●●○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

MAGIC SQUARE 6

Some of the squares of the grid for this five-by-five magic square are highlighted. Can you distribute the numbers 1 through 25 so that the sum of every horizontal, vertical and main diagonal line is equal—and only odd numbers appear in the highlighted squares?

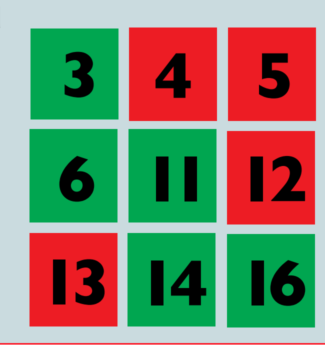
PLAYTHINK 377 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

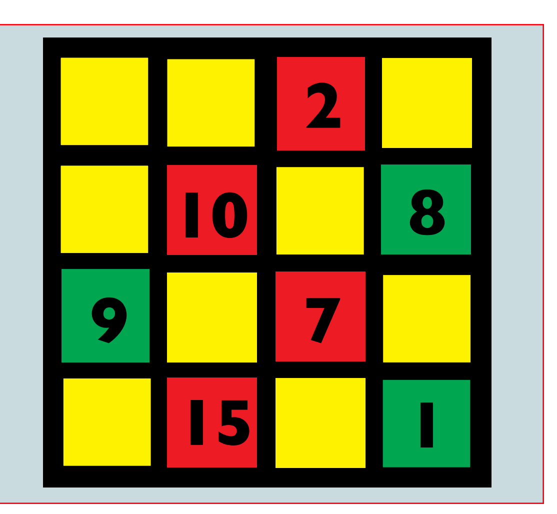
MAGIC SQUARE OF DÜRER

The German artist Albrecht Dürer engraved this magic square of order 4 in his 1514 etching *Melancholia*. It is one of many magic squares that is magic in more ways than the simple definition requires.

First, can you fill in the missing numbers (see inset) so that the sum of every horizontal, vertical and main diagonal line totals 34?

Then can you find other ways in which this square is magic?





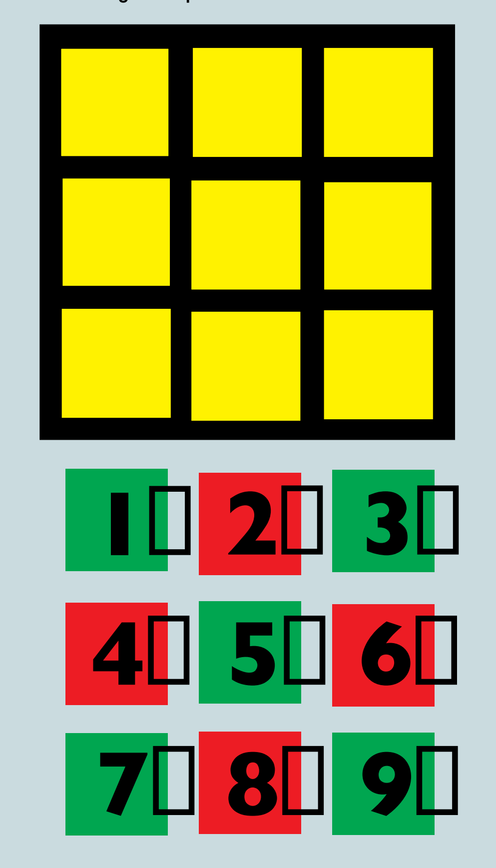
PLAYTHINK 378 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

THE LO-SHU

According to Chinese legend, the Lo-Shu dates back to at least the fifth century B.C. It is the oldest and simplest magic square.

The object of the Lo-Shu is to arrange the tiles numbered from 1 to 9 in the cells of the board so that the sum of every row, column and diagonal is the same. Not counting reflections and rotations, there is one answer.

Can you determine the sum without even solving the puzzle?

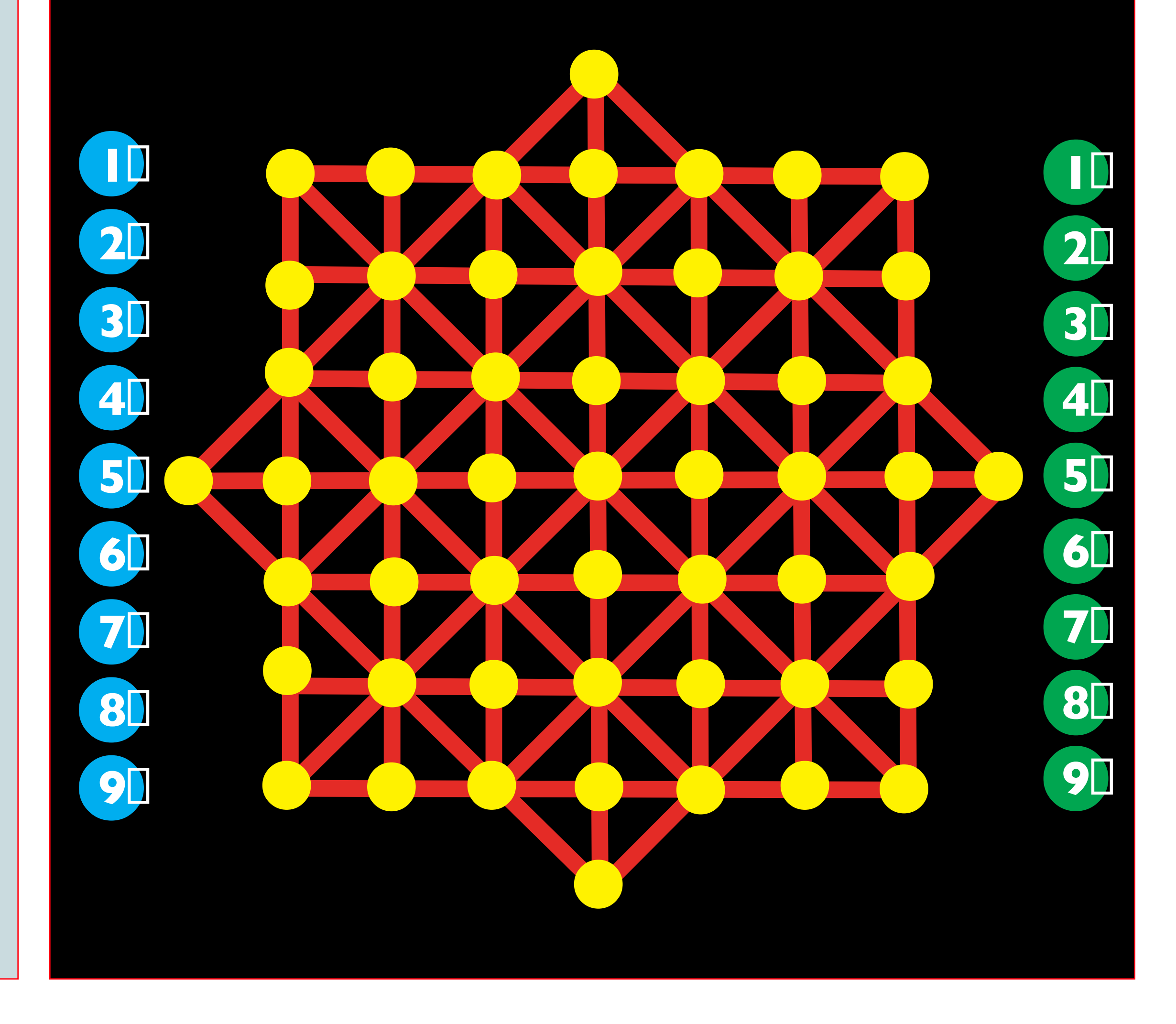


PLAYTHINK 379 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ 📄 ✂ | COMPLETION: ☐ TIME: ____

MAGIC 15 GAME

This original game was inspired by the ancient magic square. Players take turns placing their numbered markers on the game board. (You'll find it easy to make your own on a large piece of paper.) After all the markers have been placed, players take turns moving their pieces along the grid lines to adjacent empty cells; jumps are allowed, as in checkers, but a marker may jump only an opponent's marker—and only if that marker is of a lower value.

The object is to form a row of three markers that add up to 15; at least two of the three markers should be the player's color. Once a row that adds up to 15 has been made, those pieces are frozen for the rest of the game and cannot be moved. The player who makes the most rows wins.



PLAYTHINK 380 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

HINGED MAGIC SQUARE

Flipping the numbered tiles along their hinges covers some numbers and reveals others that were hidden. The back of each tile has the same number as the front; behind the tile is a number that is twice as big as the original.

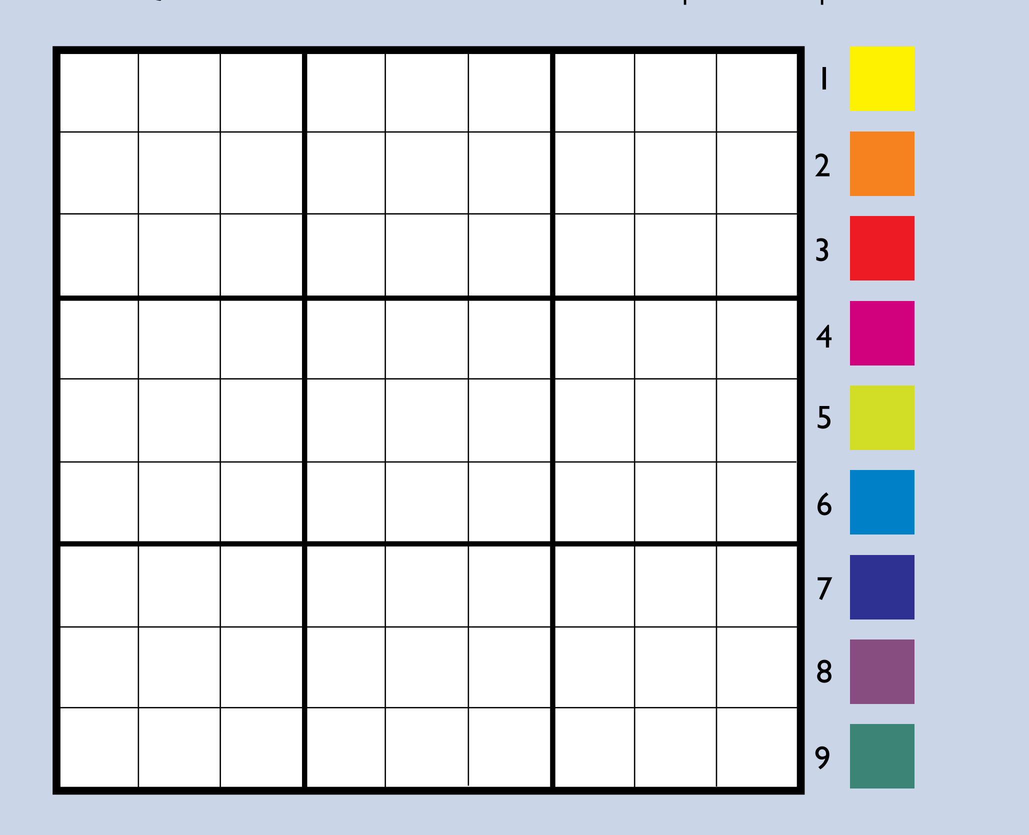
Can you flip just three numbered tiles so that the sum of every vertical, horizontal and main diagonal line equals the magic constant of 34?



PLAYTHINK 381 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

COLOR LATIN SQUARES

Color the nine-by-nine square using nine different colors so that each row, each column and each three-by-three composite square contains each color exactly once. Since the colors are numbered, you can use the numbers to help solve the puzzles.



PLAYTHINK 382 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

MONKEYS AND DONKEYS

Five monkeys and three donkeys live in a zoo. If you had to select one monkey and one donkey, how many different combinations could you choose from?









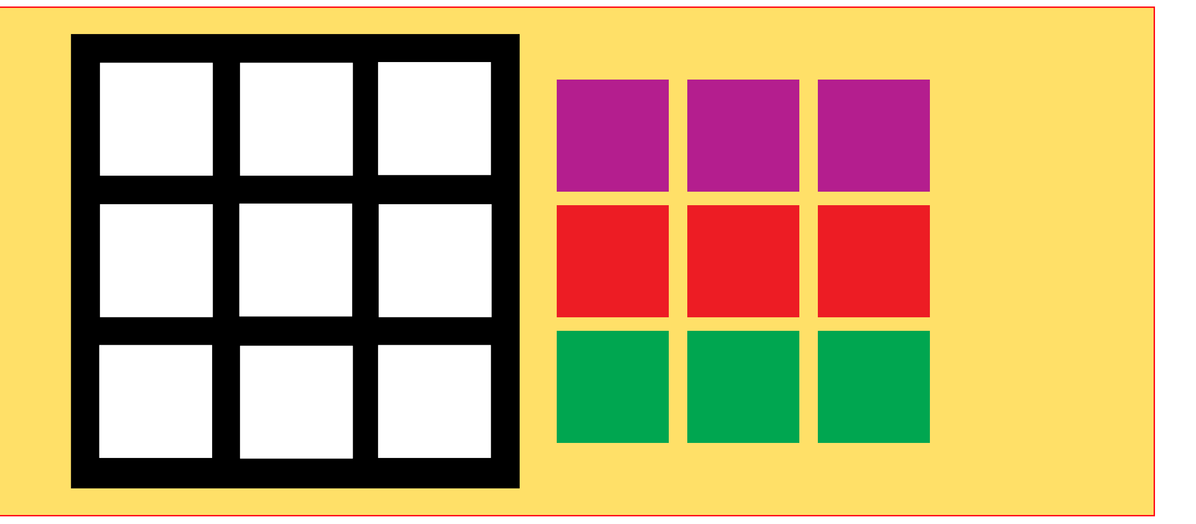

PLAYTHINK 383

DIFFICULTY: ●●○○○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

MAGIC COLOR SQUARE OF ORDER 3

Can you distribute the colored tiles across the grid so that each color appears just once in every row and column?

Can you extend the rules to account for both main diagonals? What about every diagonal?



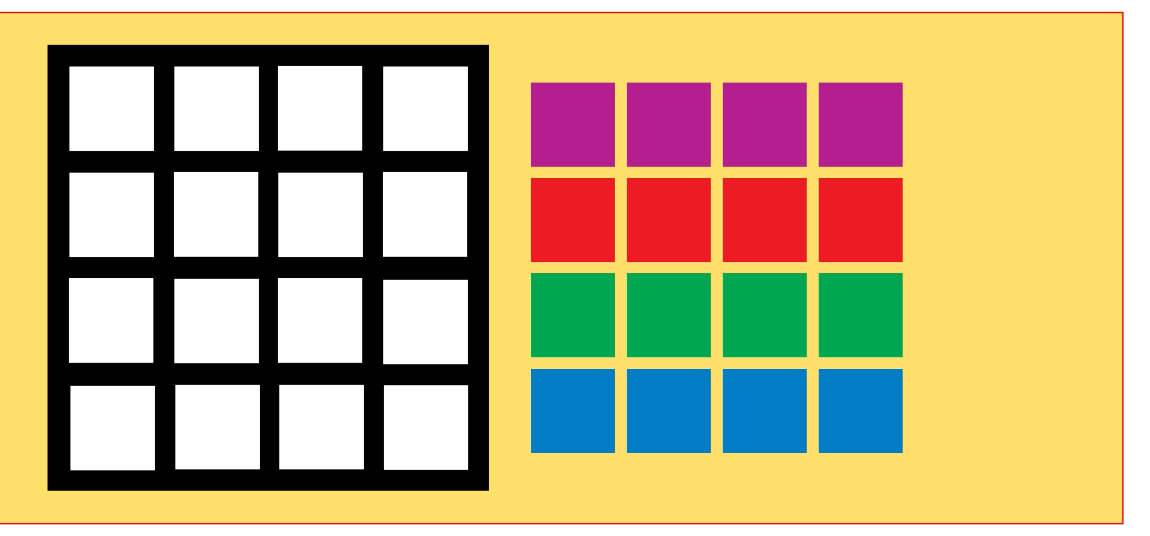
PLAYTHINK 384

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

MAGIC COLOR SQUARE OF ORDER 4

Can you distribute the colored tiles so that each color appears only once in every row and column?

Can the rule be extended to the main diagonals in this instance? To all diagonals?



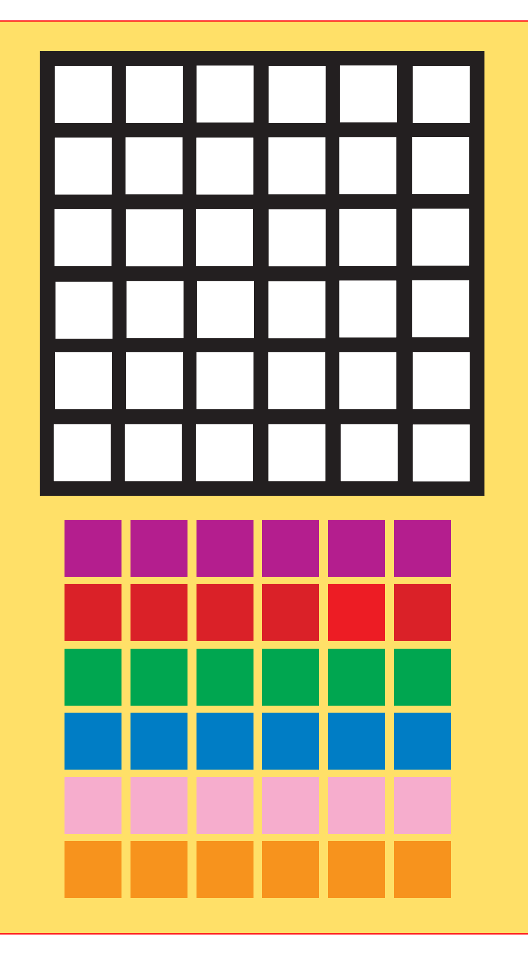
PLAYTHINK 385

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

MAGIC COLOR SQUARE OF ORDER 6

Can you distribute the thirty-six tiles so that each color appears only once in every row and column? Can you extend the rule to the two main diagonals?

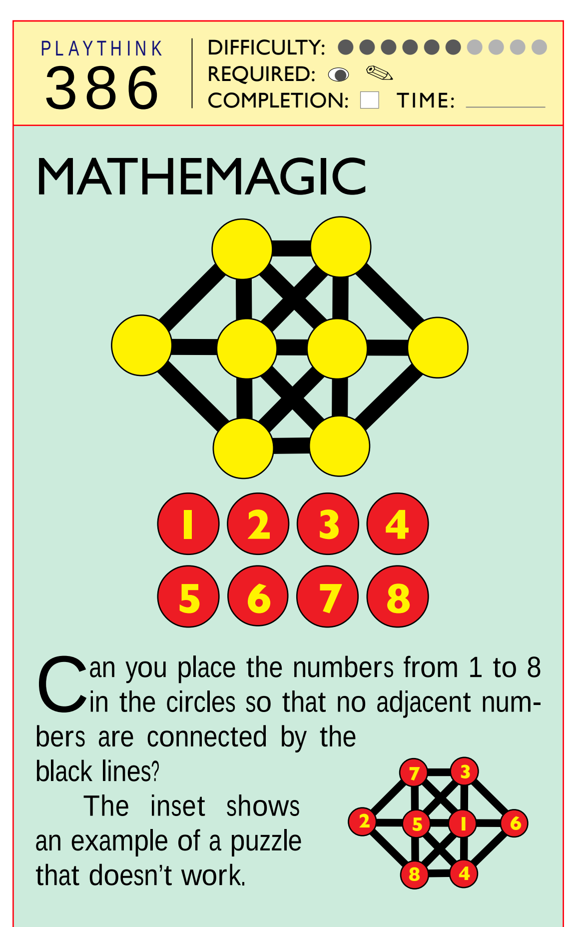
This puzzle can become a two-player game. Players take turns placing tiles on the board so that no two tiles of the same color appear in any given row or column. The last player to make a legal move wins.



PLAYTHINK 386

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

MATHEMAGIC



Can you place the numbers from 1 to 8 in the circles so that no adjacent numbers are connected by the black lines?

The inset shows an example of a puzzle that doesn't work.

Latin Squares

Near the end of his life, the great mathematician Leonhard Euler devised a new type of magic square, the Latin square. In a Latin square a number of symbols (numbers, letters, colors, etc.) are placed in a square of the same order so that each row or column contains each symbol only once. For example, a five-by-five square might contain five letters *(a, b, c, d, e)* five times each in such a way that no two *a*'s appear in the same row or column. There are also magic diagonal Latin squares, in which the same rules apply also across the two main diagonals or even across all smaller diagonals.

A further complication is found in the Greco-Latin magic square. This square consists of two Latin squares that have been superimposed so that each cell contains one element of each square, each element of one square is combined with an element of the second square only once, and each row and column contain every element from both squares. One simple illustration of such a square would be

1a, 2b, 3c
2c, 3a, 1b
3b, 1c, 2a

It is easy to see that no Greco-Latin magic square of order 2 can exist. The "Magic Color Shapes" puzzle-game (PlayThink 400) is a Greco-Latin square of order 4.

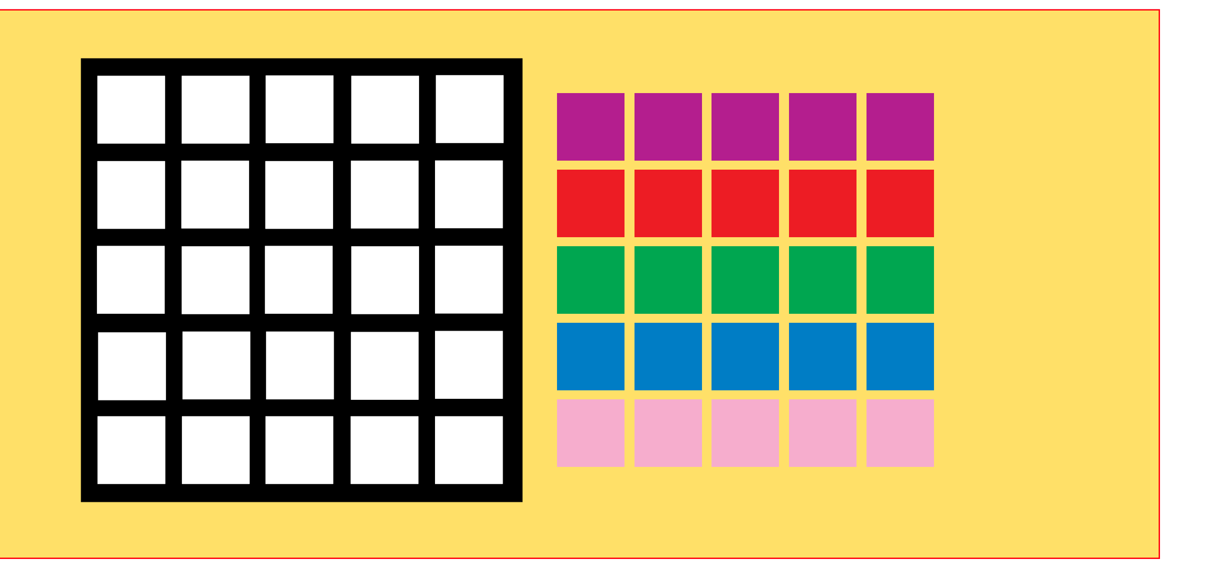
Latin and Greco-Latin magic squares are not mere diversions—they have valuable applications in experimental science. Suppose an agricultural researcher wished to test the effect of seven types of fungicides on wheat plants. He might divide an experimental field into seven parallel strips and treat each strip with a different fungicide. But such a test might be biased because of a favorable field condition in one of the plots—say, in the easternmost or southernmost strip. The best way to control for such biases is to divide the field into forty-nine plots in a seven-by-seven matrix and apply the chemicals according to the prescriptions of a Latin square. That way each fungicide is tested in every field condition. If the experiment needed to test the seven fungicides on seven different strains of wheat, then a Greco-Latin square could be applied.

In this way Euler's recreational problem has become a widely experimental design, not only in agriculture but also in biology, sociology, medicine and even marketing. The "cell" need not, of course, be a piece of land. It might be a cow, a patient, a leaf, a cage of animals, a city, a period of time and so on. The square is simply a way to combine variable elements in unique ways.

PLAYTHINK 387 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: | COMPLETION: ☐ TIME: ____

MAGIC COLOR SQUARE OF ORDER 5

Can you place the twenty-five tiles on the grid so that each color appears only once in each row and column? Again, can this be extended to the two main diagonals? What about every diagonal?



PLAYTHINK 388

DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁 ✎ 📄 ✂
COMPLETION: ☐ TIME: ______

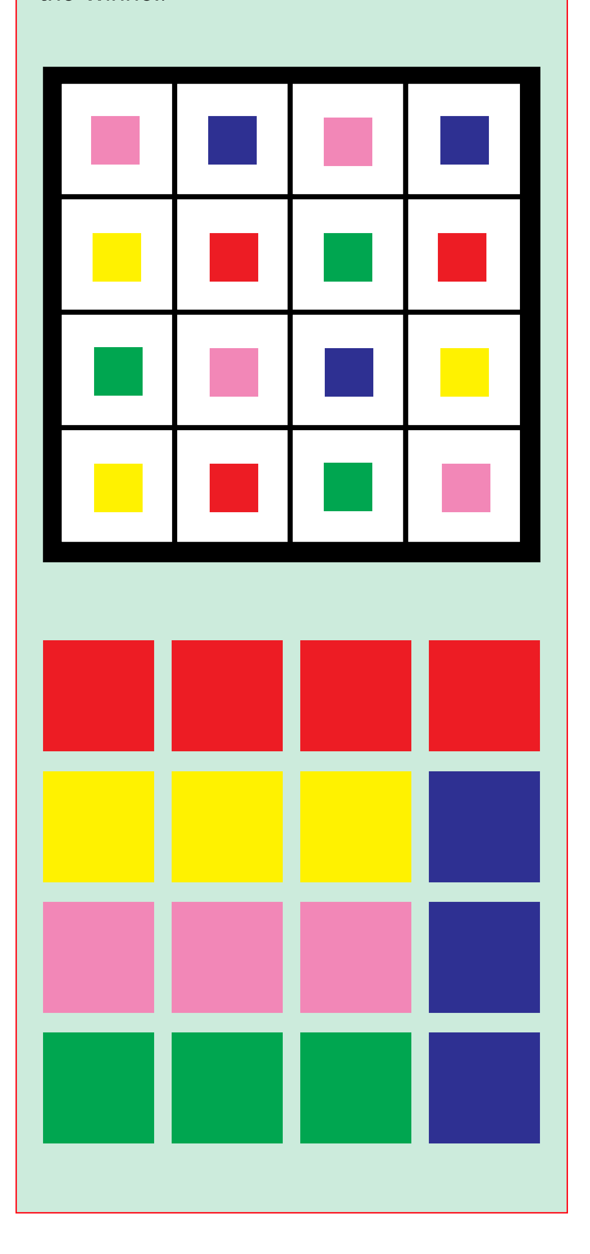
SPECTRIX

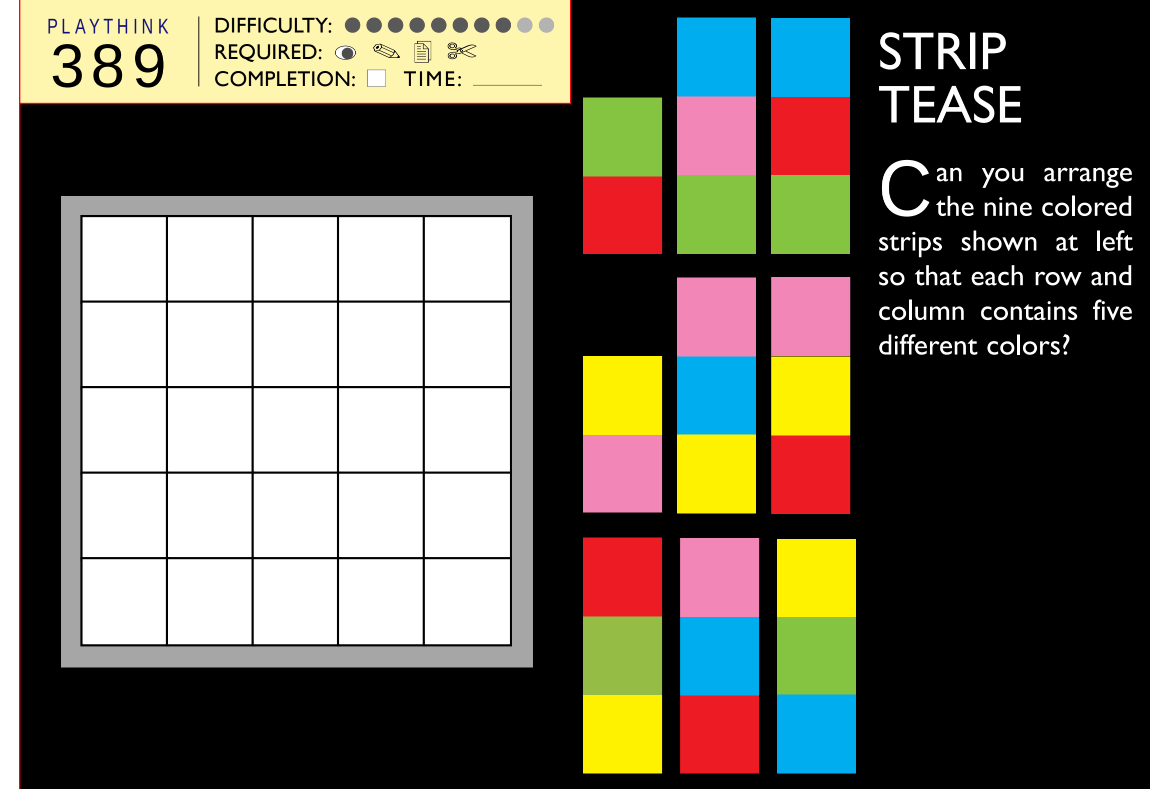
The colored tiles below can be placed one by one on the grid, but only if the following rules are observed:

- No tile can be placed on a square of the same color or horizontally, vertically or diagonally next to a square of the same color.
- Once a tile is placed on the board, that square takes on the color of the tile.
- No tile can be placed on another tile.

Can you place all sixteen tiles on the board?

The puzzle can be played as a two-person game. Players take turns placing tiles on the board in accordance with the rules above. The last person able to place a tile is the winner.





PLAYTHINK 389

DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁 ✎ 📄 ✂
COMPLETION: ☐ TIME: ______

STRIP TEASE

Can you arrange the nine colored strips shown at left so that each row and column contains five different colors?

PLAYTHINK 390

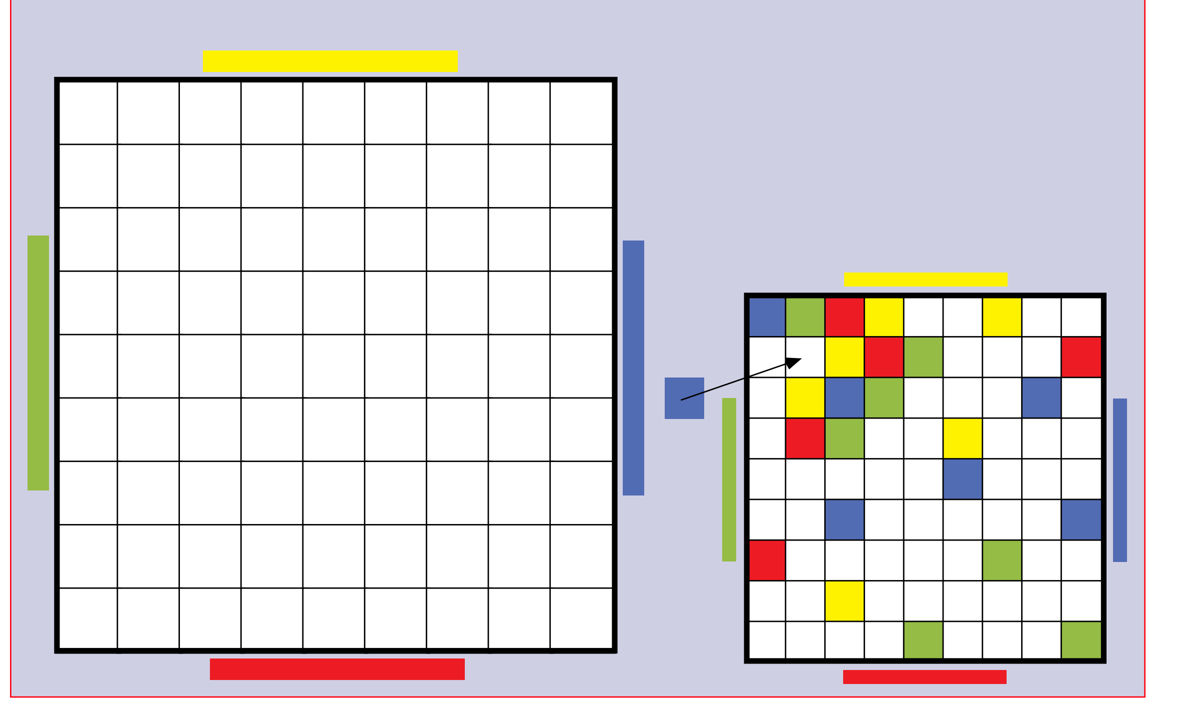
DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

FOUR-COLOR SQUARES GAME

The object of this simple but rewarding game is to create rows or columns made up of four squares of different colors. Each player controls two colors, either red and yellow or blue and green.

At each turn players place two squares, one of each color, on the board. Squares of the same color may not share a side, and no more than four squares can make a contiguous row or column.

Players receive one point for each four-color line they create; if one tile completes both a row and a column, the points are doubled. For example, in the sample game shown below, the player placing the blue square would receive four points, double the points for creating a row and a column simultaneously.



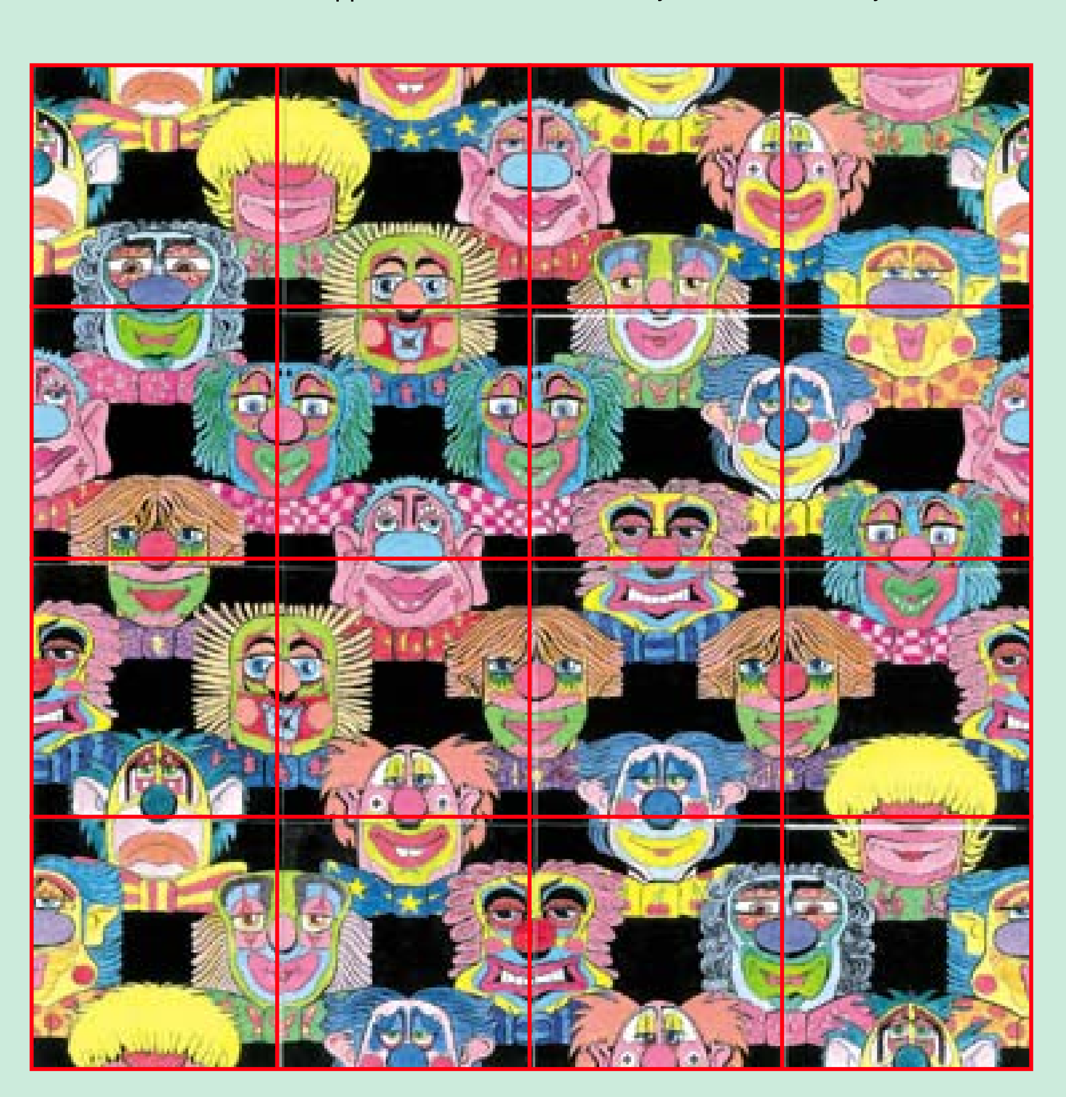
PLAYTHINK 391

DIFFICULTY: ●●●○○○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

CLOWN FUN

How many different clowns can you find in this four-by-four configuration of sixteen square tiles? Are all the clowns equally represented, or do some clowns appear more often than others? How many complete clowns are there? And how many can be shown completely in any given four-by-four arrangement?

These tiles may be copied and cut out to create tiles for many solitaire and party games. Simply use the game rules you'll find with PlayThink 123 and PlayThink 104.

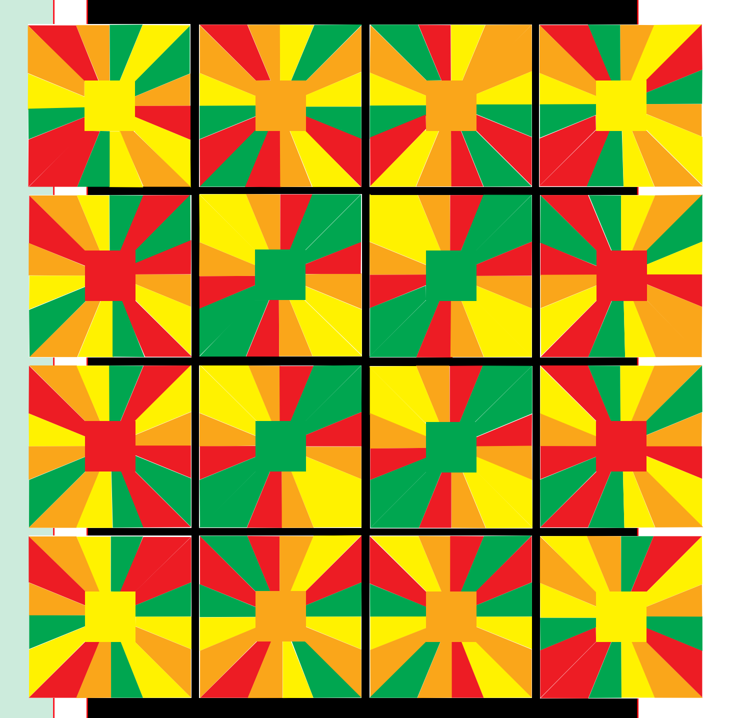


PLAYTHINK 392

DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

RADIANT SQUARES

This grid could be configured so that every edge touches an edge of the same color if only four are rotated. Can you work out which four must be turned?



PLAYTHINK 393

DIFFICULTY: ●●●●○○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

BALANCING ACROBATS

What stunt will the acrobats do next?

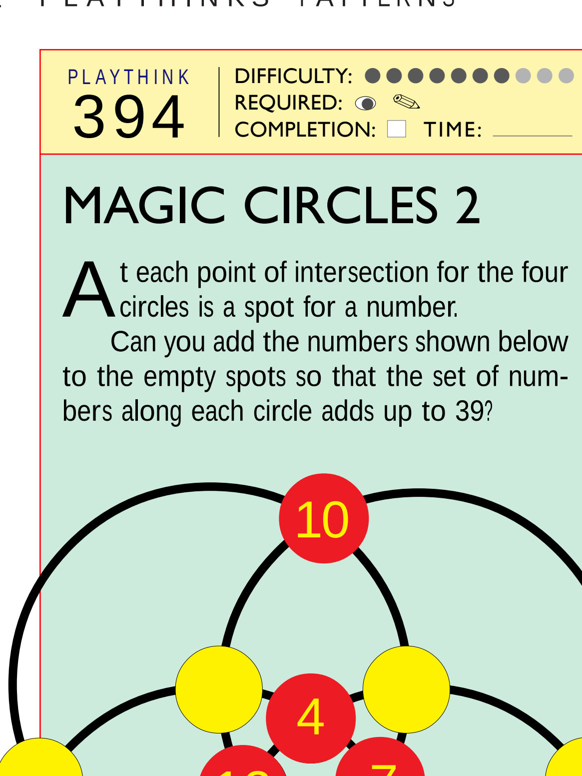


PLAYTHINK 394 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

MAGIC CIRCLES 2

At each point of intersection for the four circles is a spot for a number.

Can you add the numbers shown below to the empty spots so that the set of numbers along each circle adds up to 39?



PLAYTHINK 395 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ 📄 ✂ | COMPLETION: ☐ TIME: ____

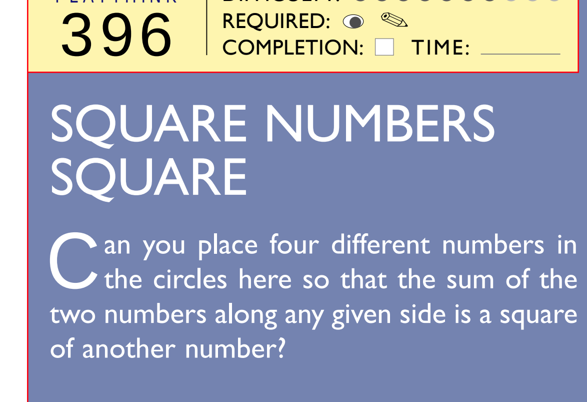
T-JUNCTIONS

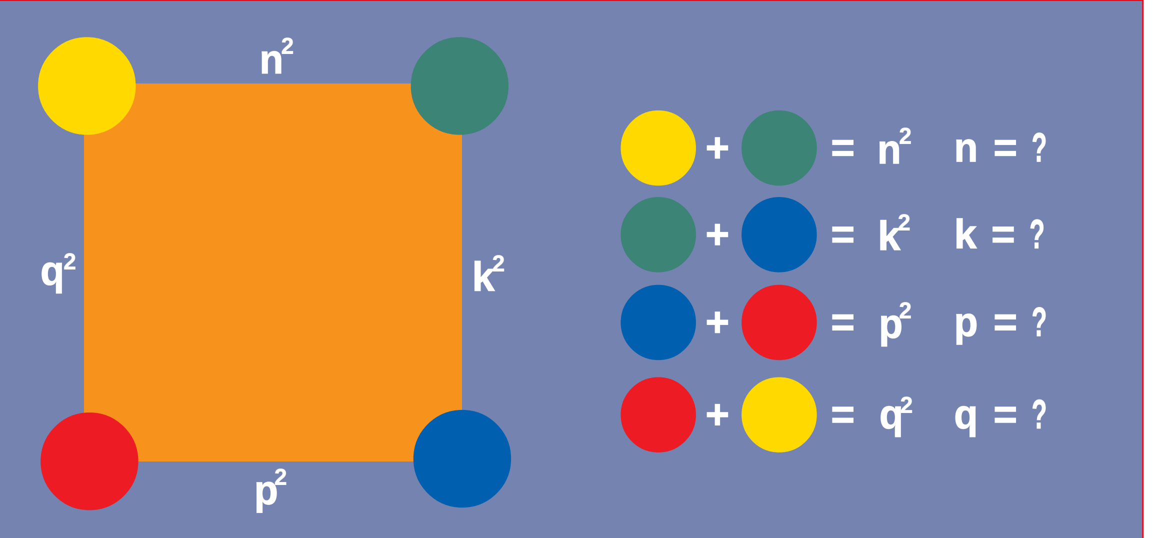
Can you place the T shapes on the colored grid in such a way that, in the resultant pattern, no color appears more than once in any given row or column?

PLAYTHINK 396 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

SQUARE NUMBERS SQUARE

Can you place four different numbers in the circles here so that the sum of the two numbers along any given side is a square of another number?

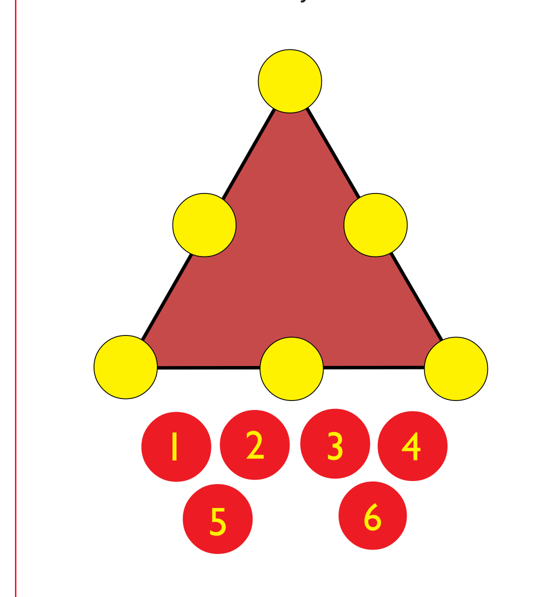




PLAYTHINK 397 | DIFFICULTY: ●●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

MAGIC TRIANGLE 1

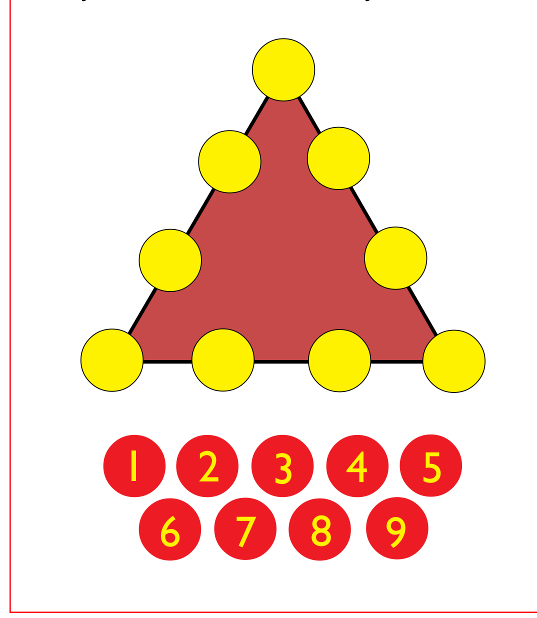
Can you place the numbers from 1 to 6 in the circles along the sides of the triangle so that three numbers on each line of circles add up to the same total? How many different solutions can you find?



PLAYTHINK 398 | DIFFICULTY: ●●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

MAGIC TRIANGLE 2

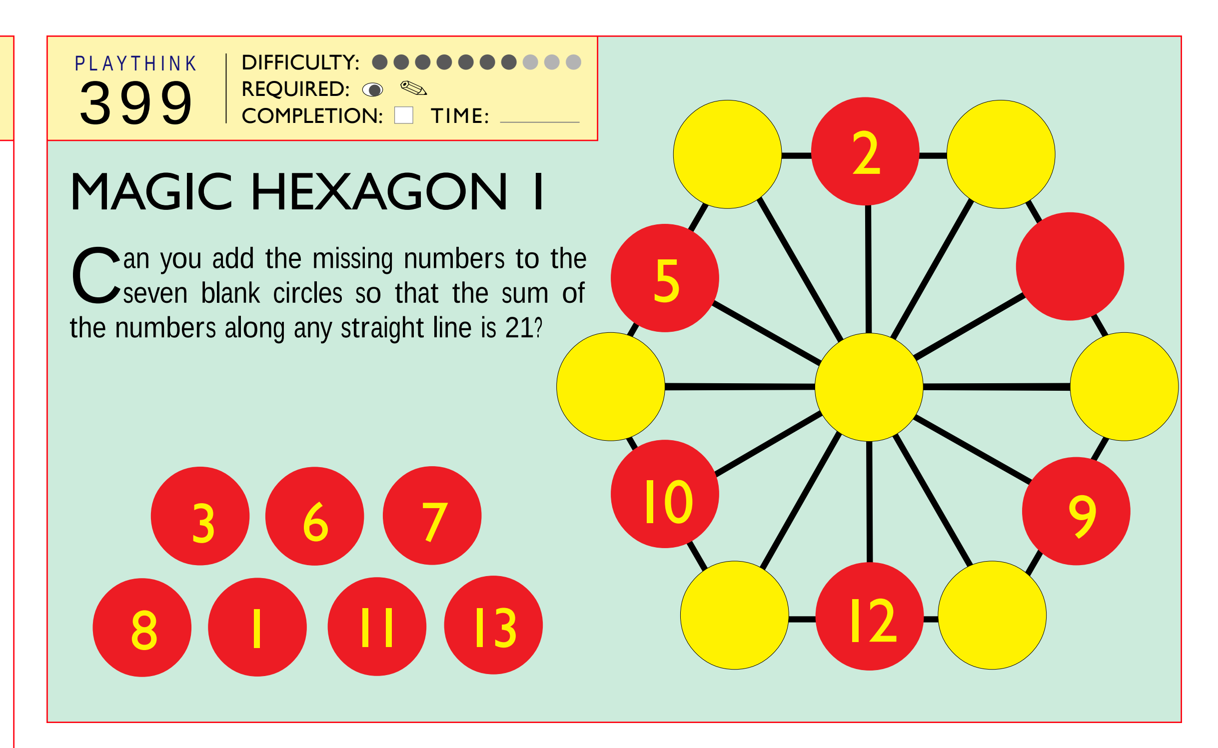
Can you place the numbers 1 through 9 in the circles along the sides of the triangles so that the four numbers in each line of circles add up to the same total? How many different solutions can you find?



PLAYTHINK 399 | DIFFICULTY: ●●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

MAGIC HEXAGON 1

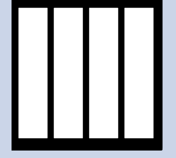
Can you add the missing numbers to the seven blank circles so that the sum of the numbers along any straight line is 21?



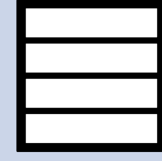
PLAYTHINK 400 | DIFFICULTY: ●●●●●●●●○○○ | REQUIRED: 👁 ✎ 📄 ✂ | COMPLETION: ☐ TIME: ____

MAGIC COLOR SHAPES

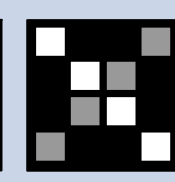
Can you arrange the sixteen colored shapes in such a way that they form more than simply a magic color square, but instead the sixteen perfect four-color, four-shape configurations illustrated in the patterns below? In other words, your answer should include four different colors and four different shapes in each of these categories:



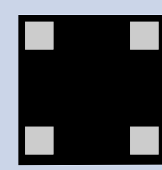
1. Four vertical columns



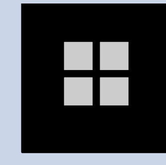
2. Four horizontal rows



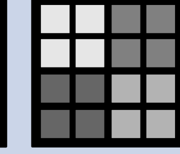
3. Two main diagonals



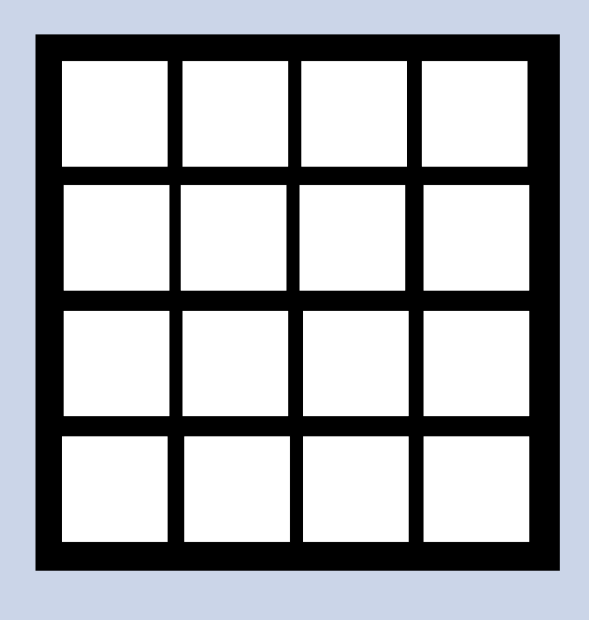
4. Four corner squares

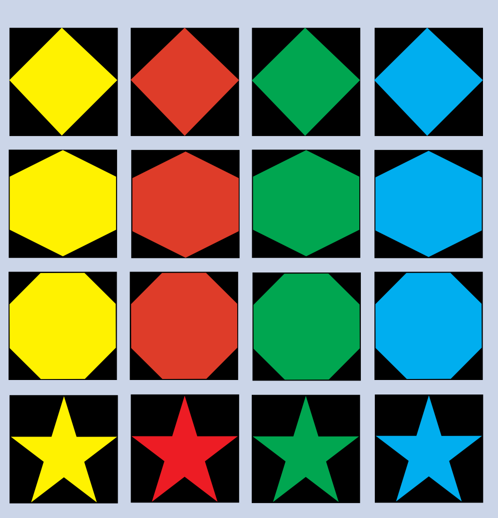


5. Four center squares



6. Four squares in each quadrant



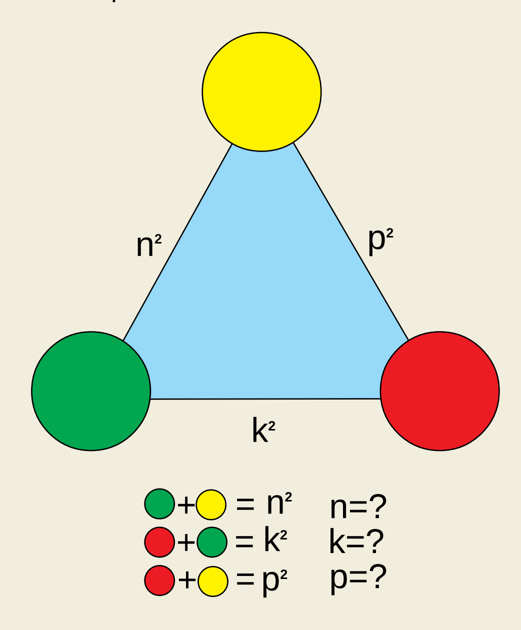


PLAYTHINK 401

DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: TIME:

SQUARE NUMBER TRIANGLE

Can you place three different numbers in the circles below so that the sum of the two numbers along any given side is equal to the square of another number?



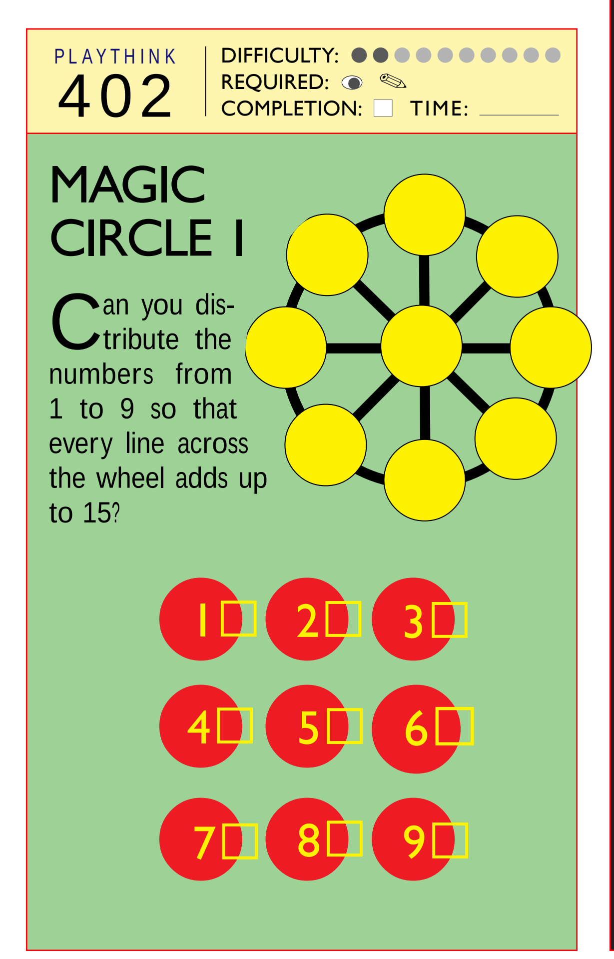
+ = n^2 n=?
+ = k^2 k=?
+ = p^2 p=?

PLAYTHINK 402

DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: TIME:

MAGIC CIRCLE I

Can you distribute the numbers from 1 to 9 so that every line across the wheel adds up to 15?

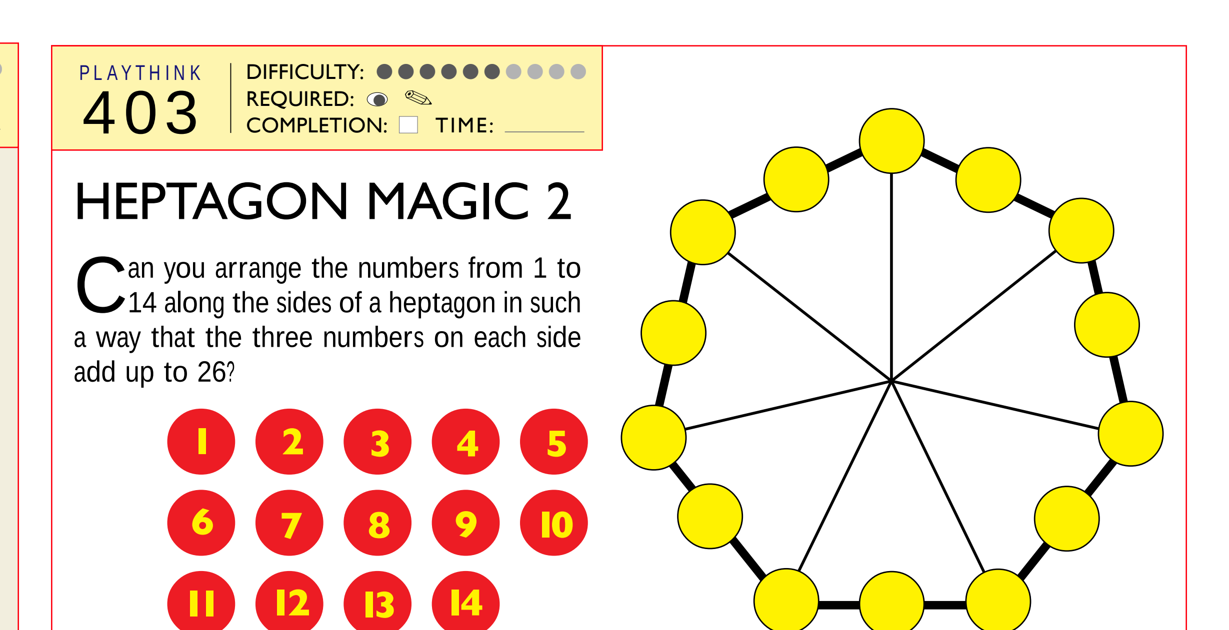


PLAYTHINK 403

DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: TIME:

HEPTAGON MAGIC 2

Can you arrange the numbers from 1 to 14 along the sides of a heptagon in such a way that the three numbers on each side add up to 26?



PLAYTHINK 404

DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: TIME:

MAGIC ALIENS SQUARE

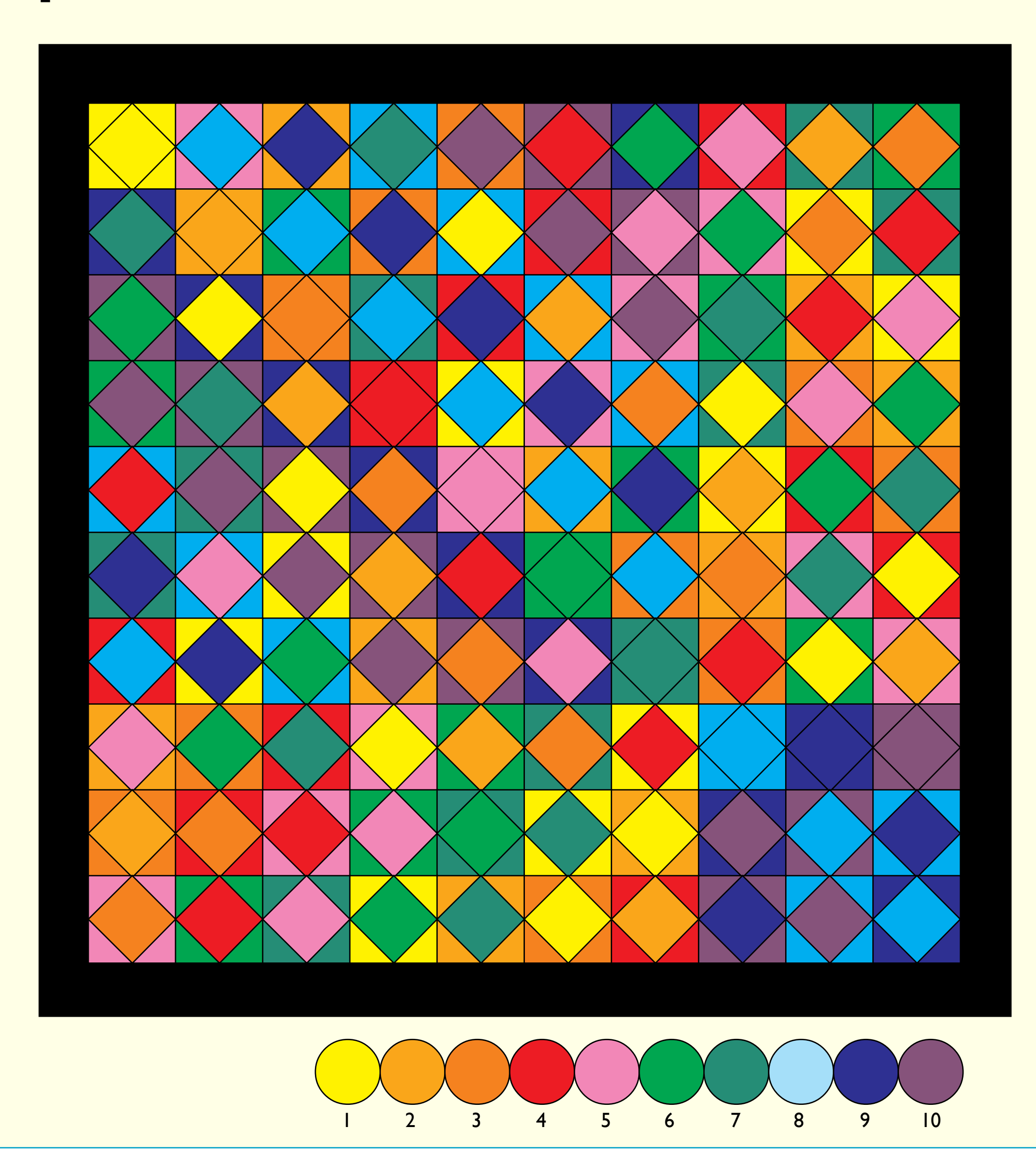
Which of the five aliens at the left will complete the magic square?



Greco-Latin Magic Square of Order 10

For many years it was thought that an order 10 Greco-Latin square was impossible. And it remained an unsolved problem even after the first computer search for an answer was conducted in 1959; after 100 hours, no solution was found. The programmers believed that a full search would take more than 100 years, but the failure reinforced the idea that no solution would ever be found.

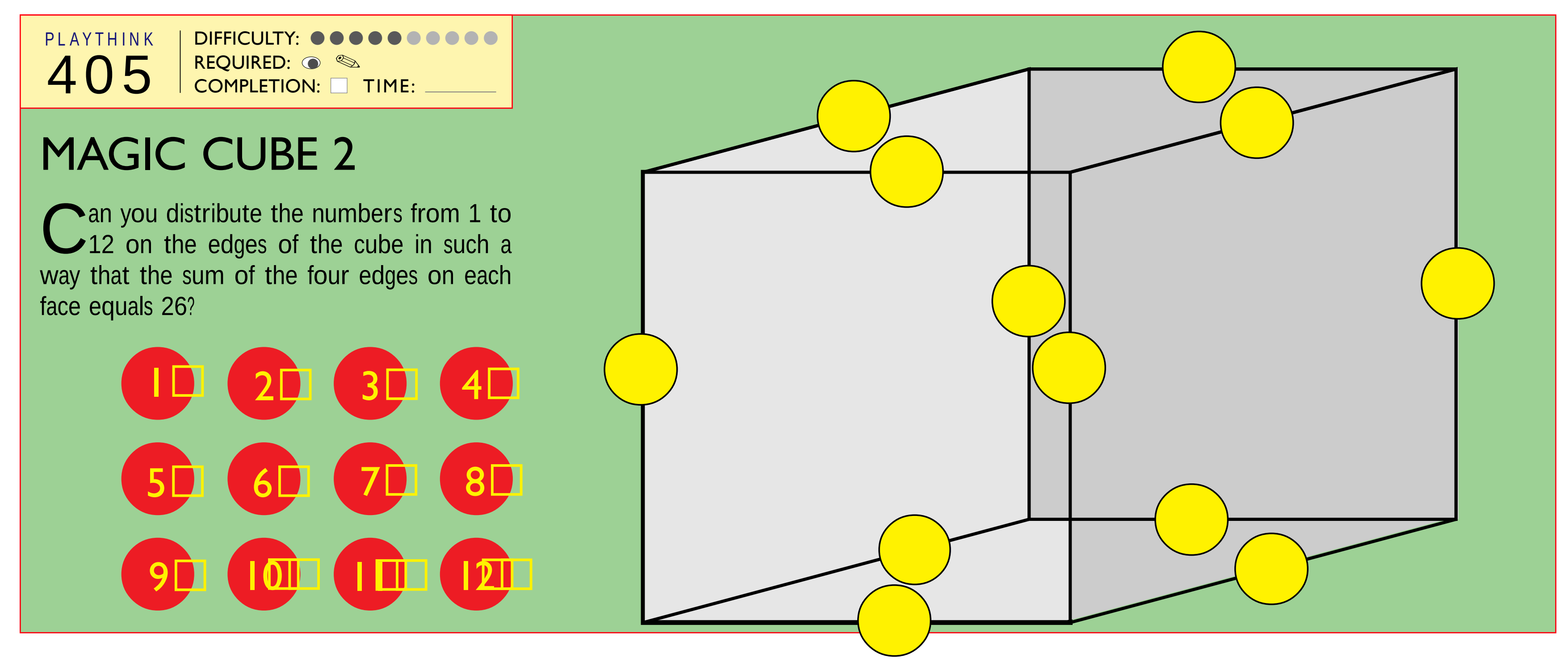
Researchers discovered a new approach in 1960, one that found a wealth of solutions not only for squares of order 10 but for squares of order 14, order 18 and greater. Illustrated here is one solution for an order 10 Greco-Latin magic square, with colors substituted for the numbers from 1 to 10.



PLAYTHINK 405 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: | COMPLETION: ☐ TIME:

MAGIC CUBE 2

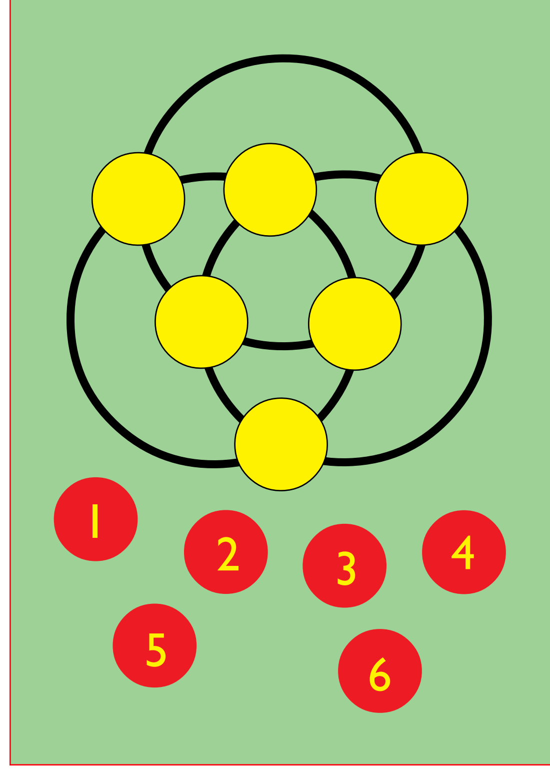
Can you distribute the numbers from 1 to 12 on the edges of the cube in such a way that the sum of the four edges on each face equals 26?



PLAYTHINK 406 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ______

MAGIC CIRCLES 3

Can you distribute the numbers from 1 to 6 on the intersections of the three circles so that the sum of the numbers on each circle is identical to the other two?



PLAYTHINK 407 | DIFFICULTY: ●●●○○○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ______

SQUARE CASCADES

The object of this game is to arrange a set of numbers so that no square possesses a value smaller than its neighbor to the right or immediately below. The solution for a square with four fields is shown. Can you place the numbers 1 to 9 on the middle game board and 1 to 16 on the game board below it according to these rules?

When completed, the effect is like water cascading down from the top left to the lower right.

4	3
2	1

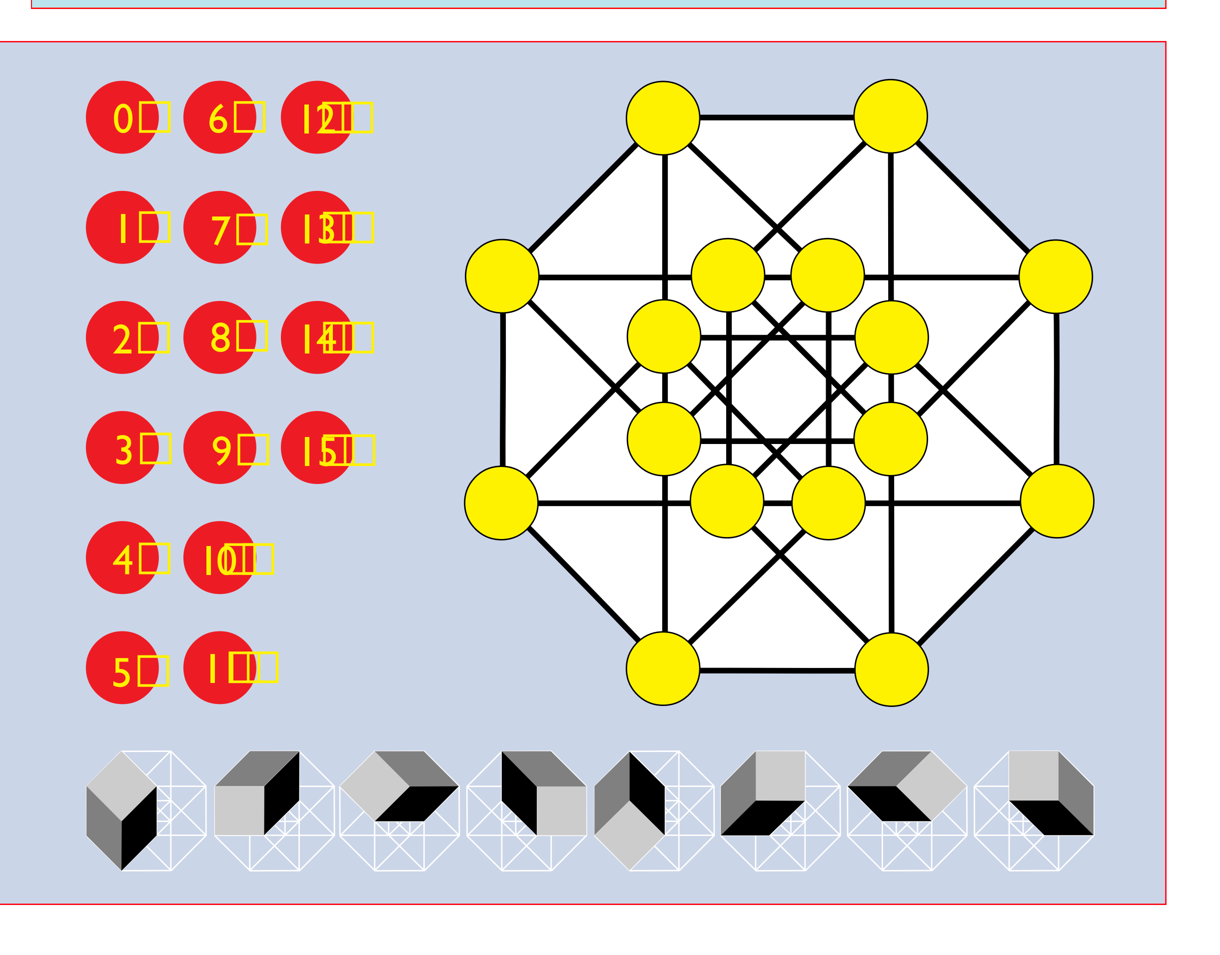
PLAYTHINK 408 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ______

HYPERCUBE

Islamic mystics first constructed the figure in this puzzle, called a tesseract. Modern mathematicians now consider it to be the two-dimensional representation of a four-dimensional hypercube.

Can the human brain really visualize four-dimensional space? Although humans are confined to three dimensions of space, it is conceivable that with the proper mathematical training, a person could develop the ability to fully visualize the tesseract.

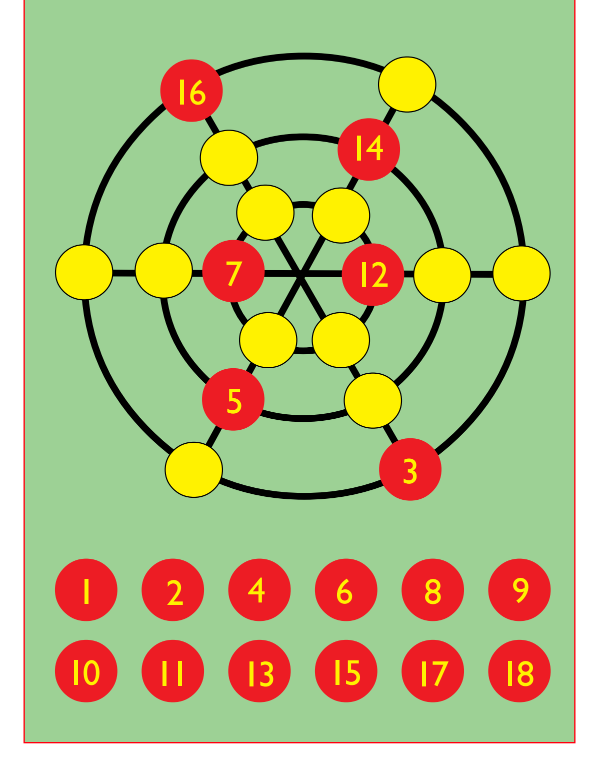
For our puzzle, can you place the numbers from 0 to 15 on the circles of the hypercube in such a way that the numbers at the corners of the "square face" of the eight cubes shown at right in perspective add up to 30?



PLAYTHINK 409
DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

MAGIC CIRCLES 4

Arrange the numbers from 1 to 18 so that the sum of any two symmetrical pairs of numbers is 19. Three pairs have already been placed. Can you place the rest?

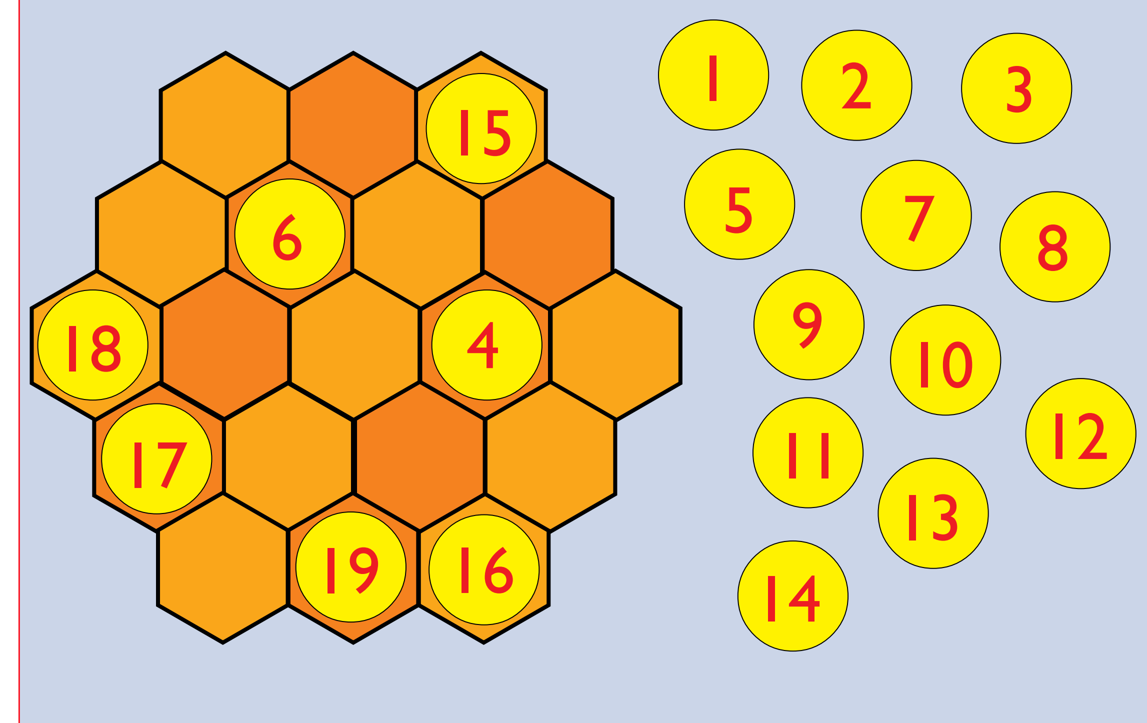


PLAYTHINK 410
DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

MAGIC HEXAGON 2

Volumes have been written about magic squares, but the "magic" can be embodied by other polygons, such as triangles, circles and hexagons. For example, can you distribute the numbers from 1 to 19 in the hexagonal game board illustrated below so that the sum of every straight line is identical? Can you work out what the magic constant must be?

To keep the puzzle from being too difficult, we've seeded some of the hexagons with numbers. You need to place only the remaining numbers.



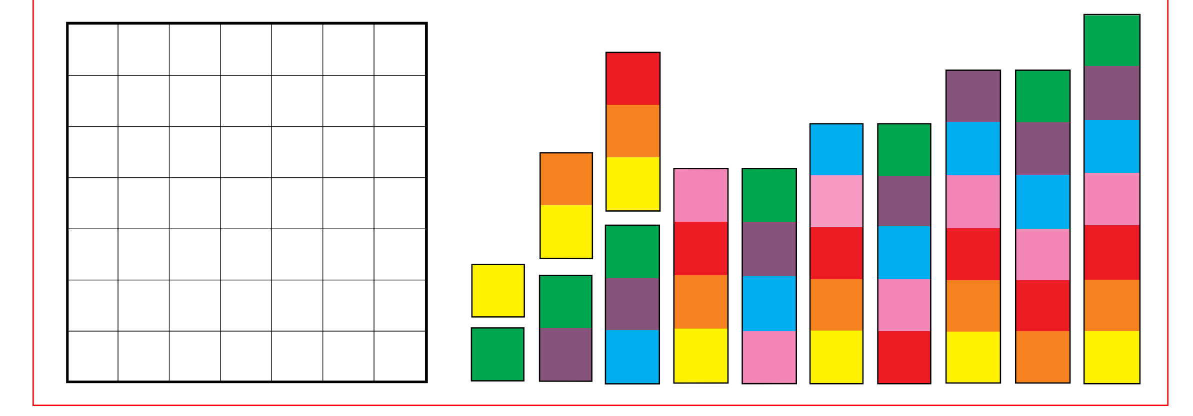
PLAYTHINK 411
DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎ 📄 ✂
COMPLETION: ☐ TIME: ____

MAGISTRIPS

The thirteen strips can be arranged into a seven-by-seven square in such a way that each horizontal row contains a single solid color.

Can you rearrange the strips so that no color appears more than once in any horizontal row? This is an easy problem with many solutions.

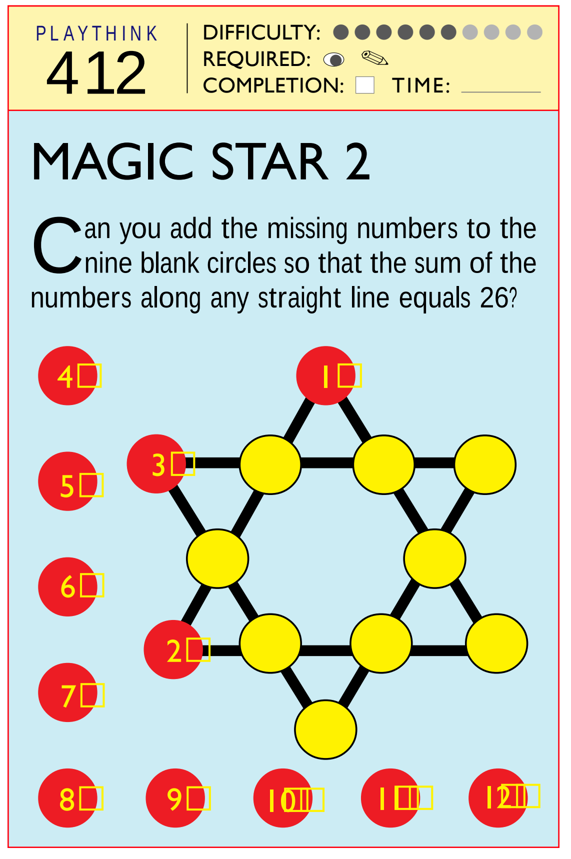
But can you then rearrange the strips again so that no color appears more than once in any row, column or diagonal line (including the small diagonals)? This puzzle can be played as a two-person game. Players take turns placing the strips on the board; the last player who can place a strip without violating the rules wins.



PLAYTHINK 412
DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

MAGIC STAR 2

Can you add the missing numbers to the nine blank circles so that the sum of the numbers along any straight line equals 26?

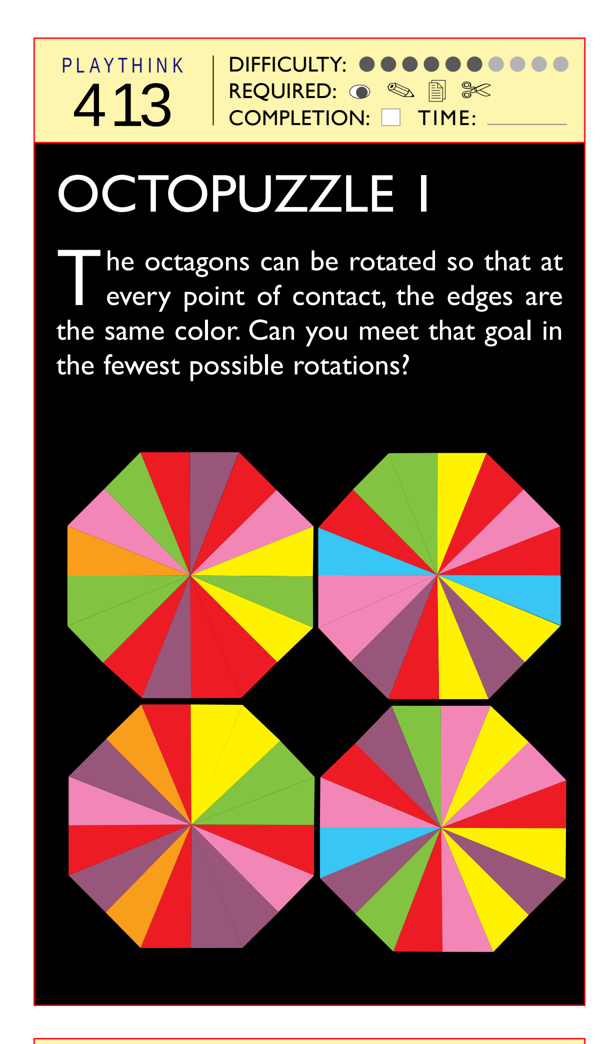


PLAYTHINK 413

DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

OCTOPUZZLE 1

The octagons can be rotated so that at every point of contact, the edges are the same color. Can you meet that goal in the fewest possible rotations?



PLAYTHINK 414

DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

OCTOPUZZLE 2

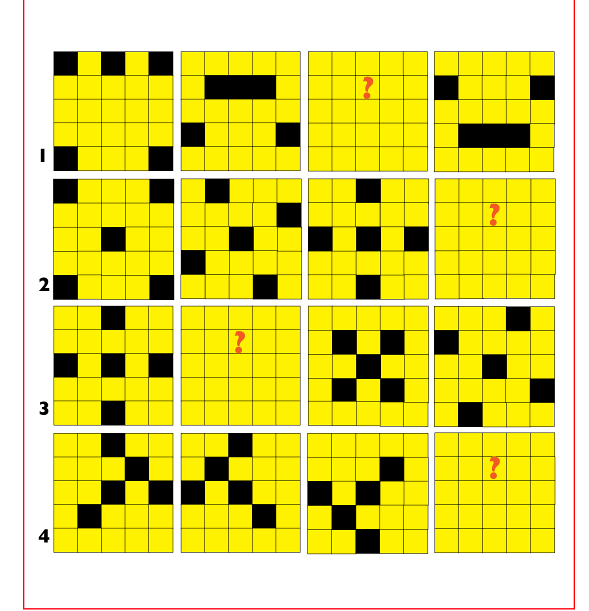
Can you rotate the nine octagons so that the edges that face each other share the same color? There are two possible solutions.

PLAYTHINK 415

DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

SQUARE DANCE

Each row is a sequence of predictable motions for five black squares, with one pattern missing from each sequence. By studying the three patterns given in each row, can you complete all four sequences?



PLAYTHINK 416

DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

GRIDS AND ARROWS

The yellow squares around the square grid must be filled with one arrow each, drawn so that the arrow points vertically, horizontally or diagonally along the grid. Can you fill in the arrows in such a way that the number of arrows pointing to each square in the grid equals the number shown in that square?

3	2	1	2	2
2	1	3	1	4
2	4	2	5	2
4	2	5	2	3
3	4	2	3	3

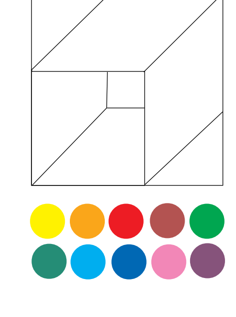
PLAYTHINK 417

DIFFICULTY: ●●●●●●●●●○
REQUIRED: 👁 📄 ✂
COMPLETION: ☐ TIME: ______

CUBES IN PERSPECTIVE 2

When a solid melts or a liquid boils, the object being heated suddenly loses much of its internal order. What was rigid becomes fluid; what was well defined is now vaporous. Such events, called phase transitions, can occur in art as well as nature.

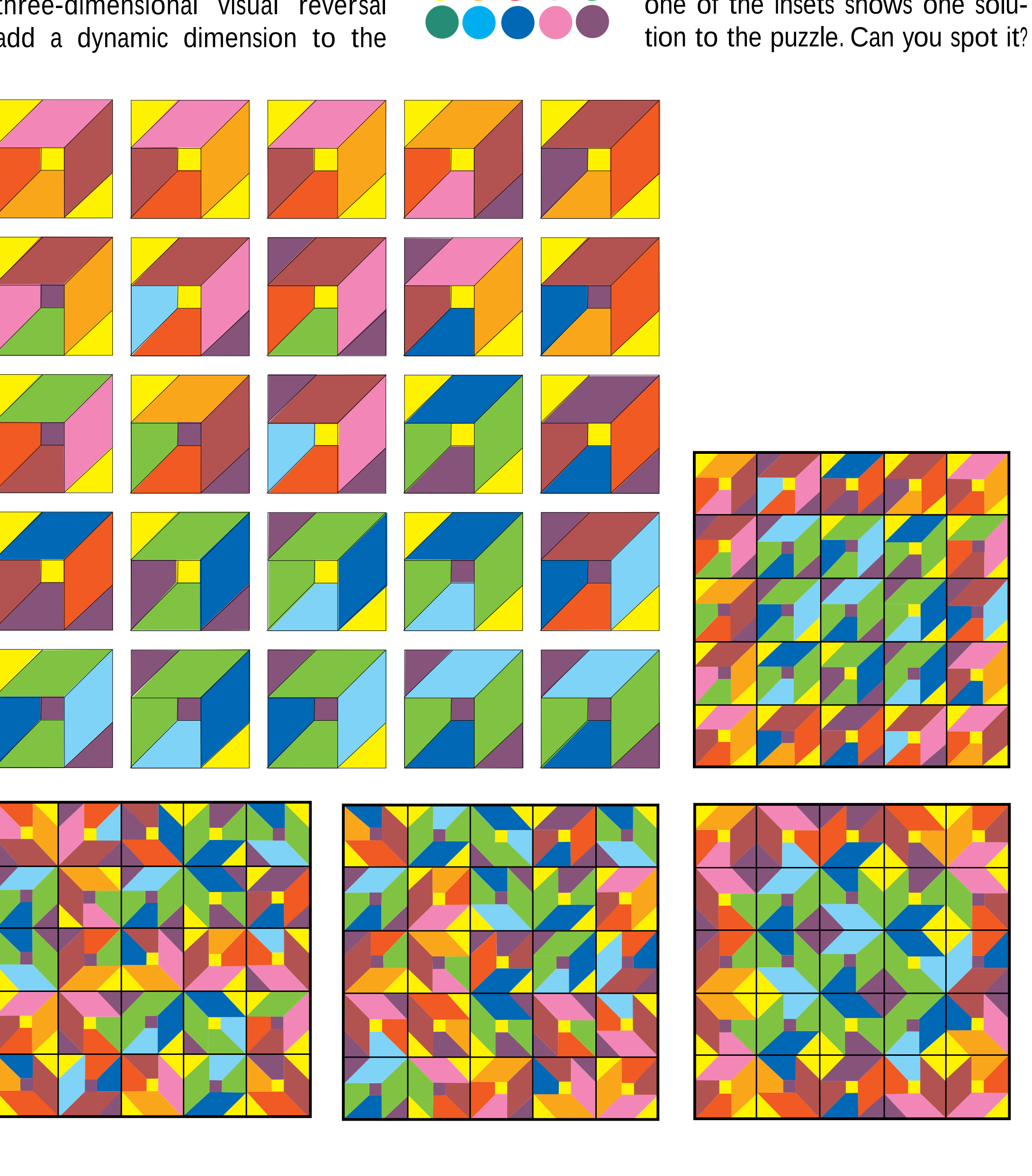
In this puzzle the domino principle achieves a similar effect. Matching the colors leads to merging the patterns of the tiles. The foreground optical illusion and three-dimensional visual reversal add a dynamic dimension to the puzzle. Indeed, among the art puzzles I've included in this book, this is one of the toughest to crack.



First, copy and cut out the twenty-five tiles and reassemble them to form a five-by-five matrix that conforms to the domino principle—colors must match along all touching sides.

The number of possible configurations is staggering: $2^{25} \times 25!$

The four compositions shown in the inset form a sequence in which the degree of order present in the pattern becomes less and less. It is, in fact, hard to believe that all these compositions are made up from the same basic elements. But one of the insets shows one solution to the puzzle. Can you spot it?



PLAYTHINK 418

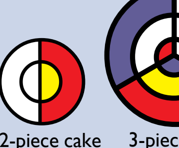
DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁 ✏
COMPLETION: ☐ TIME: ______

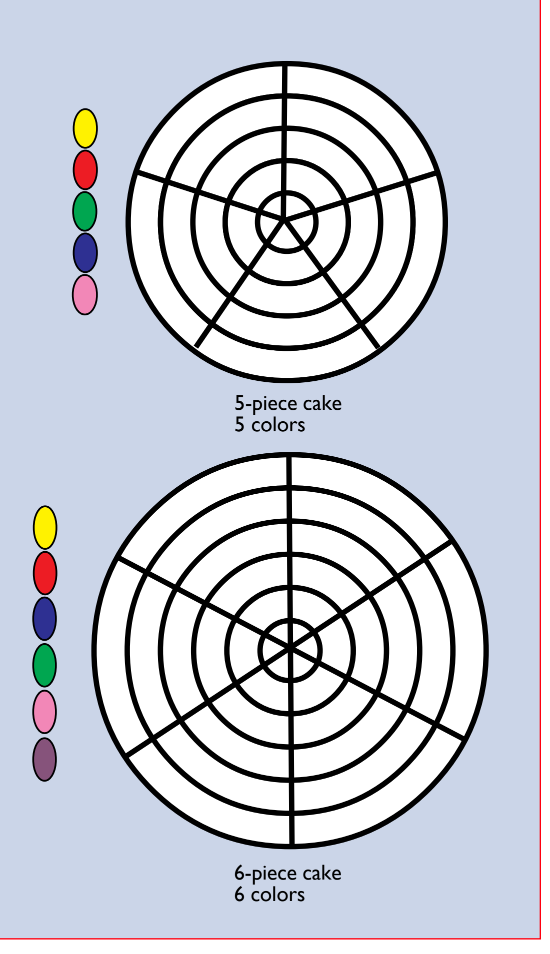
PIECE OF CAKE

The cakes below are sliced in such a way that they have the same number of concentric pieces as they have radial cuts. For example, one cake is divided into two pieces concentrically and two pieces radially, for a total of four pieces. Three radial cuts and three concentric pieces make for nine pieces.

For each cake, each slice should be filled in so that two pieces of the same color never contact each other—even across touching corners. The number of colors that can be used is equal to the number of concentric pieces: a two-cut cake can use only two colors; a three-cut cake, three colors.

As shown in the examples here, the task is impossible for a two-cut or three-cut cake. Can you make it work for a five-cut, five-color cake? How about a six-cut, six-color cake?



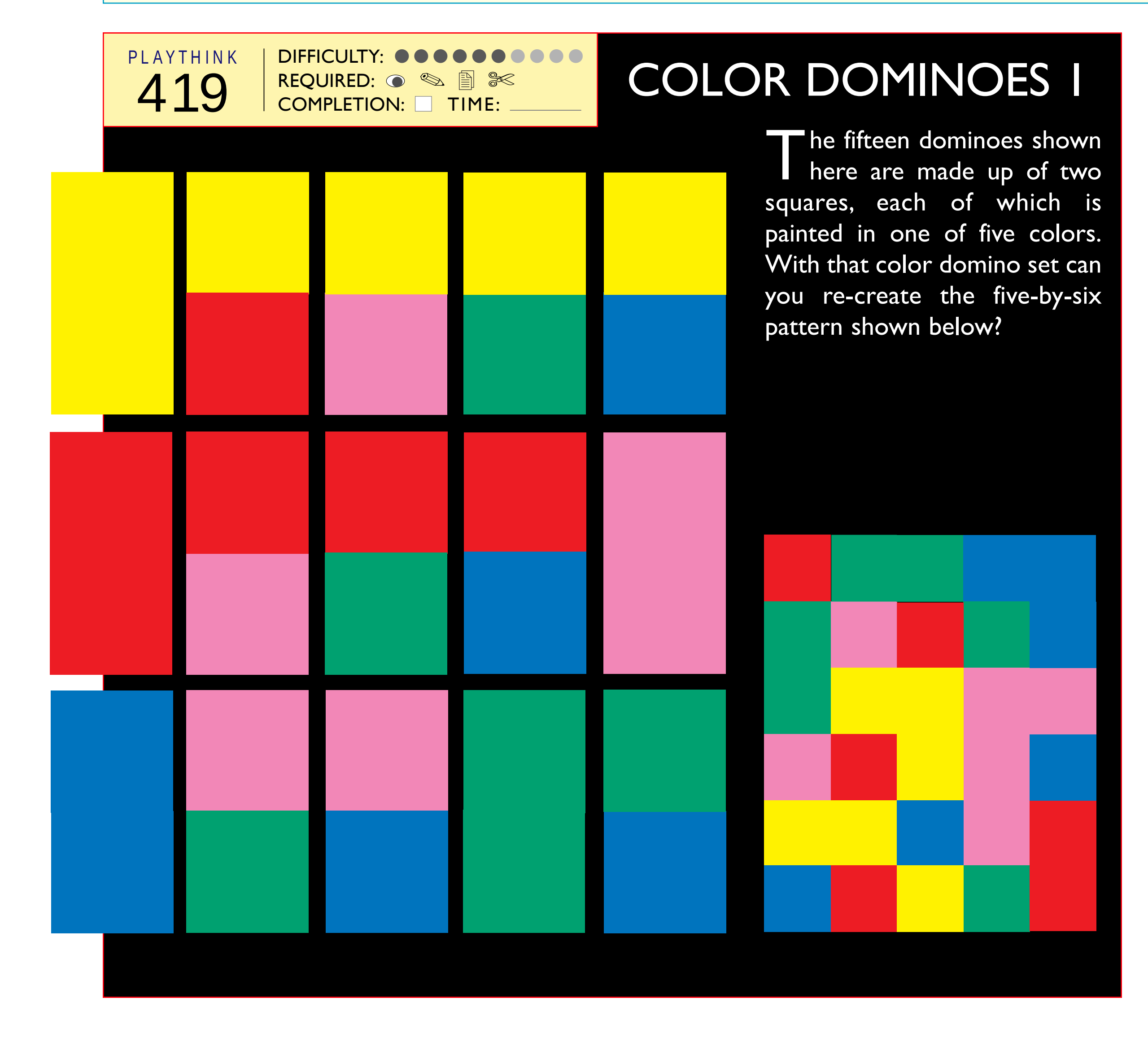


Dominoes and Combinatorial Games

Ordinary dominoes are two-by-one rectangular tiles with a different number on each end. The standard rule for playing dominoes is simple—the numbers on the adjacent ends of tiles must always match. The game of dominoes is the best-known example of a game that follows the so-called domino principle, but it is far from the only one.

The English mathematician Percy Alexander MacMahon devised a number of ingenious generalized domino games using colored polygonal dominoes that tile the plane. The set of tiles is not arbitrary: the same basic shapes or patterns are colored in all possible ways to form a complete set of tiles, no two of which are alike. (The reflections of a tile are considered to be different, but rotations are considered to be the same. This is a natural assumption because the tiles are usually colored on one side only and so cannot be turned over, but can be rotated in the plane without difficulty). The object of the games is to arrange the complete set of tiles according to the domino principle in some predetermined and pleasing pattern.

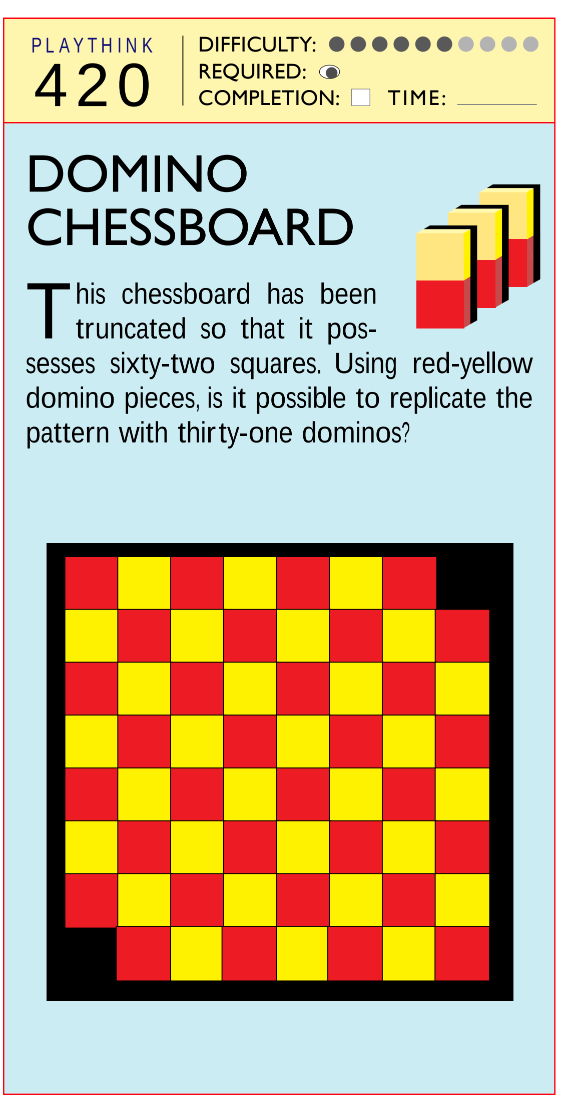
MacMahon's mathematical work was based on the theory of symmetrical functions—algebraic expressions that remain unchanged if the letters in them are permuted. For example, both a + b + c and ab + bc + ca are symmetrical functions of *a*, *b* and *c*. If the colors of a complete set of MacMahon's dominoes are permuted, we end up with exactly the same set of tiles as before. These tiles, in a sense, have a permutational symmetry.



PLAYTHINK 419 — DIFFICULTY: ●●●●●●○○○○ — REQUIRED: eye, pencil, paper, scissors — COMPLETION: ☐ TIME: ______

COLOR DOMINOES I

The fifteen dominoes shown here are made up of two squares, each of which is painted in one of five colors. With that color domino set can you re-create the five-by-six pattern shown below?



PLAYTHINK 420 — DIFFICULTY: ●●●●●●○○○○ — REQUIRED: eye — COMPLETION: ☐ TIME: ______

DOMINO CHESSBOARD

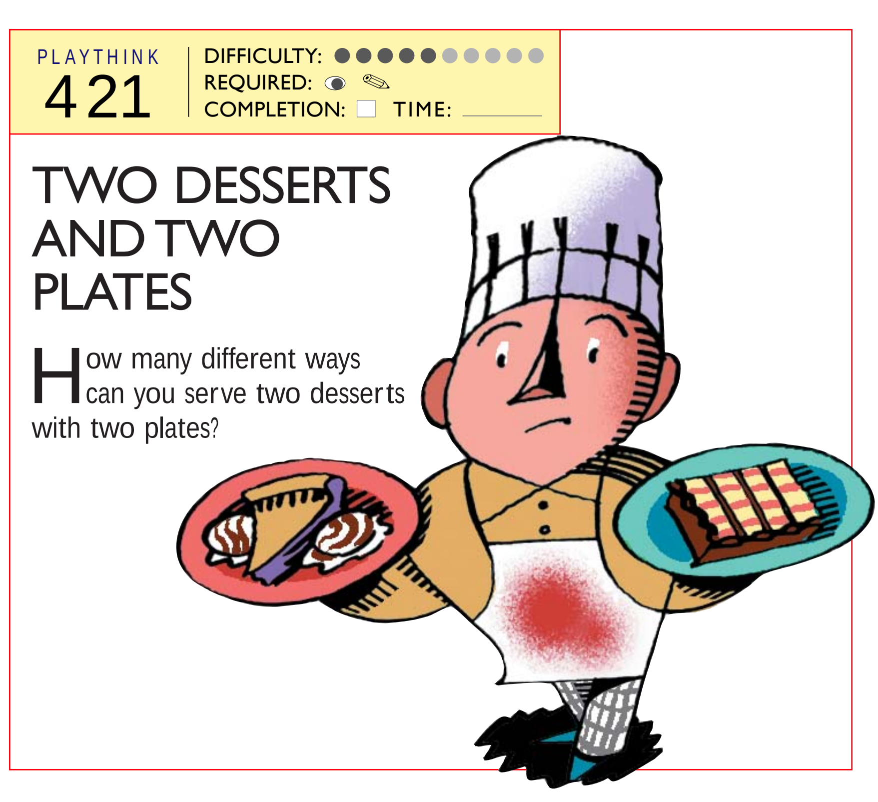
This chessboard has been truncated so that it possesses sixty-two squares. Using red-yellow domino pieces, is it possible to replicate the pattern with thirty-one dominos?

PLAYTHINK 421

DIFFICULTY: ●●●●●○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

TWO DESSERTS AND TWO PLATES

How many different ways can you serve two desserts with two plates?

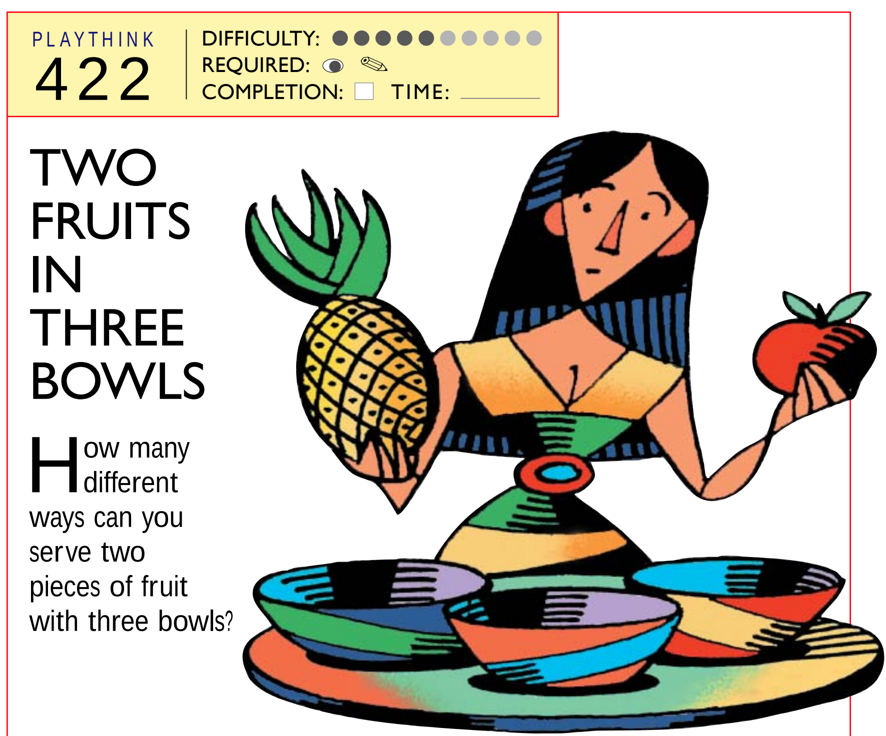


PLAYTHINK 422

DIFFICULTY: ●●●●●○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

TWO FRUITS IN THREE BOWLS

How many different ways can you serve two pieces of fruit with three bowls?



PLAYTHINK 423

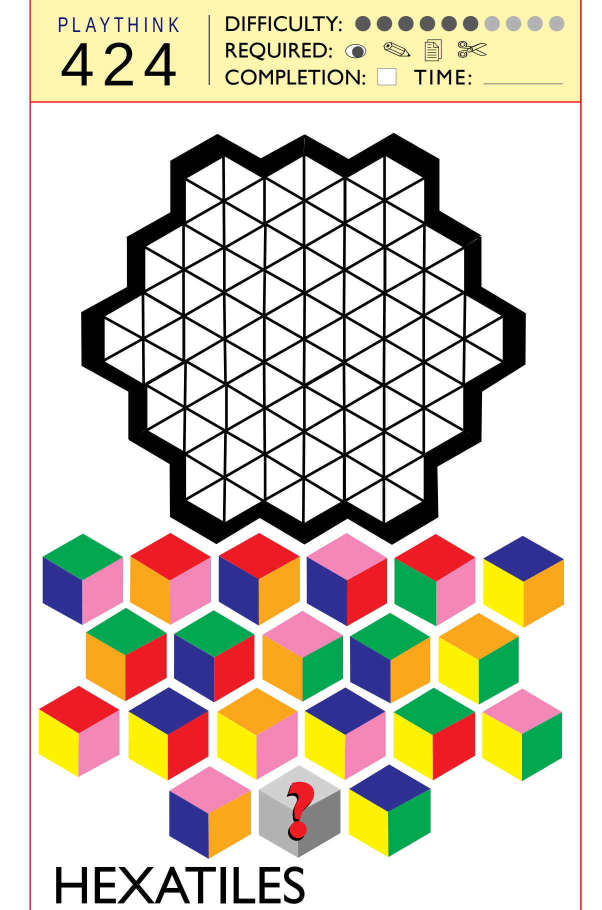
DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

COLOR DOMINOES 2

The twenty-eight dominoes shown here are made up of two squares, each painted in one of seven colors. Using that set of dominoes, can you re-create the pattern shown in the grid at top?

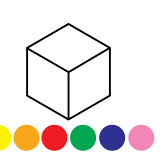
PLAYTHINK 424

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎ 📄 ✂
COMPLETION: ☐ TIME: ______



HEXATILES

Each of the hexagons shown here is divided into three fields. The fields are filled in with one of six colors, and no hexagon may have any two fields that are the same color. Following those rules, there are twenty possible hexagons (rotations and reflections don't count as being different).



Nineteen hexagons are shown. What are the colors of the missing hexagon?

Can you fit the twenty hexagons into the grid at top so that every pair of touching sides is the same color?

PLAYTHINK 425

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎ 📄 ✂
COMPLETION: ☐ TIME: ______

COLOR TRIANGLES I

For three-segmented triangles there are twenty-four permutations of four possible colors. Twenty-three are shown. Can you find the missing permutation?

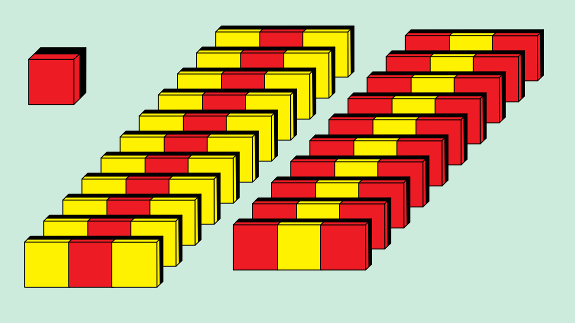
Then can you set all twenty-four triangles into the hexagon so that the colored edges of every triangle touch another edge that is the same color?

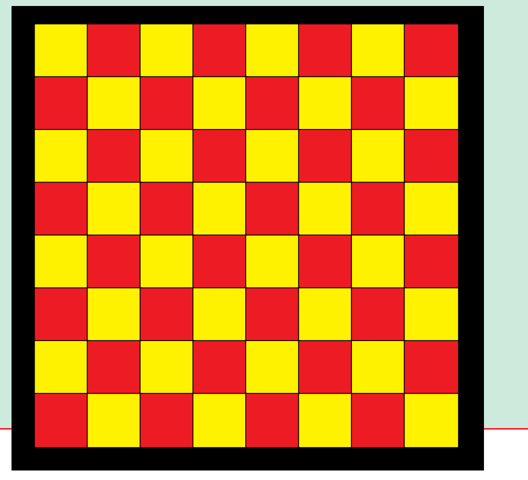
PLAYTHINK 426

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

TROMINOES AND MONOMINO

Can you cover a full chessboard with the twenty-one trominoes (dominoes made up of three squares) and one monomino shown here?



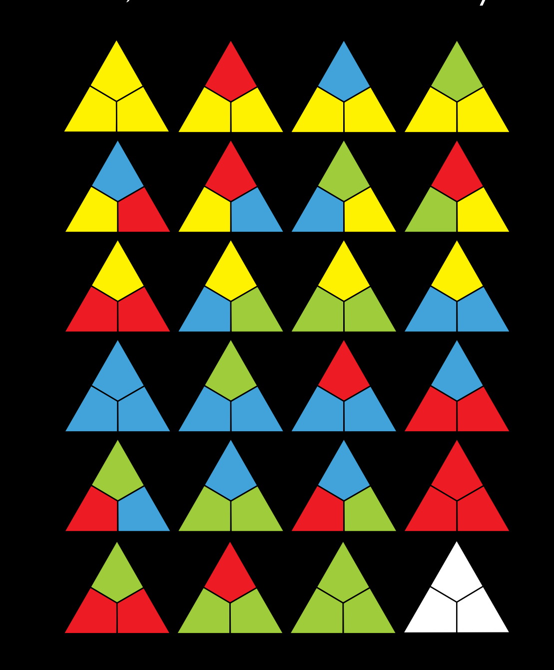


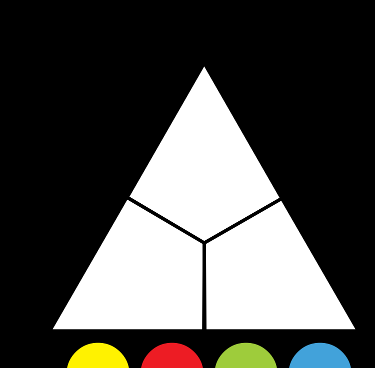
PLAYTHINK 427

DIFFICULTY: ●●●●●●●○○○○
REQUIRED: 👁 ✎ 🗐 ✄
COMPLETION: ☐ TIME: ______

COLOR TRIANGLES 2

Each of the triangles shown has three segments, each of which can be filled by one of four permitted colors. There are twenty-four permutations of four colors possible; one permutation is missing. What are the colors of the blank triangle?

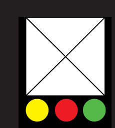




PLAYTHINK 428

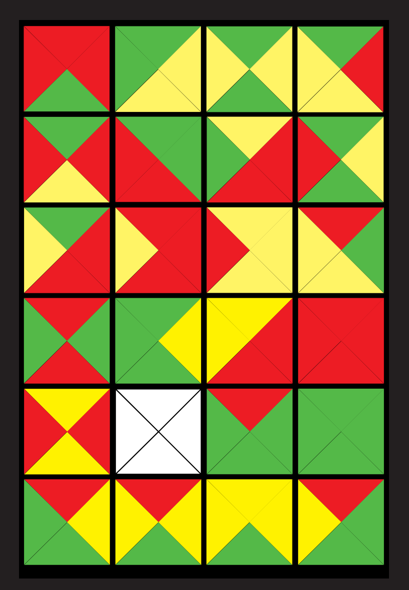
DIFFICULTY: ●●●●●●●○○○○
REQUIRED: 👁 ✎ 🗐 ✄
COMPLETION: ☐ TIME: ______

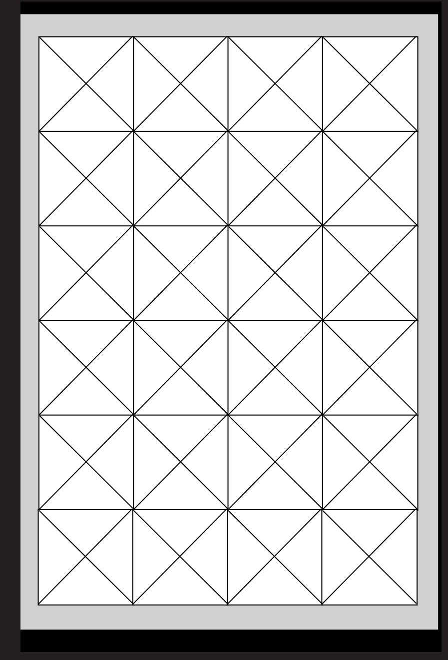
COLOR SQUARES



Each square is divided by its diagonals into four fields, each of which can be filled by one of three permitted colors. There are twenty-four permutations of three colors possible. Twenty-three are shown. What are the colors of the blank square?

Those twenty-four squares fit into a four-by-six board, shown below. Can you arrange the squares so that the outer border is all one color and the edge of each square comes into contact only with an edge of the same color?





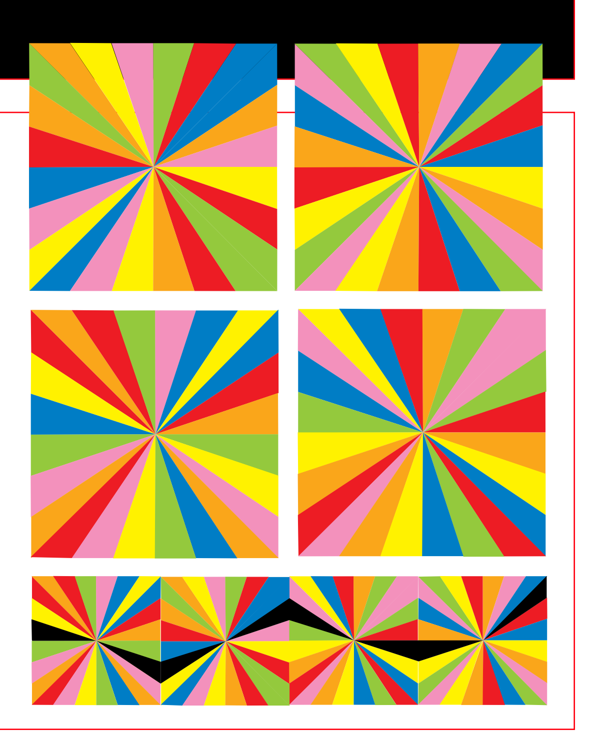
PLAYTHINK 429

DIFFICULTY: ●●●●●●●●○○○
REQUIRED: 👁 ✎ 🗐 ✄
COMPLETION: ☐ TIME: ______

COLOR CONNECTION

Each side of each square is divided into six different colors. Can you place the squares side by side in the manner shown in the inset so that one color forms a continuous zigzag through the four squares? Can you do it in less than one minute?

The puzzle will work for only one color. Which color completes the zigzag?

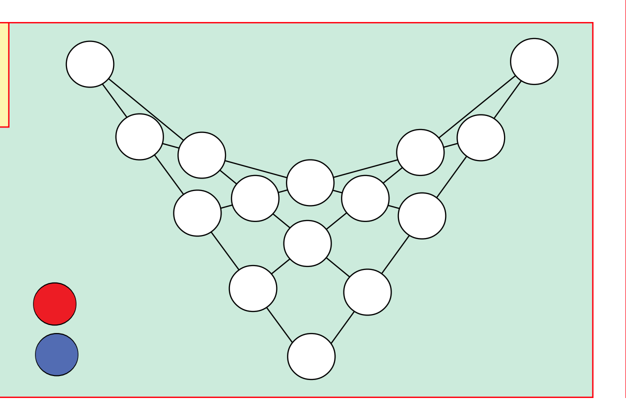


PLAYTHINK 430

DIFFICULTY: ●●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

ROWS OF COLOR

Using only red or blue, color the points of intersection one by one. Can you fill in the whole pattern without allowing any single line to have four points of the same color?



PLAYTHINK 431

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 📄 ✂
COMPLETION: ☐ TIME: ______

SPACE RESCUE: THE GAME

This matching game requires concentration, powers of association and lightning-quick reflexes. Three or more persons can play.

First, copy and cut out the sixty decoder strips on the opposite page and place them in a box. Players take turns drawing a slip out of the box and placing it in view of the others. The player who draws the slip then acts as umpire as the rest try to find the alien that matches the description on the decoder strip.

When a player spots the correct alien, he or she zooms a “space module” (also known as a finger) to the picture. If first to select the matching alien, that player receives a point.

The first player to collect five points wins the game.



ALIEN	EYES	NOSE	MOUTH	ALIEN	EYES	NOSE	MOUTH	ALIEN	EYES	NOSE	MOUTH	ALIEN	EYES	NOSE	MOUTH
1				16				31				46			
2				17				32				47			
3				18				33				48			
4				19				34				49			
5				20				35				50			
6				21				36				51			
7				22				37				52			
8				23				38				53			
9				24				39				54			
10				25				40				55			
11				26				41				56			
12				27				42				57			
13				28				43				58			
14				29				44				59			
15				30				45				60			

PLAYTHINK 432

DIFFICULTY: ●●●●●●●○○○
REQUIRED: eye, pencil, copy, scissors
COMPLETION: ☐ TIME: ______

HEXAGONS 1

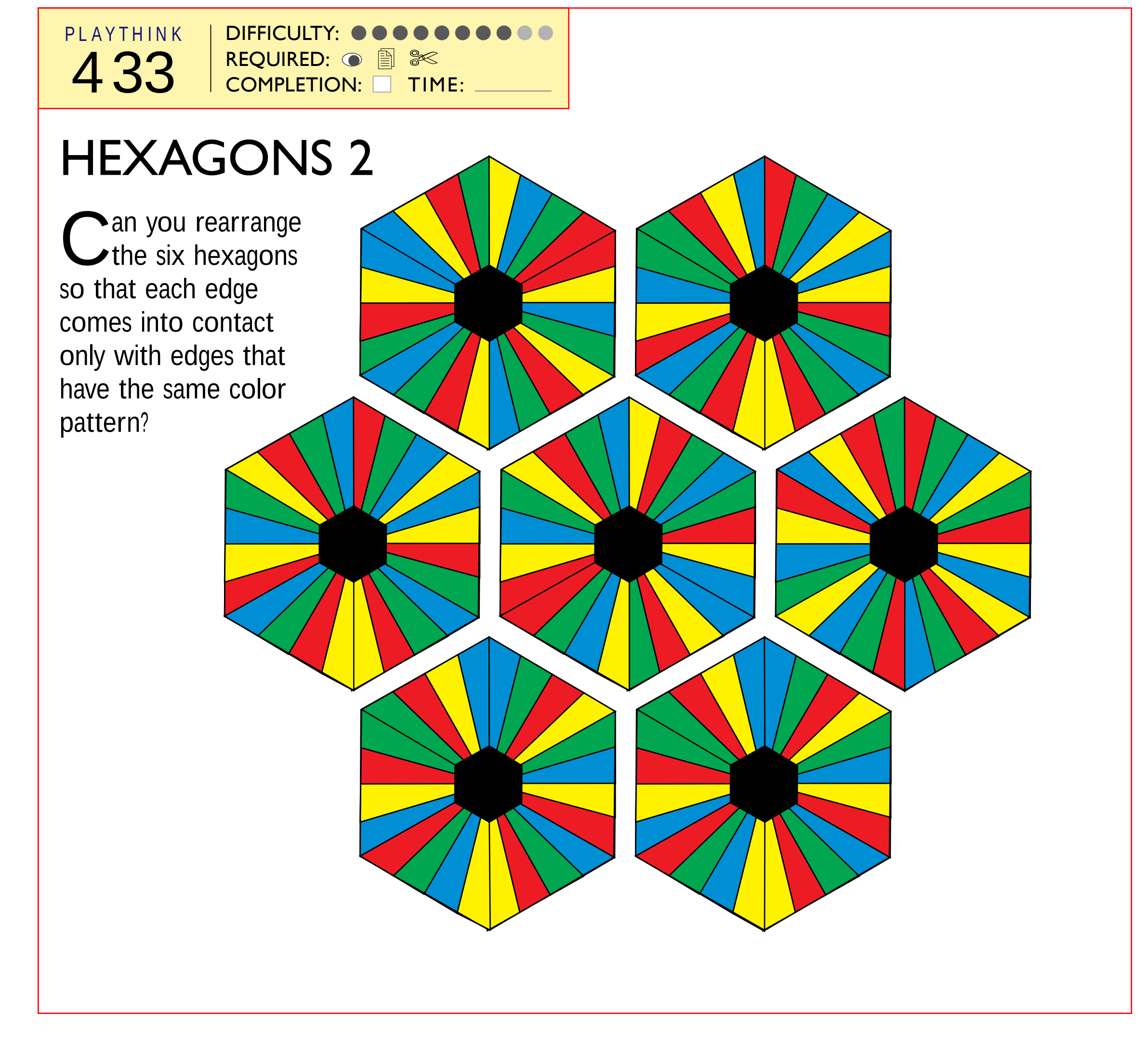
Can you arrange the six hexagons shown here into the honeycomb so that each colored edge touches another edge of the same color?

PLAYTHINK 433

DIFFICULTY: ●●●●●●●●○○
REQUIRED: eye, copy, scissors
COMPLETION: ☐ TIME: ______

HEXAGONS 2

Can you rearrange the six hexagons so that each edge comes into contact only with edges that have the same color pattern?

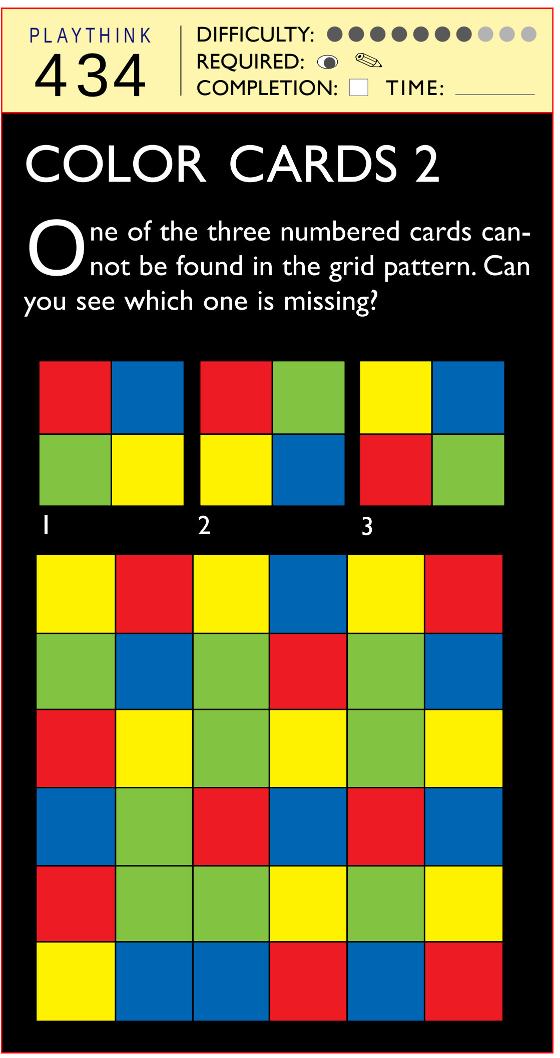


PLAYTHINK 434

DIFFICULTY: ●●●●●●●○○○
REQUIRED: eye, pencil
COMPLETION: ☐ TIME: ______

COLOR CARDS 2

One of the three numbered cards cannot be found in the grid pattern. Can you see which one is missing?





8

Dissections

Polygon Transformations

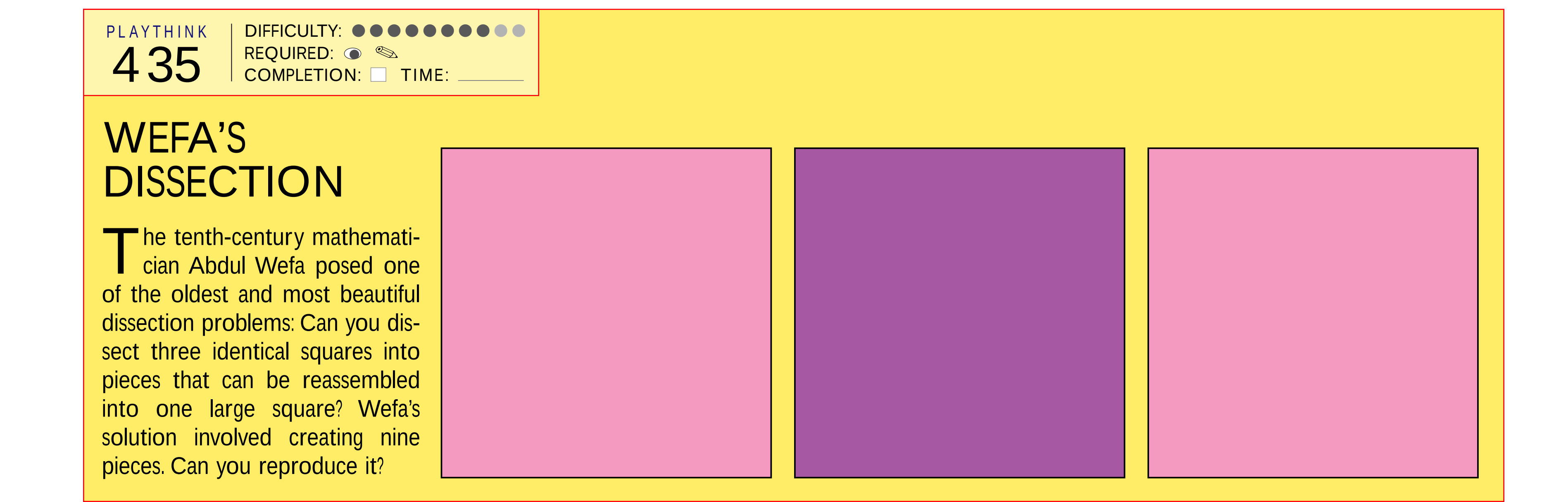
An easy way to learn about shapes is to cut them apart and reassemble the pieces to form new shapes according to simple rules. For example, if two different shapes with straight-line edges—polygons, in other words—can be assembled from the same set of pieces, then both shapes must have the same area. Conversely, any two polygons of equal area can be dissected into a finite number of pieces that can then be assembled to form either of the two original polygons. Rules like that, while simple, are useful for making calculations and predicting other relationships. The Pythagorean theorem is based on an observation of this kind.

There are many ways to divide a given shape into parts, and some of those divisions—called dissections—are particularly interesting. Although problems of dissections must have confronted humans many thousands of years ago, the earliest known systematic treatise on the subject was written by the tenth-century Persian astronomer Abdul Wefa. Only fragments of his book survive, but it contains some fascinating dissections, one of which is included below as PlayThink 435.

Dissections are found in lots of games. Jigsaw puzzles, in which the required assembly is unique, are one kind of dissection problem. Tangrams, which call for the creative reassembly of pieces, is another. Some dissection problems appear at first to do the impossible: the "Mystrix" puzzle problem (PlayThink 503) involves cutting a shape into a number of pieces, removing one of the pieces, and reassembling the remaining parts to form the original shape. It takes a keen eye to solve this paradox. But the most common use of dissections in recreational math is to find how to dissect one figure to make another in the fewest number of pieces.

In the nineteenth century mathematicians did not take problems of dissections seriously. But now there is a branch of mathematics, called dissection theory, that provides valuable insights into the solutions of many practical problems in plane and solid geometry.

In 1900 the famous mathematician David Hilbert gave an address in Paris in which he listed twenty-three unsolved mathematical problems. Many of those so-called Hilbert problems still tax our ingenuity, but the mathematician Max Dehn solved one within a year. Hilbert asked whether two polyhedral solids of equal volume can always be dissected into a system of identical pieces. Dehn proved that, unlike dissections of equal area, identical dissections of volume are not always possible. It turns out that volume is subtler than area.



PLAYTHINK 435

DIFFICULTY: ●●●●●●●●○○
REQUIRED:
COMPLETION: ☐ TIME: ______

WEFA'S DISSECTION

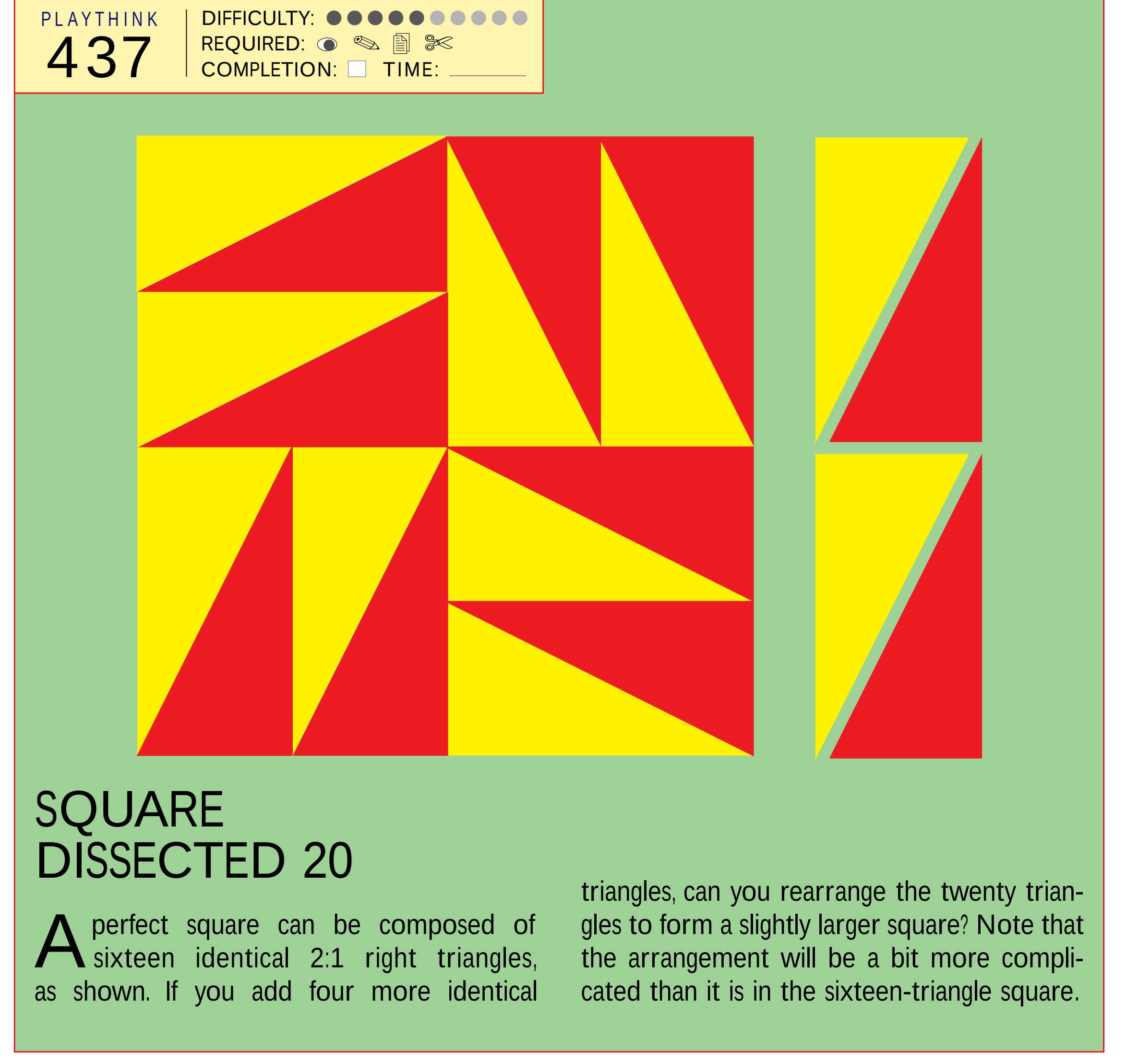
The tenth-century mathematician Abdul Wefa posed one of the oldest and most beautiful dissection problems: Can you dissect three identical squares into pieces that can be reassembled into one large square? Wefa's solution involved creating nine pieces. Can you reproduce it?

PLAYTHINK 436

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

LETTER FLIPS

Each of the eleven colored shapes can be flipped around their red mirror lines. Using nothing more than your imagination, can you work out the result of flipping all eleven shapes? Hint: the result spells out a common word.

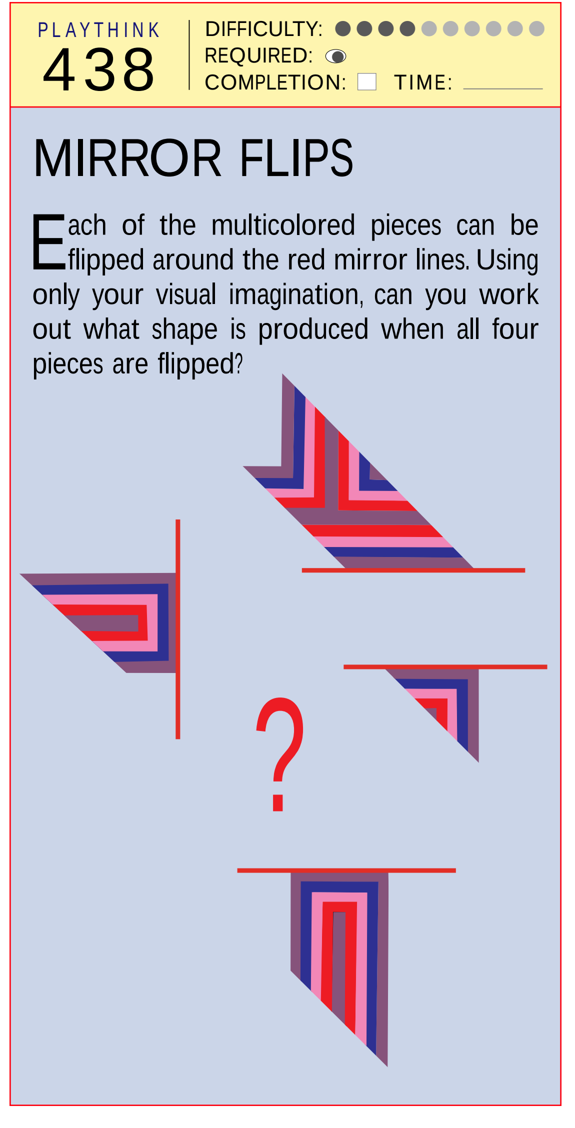


PLAYTHINK 437

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎ 📄 ✂
COMPLETION: ☐ TIME: ______

SQUARE DISSECTED 20

A perfect square can be composed of sixteen identical 2:1 right triangles, as shown. If you add four more identical triangles, can you rearrange the twenty triangles to form a slightly larger square? Note that the arrangement will be a bit more complicated than it is in the sixteen-triangle square.



PLAYTHINK 438

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

MIRROR FLIPS

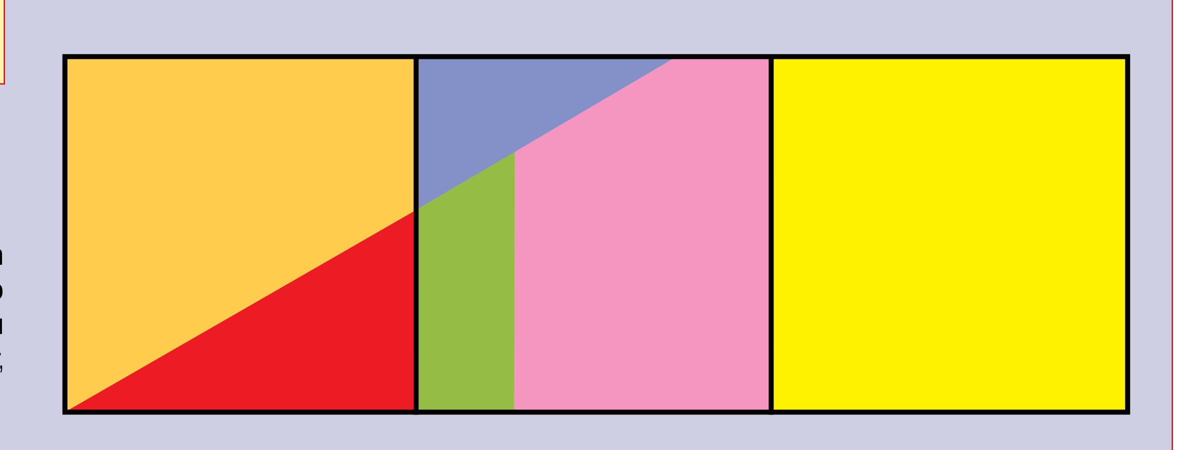
Each of the multicolored pieces can be flipped around the red mirror lines. Using only your visual imagination, can you work out what shape is produced when all four pieces are flipped?

PLAYTHINK 439

DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

THREE SQUARES INTO ONE

Two of the three identical squares shown here have been dissected—one into two pieces, the other into three. Can you rearrange the six pieces to form a larger, perfect square?

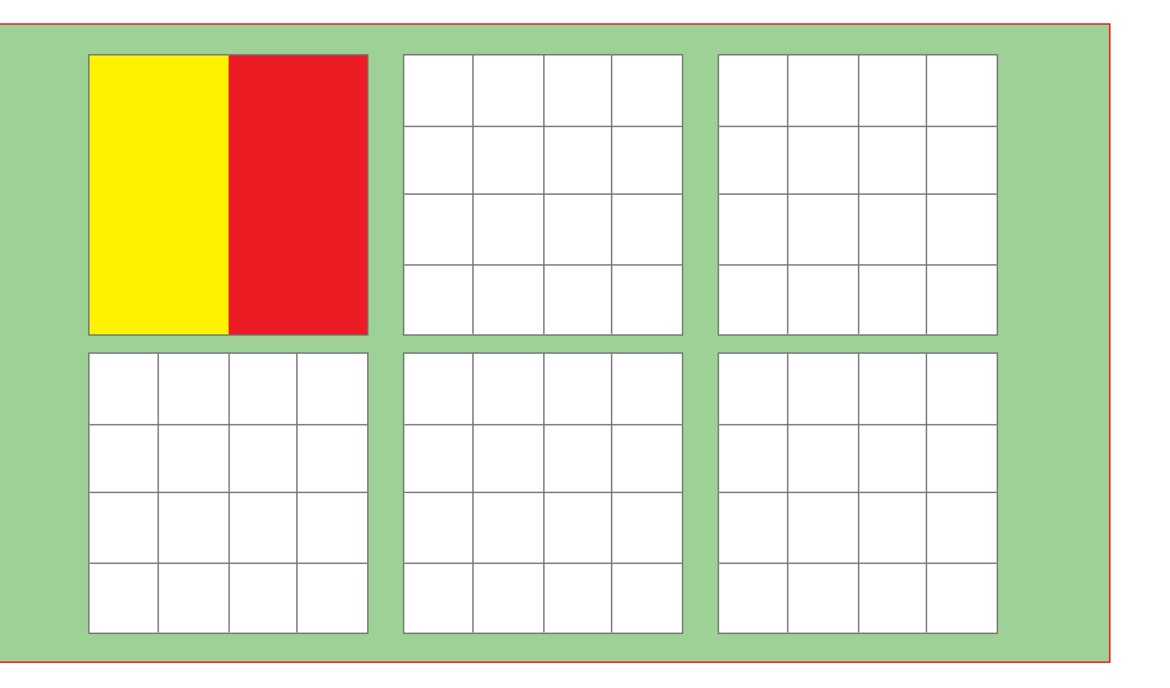


PLAYTHINK 440

DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

HALVING SQUARE

Following the grid lines provided, there are only six ways that a square can be dissected into two congruent parts, not counting rotations and reflections. One of the six is shown. Can you find the other five?

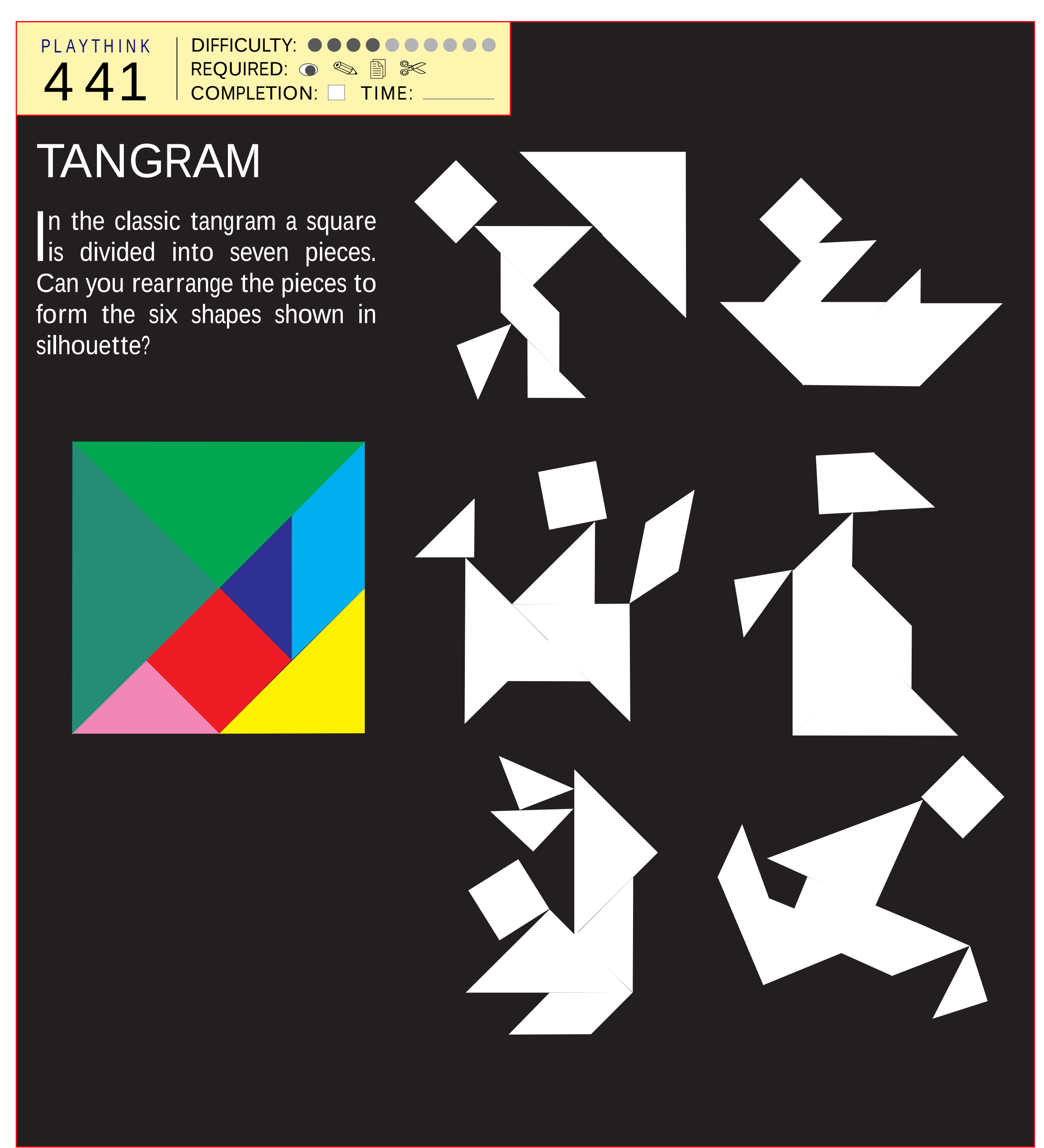


Tangrams

Cut a plane or solid figure into pieces, then fit the pieces together to form the original shape or completely new ones. That's a dissection puzzle—one of the oldest forms of recreational math. And one of the oldest dissection puzzles is the Chinese tangram. In its classic form—a square divided into seven sections—the tangram is one of the most beautiful puzzles ever constructed. From it a nearly limitless variety of pictures, both abstract and figurative, can be created. Indeed, the subtlety and richness of the tangram's combinatorial possibilities can be revealed and appreciated only after playing with the puzzle for a while. But be forewarned: The challenge can prove as addictive as it is rewarding.

Although the earliest reference to a tangram is in a Chinese book published in 1826, many believe its origins date much earlier than that. We do know that Edgar Allan Poe and Lewis Carroll were devotees; Napoleon spent endless hours in exile inventing and solving tangram problems.

There are dozens of variations of the tangram involving dissections of rectangles, circles, eggs, hearts and other shapes. After you have solved all the problems suggested here, you should try to create your own designs and figures. It is a meaningful artistic pastime that will strengthen your powers of abstract visualization.

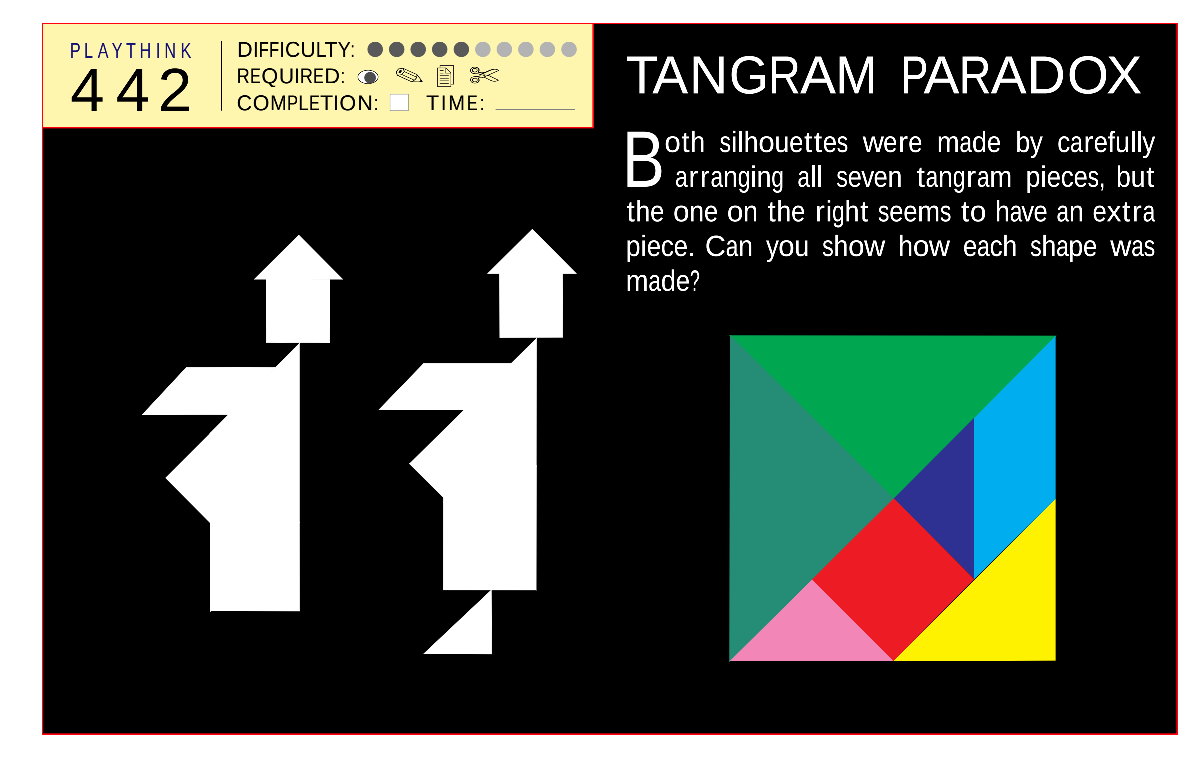


PLAYTHINK 441

DIFFICULTY: ●●●●●○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

TANGRAM

In the classic tangram a square is divided into seven pieces. Can you rearrange the pieces to form the six shapes shown in silhouette?

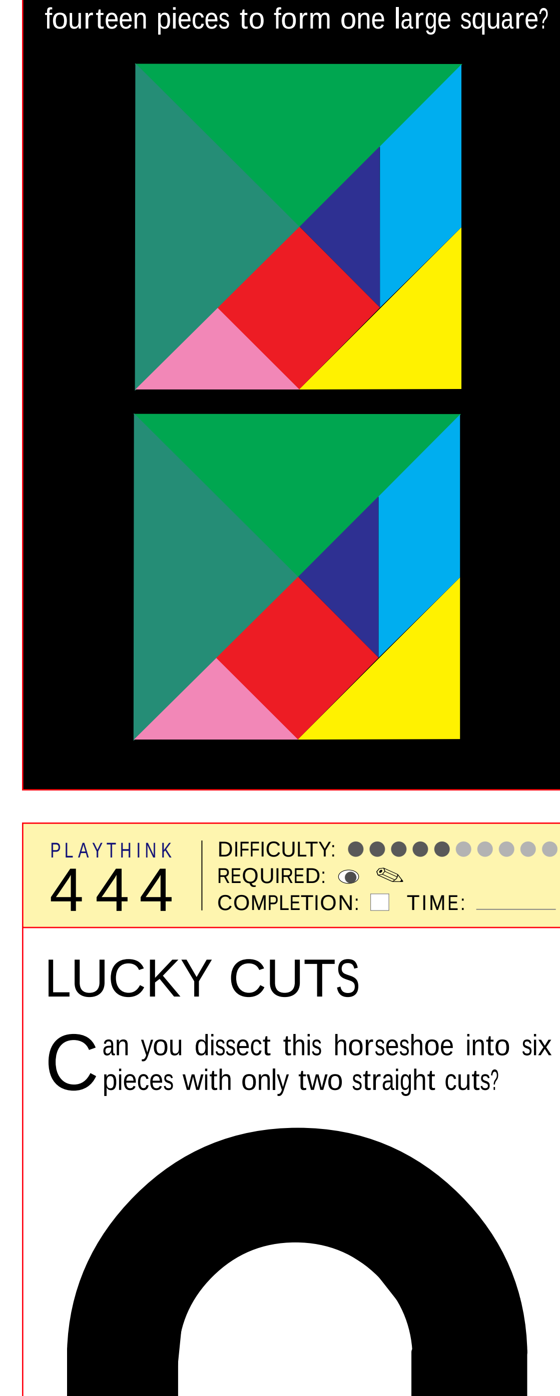


PLAYTHINK 442

DIFFICULTY: ●●●●●○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

TANGRAM PARADOX

Both silhouettes were made by carefully arranging all seven tangram pieces, but the one on the right seems to have an extra piece. Can you show how each shape was made?



PLAYTHINK 443

DIFFICULTY: ●●●●○○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

DOUBLE TANGRAM

Copy and cut out the two identical tangram sets. Can you combine all fourteen pieces to form one large square?

PLAYTHINK 444

DIFFICULTY: ●●●●●○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

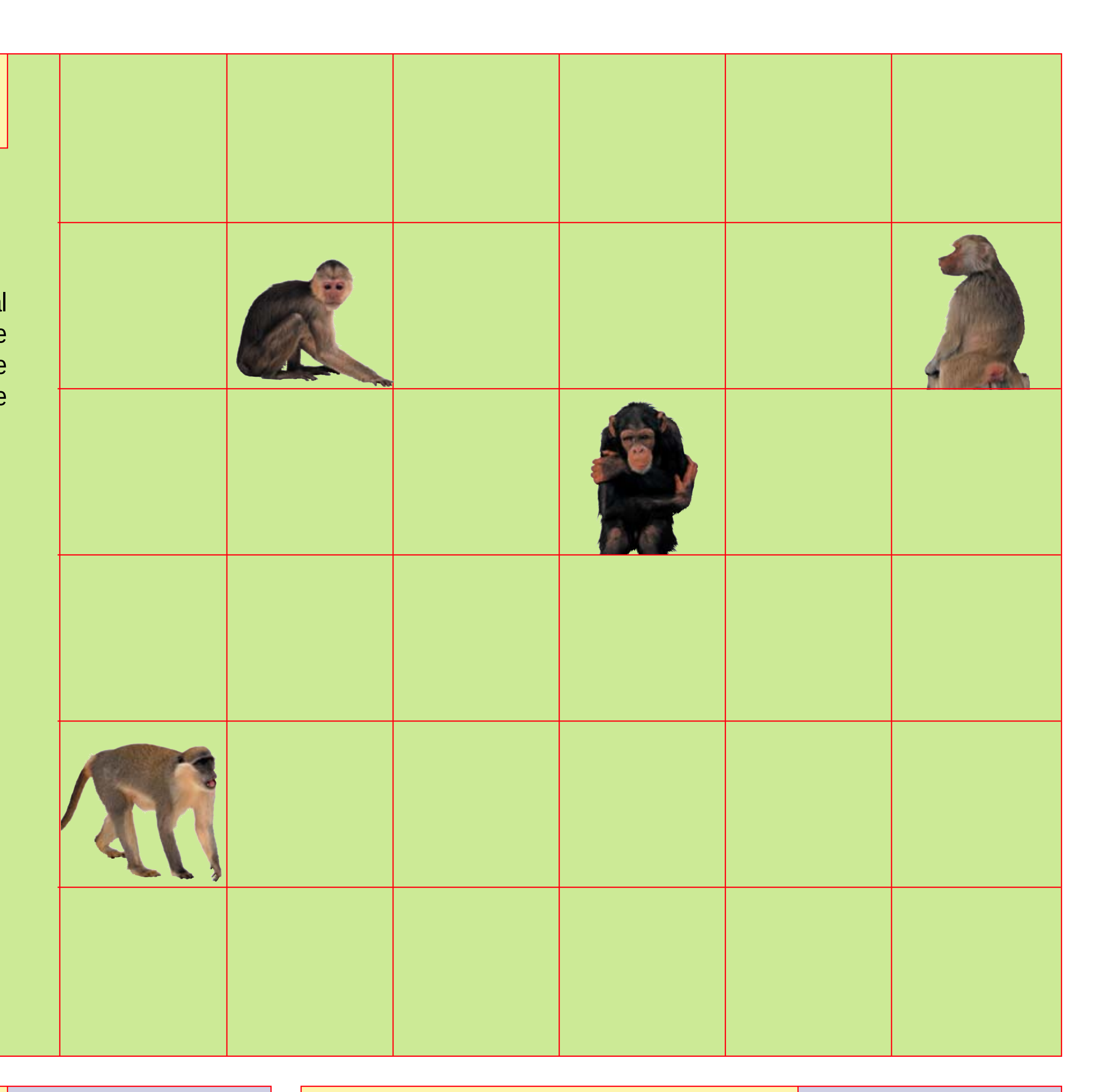
LUCKY CUTS

Can you dissect this horseshoe into six pieces with only two straight cuts?

PLAYTHINK 445 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

SEPARATING MONKEYS

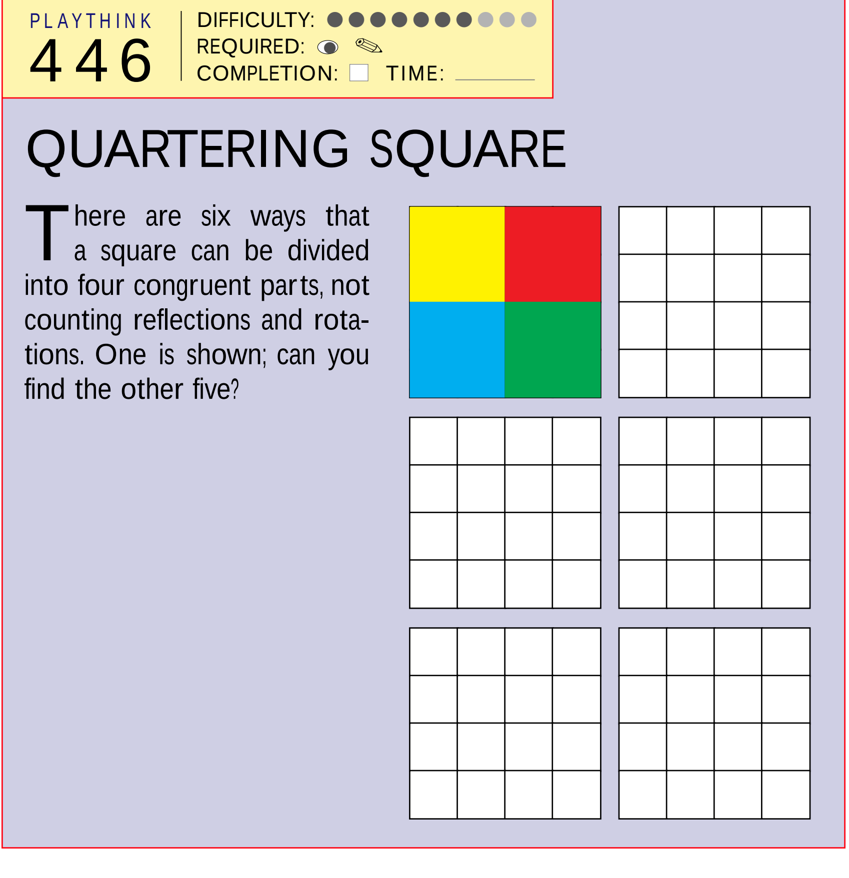
Each of the four monkeys needs an identical compartment that is fenced off from the other monkeys. Following the grid lines on the six-by-six square, can you find two possible ways of separating the monkeys?



PLAYTHINK 446 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

QUARTERING SQUARE

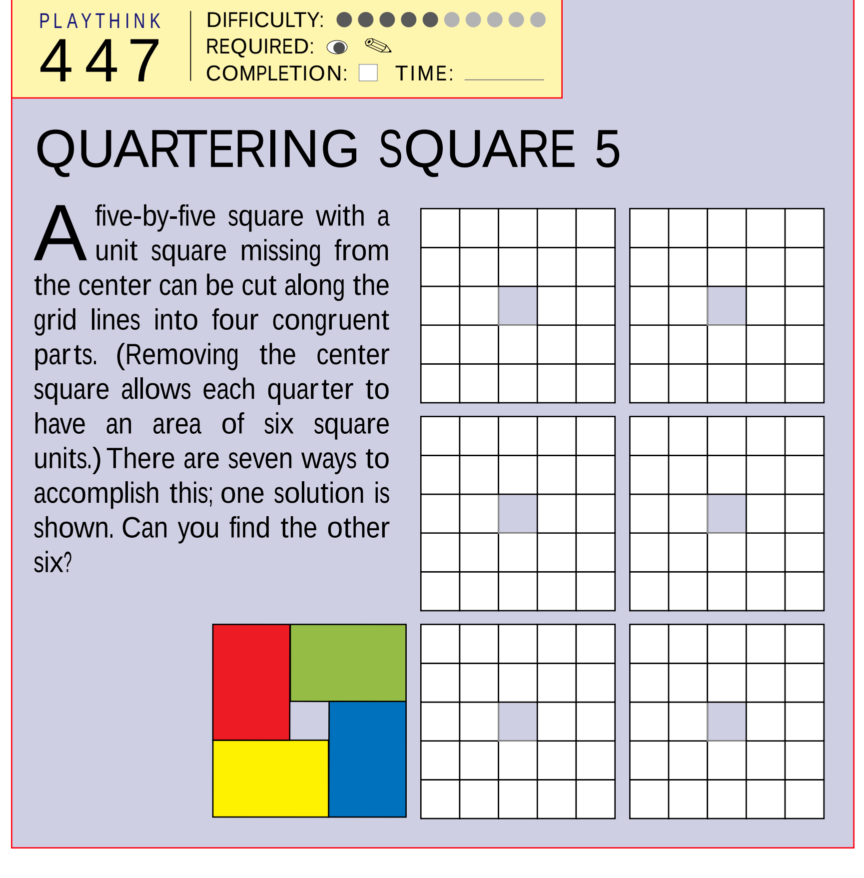
There are six ways that a square can be divided into four congruent parts, not counting reflections and rotations. One is shown; can you find the other five?



PLAYTHINK 447 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

QUARTERING SQUARE 5

A five-by-five square with a unit square missing from the center can be cut along the grid lines into four congruent parts. (Removing the center square allows each quarter to have an area of six square units.) There are seven ways to accomplish this; one solution is shown. Can you find the other six?



PLAYTHINK 448

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

FENCES

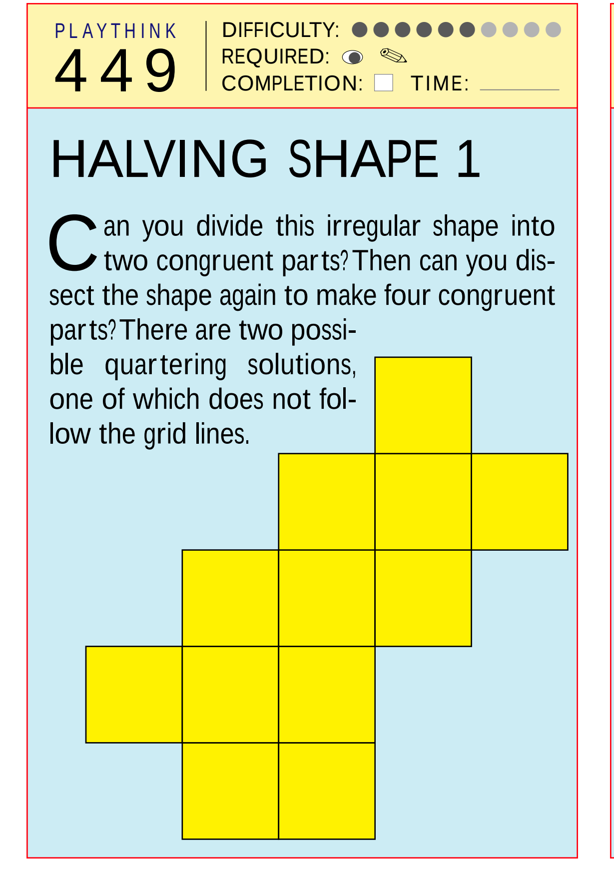
Can you erect fences along the grid lines so that each of the four types of animals has a pen that is identical in size and shape?

PLAYTHINK 449

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

HALVING SHAPE 1

Can you divide this irregular shape into two congruent parts? Then can you dissect the shape again to make four congruent parts? There are two possible quartering solutions, one of which does not follow the grid lines.

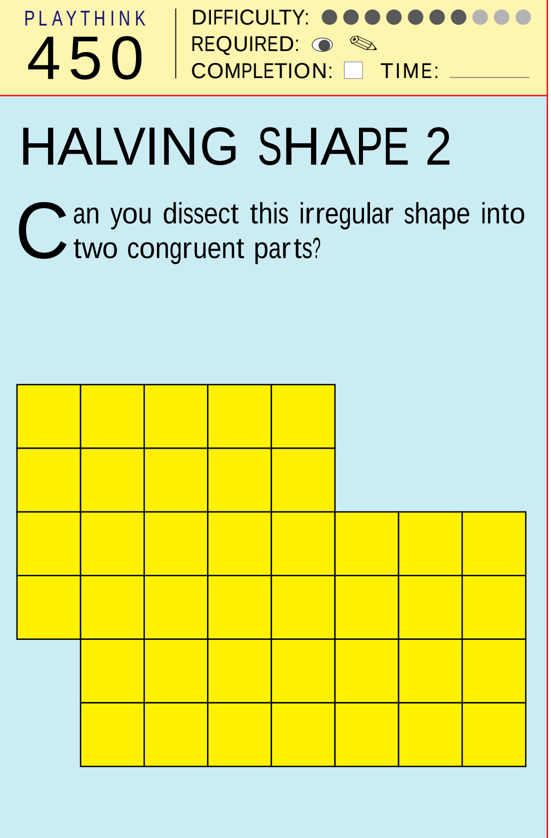


PLAYTHINK 450

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

HALVING SHAPE 2

Can you dissect this irregular shape into two congruent parts?

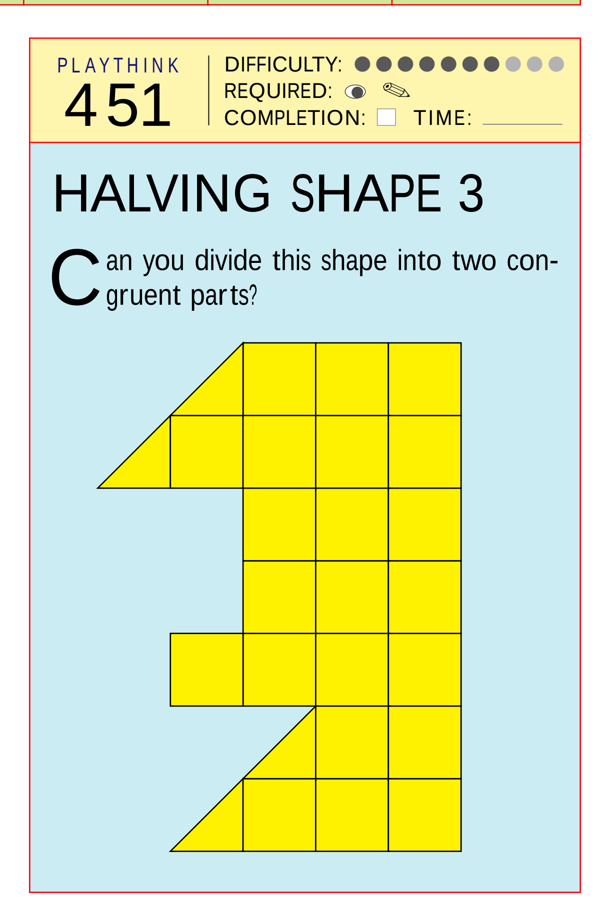


PLAYTHINK 451

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

HALVING SHAPE 3

Can you divide this shape into two congruent parts?



Not So Simple

A popular type of puzzle involves dividing a given shape into two, three, four or even more equal parts. In some cases *equal* means simply of equal area; in other cases the pieces must be congruent—that is, exactly identical in size and shape. One might think such problems would be easy to solve, but they often prove challenging in spite of their underlying simplicity.

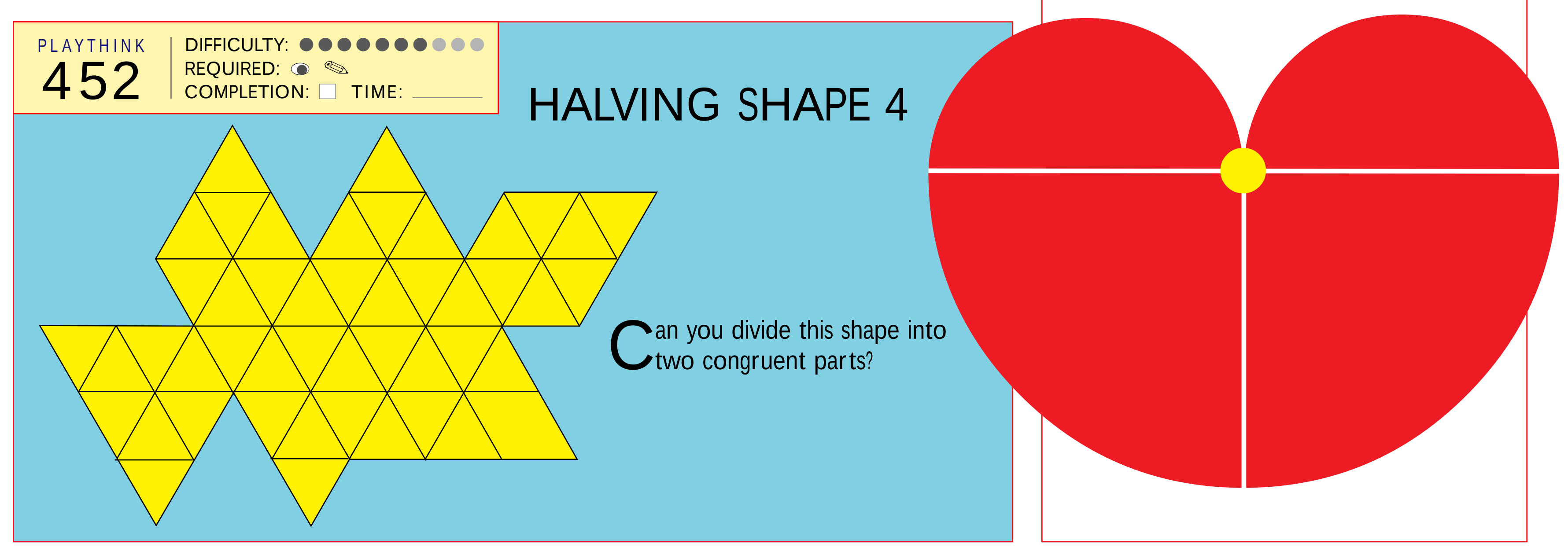
PLAYTHINK 453
DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

HALVING HEART

Examine the diagram of the heart-shaped figure below. Can you work out which line through the yellow point divides the perimeter of the shape into two equal parts?

PLAYTHINK 452
DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

HALVING SHAPE 4



Can you divide this shape into two congruent parts?

PLAYTHINK 454
DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

QUARTERING SHAPE 1

Can you help Simon divide his L-shaped carpet into four congruent parts?

PLAYTHINK 455
DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

QUARTERING SHAPE 2

Can you divide this shape into four congruent parts?

PLAYTHINK 456
DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

QUARTERING SHAPE 3

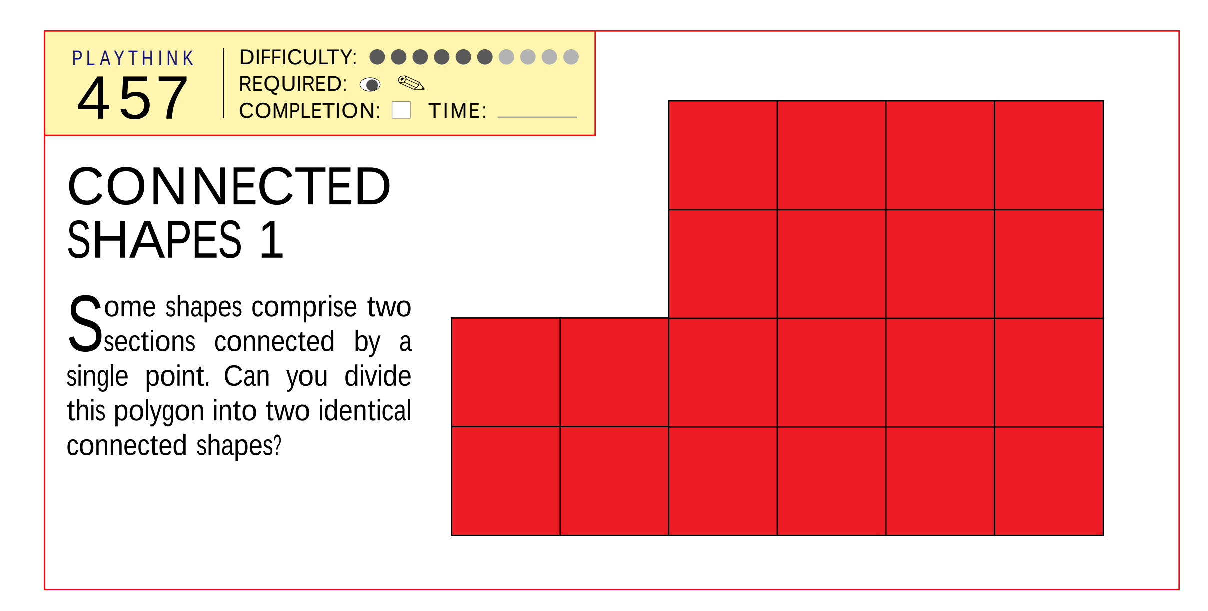
Anna must divide this trapezoidal shape into four congruent parts. Can you show her how to do it?

PLAYTHINK 457

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

CONNECTED SHAPES 1

Some shapes comprise two sections connected by a single point. Can you divide this polygon into two identical connected shapes?



PLAYTHINK 458

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

CONNECTED SHAPES 2



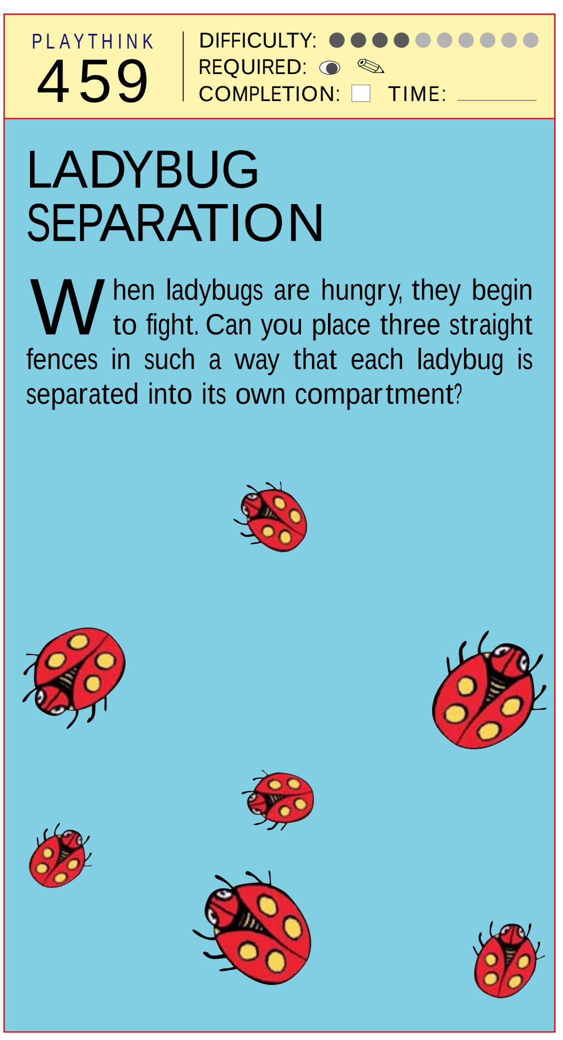
This nonconvex polygon is divided into twenty-four identical triangles, each of which is filled in with one of four colors. Can you rearrange the triangles within the boundary of the polygon to make four identical connected shapes? Each of the shapes should be of one color, and the shapes can be counted as identical even if they are reflections or rotations of one another.

PLAYTHINK 459

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

LADYBUG SEPARATION

When ladybugs are hungry, they begin to fight. Can you place three straight fences in such a way that each ladybug is separated into its own compartment?

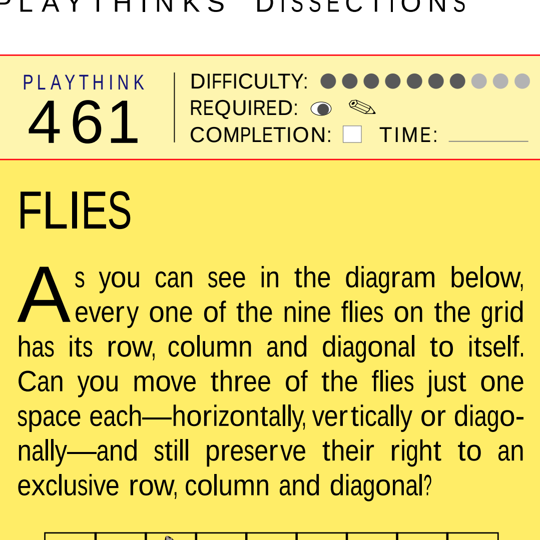


PLAYTHINK 460

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎ 🗎 ✂
COMPLETION: ☐ TIME: ______

GREEK CROSS CUT

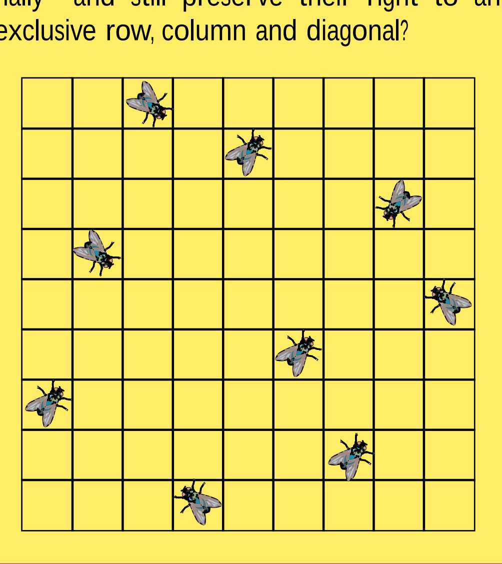
This figure can be divided into two identical parts in such a way that when the pieces are rearranged, they form a perfect Greek cross. Can you work out how this can be done?



PLAYTHINK 461
DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

FLIES

As you can see in the diagram below, every one of the nine flies on the grid has its row, column and diagonal to itself. Can you move three of the flies just one space each—horizontally, vertically or diagonally—and still preserve their right to an exclusive row, column and diagonal?





PLAYTHINK 462
DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

GREEK CROSS INTO SQUARES

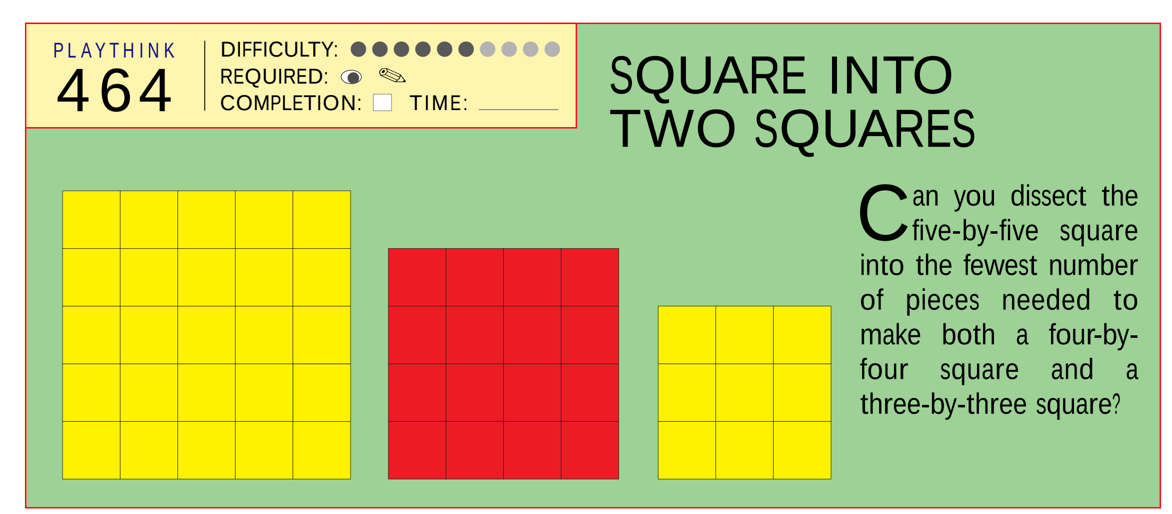
Can you dissect this Greek cross into nine pieces that can fit together to form either five small squares or one large one?

PLAYTHINK 463
DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 🗎 ✂
COMPLETION: ☐ TIME: ______

STAR TO RECTANGLE

The six-pointed star has been dissected into six pieces. Can you reassemble the pieces to form a rectangle?





PLAYTHINK 464
DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

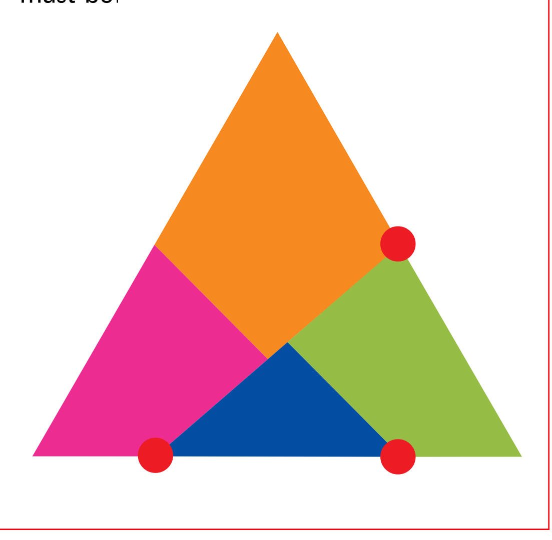
SQUARE INTO TWO SQUARES

Can you dissect the five-by-five square into the fewest number of pieces needed to make both a four-by-four square and a three-by-three square?

PLAYTHINK 465
DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

HINGED TRIANGLE

This equilateral triangle has been dissected into four parts. Hinges, marked in red, connect the parts to each other. If you leave the blue piece fixed and swing the others around their hinges, you can rearrange the pieces to form a new shape. Can you work out what that new shape must be?



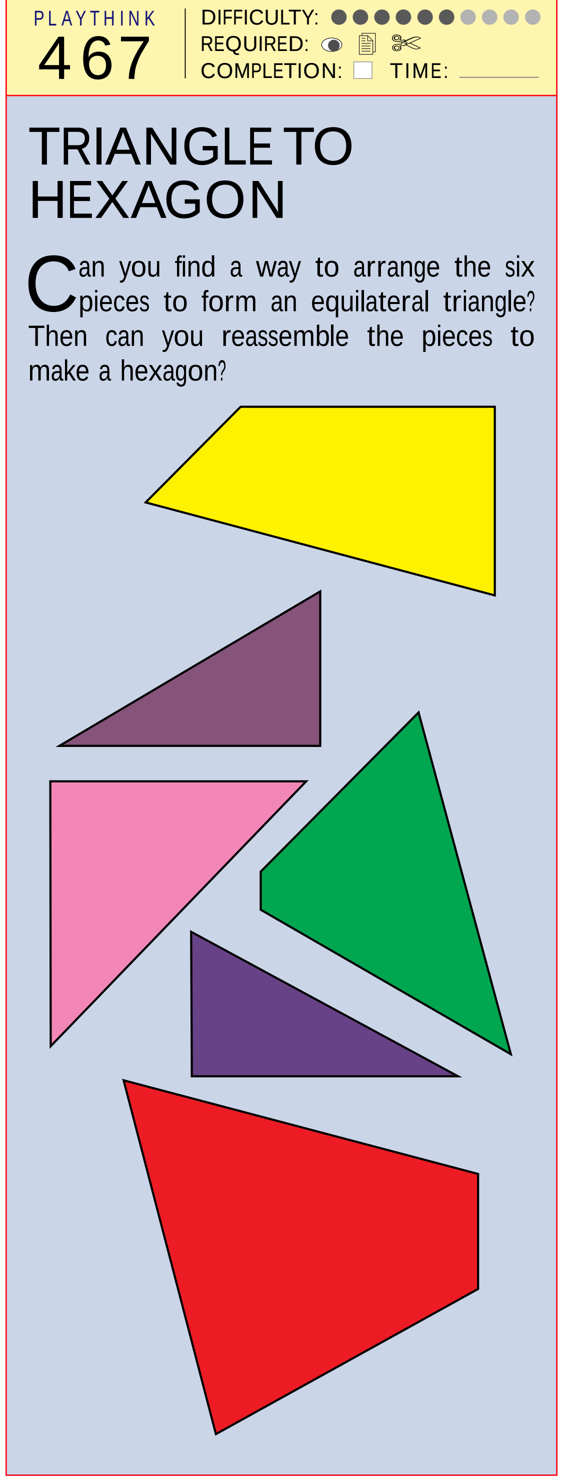
PLAYTHINK 466

DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

SUBMARINE NET

Navy frogmen must cut a path through the enemy net to allow the submarine to pass through, but they have only enough time to cut through the ropes of the net; the knots are too thick.

Can you find the path from the top to the bottom that will require the fewest cuts?

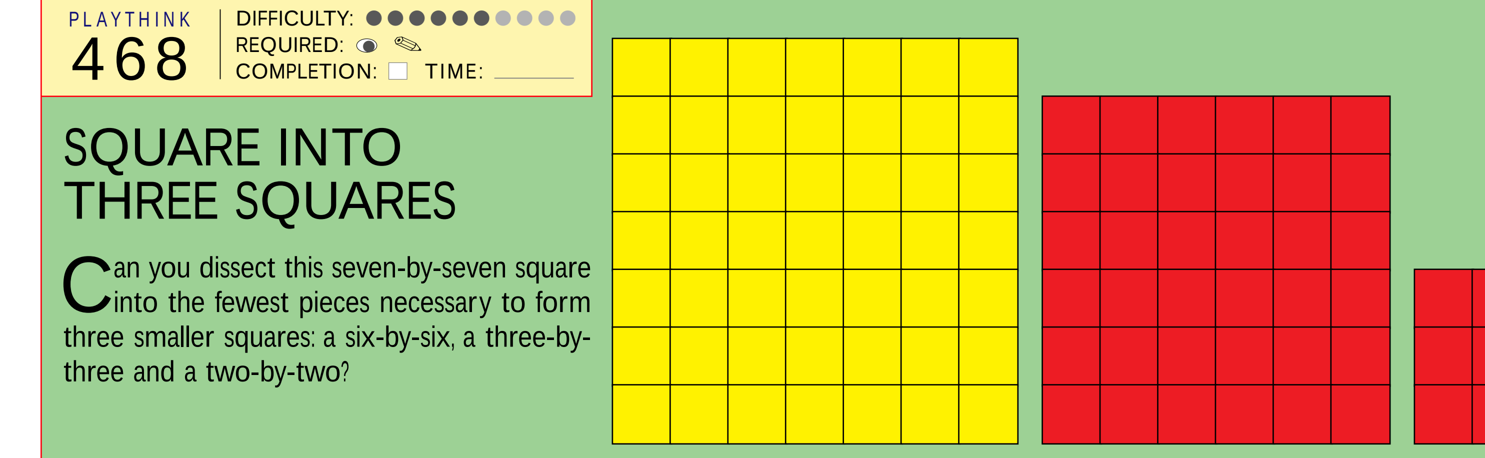


PLAYTHINK 467

DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

TRIANGLE TO HEXAGON

Can you find a way to arrange the six pieces to form an equilateral triangle? Then can you reassemble the pieces to make a hexagon?



PLAYTHINK 468

DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

SQUARE INTO THREE SQUARES

Can you dissect this seven-by-seven square into the fewest pieces necessary to form three smaller squares: a six-by-six, a three-by-three and a two-by-two?

Pythagorean Theorem

The ancient geometric theorem attributed to Pythagoras is one of the few theorems that almost everybody has at least a nodding acquaintance with. It concerns the relationships between the two short sides of a right triangle and the long side, or hypotenuse.

It is as famous in words—the square of the hypotenuse of any right triangle is equal to the sum of the squares of the other two sides—as it is in symbols:

$$a^2 + b^2 = c^2$$

in which *a* and *b* are the lengths of the two short sides, and *c* is the length of the hypotenuse.

But what does this actually mean?

In numerical terms it means that we may construct right-angled triangles by using any three lengths *a*, *b*, *c* that satisfy the Pythagorean condition.

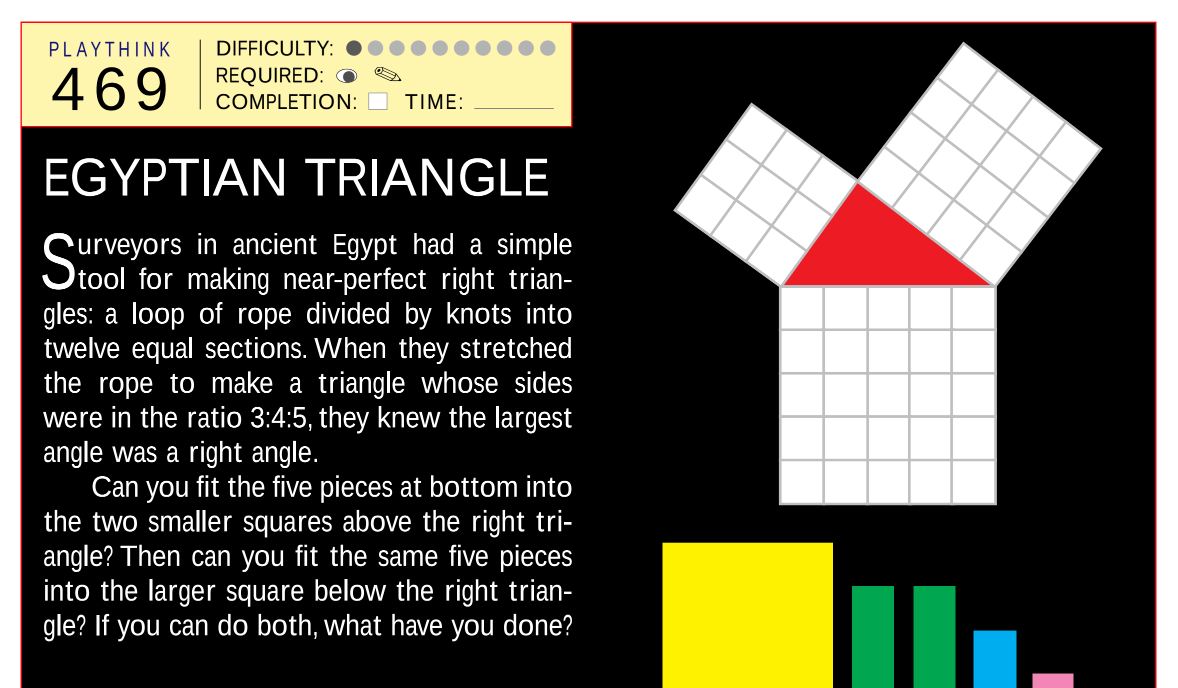
For example, since

$$3^2 + 4^2 = 5^2$$

a triangle with sides 3, 4 and 5 is necessarily right-angled. The surveyors of ancient Egypt, it is said, knew of this relationship and divided a rope into twelve equal parts by knots to form the so-called Egyptian triangle, which they used to construct nearly perfect right angles.

There are many other whole-number Pythagorean triplets: 5-12-13 and 8-15-17, to name two. The general rule for finding all Pythagorean triplets is known and was one of the first results to be obtained in the theory of Diophantine equations—that is, equations solved using only whole numbers. This is a surprising link between geometry and the theory of numbers.

Geometrically, the Pythagorean theorem asserts an equality of areas. A square whose side is laid against the hypotenuse of a right triangle has exactly the same area as two squares laid against the other two sides combined. One interesting problem (see PlayThink 469) demonstrates this directly, by finding a way to cut up the two smaller squares into pieces that can be reassembled to form the larger square. An alternative and very beautiful solution to this problem, known as Perigal's dissection (see PlayThink 470), leaves the smallest square intact and cuts the middle-sized square into four pieces of the same shape and size.

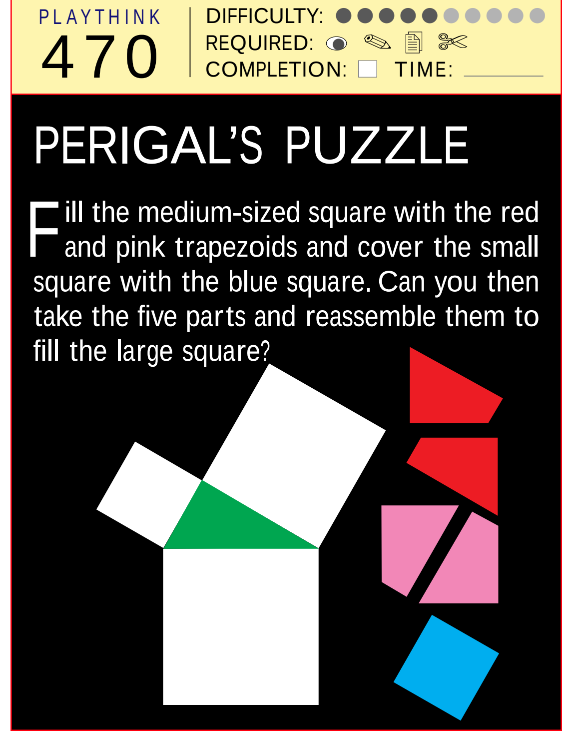


PLAYTHINK 469 | DIFFICULTY: ●○○○○○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

EGYPTIAN TRIANGLE

Surveyors in ancient Egypt had a simple tool for making near-perfect right triangles: a loop of rope divided by knots into twelve equal sections. When they stretched the rope to make a triangle whose sides were in the ratio 3:4:5, they knew the largest angle was a right angle.

Can you fit the five pieces at bottom into the two smaller squares above the right triangle? Then can you fit the same five pieces into the larger square below the right triangle? If you can do both, what have you done?



PLAYTHINK 470 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ 📄 ✂ | COMPLETION: ☐ TIME: ____

PERIGAL'S PUZZLE

Fill the medium-sized square with the red and pink trapezoids and cover the small square with the blue square. Can you then take the five parts and reassemble them to fill the large square?

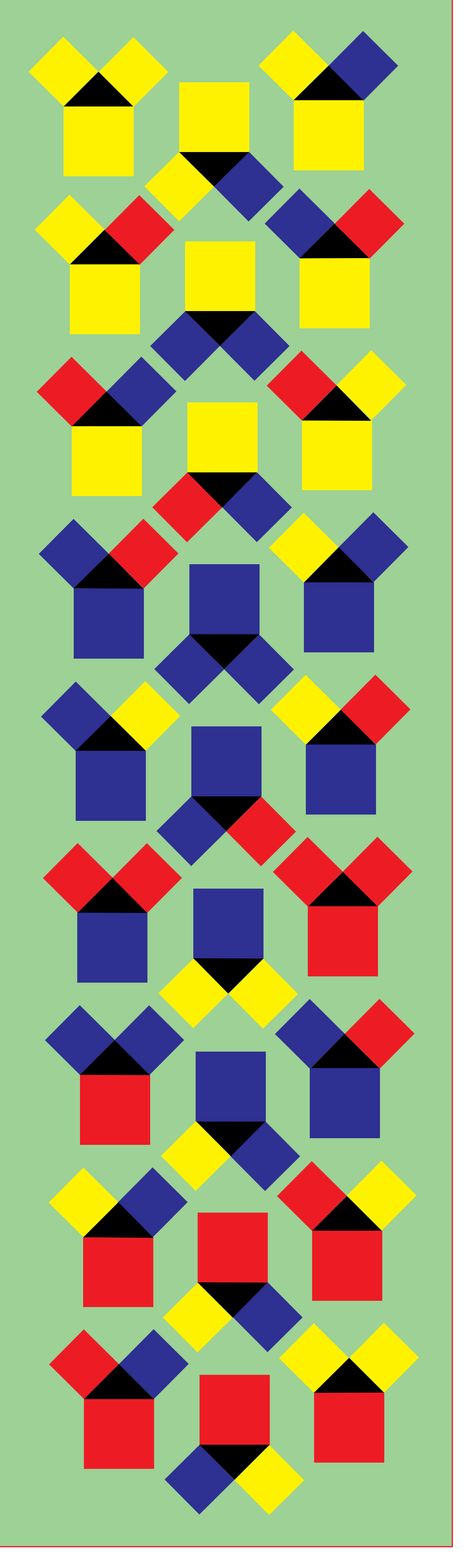
PLAYTHINK 471

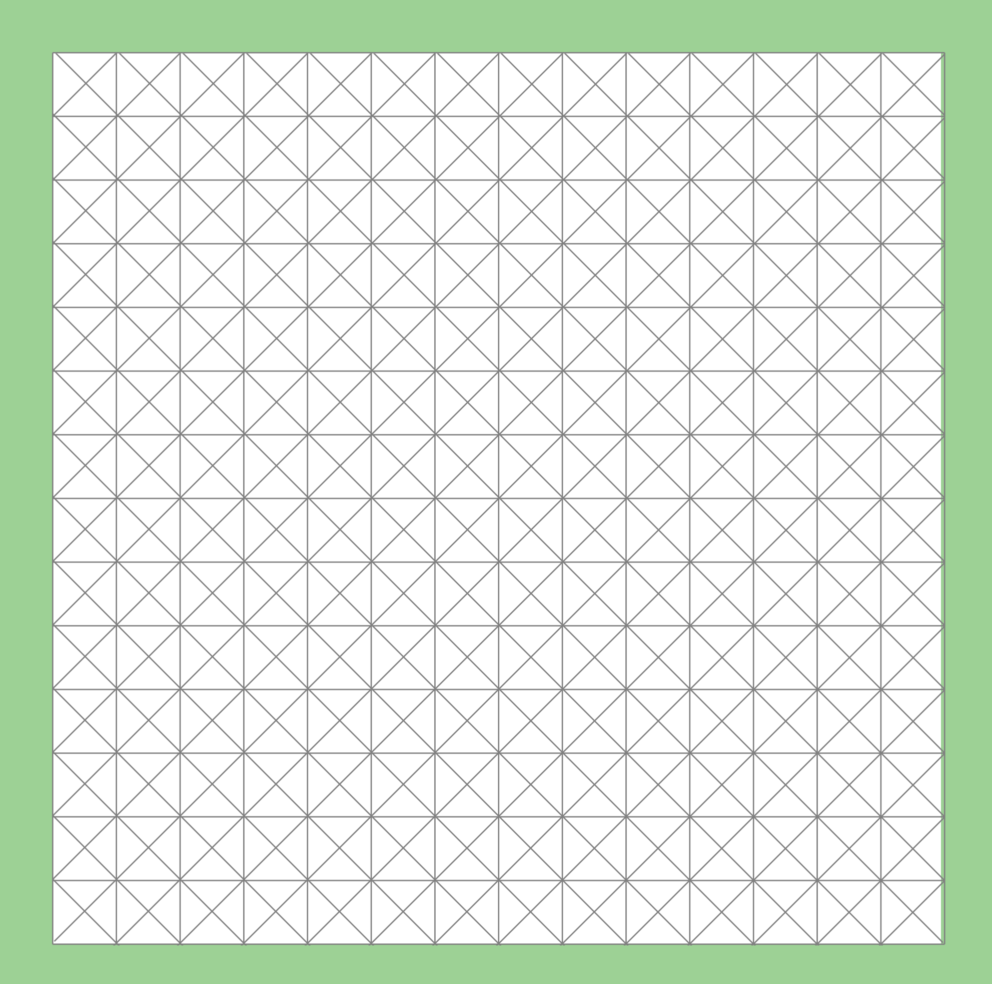
DIFFICULTY: ●●●●●○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

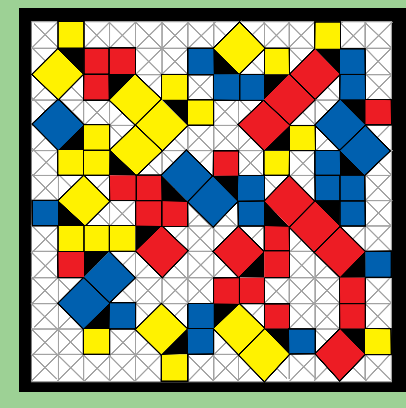
PYTHAGORINO

This novel three-player domino game is based on a shape derived from the Pythagorean theorem: three squares arranged around an isosceles right triangle. The twenty-seven Pythagorean playing pieces come in various combinations of three colors—red, blue and yellow.

The pieces are placed one at a time on the grid so that the smaller squares align with the grid and the larger squares align with the diagonals. Players are assigned a color and take turns placing pieces on the grid in much the same way standard dominoes are played: adjacent squares—either small or large—must be the same color. See the example below for a demonstration of this. A player gets a point for every big square of his or her color that forms an unbroken strip consisting of at least two squares of that color. The play continues until no additional pieces can be placed. The player with the most points wins; in a tie, the win goes to the player whose points were made with the fewest number of strips.







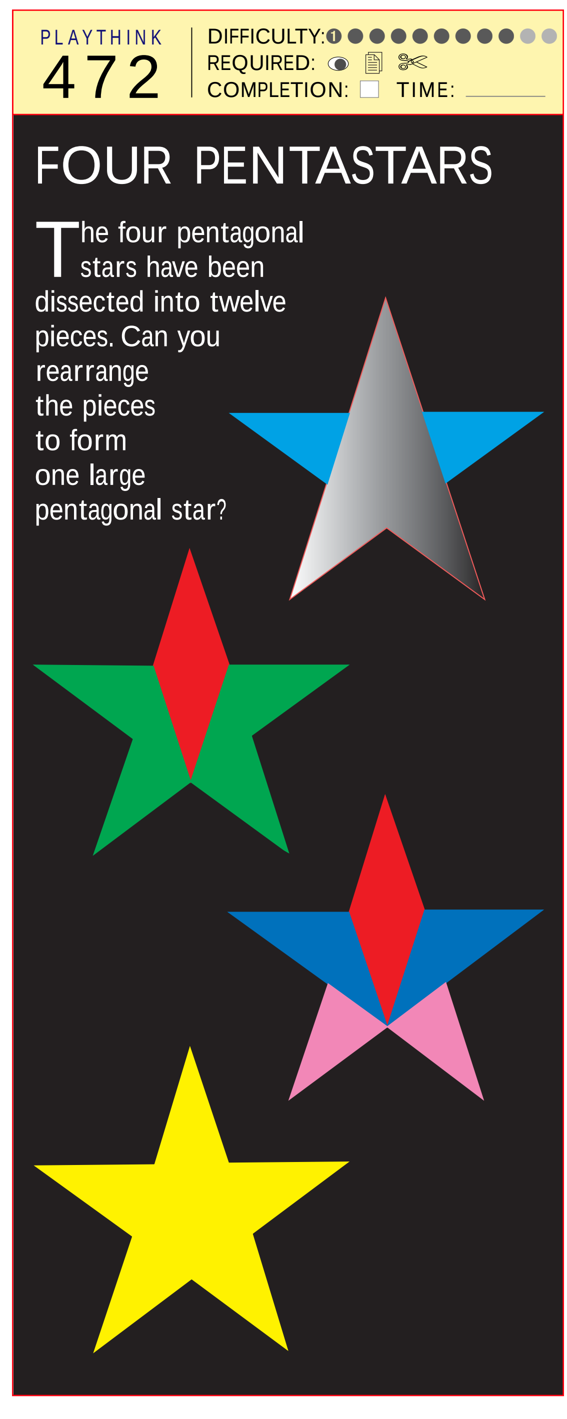
SAMPLE GAME OF PYTHAGORINO

The score of this game was red 6, blue 6 and yellow 5. Red wins the tie because the large red squares were linked using fewer strips (just two) than the blue ones. Notice that isolated large squares do not score points.

PLAYTHINK 472 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 📄 ✂ | COMPLETION: ☐ TIME: ____

FOUR PENTASTARS

The four pentagonal stars have been dissected into twelve pieces. Can you rearrange the pieces to form one large pentagonal star?



PLAYTHINK 473 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

TRIANGLE TO STAR

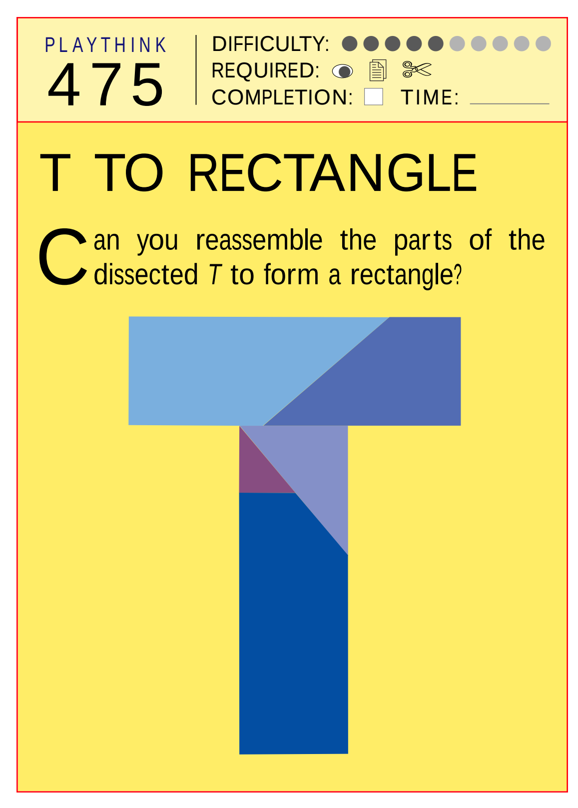
This equilateral triangle is composed of twenty-four identical right triangles. Just by looking at it, can you work out how to rearrange the pieces to form a perfect six-pointed star?

PLAYTHINK 474 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 📄 ✂ | COMPLETION: ☐ TIME: ____

PENTAGONAL STARS

These five pentagonal stars have been dissected into eighteen pieces. The pieces can be rearranged to form a large pentagonal star. Can you work out how that can be accomplished?



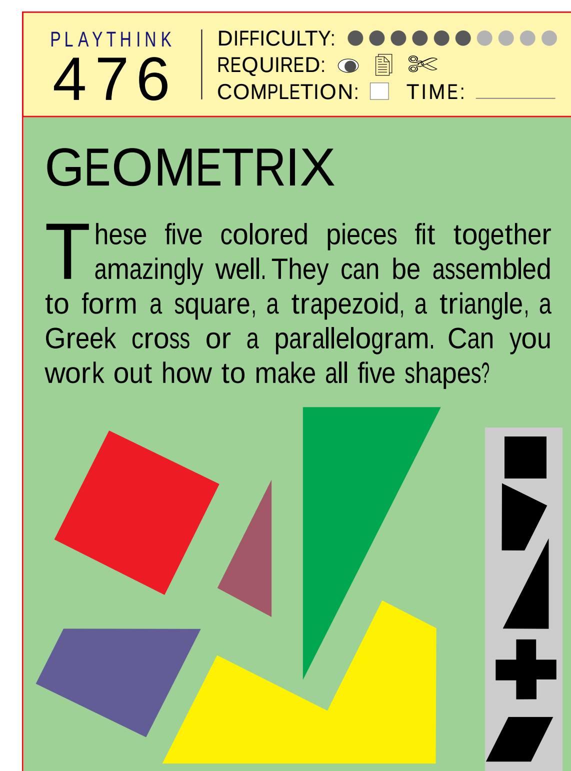


PLAYTHINK 475

DIFFICULTY: ●●●●●○○○○○
REQUIRED:
COMPLETION: ☐ TIME:

T TO RECTANGLE

Can you reassemble the parts of the dissected *T* to form a rectangle?

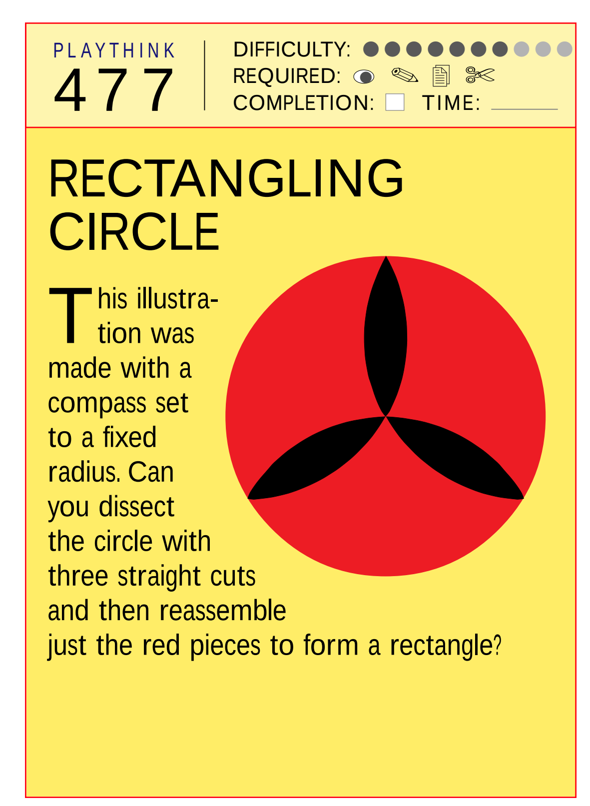


PLAYTHINK 476

DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME:

GEOMETRIX

These five colored pieces fit together amazingly well. They can be assembled to form a square, a trapezoid, a triangle, a Greek cross or a parallelogram. Can you work out how to make all five shapes?

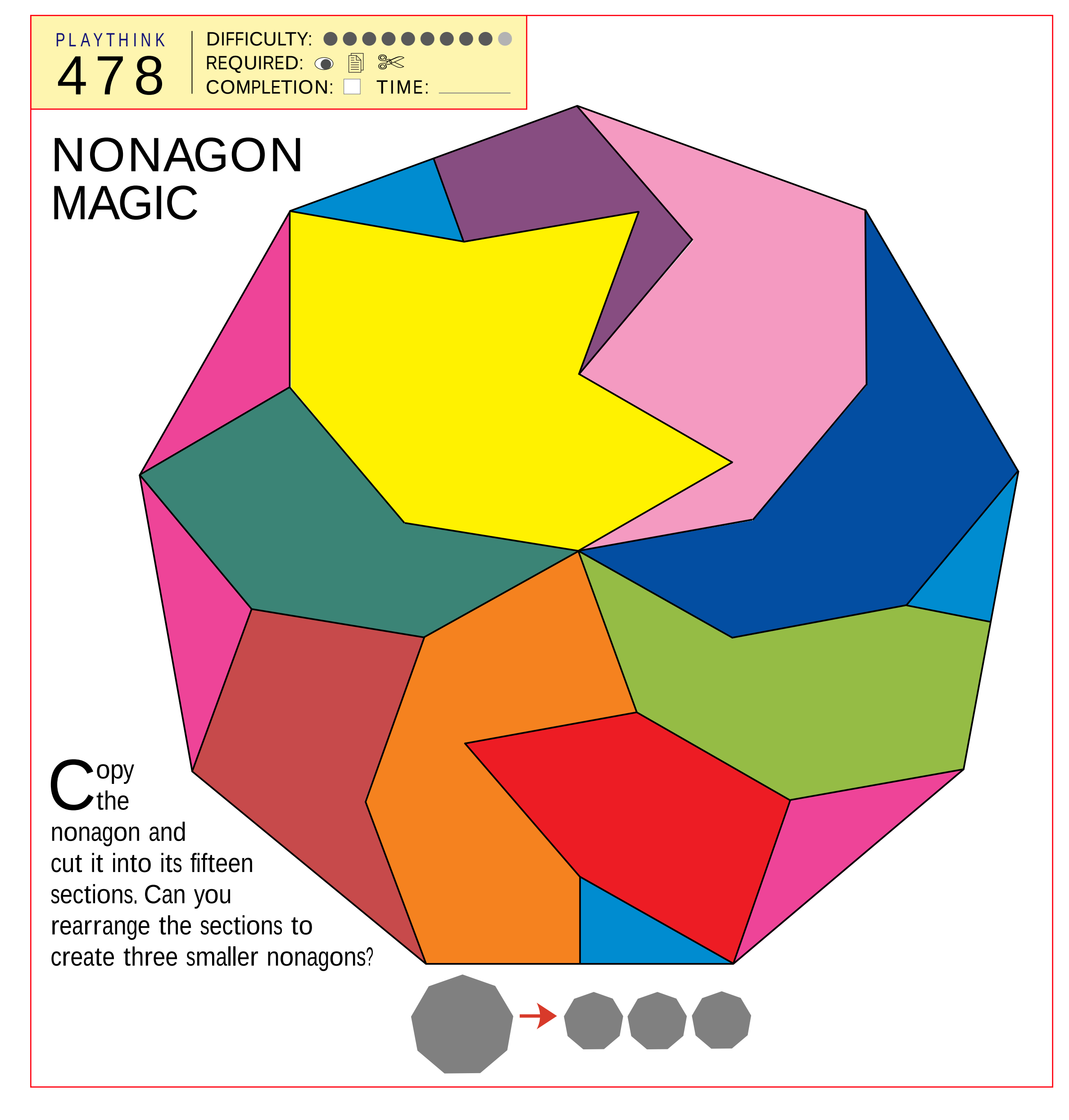


PLAYTHINK 477

DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME:

RECTANGLING CIRCLE

This illustration was made with a compass set to a fixed radius. Can you dissect the circle with three straight cuts and then reassemble just the red pieces to form a rectangle?



PLAYTHINK 478

DIFFICULTY: ●●●●●●●●●○
REQUIRED:
COMPLETION: ☐ TIME:

NONAGON MAGIC

Copy the nonagon and cut it into its fifteen sections. Can you rearrange the sections to create three smaller nonagons?



PLAYTHINK 479

DIFFICULTY: ●●●●●●●●●○
REQUIRED:
COMPLETION: ☐ TIME:

PENTAGONAL STAR

Copy the five-pointed star and cut it into its seventeen sections. Can you rearrange the parts to form four identical decagons (regular polygons with ten sides)?

Star dissection transformation puzzles are probably the prettiest and most intriguing of all geometrical dissections. When they are designed to require the fewest possible number of pieces, they often possess remarkable symmetries and beauty.

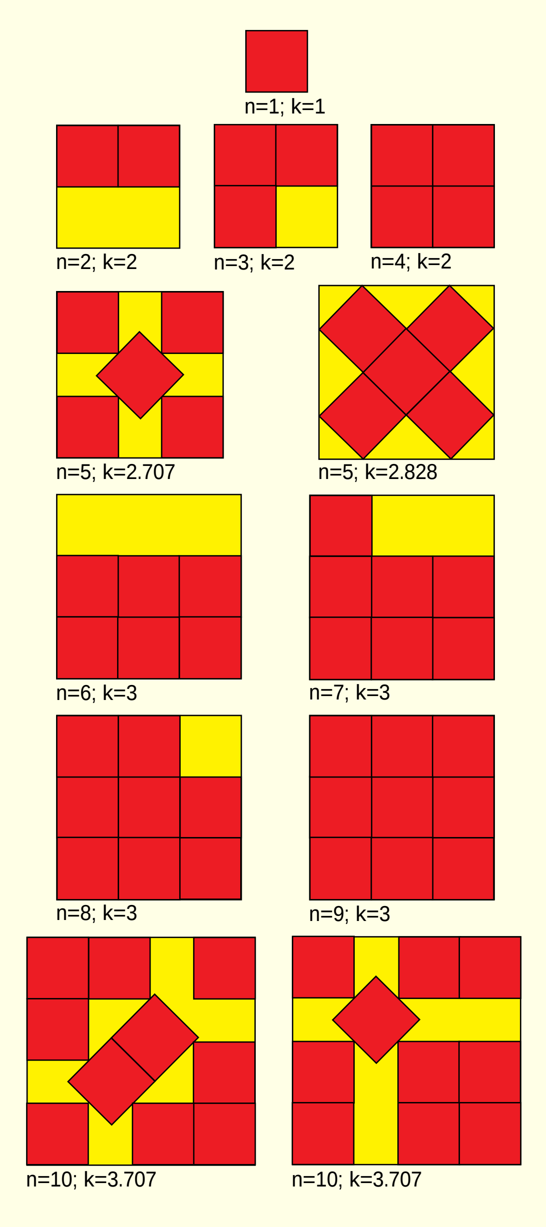
Packing Shapes

"Wheels within wheels" is a common phrase, but what about squares within squares? Suppose you have a number of identical squares to pack inside a larger square. What is the smallest size that the large box has to be to fit a given number of smaller squares without overlaps? If the smaller squares are not allowed to tilt, the problem is trivial. Allowing tilting adds to the difficulty, but it also allows more efficient solutions to emerge.

For one to four squares, tilting provides no advantage. But in order to pack five squares into a larger square without tilting, you must use a square with sides three times as large as the packed squares. Tilt the five squares into a cross, and the large square need be only 2.828 units on a side. A more efficient packing can be obtained if only the central square is tilted; then the larger square has side 2.707.

For n = 6, 7, 8 or 9, the untilted solutions are again as efficient as any other. But when you must pack ten squares, tilting provides a better solution, although no one yet knows if the solutions so far obtained are the best or whether some ingenious packing can be made to do it better.

As the number of squares becomes large, the task of proving that a given packing is minimal becomes increasingly difficult, except in cases in which the number of squares is itself a square—that is, 9, 16, 25 and so on. There are many other packing problems, most of which are equally baffling, especially those that allow irregular packing. An important example is the packing of circles in the plane. The analogous problem of spheres packed into space poses an even more severe problem. The densest regular packing is known, but whether any irregular packing would be better is still a mystery. Most mathematicians don't think they'll find a better solution, but that remains unproved.



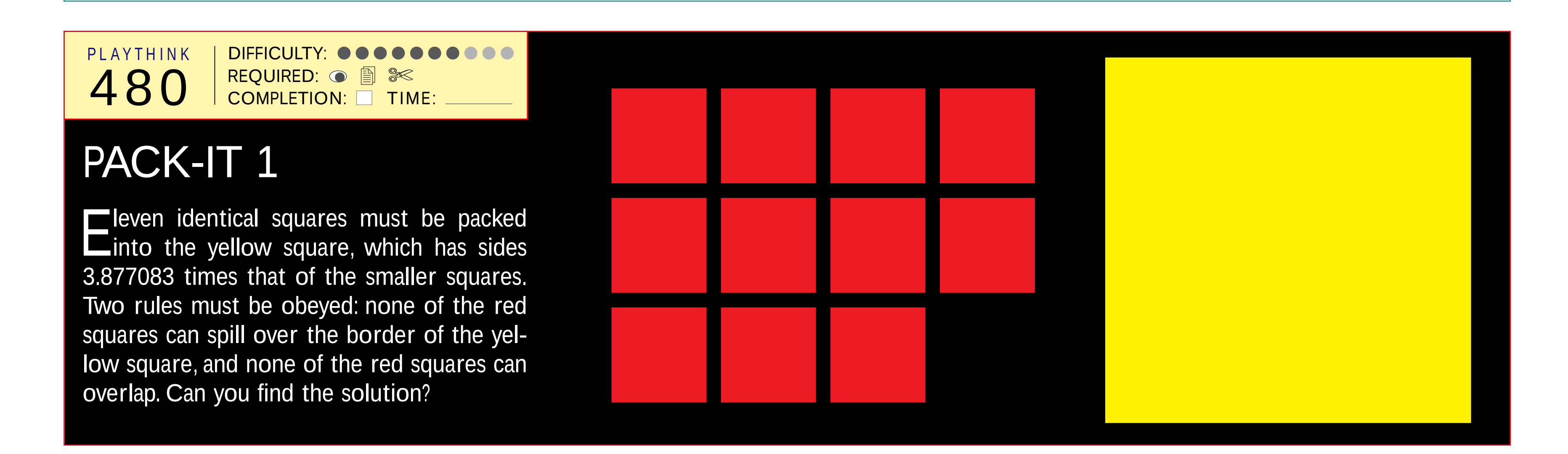
PACKING UNIT SQUARES

The best results for packing unit squares into a larger square are shown; the solutions range from one to ten squares.

PLAYTHINK 480 DIFFICULTY: ●●●●●●●○○○ REQUIRED: COMPLETION: ☐ TIME: ____

PACK-IT 1

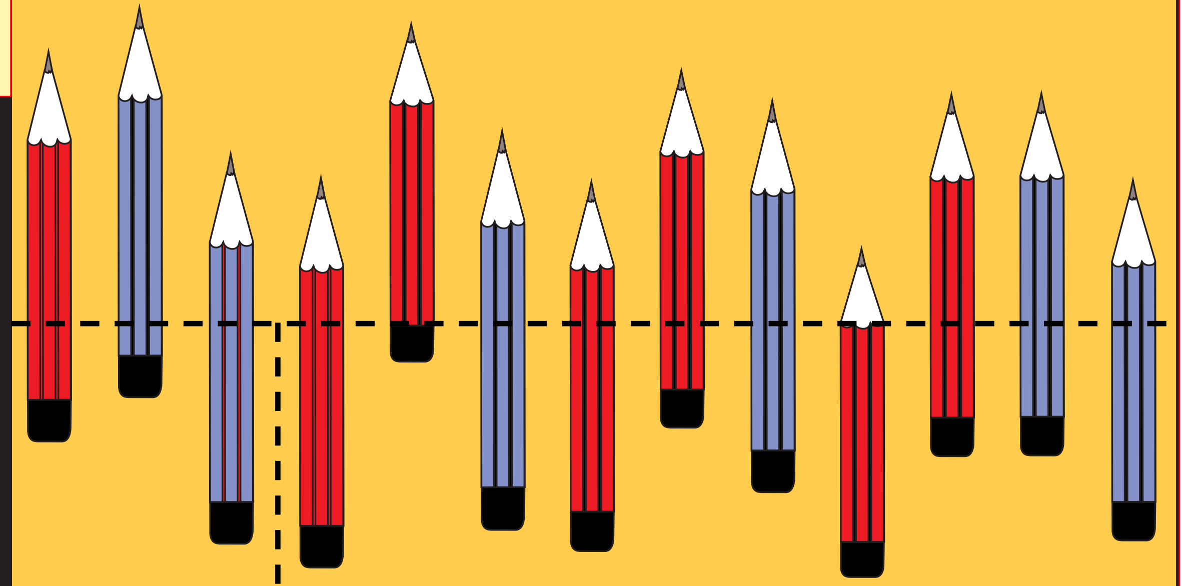
Eleven identical squares must be packed into the yellow square, which has sides 3.877083 times that of the smaller squares. Two rules must be obeyed: none of the red squares can spill over the border of the yellow square, and none of the red squares can overlap. Can you find the solution?



PLAYTHINK 481 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

DISAPPEARING PENCIL

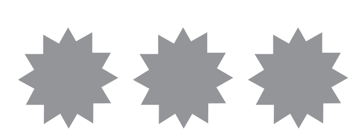
There are seven red pencils and six blue pencils in this drawing. Will cutting along the line and swapping the lower left and lower right parts of the figure have any effect on what you see?



PLAYTHINK 482 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

STAR PUZZLE

This twelve-pointed star has been dissected into twenty-four sections. Can you rearrange the pieces to make three smaller twelve-pointed stars?



PLAYTHINK 483 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

TWELVE-POINTED STAR

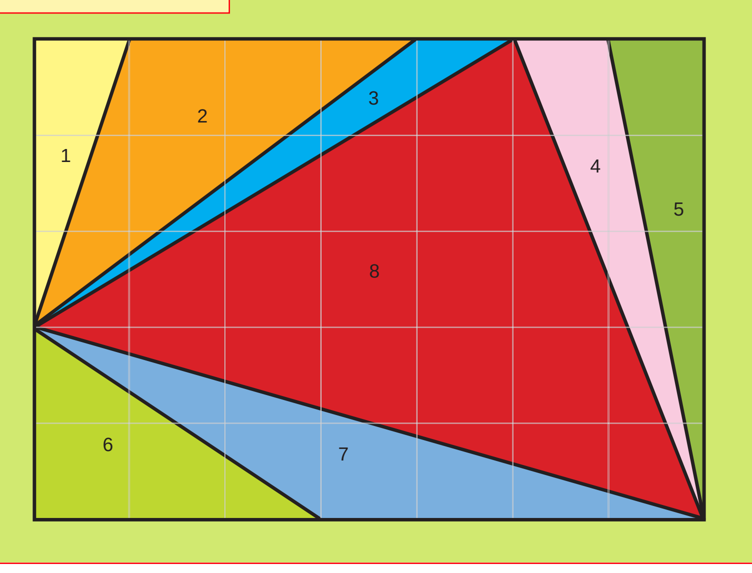
Copy this twelve-pointed star and cut it into its twenty-four sections. Can you rearrange the parts to make three smaller twelve-pointed stars?



PLAYTHINK 484 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

DISSECTED PLOT

Just by looking at the dissected rectangular plot, can you tell the size of each region in terms of the unit squares of the grid? Which part of the plot is bigger: the large red triangle, or all the rest combined?



Squared Squares and Rectangles

Mathematicians look for order everywhere. And when they find it, they like to give expression to their enthusiasm by defining numbers, squares, rectangles, triangles and parallelograms as "perfect."

In 1934 the famous Hungarian mathematician Paul Erdös posed this dissection problem: *Can a square or rectangle be subdivided into smaller squares of which no two are alike?* Such squares or rectangles are called perfect, or squared. (Squares or rectangles in which some of the element squares are identical are, not surprisingly, called imperfect.) Erdös concluded that such a square is impossible, probably influenced by the easily proved fact that one cannot dissect a cube into smaller cubes in which no two are identical. The best one could achieve, Erdös believed, was to dissect a rectangle into smaller squares no two of which are alike.

For many years Erdös appeared to be right. But then a team of mathematicians exploiting an analogy with the theory of electrical circuits found such a perfect square. Their square, which was made up of twenty-four different squares of consecutive sizes, was the longtime record holder for smallest perfect square. But in 1978 the Dutch mathematician A. J. W. Duijvestijn found a better solution—one that required only twenty-one element squares (see page 185).

PLAYTHINK 485 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ 📄 ✂ | COMPLETION: ☐ TIME: ____

SMALLEST SQUARED RECTANGLE

No rectangle has been found that can be divided into fewer than nine squares of different sizes—a so-called perfect rectangle. The smallest such rectangle is composed of the squares, shown below, with sides of 1, 4, 7, 8, 9, 10, 14, 15 and 18 units. From those elements, can you form a perfect rectangle?

A Strange Coincidence

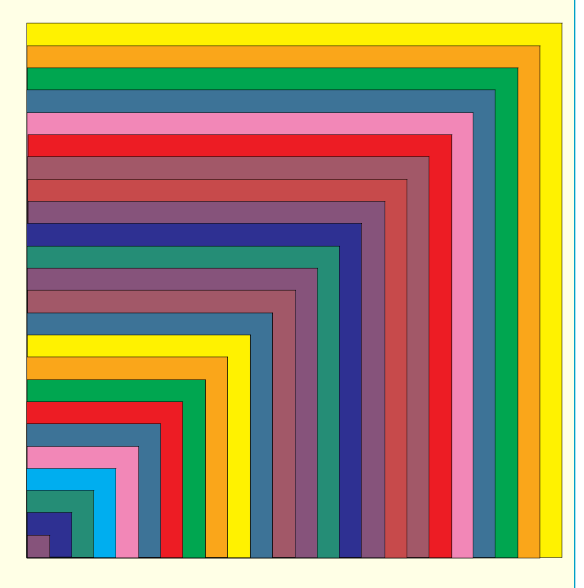
By now you have played with puzzles based on packing identical shapes, circles and squares. And you have begun to think about packing nonidentical squares. One possibility that springs to mind is to use consecutive squares of sides 1, 2, 3 ,4 . . . and so on, up to some particular limit. Is there a square that can be cut into such a system of smaller squares?

If the squares are to fill the large square completely, they cannot be placed at a tilt. So the outer square must have a side that is a whole number. Therefore, the total area of the system of small squares must itself be a square.

Summing the first few consecutive squares isn't very helpful.

$$1^2 + 2^2 + 3^2 + 4^2 + \ldots + 24^2 = 4{,}900 = 70^2$$

In fact, this is the only sum of consecutive squares that results in a square for the total. (The demonstration is a difficult exercise in the theory of numbers and was an unsolved problem for a considerable time.) That raises a geometrical problem that is one of the most beautiful puzzles in recreational geometry: Can one pack the first twenty-four consecutive squares into a seventy-by-seventy square? If you want to try it, see PlayThink 486.

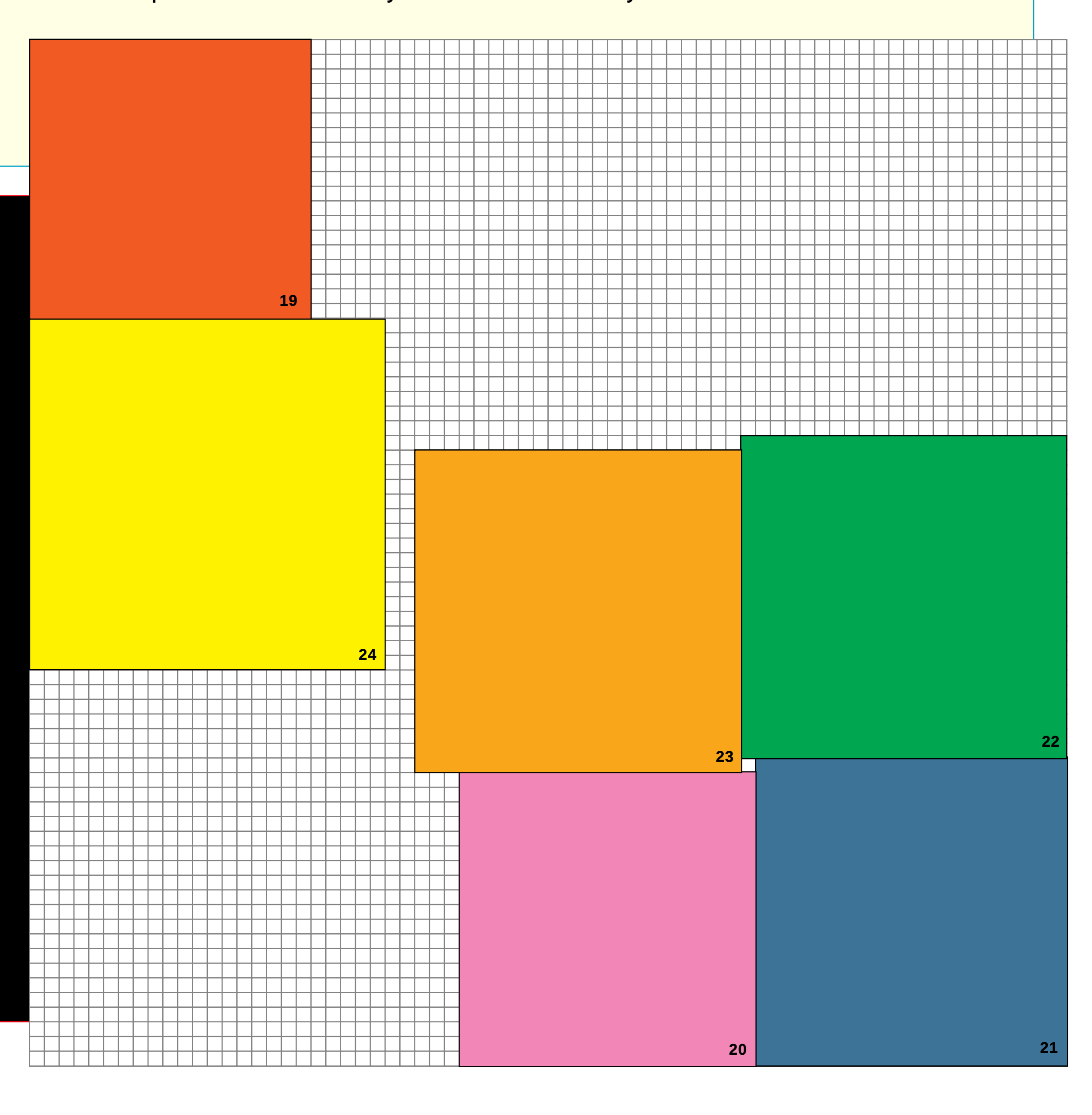


PLAYTHINK 486 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

SQUARE INFINITY

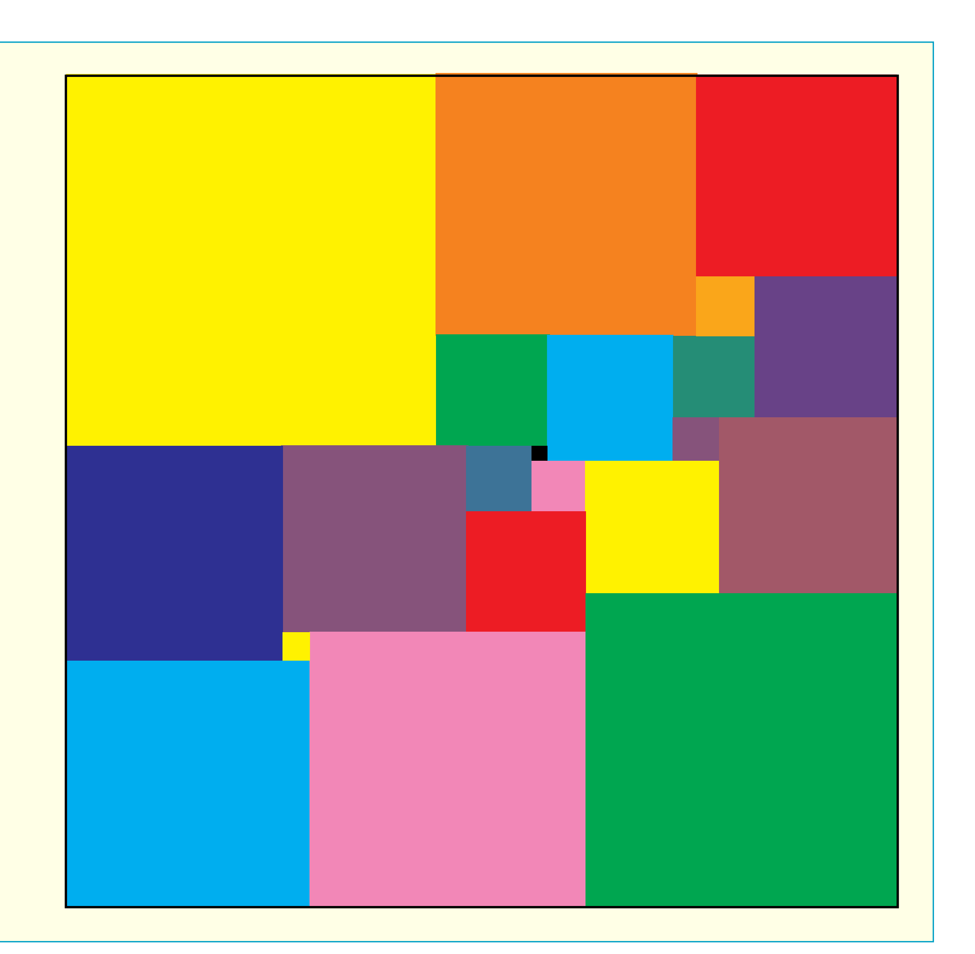
Twenty-four squares with sides ranging from 1 to 24 units have a total area of 4,900 square units. The seventy-by-seventy game board shown at right also has an area of 4,900 square units. Can you cover the board with the twenty-four squares without overlap? To give you a head start, the largest squares have been placed.

Is there a smaller number of consecutive squares that add up to a square number?



Squared Square

A square that is composed of smaller squares of different sizes is called a perfect square. (The smaller squares should all have sides that are whole numbers.) The smallest known perfect square is made up of twenty-one squares; those squares have sides of 2, 4, 6, 7, 8, 9, 11, 15, 16, 17, 18, 19, 24, 25, 27, 29, 33, 35, 37, 42 and 50 units. The diagram here shows how those squares are put together to make one larger square with sides of 112 units.



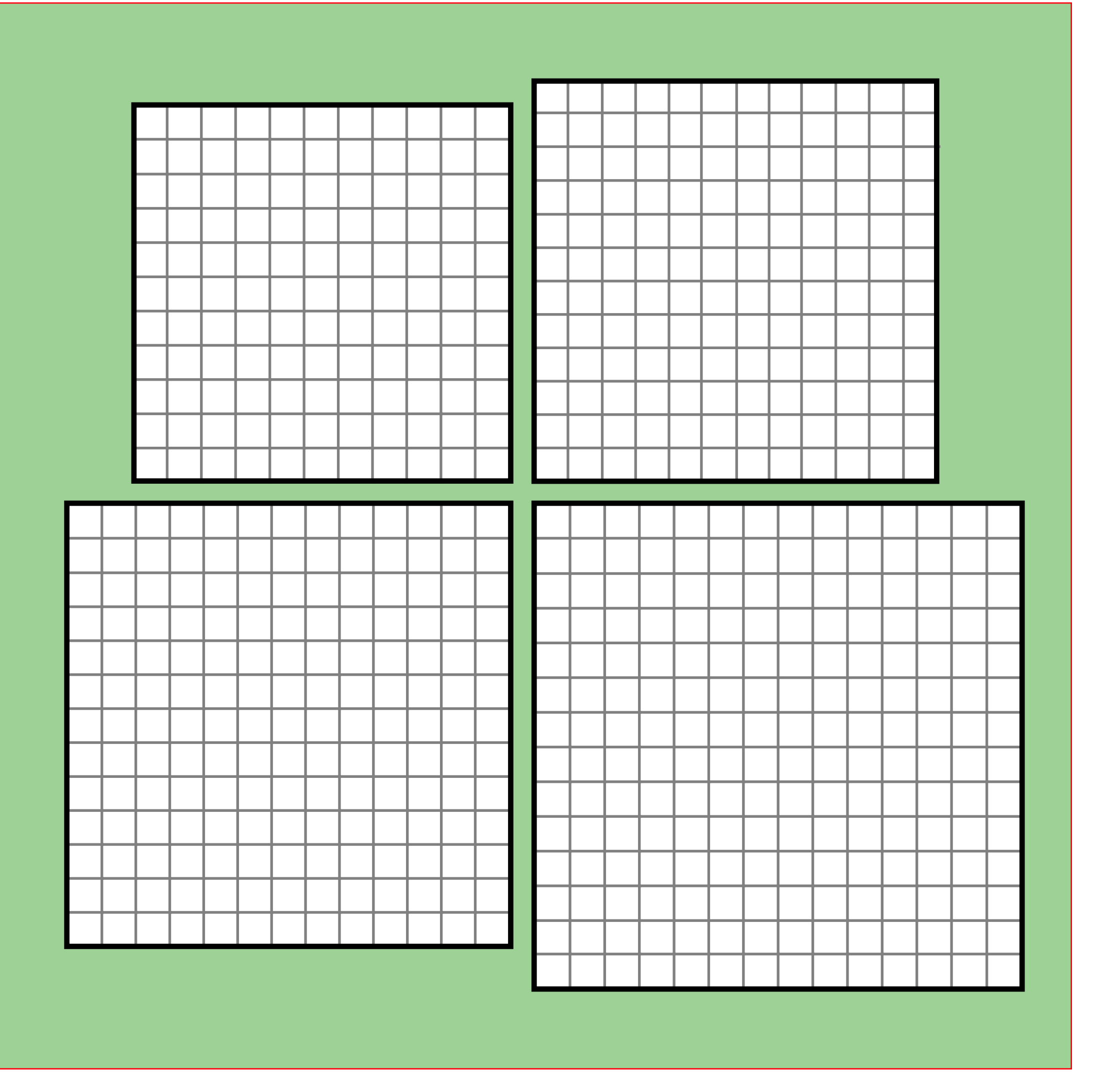
PLAYTHINK 487

DIFFICULTY: ●●●●●○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

IMPERFECT SQUARE

Squares that have been divided into smaller squares, with two or more squares being of identical size, are called imperfect squares. For example, a three-by-three square can be dissected into one two-by-two square and five one-by-one squares—a total of six pieces. You might try dividing a four-by-four square into one three-by-three square and seven one-by-one squares, but the minimal solution will involve just four two-by-two squares.

In general, squares with sides that are of even-numbered lengths are easy to form as imperfect squares; those with sides that are of odd-numbered lengths are more complicated. To see how this is so, dissect these squares, with sides of 11, 12, 13 and 14 units, into imperfect squares with the least number of pieces.

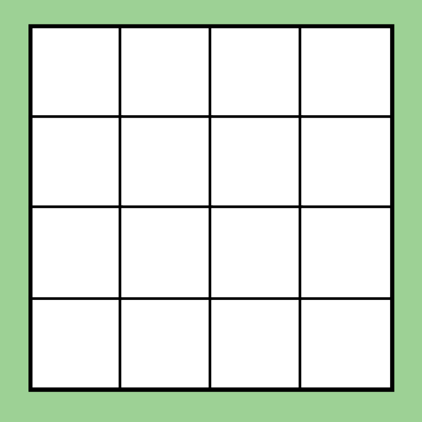


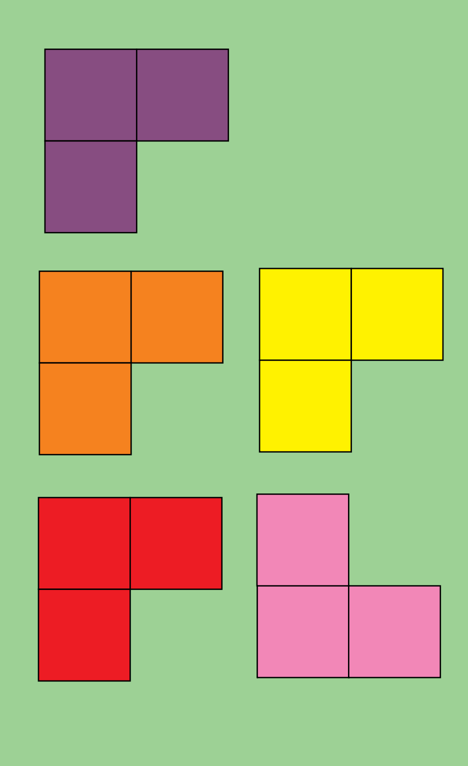
PLAYTHINK 488

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

UNCOVERED SQUARE

If you try to fit the five chevron shapes onto the four-by-four board, one square will always be uncovered. After all, the five chevrons each cover an area of three units, and the board has an area of sixteen units. But can that uncovered square be anywhere on the board?



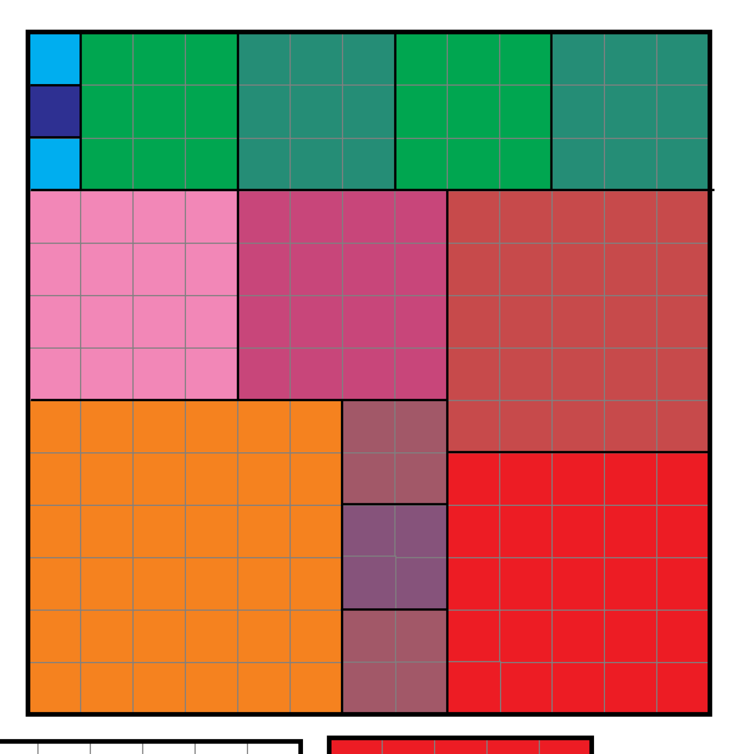


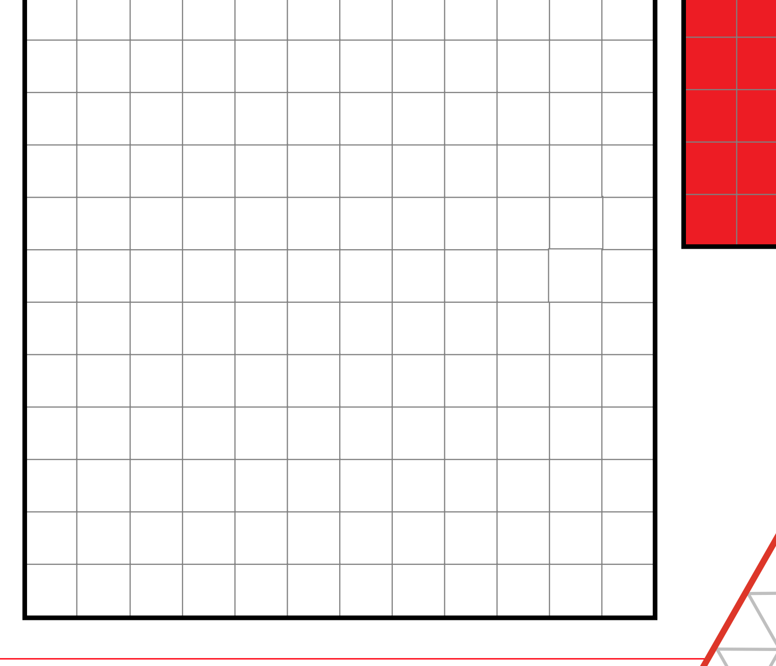
PLAYTHINK 489

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

IMPERFECT SQUARE SPLIT

Fifteen squares can form an imperfect thirteen-by-thirteen square, as shown here. If you remove one of the five-by-five squares, can you reassemble the remaining squares to form a twelve-by-twelve perfect square?



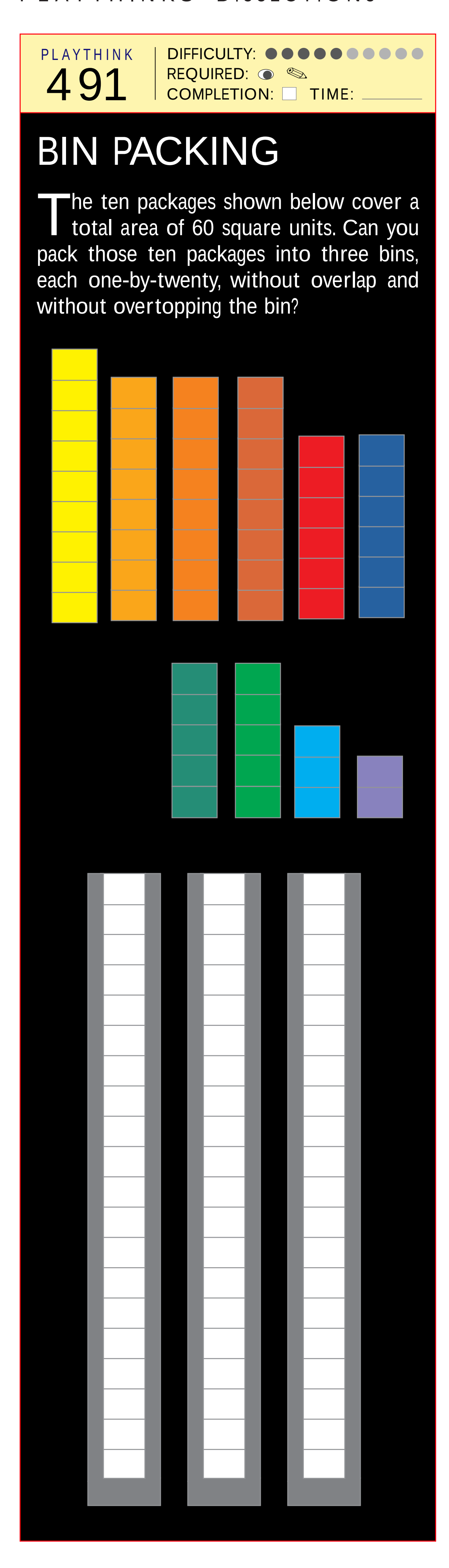


PLAYTHINK 490

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

IMPERFECT TRIANGLE

Using the triangular grid as a guide, divide this equilateral triangle with sides 11 units long into smaller-integer triangles. What is the smallest number of such triangles that will completely cover the figure?



PLAYTHINK 491 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

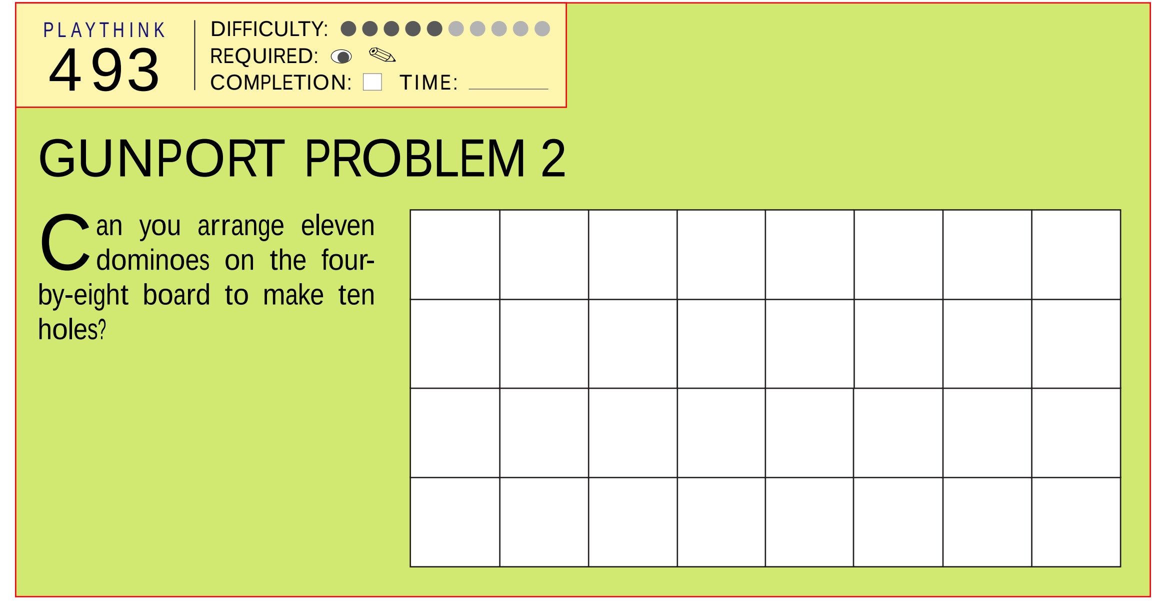
BIN PACKING

The ten packages shown below cover a total area of 60 square units. Can you pack those ten packages into three bins, each one-by-twenty, without overlap and without overtopping the bin?

PLAYTHINK 492 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

GUNPORT PROBLEM 1

Many interesting problems have been built around blocks whose sides, like dominoes, possess a 2:1 ratio. One such puzzle is the gunport problem, in which one must find a way to construct the most one-by-one holes with two-by-one blocks. Can you arrange ten two-by-one blocks on a four-by-eight grid to make eight holes, each of which is one-by-one?



PLAYTHINK 493 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

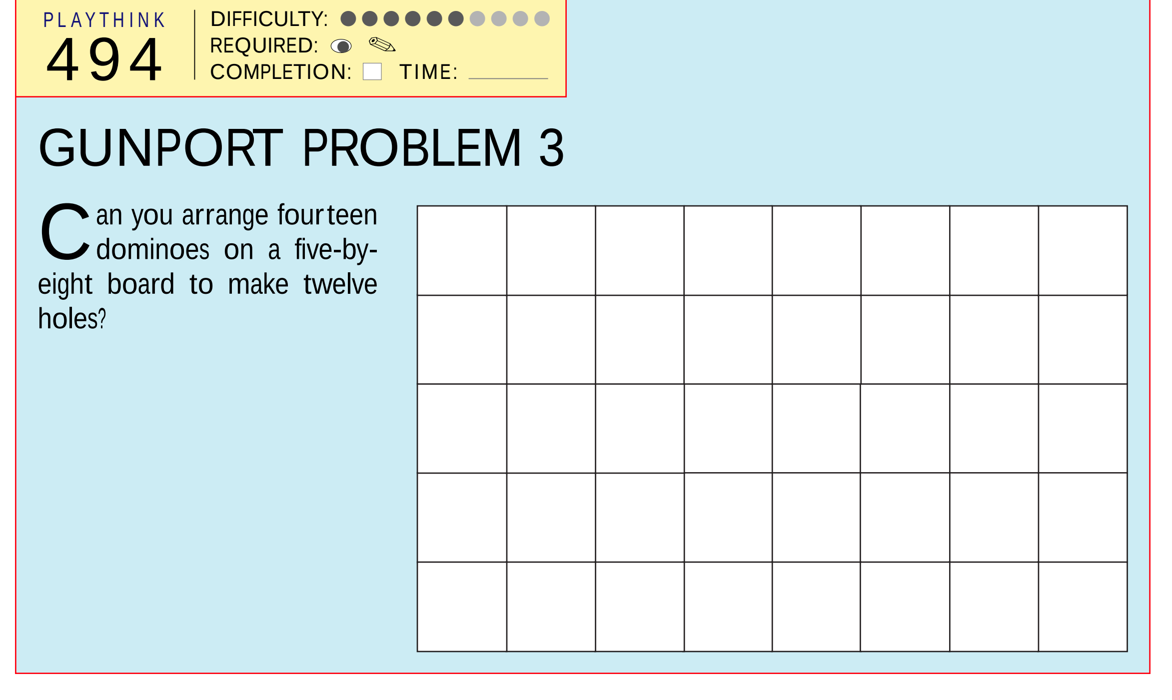
GUNPORT PROBLEM 2

Can you arrange eleven dominoes on the four-by-eight board to make ten holes?

PLAYTHINK 494 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

GUNPORT PROBLEM 3

Can you arrange fourteen dominoes on a five-by-eight board to make twelve holes?

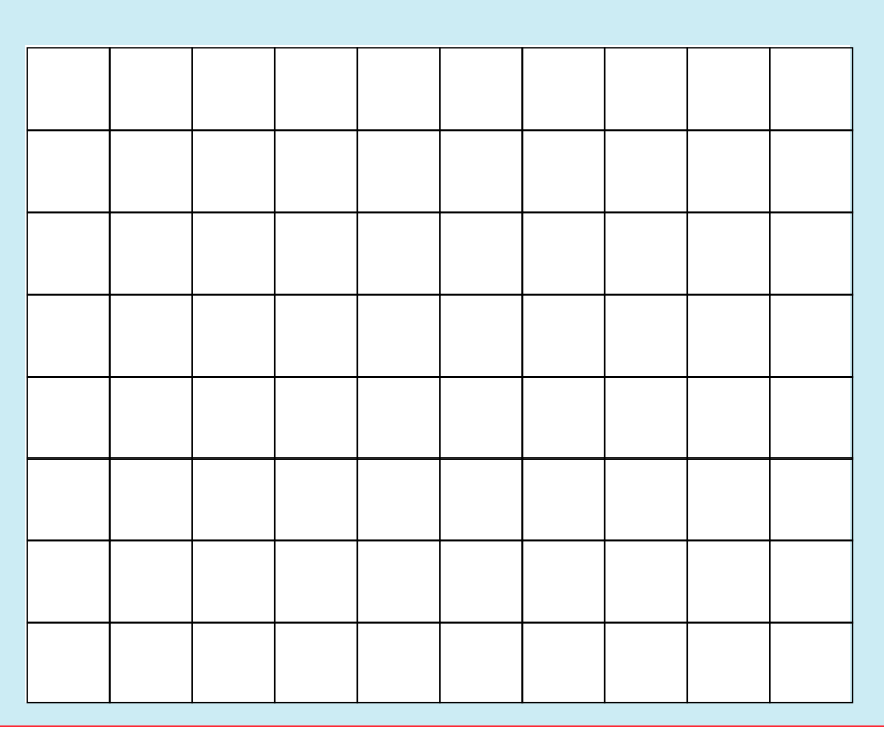


PLAYTHINK 495

DIFFICULTY: ●●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

GUNPORT PROBLEM 4

Can you arrange twenty-seven dominoes on an eight-by-ten board to make twenty-six holes?

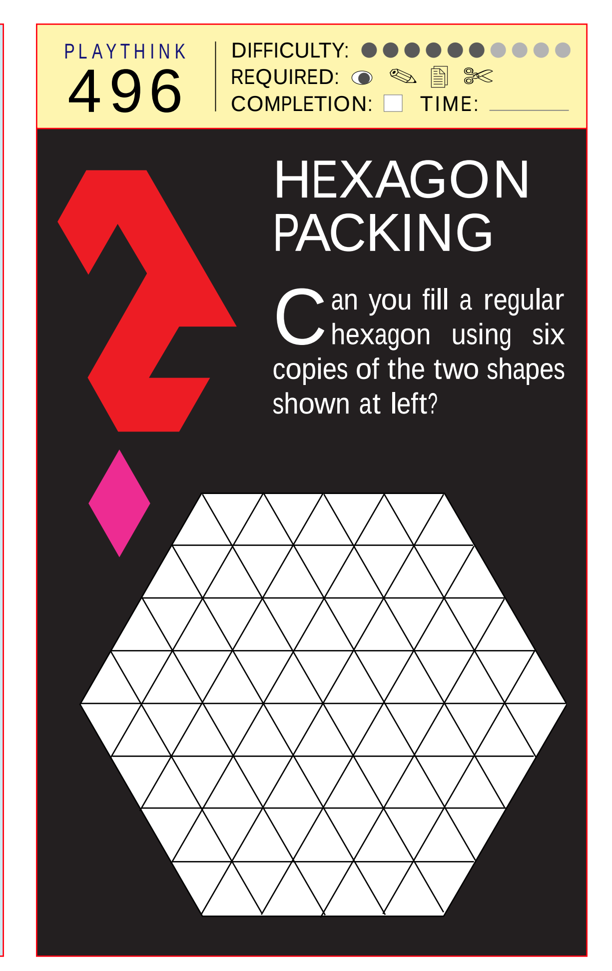


PLAYTHINK 496

DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

HEXAGON PACKING

Can you fill a regular hexagon using six copies of the two shapes shown at left?



Vanishing Pieces

Most optical tricks and perceptual illusions fail to hold our attention because the secret of their trickery becomes obvious fairly quickly. But a remarkable group of images known as “geometrical paradoxes” are so subtle that they continue to intrigue and surprise even after their workings have been explained.

Geometrical paradoxes involve separating and rearranging parts of a total length or area. After reassembling the figure in what seems to be its entirety, a portion of the original figure is left over.

The explanation lies in what the great American puzzle genius Martin Gardner calls the principle of concealed distribution. The eye has a great tolerance for subtle alterations in the rearranged version. Tiny increases in the gaps between the parts or in the lengths of the reassembled pieces go unnoticed, so people believe both must have the same area or length.

Sam Loyd, the greatest American puzzle creator (and the inventor of Parcheesi), was the originator of the most famous puzzle in this group: “Get Off the Earth” (a variation of which you can try; see PlayThink 481). Invented in 1896, it involves two disks attached at their common center. In one orientation the disks show parts of thirteen warriors standing on the planet. But when the top disk is rotated a bit, one of the warriors disappears. The puzzle caused such a sensation that it was used as part of a publicity campaign for William McKinley’s presidential bid.

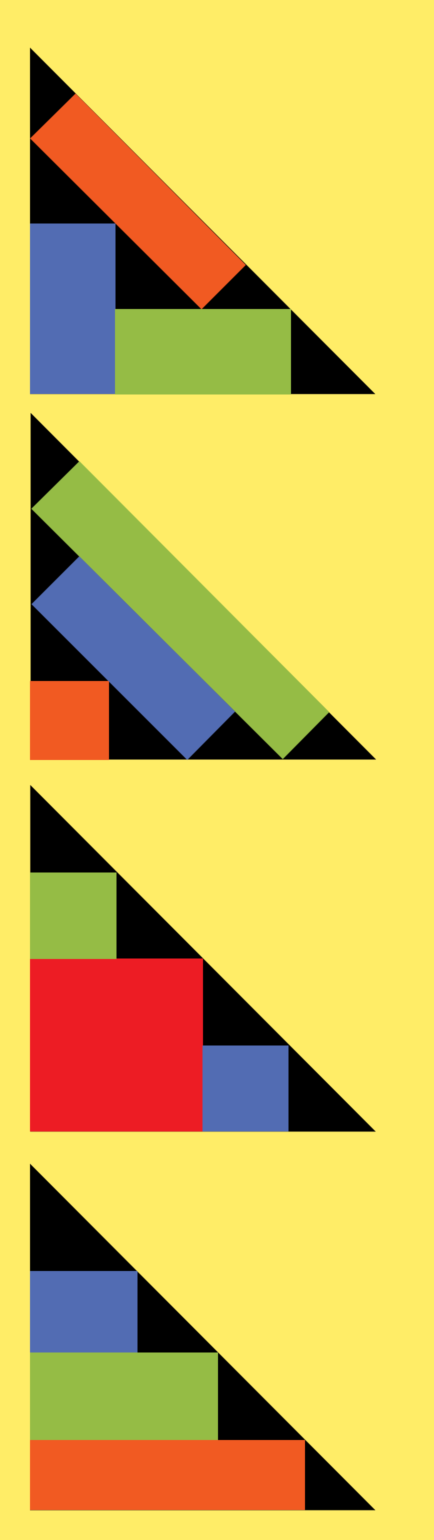
Over the years the Canadian illusionist Melville Stover and many others have perfected the art, creating subtle variations of the principle and loads of exciting puzzles. Some crooks also used the method of concealed distribution—to convert fourteen $100 bills into fifteen by cutting each into two parts and gluing one part to the next. Although the effect was subtle, it was noticeable—and quite illegal.

PLAYTHINK 497

DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

RECTANGLES IN TRIANGLE

Four examples of right isosceles triangles partially filled with squares or rectangles are shown below. Just by looking at them, can you tell in which examples the shapes cover the greatest proportion of the triangle?

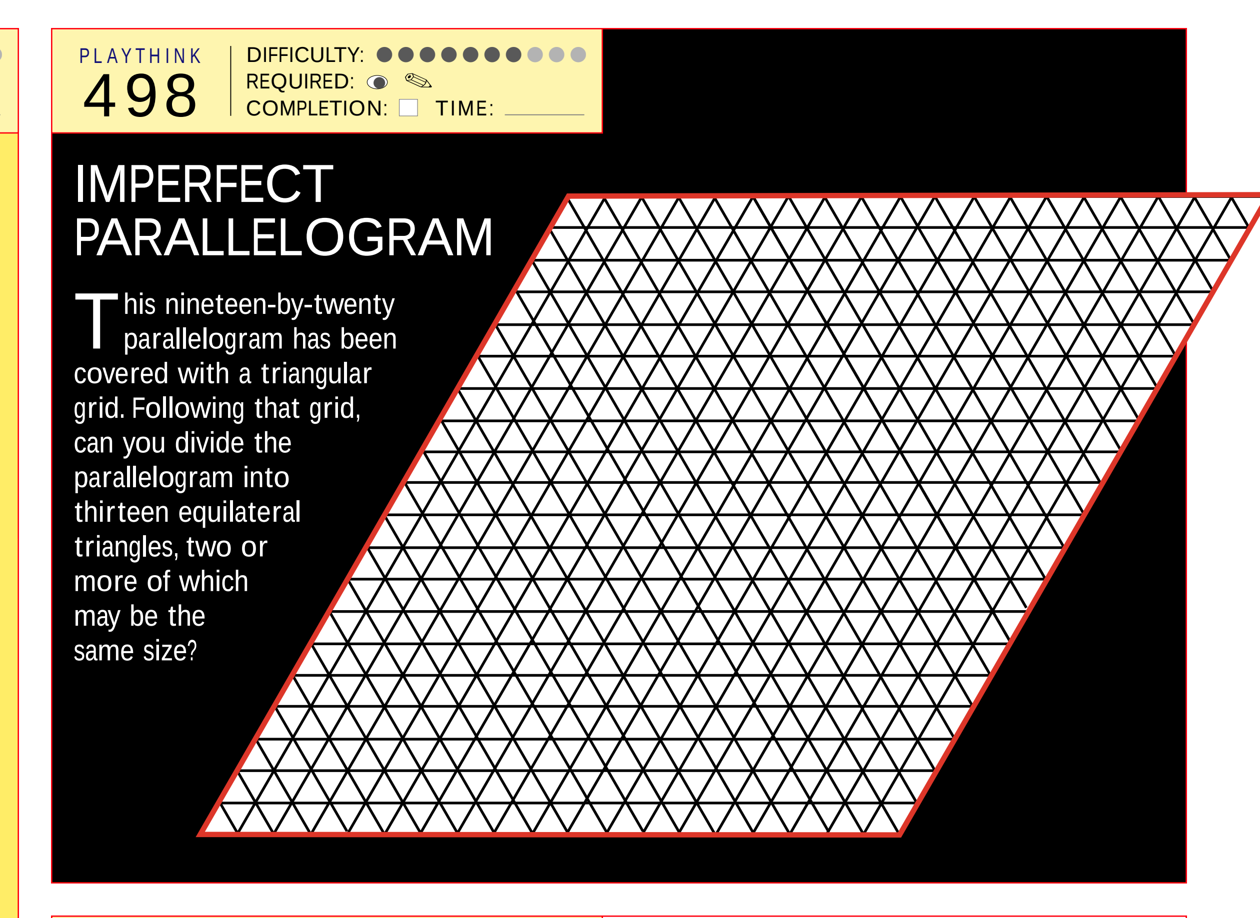


PLAYTHINK 498

DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

IMPERFECT PARALLELOGRAM

This nineteen-by-twenty parallelogram has been covered with a triangular grid. Following that grid, can you divide the parallelogram into thirteen equilateral triangles, two or more of which may be the same size?



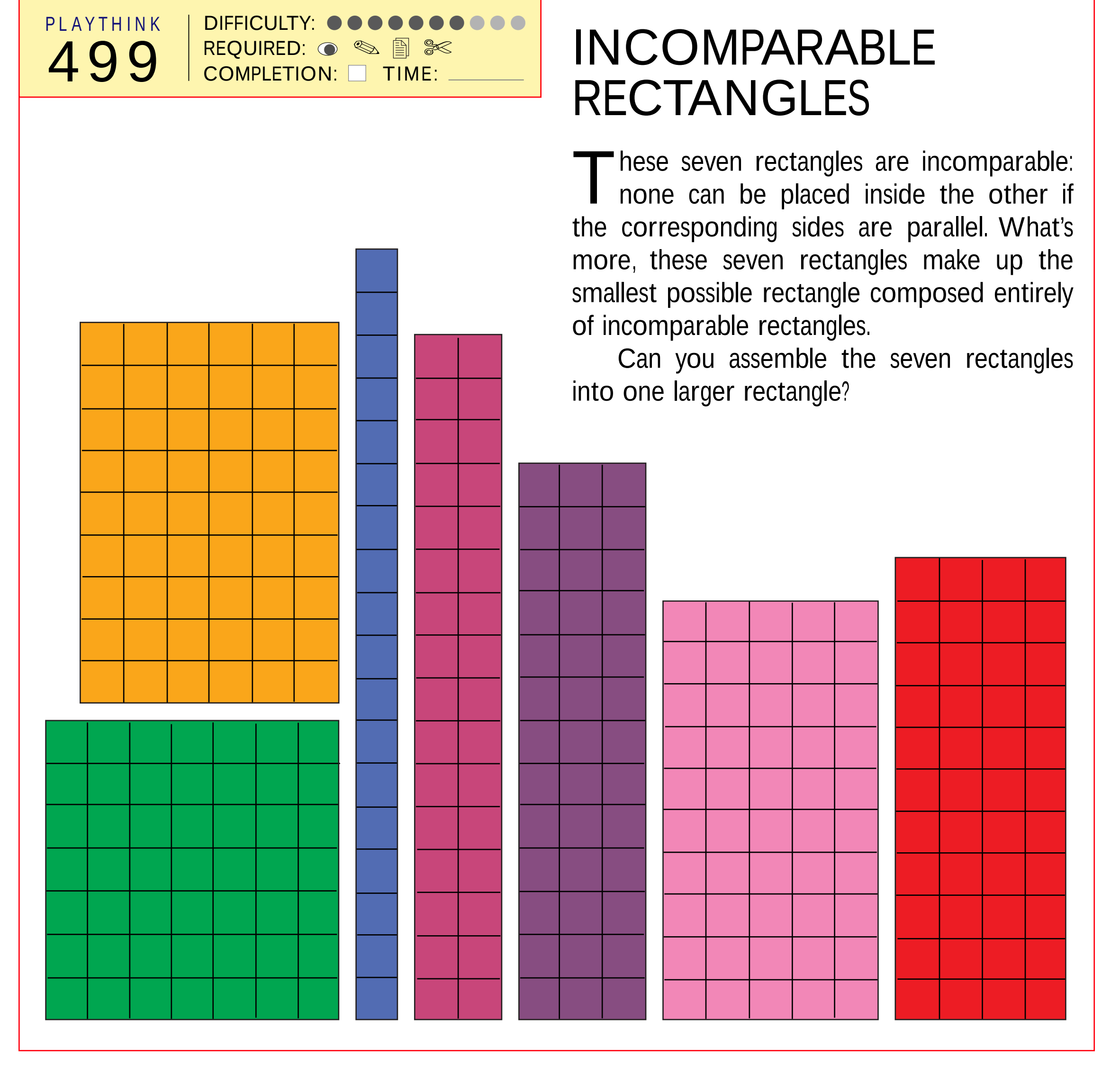
PLAYTHINK 499

DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

INCOMPARABLE RECTANGLES

These seven rectangles are incomparable: none can be placed inside the other if the corresponding sides are parallel. What's more, these seven rectangles make up the smallest possible rectangle composed entirely of incomparable rectangles.

Can you assemble the seven rectangles into one larger rectangle?



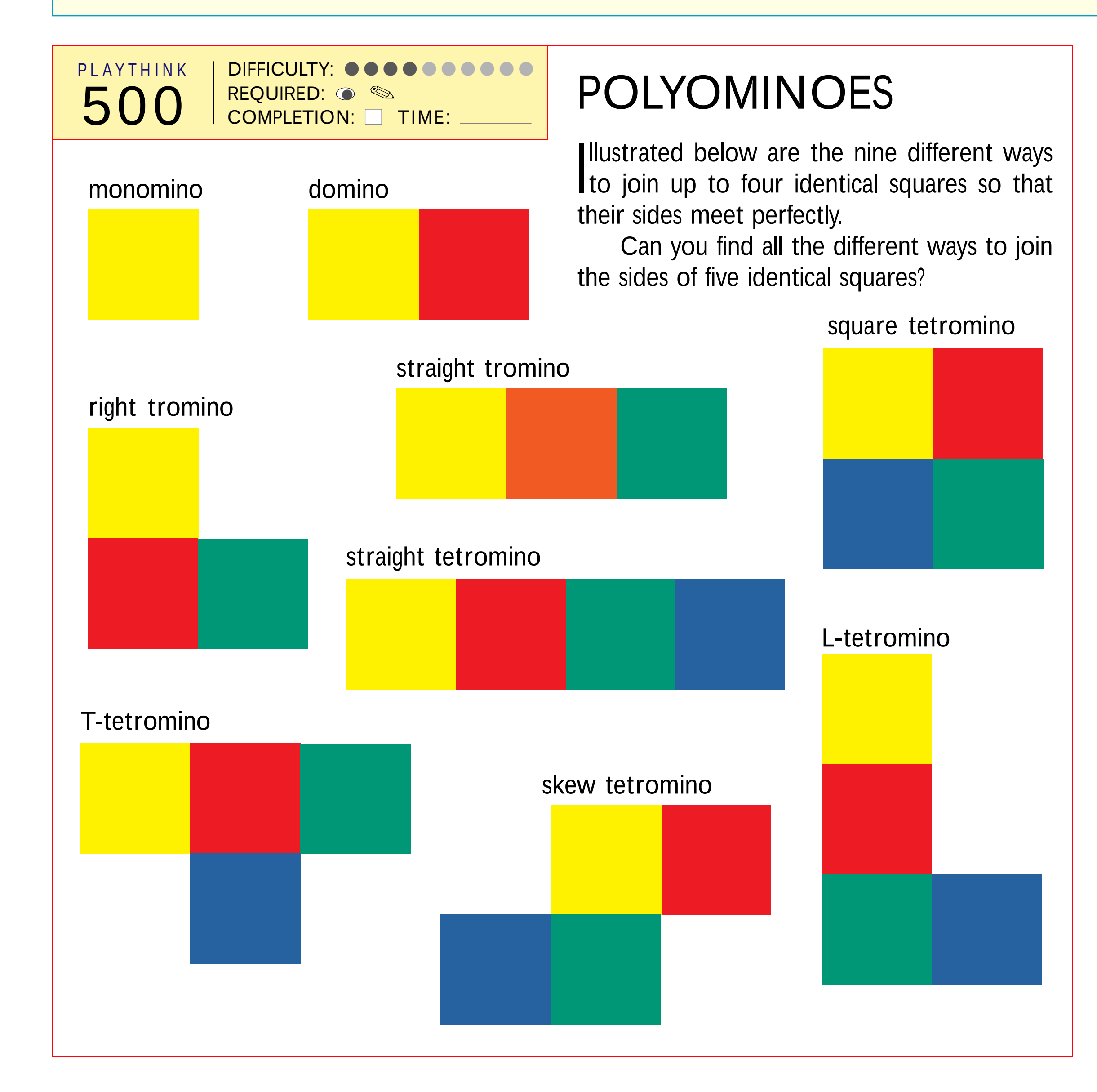
Polyominoes

Dominoes are the playing pieces, or tiles, of a centuries-old game. The tiles are made up of two unit squares joined along a common edge, and each square is marked with an independent number of dots. But mathematicians—recreational and otherwise—have elaborated on the basic domino shape by adding successively more unit squares. The results—three-square trominoes, four-square tetrominoes, five-square pentominoes and the like—are collectively known as polyominoes.

The first polyomino problem appeared in 1907. Now no mention of [combinatorics] and puzzles can be made without a reference to polyominoes, and especially to pentominoes, on which volumes have been written.

The popularity of these shapes, both as a form of mathematical recreation and as educational tools, owes much to two men: Solomon Golomb, who invented them in 1953, and Martin Gardner, who has introduced beautiful puzzles, games and problems based on them to wide audiences.

It's fun to think about the different polyominoes that can be constructed from a certain number of unit squares. For instance, the domino has but one possible shape, and the tromino just two. But there are 5 tetrominoes, 12 pentominoes and 12 hexominoes (six-squared polyominoes). After that, the numbers rise steeply: 108 heptominoes and 369 octominoes.

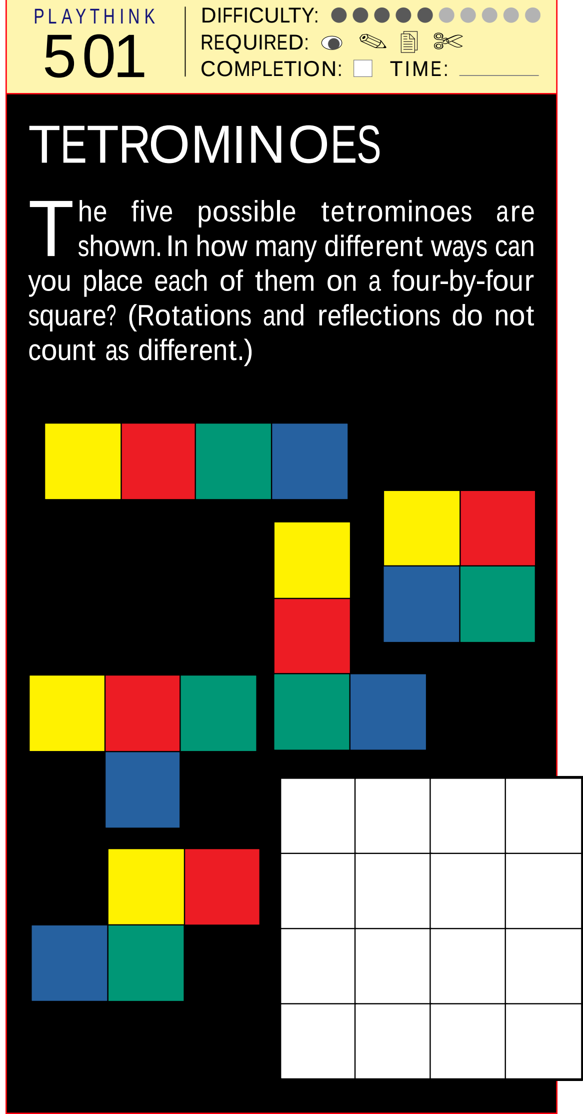


PLAYTHINK 500 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ____

POLYOMINOES

Illustrated below are the nine different ways to join up to four identical squares so that their sides meet perfectly.

Can you find all the different ways to join the sides of five identical squares?



PLAYTHINK 501 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ____

TETROMINOES

The five possible tetrominoes are shown. In how many different ways can you place each of them on a four-by-four square? (Rotations and reflections do not count as different.)

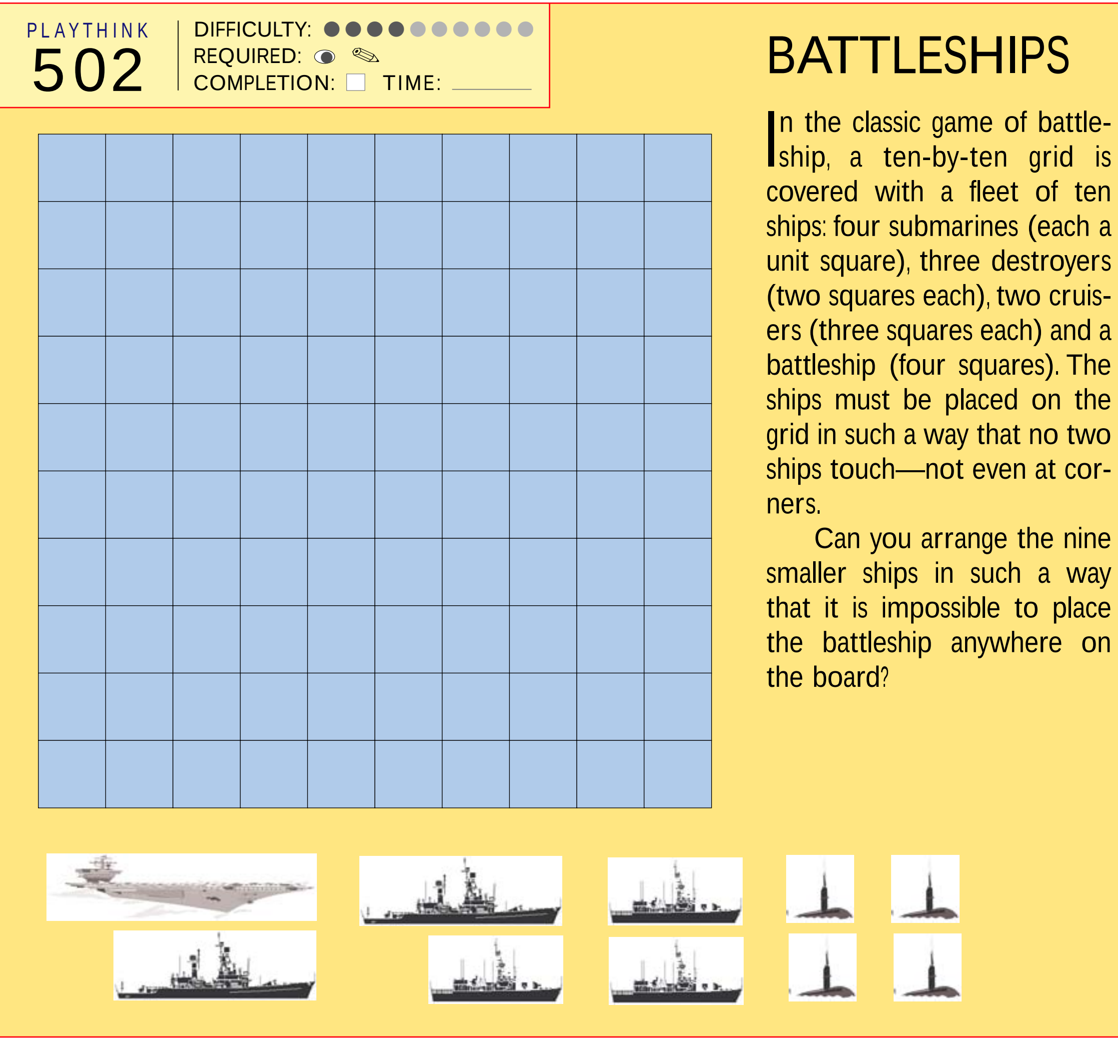
PLAYTHINK 502

DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ______

BATTLESHIPS

In the classic game of battleship, a ten-by-ten grid is covered with a fleet of ten ships: four submarines (each a unit square), three destroyers (two squares each), two cruisers (three squares each) and a battleship (four squares). The ships must be placed on the grid in such a way that no two ships touch—not even at corners.

Can you arrange the nine smaller ships in such a way that it is impossible to place the battleship anywhere on the board?



PLAYTHINK 504

DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ______

TRIANGLE FITTING

How many of the small figures can you place in the larger one without overlap?



PLAYTHINK 503

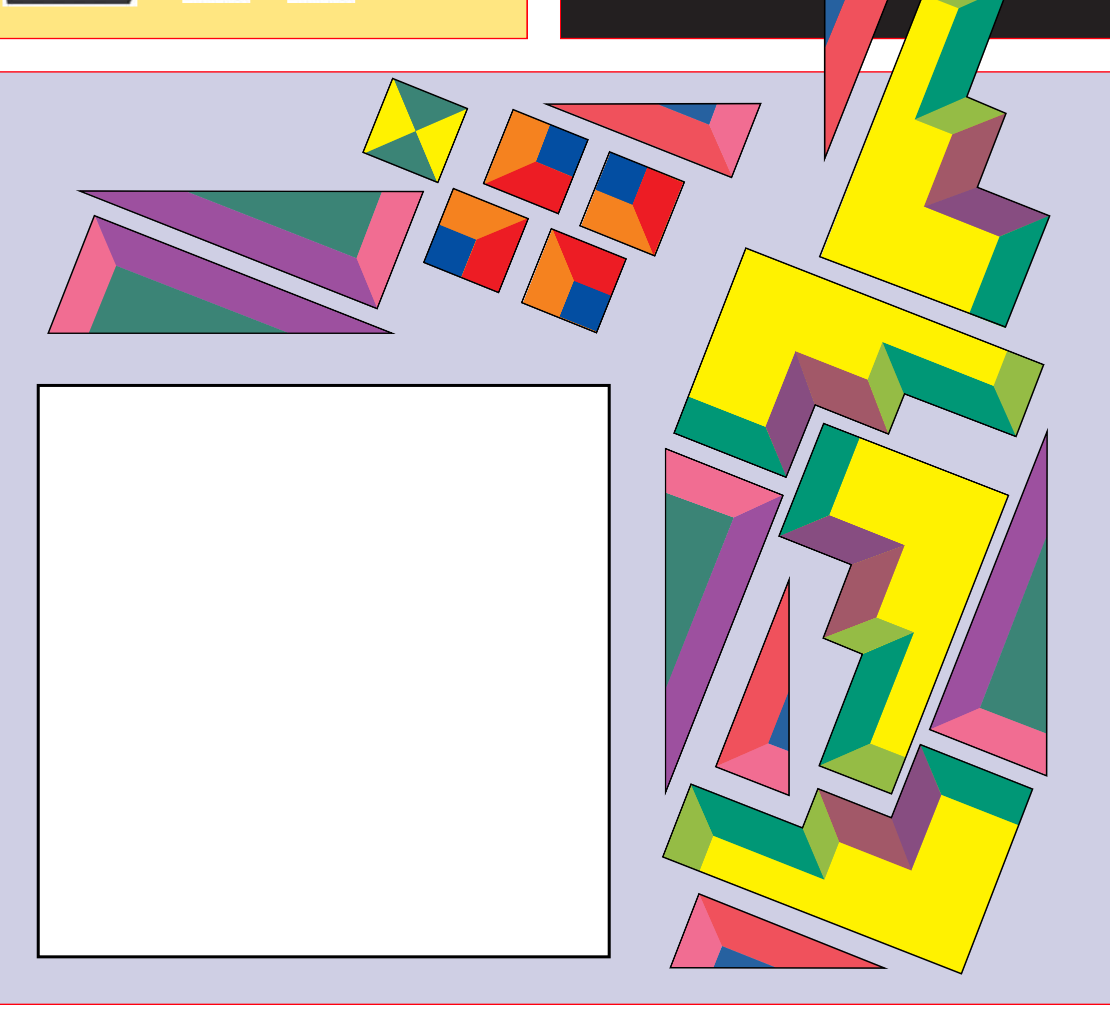
DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ______

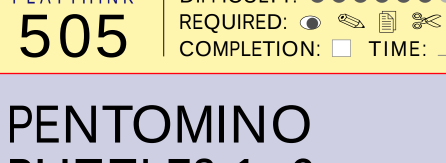
MYSTRIX: THE DISAPPEARING SQUARE PUZZLE

Ever thought you were the center of attention, only to find that no one noticed when you were missing? This puzzle offers the same weird effect in geometric form: you can remove a central piece from a dissection and never notice it's gone.

No sleight of hand or hypnosis is necessary to pull off this feat of geometric magic. Simply copy and cut out all seventeen pieces. Use all the parts to completely cover the white square at right. Then remove the small green and yellow square and reassemble the remaining pieces on the white square. You'll find that you can cover the area again with virtually the same pattern!

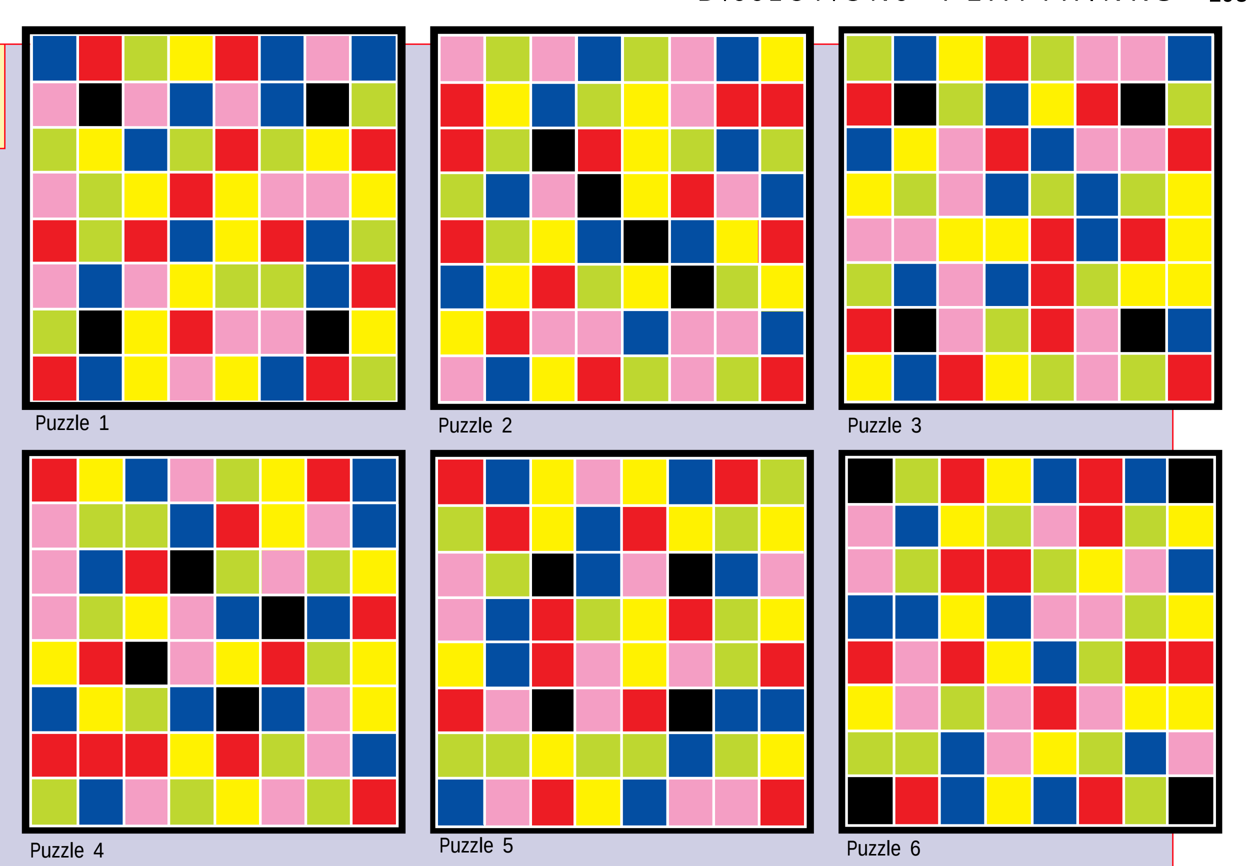
Why doesn't the extra square make a difference?





PENTOMINO PUZZLES 1–6

Using the set of color pentominoes you can find with PlayThink 508, can you find all twelve pentominoes in each eight-by-eight grid? (The squares that are not covered are shown in black.) Note that reflections of the pieces are allowed. Once you've found all the positions, draw an outline around each pentomino.

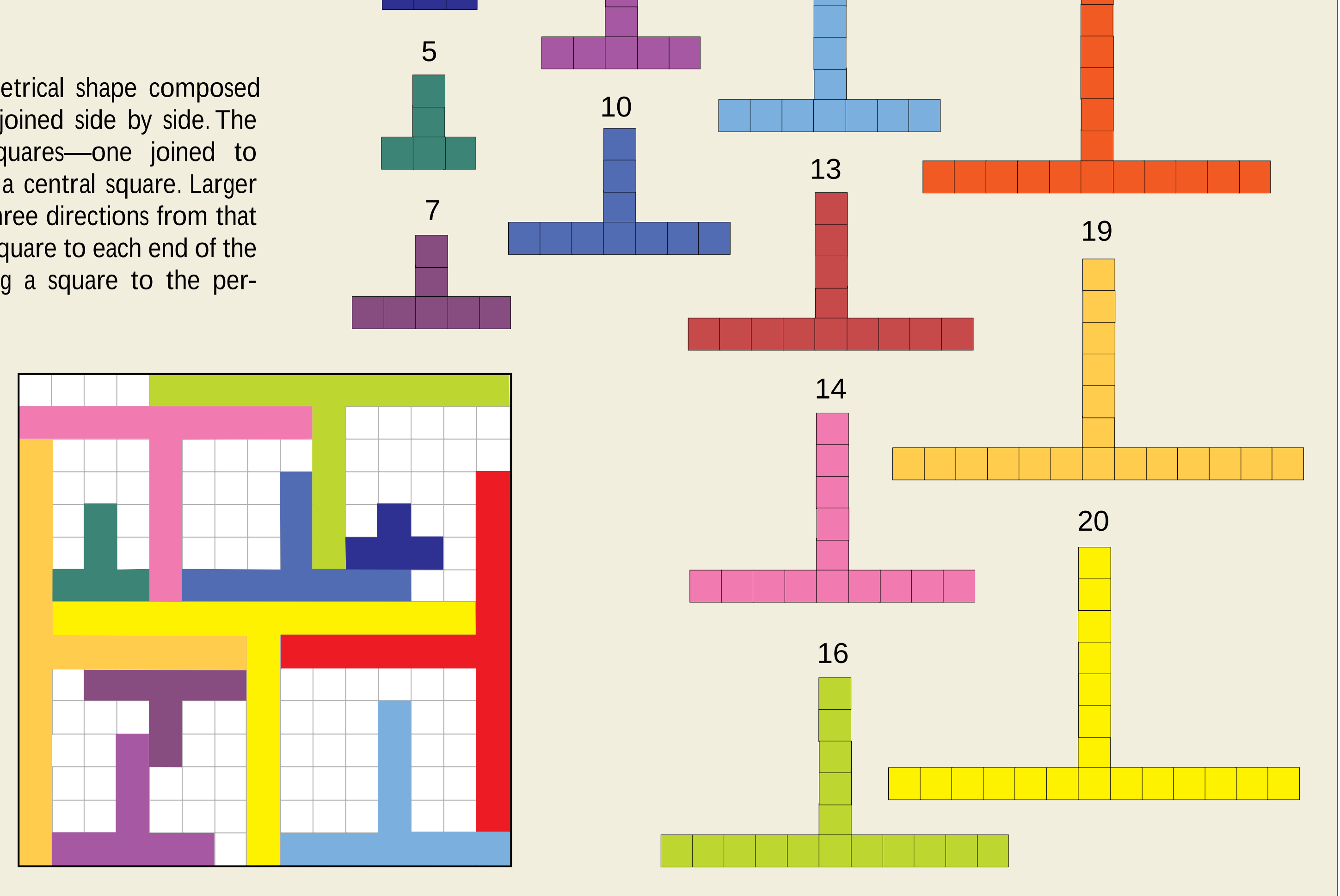




T-TILES

A T-tile is a symmetrical shape composed of unit squares joined side by side. The smallest has four squares—one joined to three of the sides of a central square. Larger ones are built up in three directions from that junction: by adding a square to each end of the crossbar or by adding a square to the perpendicular arm.

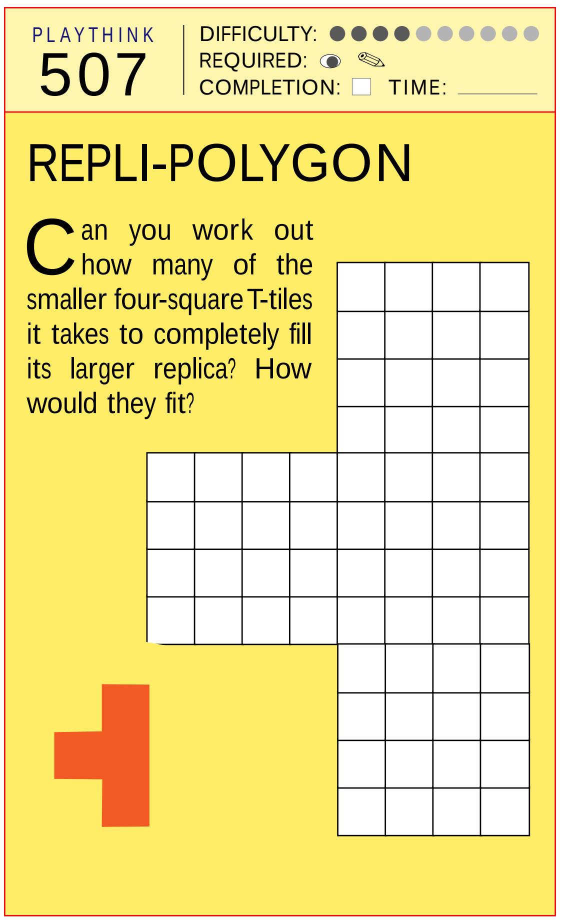
The first twelve nonidentical T-tiles are shown. Can you fit them all onto a fifteen-by-fifteen grid without overlap? A sample attempt that failed to fit the thirteen-square tile is shown here.



PLAYTHINK 507 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

REPLI-POLYGON

Can you work out how many of the smaller four-square T-tiles it takes to completely fill its larger replica? How would they fit?

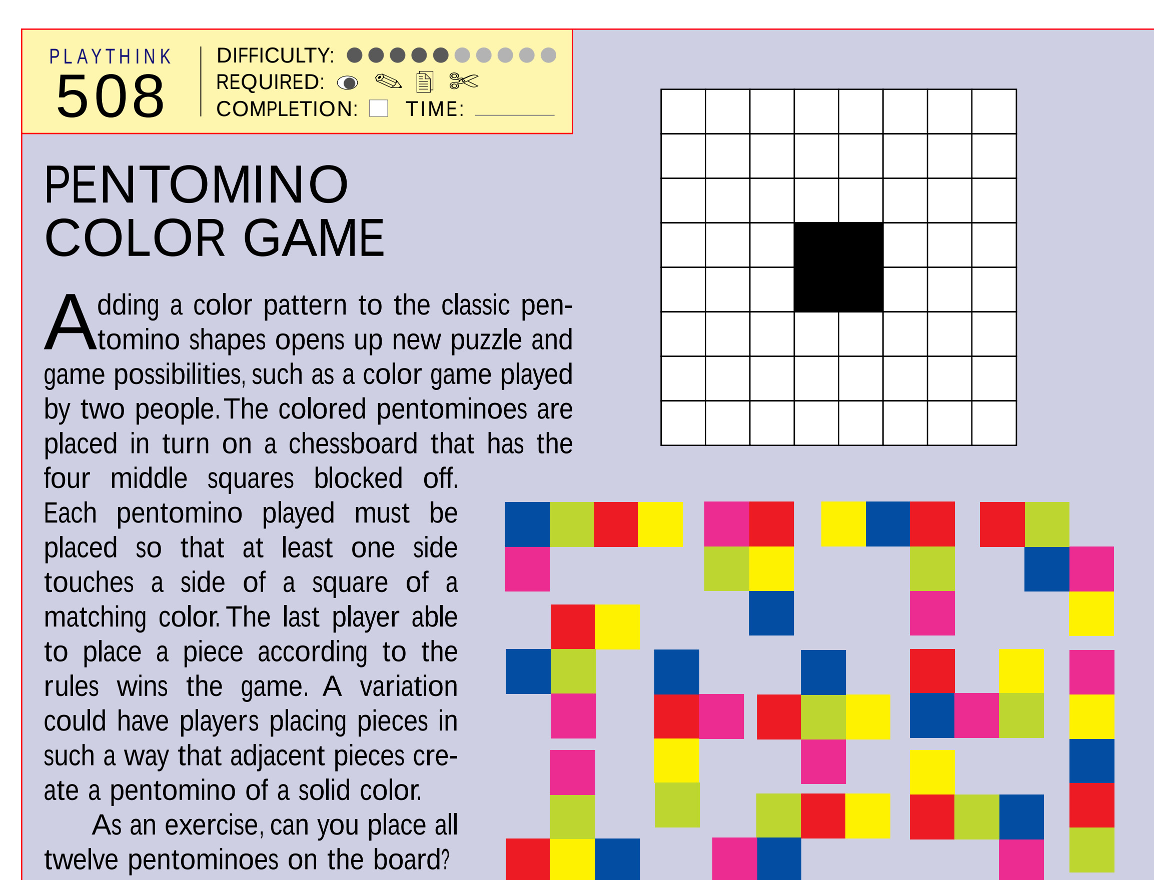


PLAYTHINK 508 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

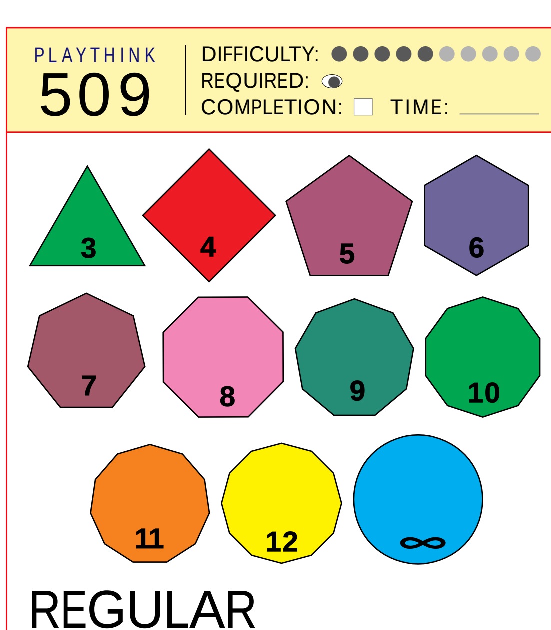
PENTOMINO COLOR GAME

Adding a color pattern to the classic pentomino shapes opens up new puzzle and game possibilities, such as a color game played by two people. The colored pentominoes are placed in turn on a chessboard that has the four middle squares blocked off. Each pentomino played must be placed so that at least one side touches a side of a square of a matching color. The last player able to place a piece according to the rules wins the game. A variation could have players placing pieces in such a way that adjacent pieces create a pentomino of a solid color.

As an exercise, can you place all twelve pentominoes on the board?



PLAYTHINK 509 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____



REGULAR TESSELLATIONS

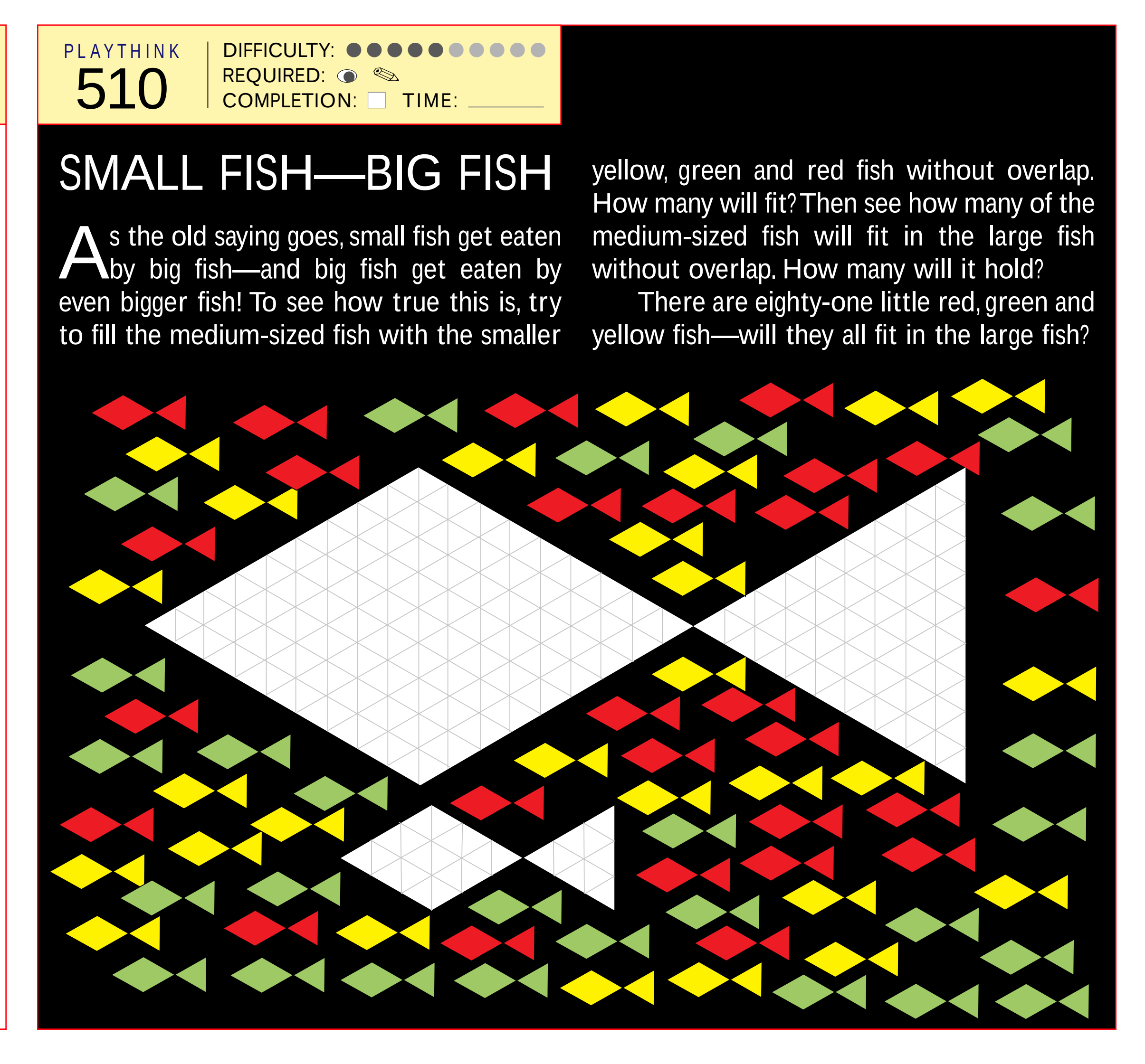
A regular tessellation is a mosaic made up of identical regular polygons that completely fill a plane. There is an infinite number of regular polygons—from the equilateral triangle and the square up to the circle, which may be considered a regular polygon with an infinite number of sides. Can you work out how many of those regular polygons are capable of tessellating a plane?

PLAYTHINK 510 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

SMALL FISH—BIG FISH

As the old saying goes, small fish get eaten by big fish—and big fish get eaten by even bigger fish! To see how true this is, try to fill the medium-sized fish with the smaller yellow, green and red fish without overlap. How many will fit? Then see how many of the medium-sized fish will fit in the large fish without overlap. How many will it hold?

There are eighty-one little red, green and yellow fish—will they all fit in the large fish?




1
2
4
6
?
4
8
?
?
6
?
8
Numbers
9

Numbers and Sequences

Throughout history people have held that numbers, especially certain numbers, possess special powers. Some mystics used the numbers they found in names and words to weave ingenious patterns to explain everything. Others believed numbers helped them conjure spirits, perform witchcraft and predict the end of the world. Even today some people believe that certain telephone numbers are lucky or that the combination 666 is a symbol of great evil.

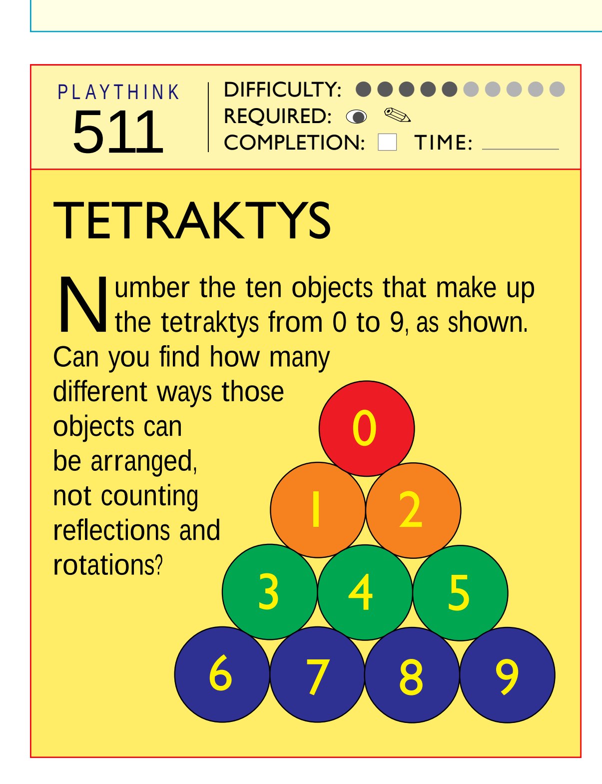
Numbers revealed the patterns of the universe to the ancients. And they still do: the British astronomer Martin Rees titled his masterful book describing the quest for a final theory of physics *Six Numbers*.

Nature is mathematics. Look at spirals and the golden ratio, at fractals and the periodic table of elements. Nature can almost always be described with a simple formula—not because man has invented mathematics to do so but because of some hidden mathematical aspect of nature itself.

Numbers are also symbols—a quick way of writing or talking about objects. Instead of showing a handful of fingers and saying, "I want this many," early humans found it easier to say, "I want five"—especially when they wanted to indicate more things than could be counted on fingers and toes.

Furthermore, like the objects they can be imagined to represent, numbers can also form patterns. Indeed, though numbers are often considered individual entries, they can be presented as a sequence, enabling us to observe the tendencies of a pattern as a whole. Over the centuries number sequences have helped mathematicians and scientists to interpret patterns found in nature, like the famous Fibonacci sequence (see PlayThink 551), a purely mathematical creation that was later found to match the growth of many natural forms.

Similarly, though mathematics was originally thought of as the study of numbers, it is now defined as the science of patterns, whether they are made with numbers, colors, shapes or anything else. The simplest kind of a pattern, a sequence, is just a list of numbers following a certain order; a more advanced pattern, called a series, is the sum of the numbers in a sequence. Recognizing the pattern behind a sequence or a series enables you to predict every other member in the group. But to see the pattern, you must first understand how things are organized.

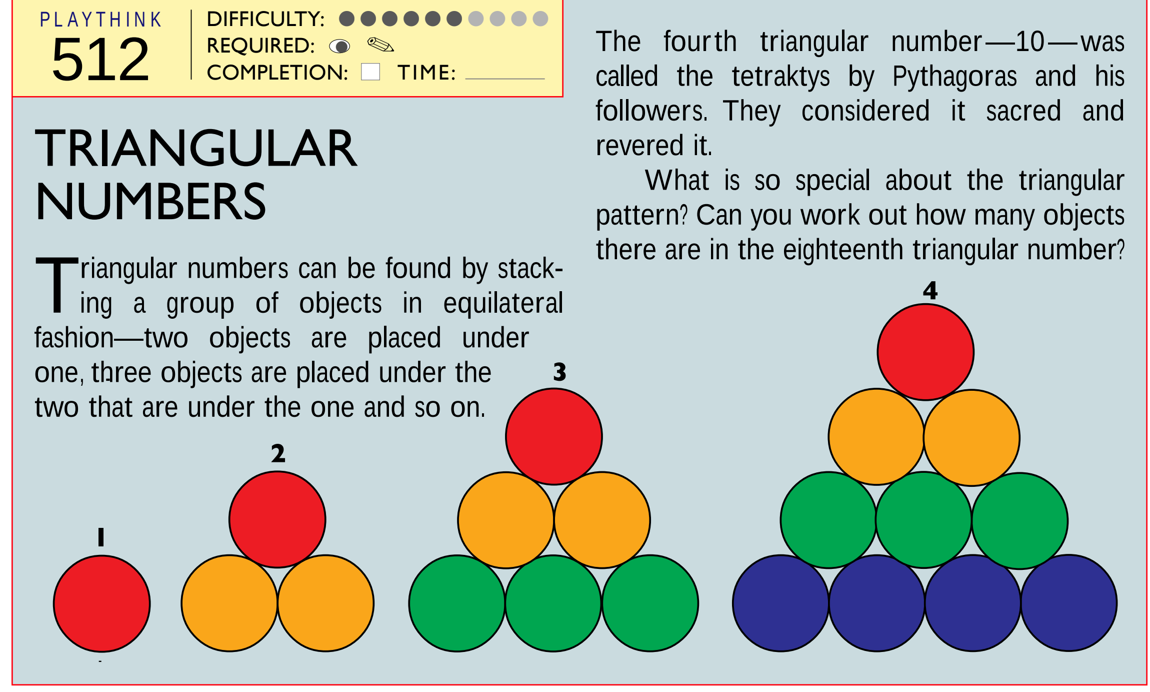


PLAYTHINK 511

DIFFICULTY: ●●●●●○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

TETRAKTYS

Number the ten objects that make up the tetraktys from 0 to 9, as shown. Can you find how many different ways those objects can be arranged, not counting reflections and rotations?



PLAYTHINK 512

DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

TRIANGULAR NUMBERS

Triangular numbers can be found by stacking a group of objects in equilateral fashion—two objects are placed under one, three objects are placed under the two that are under the one and so on.

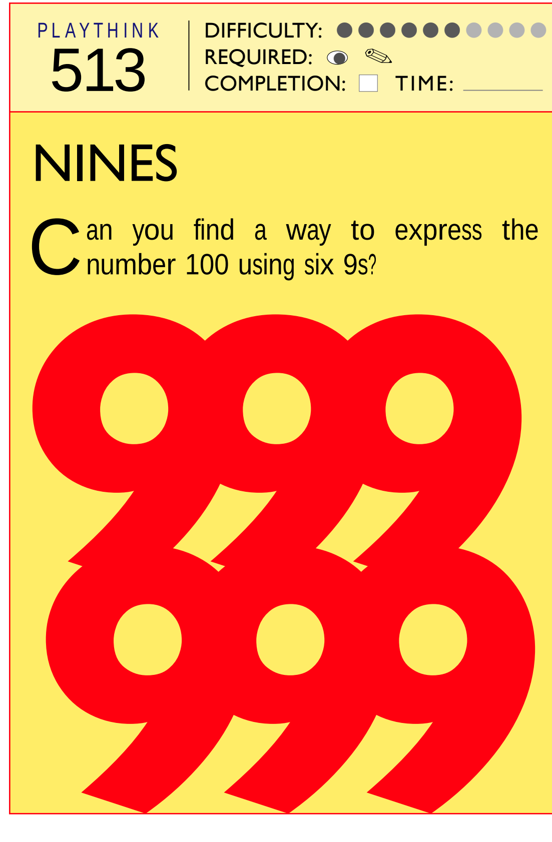
The fourth triangular number—10—was called the tetraktys by Pythagoras and his followers. They considered it sacred and revered it.

What is so special about the triangular pattern? Can you work out how many objects there are in the eighteenth triangular number?

PLAYTHINK 513 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

NINES

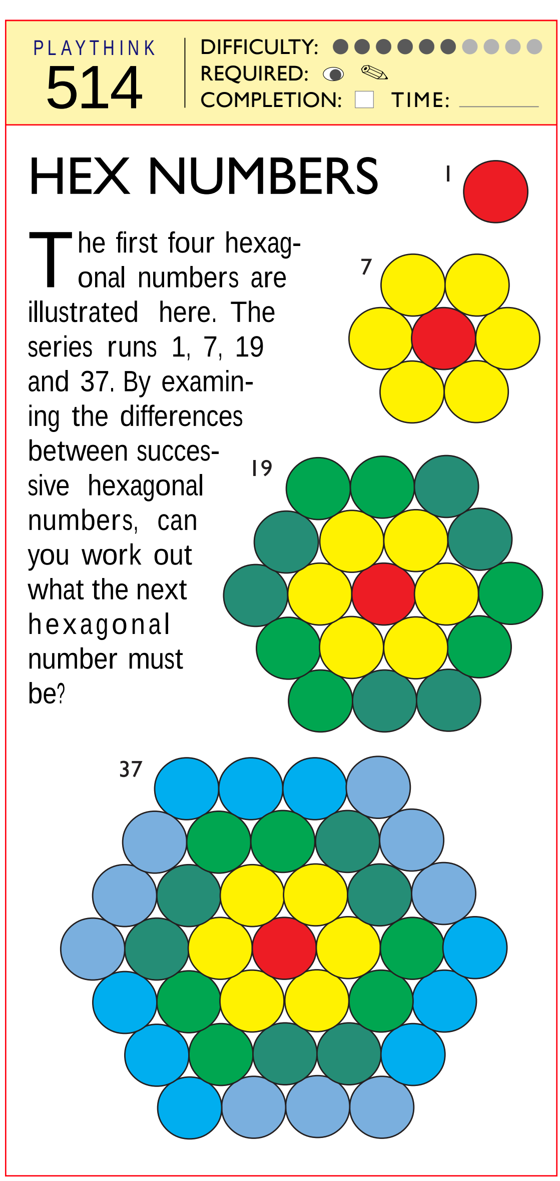
Can you find a way to express the number 100 using six 9s?



PLAYTHINK 514 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

HEX NUMBERS

The first four hexagonal numbers are illustrated here. The series runs 1, 7, 19 and 37. By examining the differences between successive hexagonal numbers, can you work out what the next hexagonal number must be?



PLAYTHINK 515 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

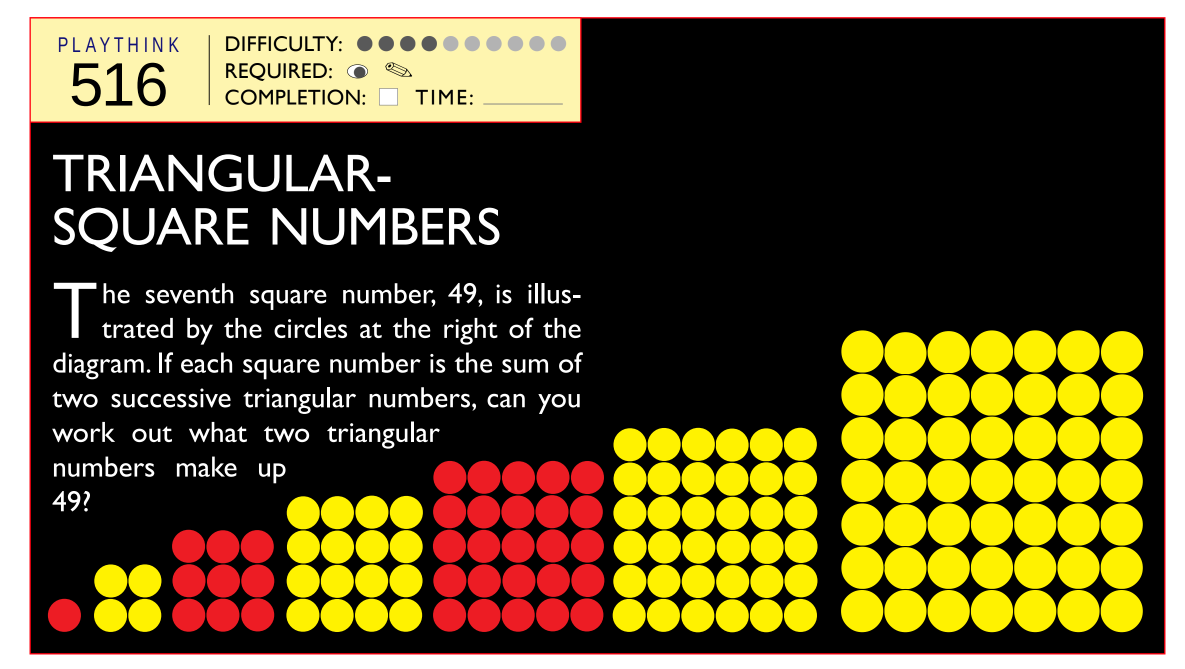
SQUARE NUMBERS

A number that is multiplied by itself is called a square. The first six square numbers are illustrated below in figurate form. Can you continue the sequence by examining the difference in value between successive squares? What is the seventh square?

PLAYTHINK 516 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

TRIANGULAR-SQUARE NUMBERS

The seventh square number, 49, is illustrated by the circles at the right of the diagram. If each square number is the sum of two successive triangular numbers, can you work out what two triangular numbers make up 49?

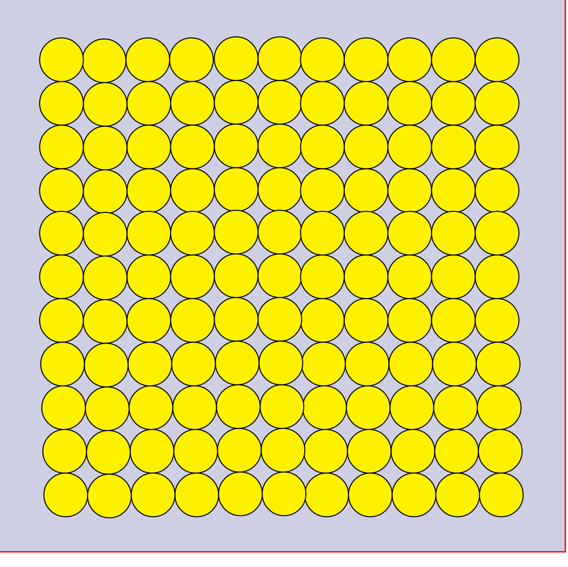


PLAYTHINK 517 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

TRIANGULAR NUMBERS—ODD SQUARES

The study of figurate numbers belongs to a branch of number theory called Diophantine analysis, a field that specializes in finding integral solutions of equations. The following puzzle is derived from that field of study.

The eleventh square number can be depicted as 121 objects in an eleven-by-eleven array. Diophantine analysis has shown that every odd square number is equal to eight times a triangular number, plus one. Can you work out the triangular number that plugs into that equation to make 121?



PLAYTHINK 518 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

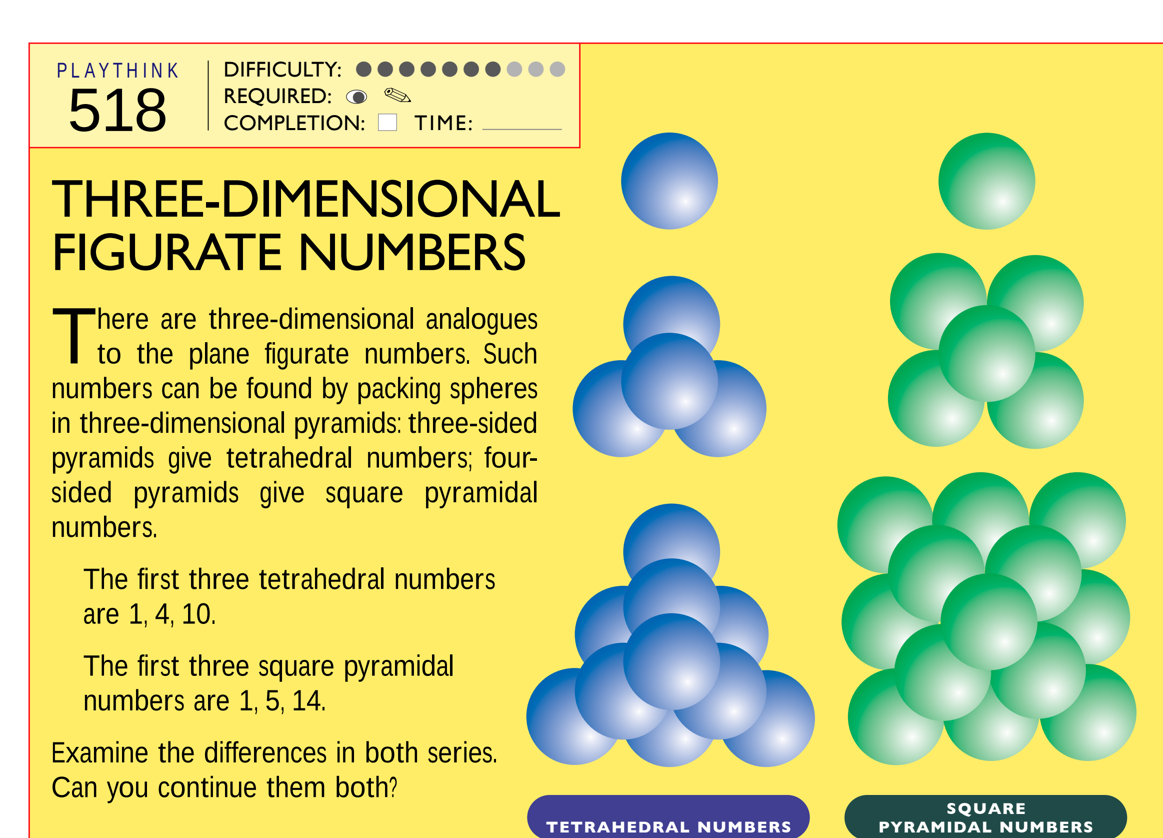
THREE-DIMENSIONAL FIGURATE NUMBERS

There are three-dimensional analogues to the plane figurate numbers. Such numbers can be found by packing spheres in three-dimensional pyramids: three-sided pyramids give tetrahedral numbers; four-sided pyramids give square pyramidal numbers.

The first three tetrahedral numbers are 1, 4, 10.

The first three square pyramidal numbers are 1, 5, 14.

Examine the differences in both series. Can you continue them both?



PLAYTHINK 519 | DIFFICULTY: ●●●○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

COUNTING SHEEP

Without counting, can you tell whether there are more sheep moving to the right or to the left?

PLAYTHINK 520 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

FORTY TOTAL

Consider the numbers from 1 to 40, inclusive. Imagine trying to express each of those numbers as a combination of other numbers that are added or subtracted together—for example, 3 can be 1 + 2 or it can be 4 – 1.

Can you find four numbers that, either singly or combined with some or all of the other three numbers, can express every number from 1 to 40? In each combination, however, any given number can appear only once—for example, 5 + 5 is not allowed. To check your answer, fill in the table below with the various combinations.

	=	1		=	21
	=	2		=	22
	=	3		=	23
	=	4		=	24
	=	5		=	25
	=	6		=	26
	=	7		=	27
	=	8		=	28
	=	9		=	29
	=	10		=	30
	=	11		=	31
	=	12		=	32
	=	13		=	33
	=	14		=	34
	=	15		=	35
	=	16		=	36
	=	17		=	37
	=	18		=	38
	=	19		=	39
	=	20		=	40

PLAYTHINK 521

DIFFICULTY: ●●●●●●●○○○

REQUIRED: 👁

COMPLETION: ☐ TIME: ______

COUNTING GAUSS

When Carl Friedrich Gauss was six years old (back in 1783), his schoolteacher asked the students to add up all the numbers from 1 to 100.

Unfortunately for the teacher, who was hoping to keep the class occupied, it took young Gauss only a few seconds to work out the answer. He had spotted a pattern in the sequence and could provide the answer via a simple operation that he performed in his head. Of course, with a mind like that, it didn't take very long for Gauss to become one of Germany's most celebrated mathematicians and scientists.

Can you figure out what Gauss did to come up with the answer?



PLAYTHINK 522

DIFFICULTY: ●●●●○○○○○○

REQUIRED: 👁 ✎

COMPLETION: ☐ TIME: ______

SUM FIFTEEN

In this game each player selects a color (red or green) and then takes turns coloring one number at a time with it. The player who is first to color in three numbers that add up to exactly 15 wins.

Can you work out the best strategy for this game?

1 2 3 4 5 6 7 8 9

PLAYTHINK 523

DIFFICULTY: ●●●○○○○○○○

REQUIRED: 👁 ✎

COMPLETION: ☐ TIME: ______

LAGRANGE'S THEOREM

A famous theory of numbers states that every whole number can be expressed as the sum of, at most, four squares. This can be demonstrated graphically: Examine these two rectangles, one with 12 square units and one with 15 square units. Can you show how those rectangles are each composed of four smaller squares?

12

15

PLAYTHINK 524

DIFFICULTY: ●●●●●○○○○○

REQUIRED: 👁 ✎

COMPLETION: ☐ TIME: ______

IRRATIONAL

The ancient Greeks believed that any length or area could be expressed as the fraction of two whole numbers. Even a number as unusual as 1.000390625 could be written simply as the fraction 2,561/2,560. Such fractions are called rational numbers.

Pythagoras and his followers were preoccupied with right triangles, and their deep study led them to attempt to measure the hypotenuse of the simplest right triangle of them all: one in which both legs are of equal length. That research, however, resulted in an unexpected and disturbing answer.

Can you determine the length of a right triangle in which both legs are 1 unit long? Was it possible for the Pythagoreans to measure this length exactly?

?

1

1

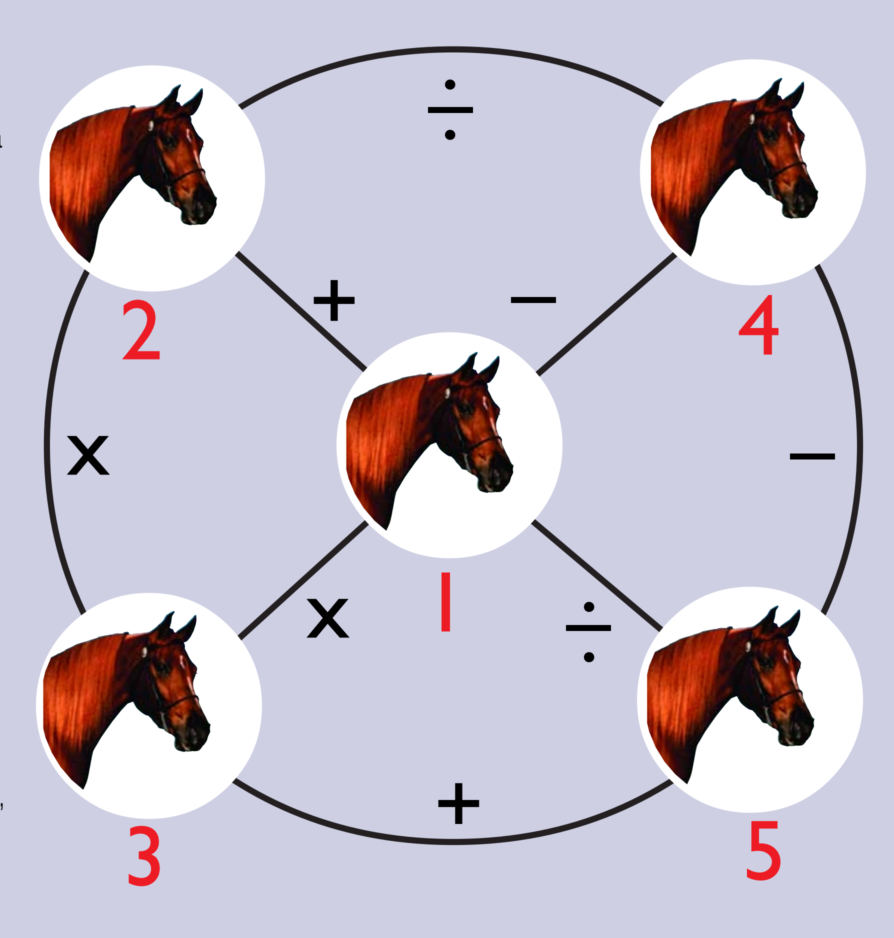
PLAYTHINK 525

DIFFICULTY: ●●●○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

HORSE COUNT

Each horse has a numerical value from 1 to 5, and most of the pairs of horses are connected by a line accompanied by an arithmetic operator: +, –, x, or ÷.

Can you connect the horses in such a way that the mathematical operation of its path provides the maximum total? One possibility, 2 x 3 + 5 ÷ 1 – 4, gives a total of 7, which is not the maximum.



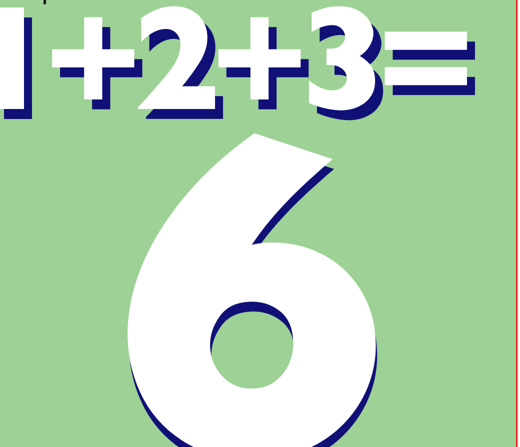
PLAYTHINK 528

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

PERFECT NUMBERS

A perfect number is the sum of all the factors that divide evenly into it—including 1 but excluding the number itself. The first perfect number is 6, which is divisible by 3, 2 and 1 and is the sum of 1, 2 and 3.

So far, thirty-eight perfect numbers have been found. Can you work out what the second perfect number is?



PLAYTHINK 526

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

ODD SUM

Can you find five odd numbers that add up to 100?

What about six odd numbers that add up to 100?

	?
?	?
+ ?	?
+ ?	?
+ ?	?
+ ?	?
100	100

PLAYTHINK 527

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

APPLE PICKERS

If five apple pickers can pick five apples in five seconds, how many apple pickers would it take to pick sixty apples a minute?



PLAYTHINK 529

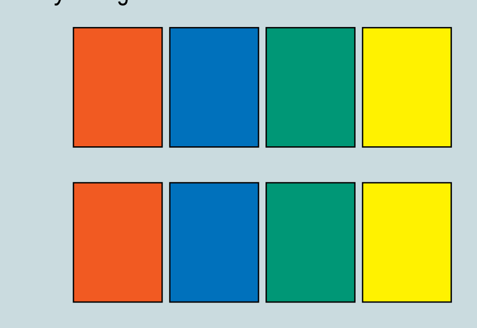
DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

STACKING ORDER

You are asked to stack these eight blocks according to four simple rules:

1. Just one block must lie between the two red blocks.
2. Two blocks must lie between the pair of blue blocks.
3. Three blocks must separate the pair of green blocks.
4. Four blocks must separate the pair of yellow blocks.

Can you figure out how to do it?



PLAYTHINK 530

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 📄 ✂
COMPLETION: ☐ TIME: ______

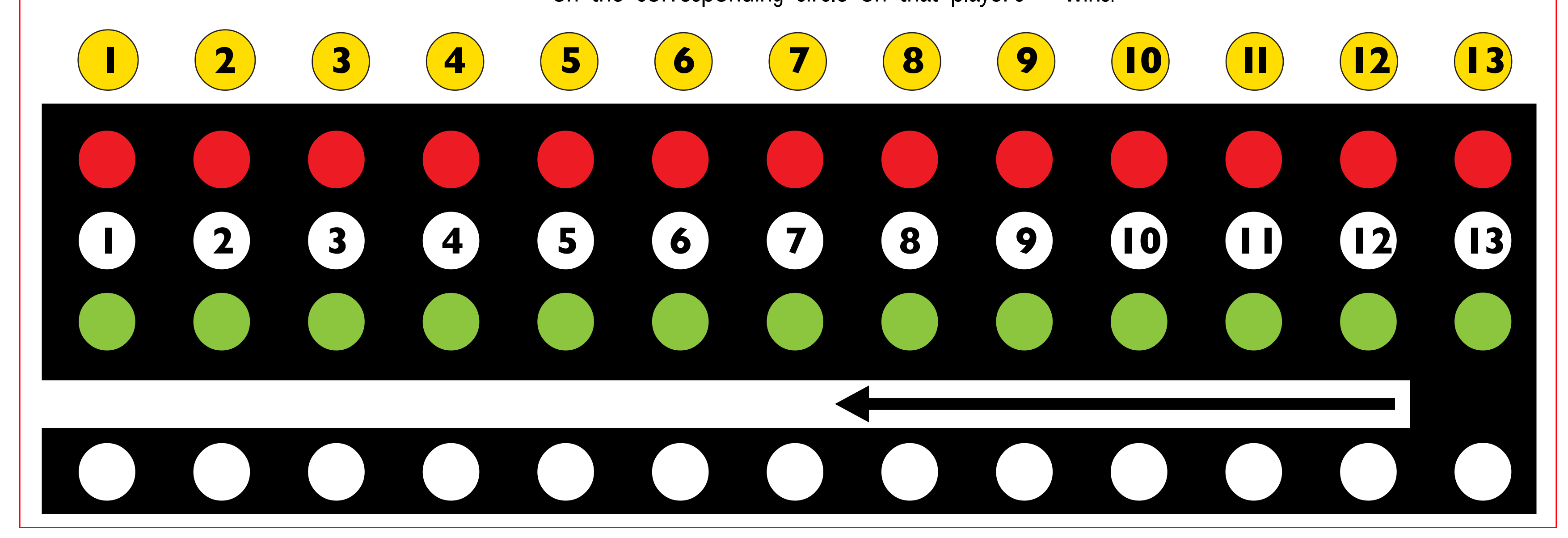
NUMBER STRIP

■ *A Two-Player Memory Game*

Here's a simple game to test your memory for numbers.

Distribute the thirteen tiles face down in random order on the upper part of the game board. The game begins with the search for the number 1 and continues consecutively. Players choose to be red or green, and then take turns picking up one tile at a time. If the number on the tile matches the number being searched for (that is, 1 followed by 2, followed by 3, etc.), the tile is turned over and placed on the corresponding circle on that player's side of the strip. (In other words, place the tile on the small red circle if you're the red player or on the small green circle if you're the green player.) If the selected tile is not a match, then the tile is placed face down at the first blank space on the bottom row, starting at the right.

Each time a player makes a match, he or she gets to take another turn, picking tiles from either the top or bottom row—always searching for the next consecutive number. The player who matches the most numbers wins.



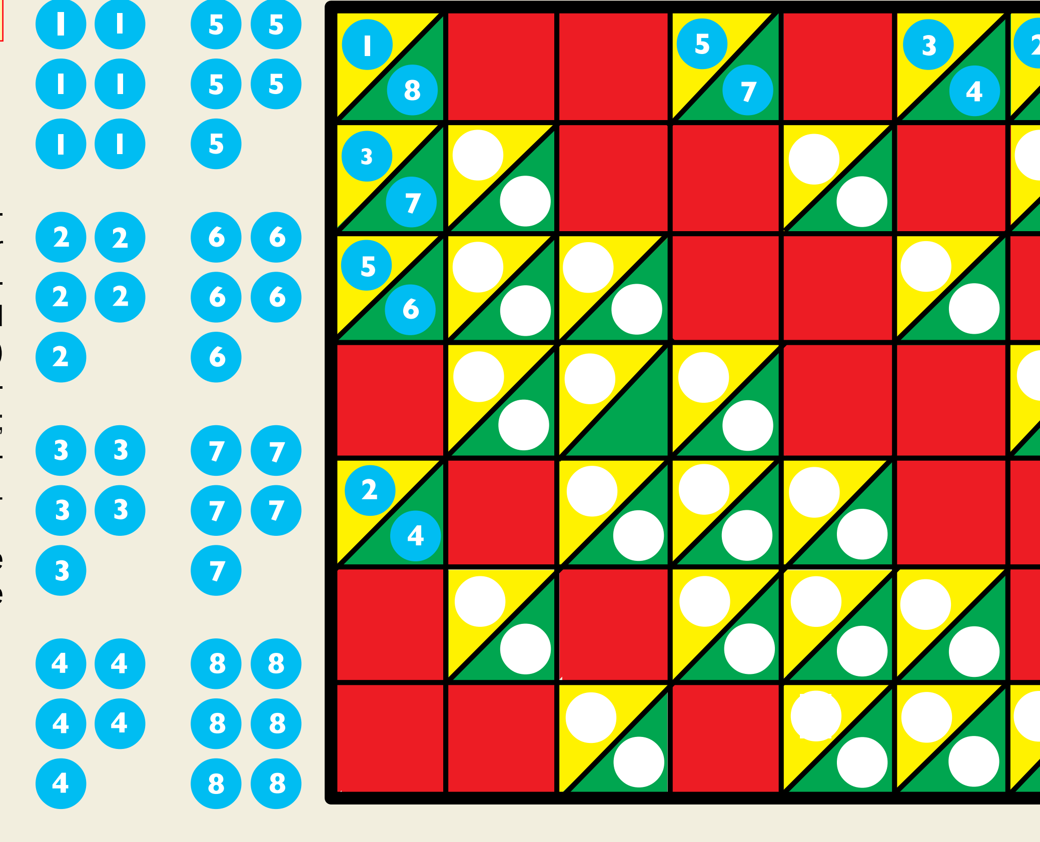
PLAYTHINK 531

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

PAIRING FIELDS GAME

The numbers from 1 to 8 must be distributed on the grid. Each number can appear only once in each column and row, and numbers can be entered only in the yellow and green cells. (The red cells must remain blank.) One additional rule: Each specific pair of numbers can appear only once on the grid; because the 1-8 pair was used in the top left-hand corner, neither the 1-8 pair nor the 8-1 pair can be used again.

The top row and left-hand column have been filled in for you. Can you complete the grid?



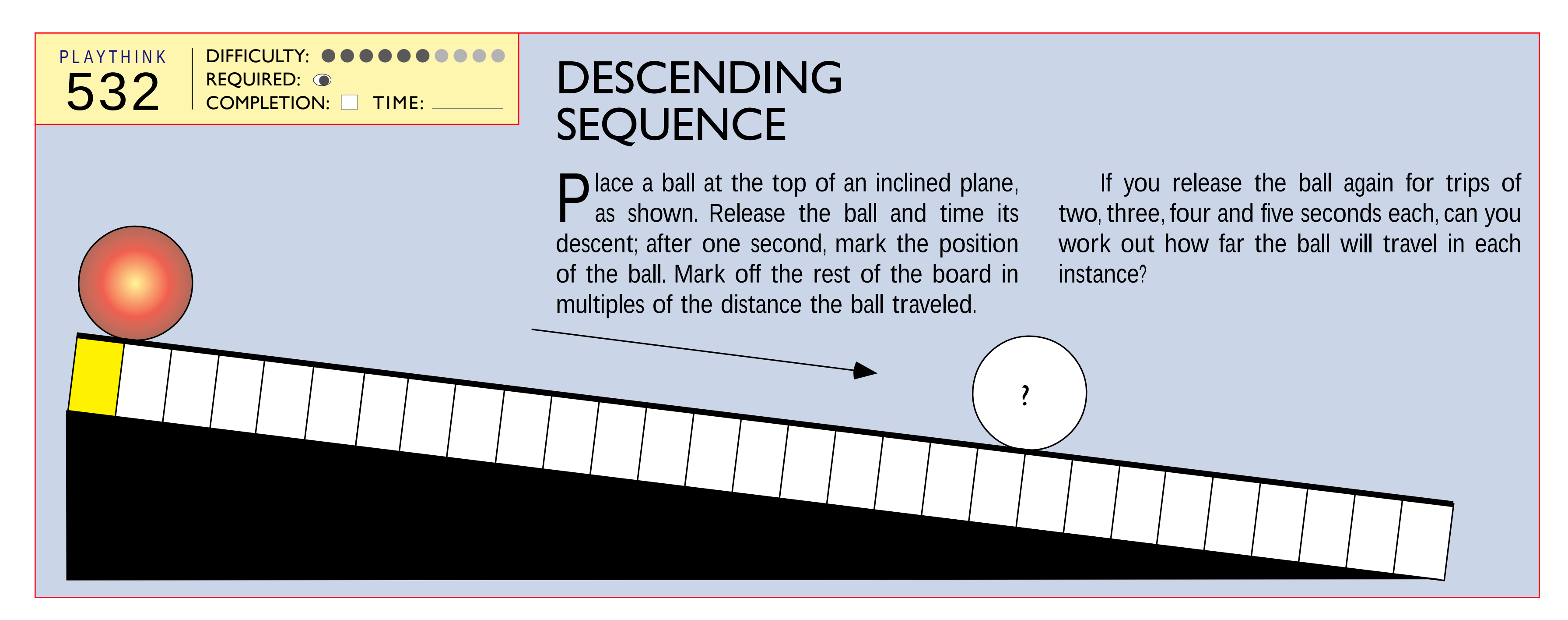
PLAYTHINK 532

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

DESCENDING SEQUENCE

Place a ball at the top of an inclined plane, as shown. Release the ball and time its descent; after one second, mark the position of the ball. Mark off the rest of the board in multiples of the distance the ball traveled.

If you release the ball again for trips of two, three, four and five seconds each, can you work out how far the ball will travel in each instance?



Perfect Numbers

The Pythagoreans were obsessed with holding numbers to moral standards; to them a perfect number was the sum of all the smaller numbers that divided into it exactly, including 1 but excluding the number itself. The first perfect number was easy to find: the factors of 6 (excluding 6 itself) are 1, 2 and 3, which add up to 6.

Very few numbers have that characteristic. The factors for the number 12, for example, are 1, 2, 3, 4, 6 and 12. The sum of those numbers, excluding 12, is 16, so 12 is not a perfect number. In fact, the ancient Greeks discovered only the first four of those rare numbers: 6, 28, 496 and 8,128.

Over a millennium passed before the fifth perfect number—33,550,336—was discovered in 1460. Euler found another perfect number, one with nineteen digits, in 1782. The irony is that we know a great many perfect numbers today because of a formula discovered by Euclid.

In his book *Elements*, Euclid proved that if $2^n - 1$ is a prime number, then $2^{n-1}(2^n - 1)$ is a perfect number. Euclid's formula gives only even perfect numbers, and it is uncertain that any odd perfect numbers exist. Thus far none have been found up to 10^{200}.

Other curious properties of perfect numbers abound. All perfect numbers, for instance, are triangular. And the end digits of perfect numbers present a tantalizing mystery because every known perfect number ends in either 6 or 28, preceded by an odd number. The series of terminal digits for the twenty-three known perfect numbers is 6–8–6–8–6–6–8–8–6–6–8–8–6–8–8–8–6–6–6–8–6–6–6. The sequence contains infuriating hints of order. For example, if we partition the series into triplets, starting at the left, no triplet contains three of a kind. Are the digits trying to tell us something, or is this simply a coincidence waiting to be debunked?

Numbers whose divisors add up to less than themselves are called deficient; those numbers whose divisors add up to more than themselves are called abundant. The smallest abundant number is 12.

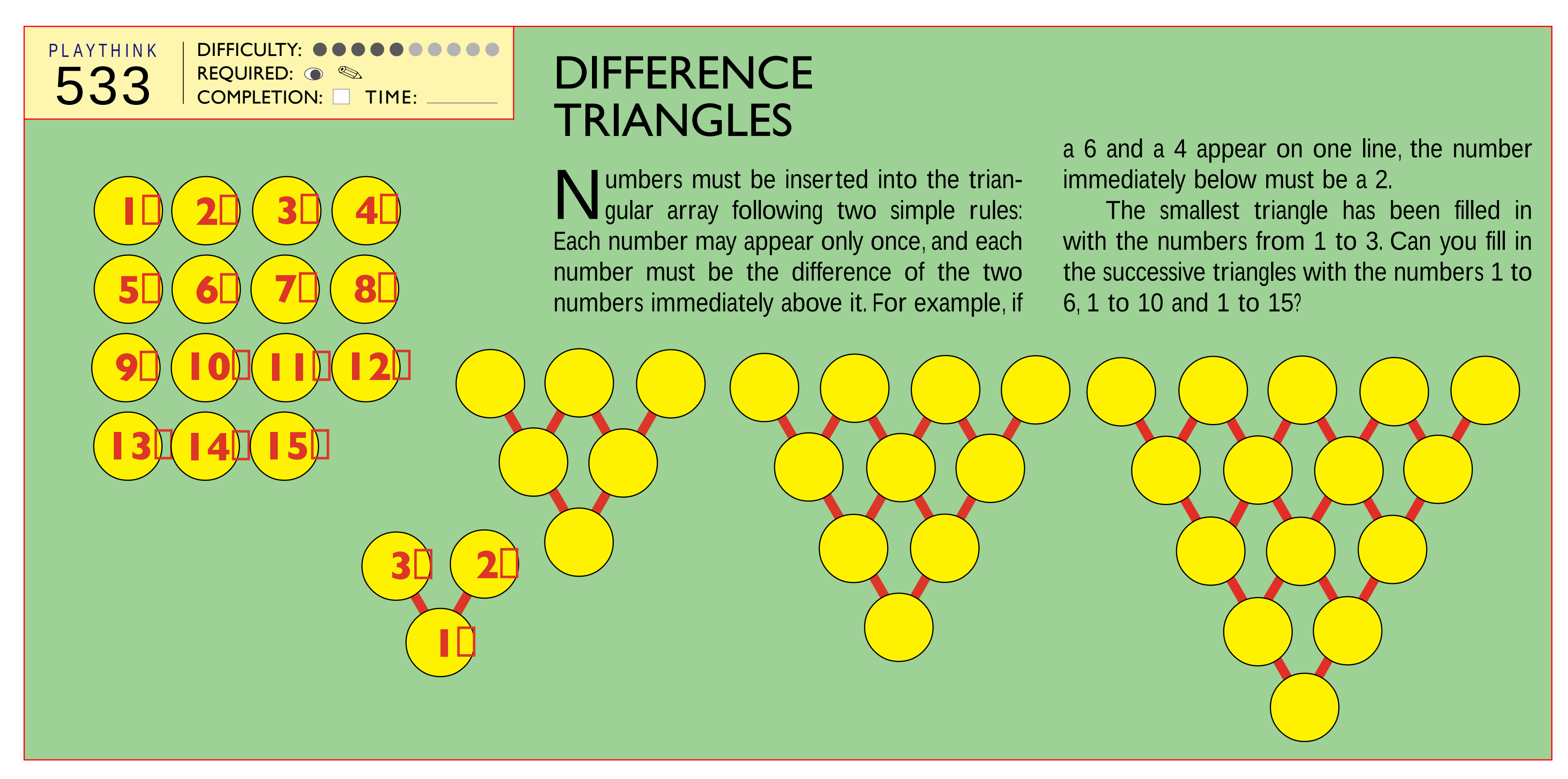
Numerologists have attributed special significance to perfect numbers; students of the Bible have noted that the first two perfect numbers are embedded in the structure of the universe. After all, God created the universe in six days, and the moon circles the earth every twenty-eight days.

PLAYTHINK 533 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

DIFFERENCE TRIANGLES

Numbers must be inserted into the triangular array following two simple rules: Each number may appear only once, and each number must be the difference of the two numbers immediately above it. For example, if a 6 and a 4 appear on one line, the number immediately below must be a 2.

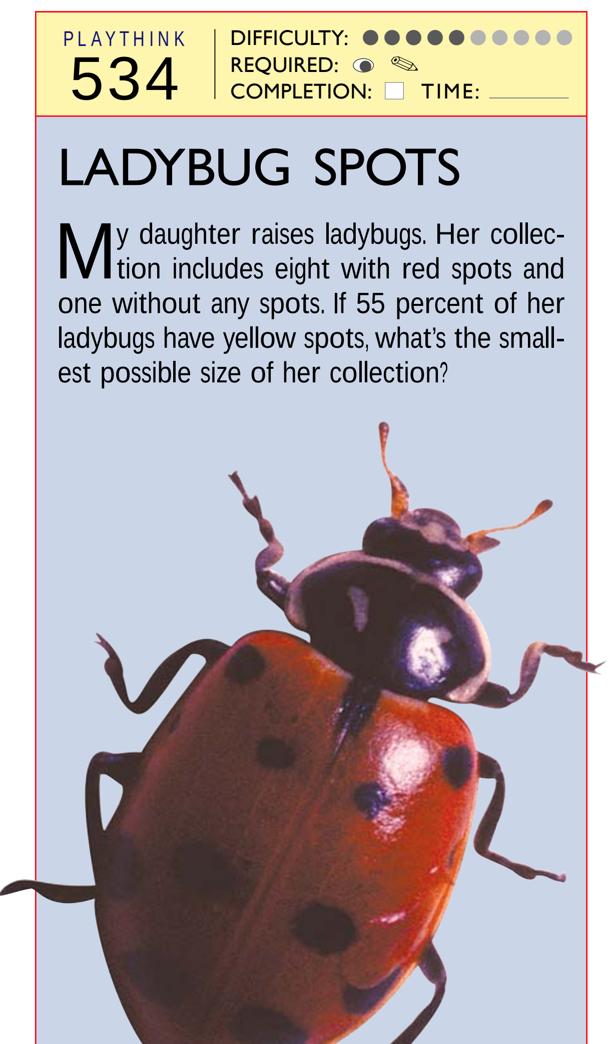
The smallest triangle has been filled in with the numbers from 1 to 3. Can you fill in the successive triangles with the numbers 1 to 6, 1 to 10 and 1 to 15?



PLAYTHINK 534 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

LADYBUG SPOTS

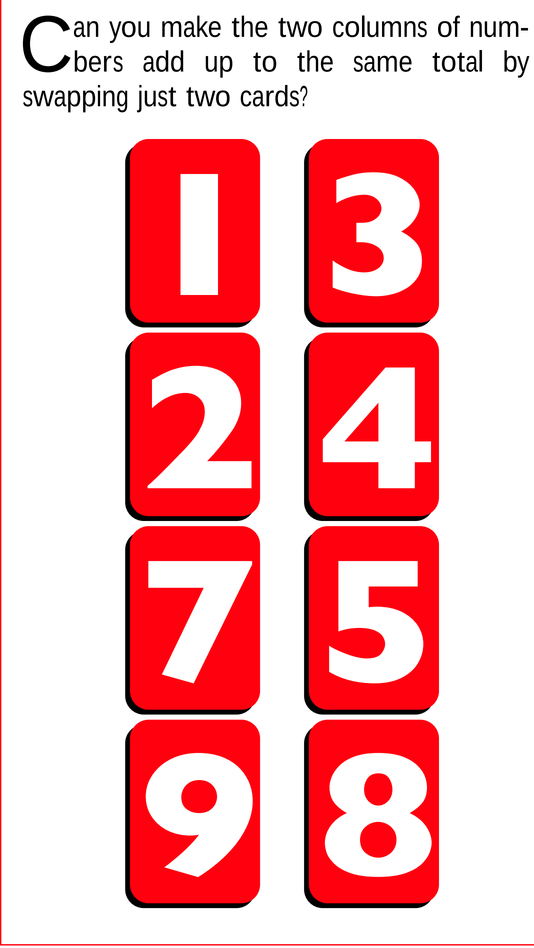
My daughter raises ladybugs. Her collection includes eight with red spots and one without any spots. If 55 percent of her ladybugs have yellow spots, what's the smallest possible size of her collection?



PLAYTHINK 535 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

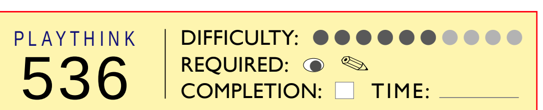
EIGHT CARDS

Can you make the two columns of numbers add up to the same total by swapping just two cards?

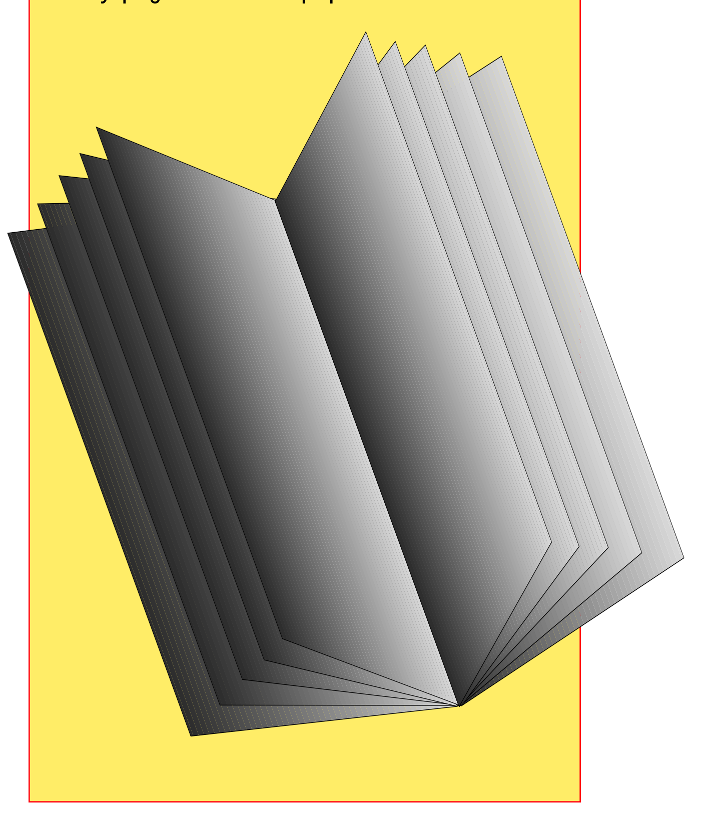


PLAYTHINK 536 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

PAGE NUMBERS



You pull out a page from a newspaper and find that pages 8 and 21 are on the same sheet. From that, can you tell how many pages the newspaper has?



Number Cards

Number cards are a bit like families: every member is unique, yet each one has some feature that is strongly reminiscent of another. In every set of number cards, every number appears twice and no pair of numbers appears together more than once.

The simplest number card set has three cards, each with two numbers. The numbers are distributed 1-2, 1-3 and 2-3. Although each number appears only twice, every card possesses exactly one number in common with any other card. In a set of four number cards, then, each card has three numbers, so that the numbers on one card are distributed one each to the other three.

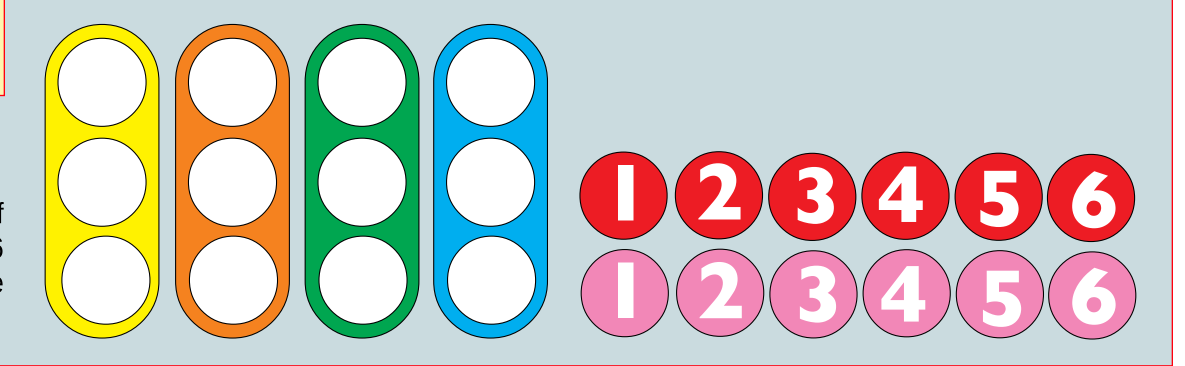
Examine the numbers used in the four-, five- and six-card sets found in the following puzzles. Can you see why it takes forty-two numbers to make a seven-card set?

PLAYTHINK 537

DIFFICULTY: ●●○○○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

NUMBER CARDS 1

Can you fill in the three blanks on each of the four cards with numbers from 1 to 6 so that any given pair of cards has exactly one number in common?

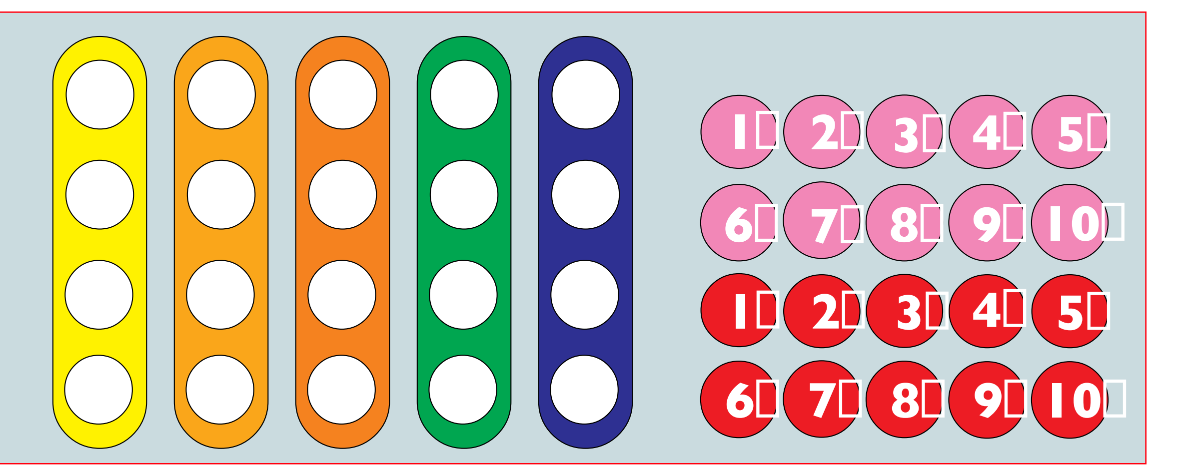


PLAYTHINK 538

DIFFICULTY: ●●●○○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

NUMBER CARDS 2

Can you fill in each of the four blanks on all five cards with a number from 1 to 10 in such a way that every number appears only twice and every pair of cards has exactly one number in common?

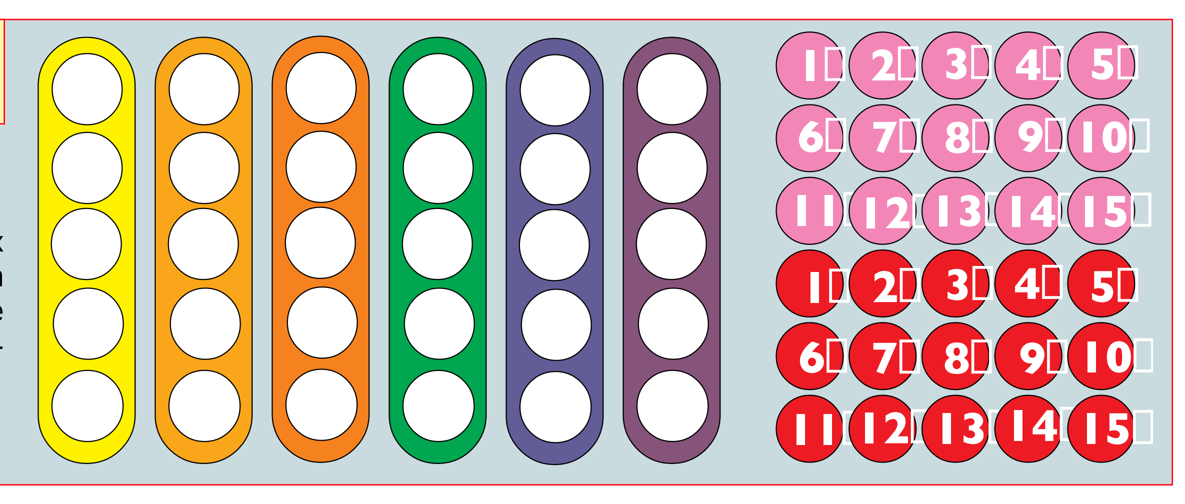


PLAYTHINK 539

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

NUMBER CARDS 3

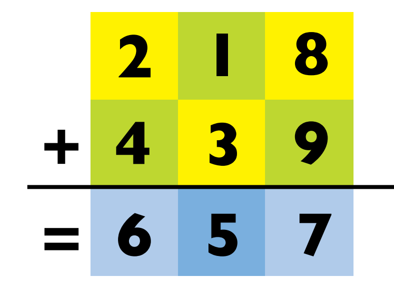
Can you fill in each of the spaces on the six cards with a number from 1 to 15 in such a way that each number appears only twice and every pair of cards has exactly one number in common?



PLAYTHINK 540 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

SUM SQUARES

The first nine digits are arranged in a square, as shown below, so that the number formed on the first line can be added to the number on the second line to make the number on the third line. Can you make another square that adds up in that way?



PLAYTHINK 541 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

TEN-DIGIT NUMBERS

How many different ten-digit numbers can be written with the digits 0 to 9? (Starting a number with 0 is not allowed.)

1,234,567,890

PLAYTHINK 543 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

PERSISTENCE OF NUMBERS

One property of numbers is their persistence. Take the number 723 as an illustration: if you multiply the digits 7, 2 and 3 together, the product is 42; multiply 4 and 2 together to get 8. Because this operation takes two steps to reach a single-digit number, the persistence of 723 is 2.

What is the smallest number of persistence? What are the smallest numbers that lead to persistences of 2, 3 and 4?

723

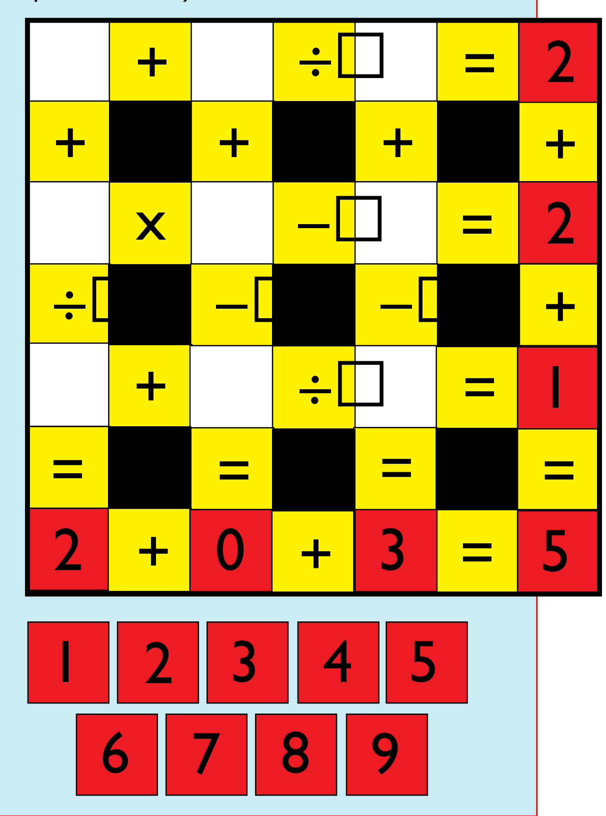
7x2x3=42

4x2=8

PLAYTHINK 544 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

ARITHMAGIC SQUARE

Can you fill in the blanks with the numbers from 1 to 9 so that each mathematical equation is correct? (The operations read from left to right and from top to bottom.)



PLAYTHINK 542 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

FRIEZE NUMBER PATTERN

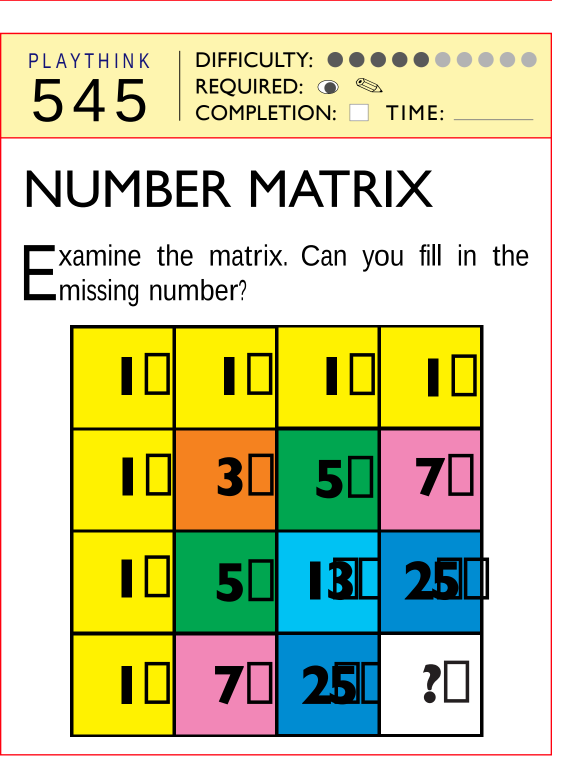
Look closely at the number pattern. Can you discover the simple rule that created the pattern? What number should fill the space outlined in red?

0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 1 1 1 1 1 1 1 1 1 1
1 2 2 3 1 2 4 1 2 2 3 1
3 1 3 5 2 1 7 3 1 3 5 2 1
2 1 7 3 1 3 [ ] 2 1 7 3 1
3 1 2 4 1 2 2 3 1 2 4 1 2
1 1 1 1 1 1 1 1 1 1 1 1
0 0 0 0 0 0 0 0 0 0 0 0 0

PLAYTHINK 545 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

NUMBER MATRIX

Examine the matrix. Can you fill in the missing number?



PLAYTHINK 546
DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

NUMBER 4 MAGIC

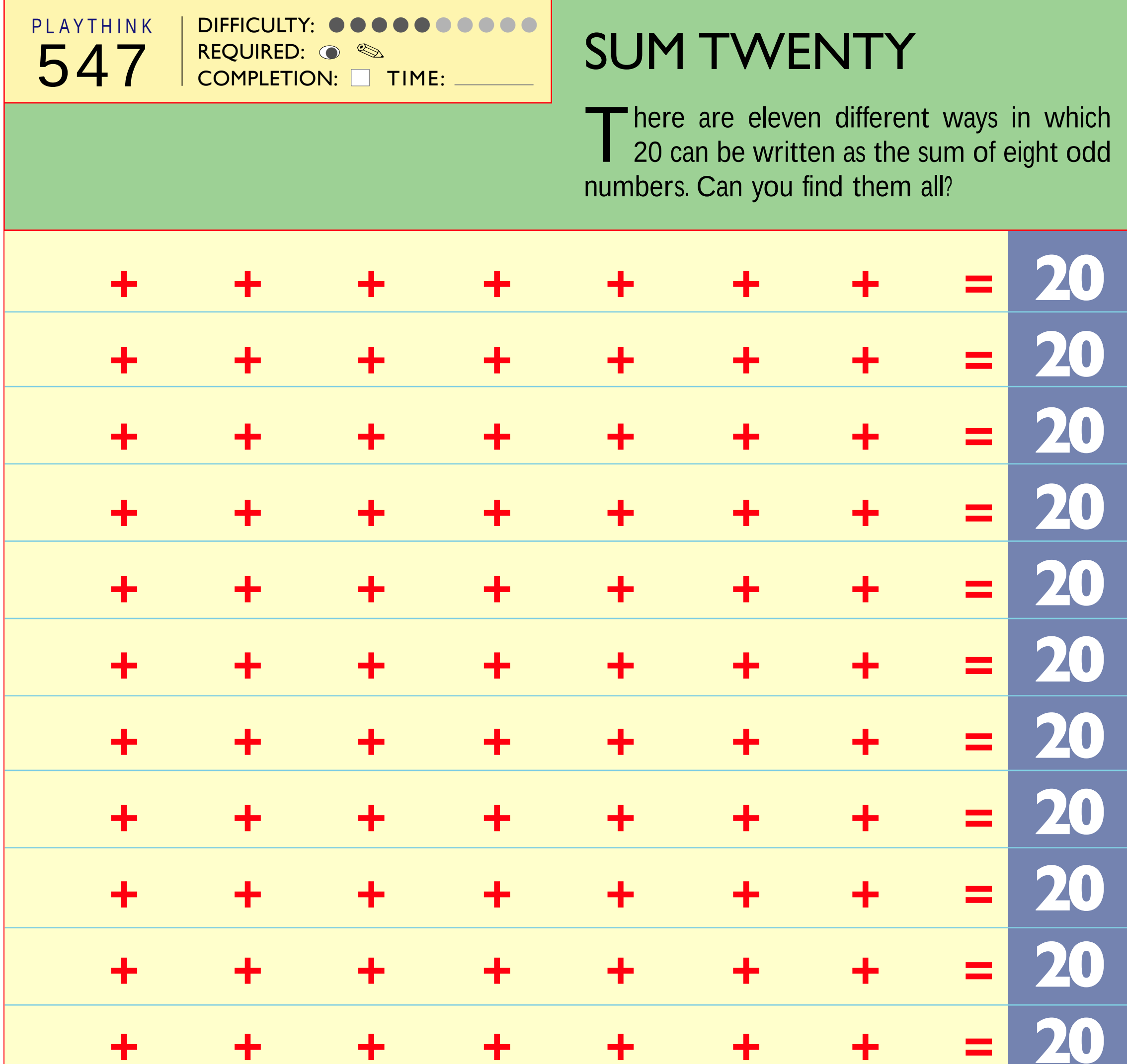
This problem is more than 100 years old and has been revived in many different variations.

Can you express each number from 0 to 10 using only combinations of the number 4? You are allowed to use any of the basic mathematical operations (addition, subtraction, multiplication, division and grouping with parentheses), and you may employ as many fours as needed. But try to find the most compact expression for each number.

PLAYTHINK 547
DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

SUM TWENTY

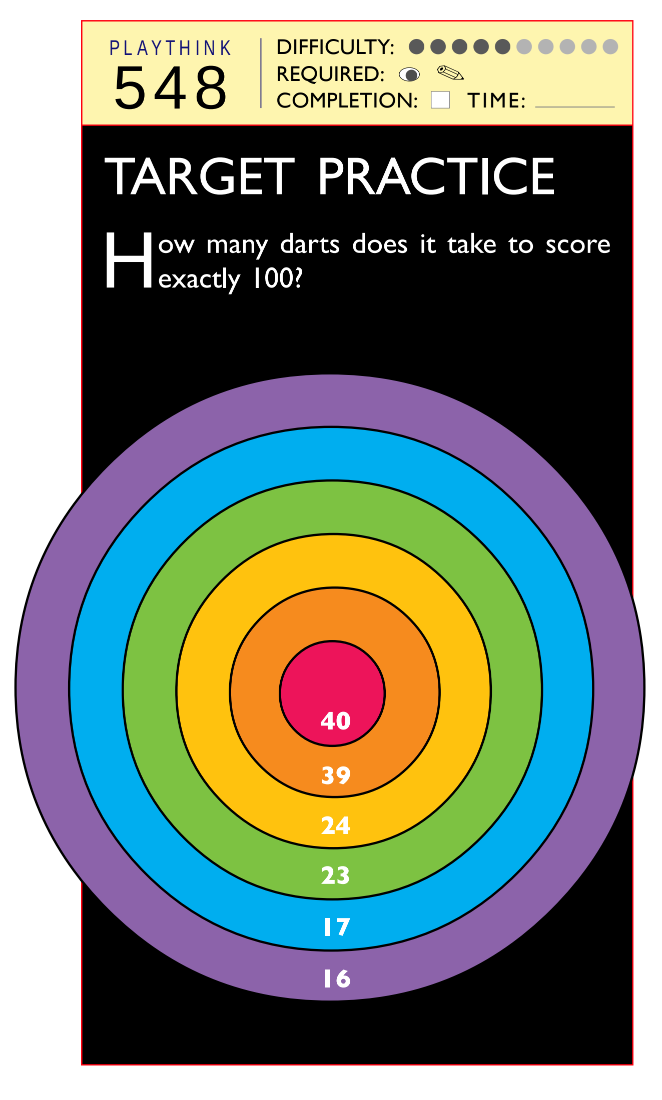
There are eleven different ways in which 20 can be written as the sum of eight odd numbers. Can you find them all?



PLAYTHINK 548
DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

TARGET PRACTICE

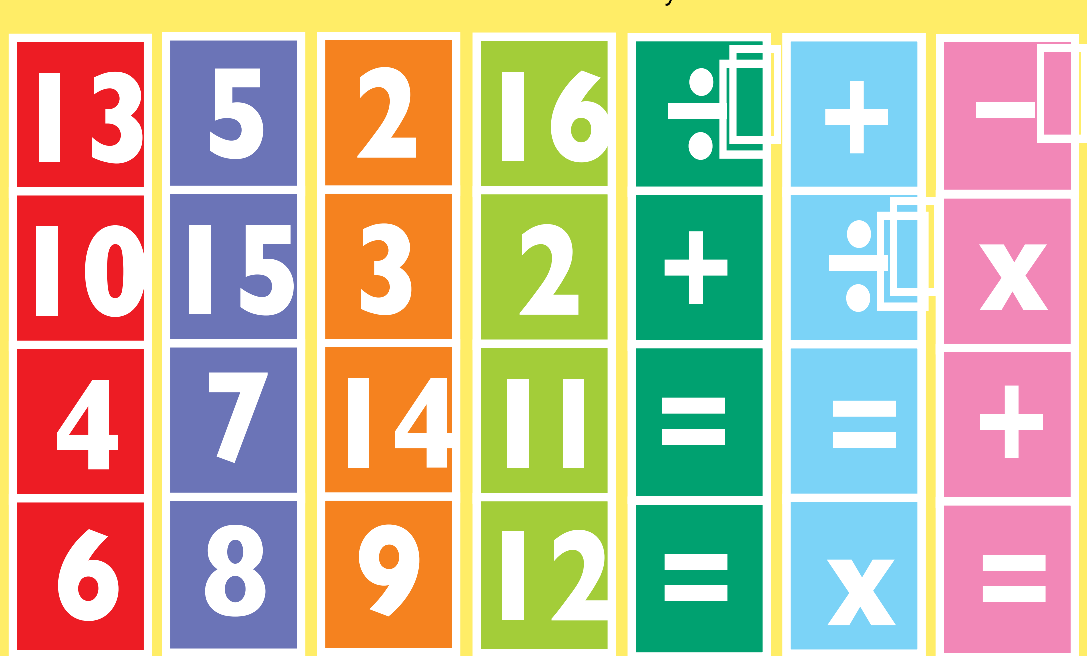
How many darts does it take to score exactly 100?

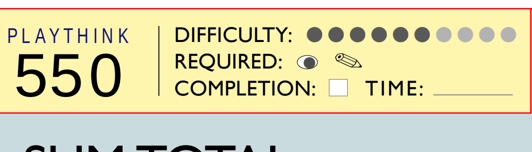


PLAYTHINK 549
DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

NUMBER STRIPS

Can you rearrange the order of the seven strips so that each row contains a correct mathematical statement? Note that the strips containing operators can be inverted if necessary.



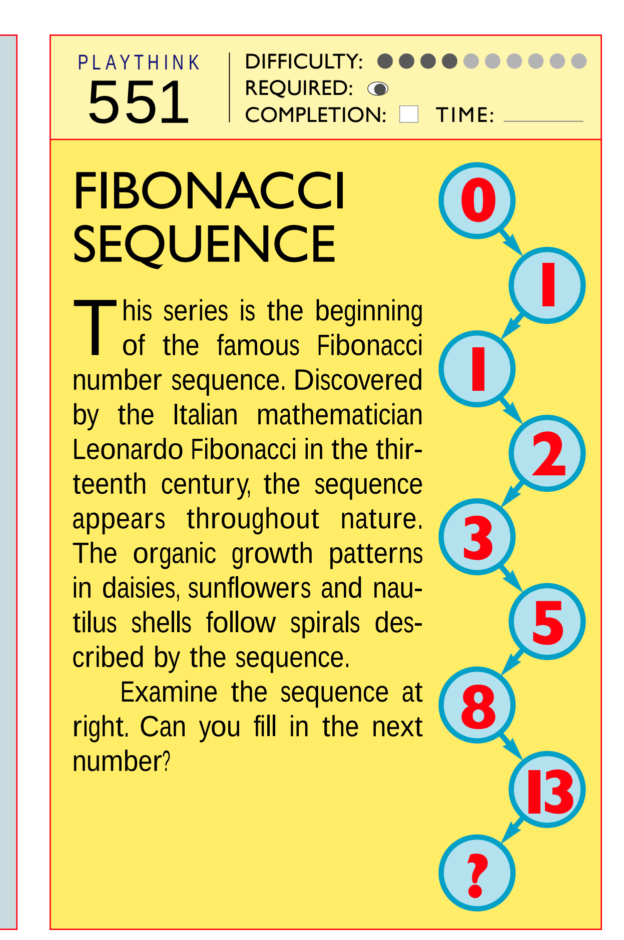


SUM TOTAL

Both sums below are composed of the same number of digits from 1 to 9. Can you tell which sum is greater?

	9	8	7	6	5	4	3	2	1
+		8	7	6	5	4	3	2	1
+			7	6	5	4	3	2	1
+				6	5	4	3	2	1
+					5	4	3	2	1
+						4	3	2	1
+							3	2	1
+								2	1
+									1

	1	2	3	4	5	6	7	8	9
+	1	2	3	4	5	6	7	8	
+	1	2	3	4	5	6	7		
+	1	2	3	4	5	6			
+	1	2	3	4	5				
+	1	2	3	4					
+	1	2	3						
+	1	2							
+	1								



PLAYTHINK 551
DIFFICULTY: REQUIRED: COMPLETION: TIME:

FIBONACCI SEQUENCE

This series is the beginning of the famous Fibonacci number sequence. Discovered by the Italian mathematician Leonardo Fibonacci in the thirteenth century, the sequence appears throughout nature. The organic growth patterns in daisies, sunflowers and nautilus shells follow spirals described by the sequence.

Examine the sequence at right. Can you fill in the next number?

PLAYTHINK 552
DIFFICULTY: REQUIRED: COMPLETION: TIME:

NONCONSECUTIVE DIGITS

How many two-digit numbers possess no consecutive digits?

10, 11, 13, 14, . . .

PLAYTHINK 553
DIFFICULTY: REQUIRED: COMPLETION: TIME:

MISSING LINKS

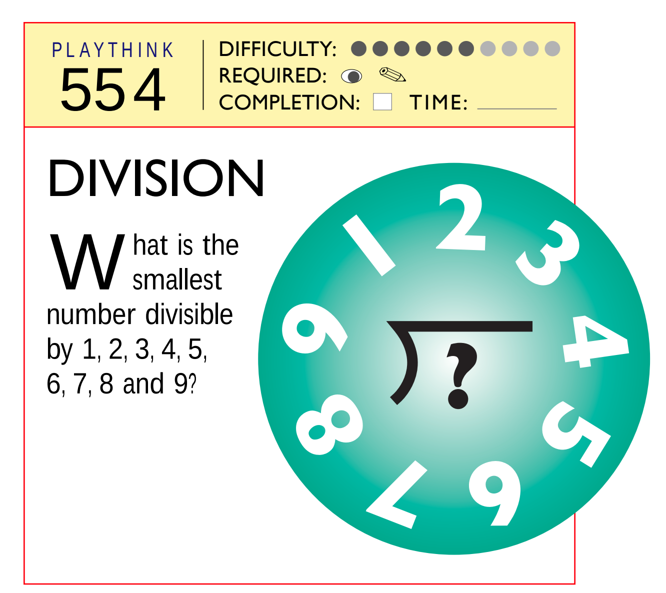
The numbers below are part of an equation in which all the plus or minus signs have been stripped out. What's more, two of the digits are actually part of a two-digit number. Can you work out the correct form of the equation?

1 2 3 4 5 6 7 8 9=100

PLAYTHINK 554
DIFFICULTY: REQUIRED: COMPLETION: TIME:

DIVISION

What is the smallest number divisible by 1, 2, 3, 4, 5, 6, 7, 8 and 9?

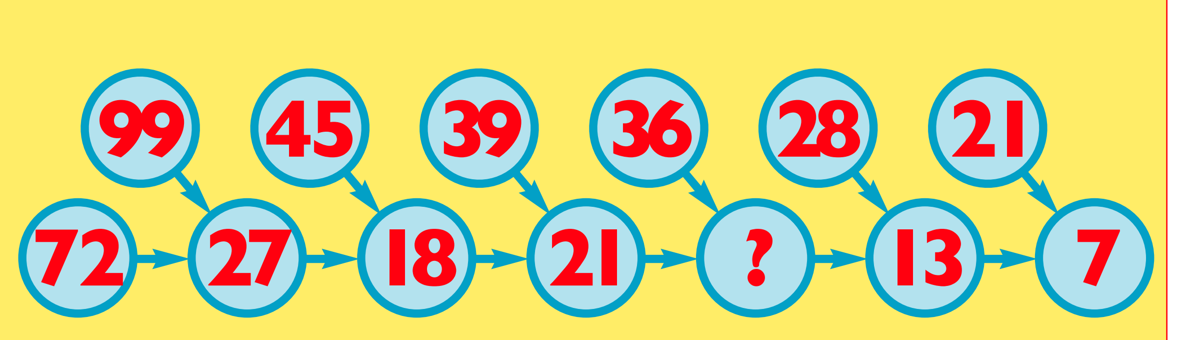


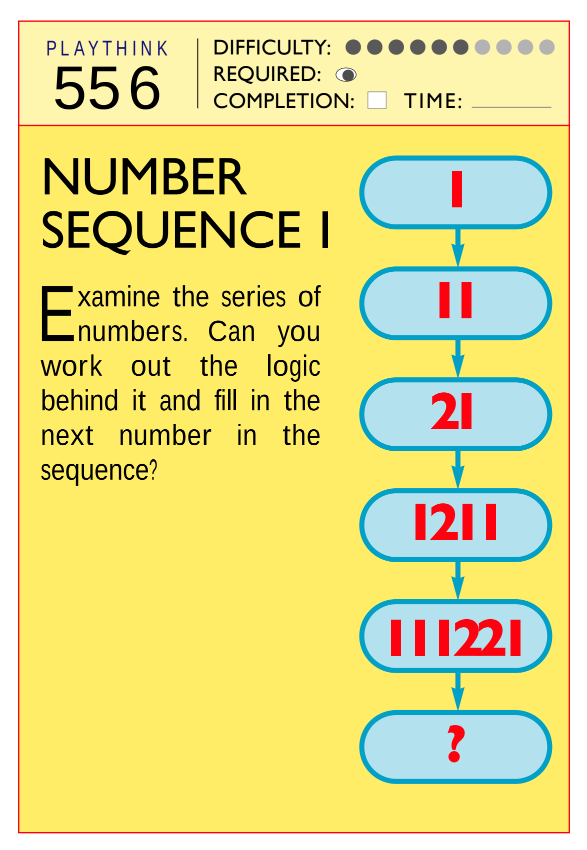
PLAYTHINK 555
DIFFICULTY: REQUIRED: COMPLETION: TIME:

NOB'S TRICKY SEQUENCE

Nob Yoshigahara discovered this beautiful number sequence, and there is no misprint: the last circle should contain a 7, not an 8.

Can you work out the logic behind the sequence and fill in the missing number?

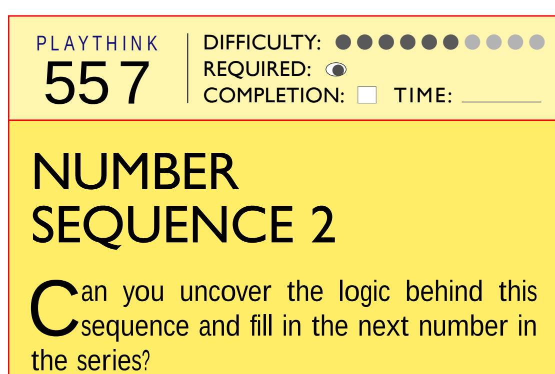




PLAYTHINK 556 DIFFICULTY: ●●●●●●○○○○ REQUIRED: 👁 COMPLETION: ☐ TIME: ____

NUMBER SEQUENCE 1

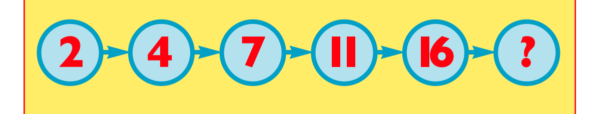
Examine the series of numbers. Can you work out the logic behind it and fill in the next number in the sequence?

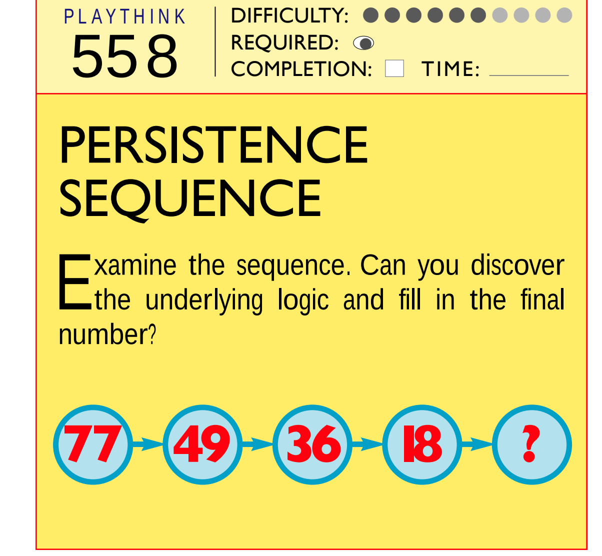


PLAYTHINK 557 DIFFICULTY: ●●●●●●○○○○ REQUIRED: 👁 COMPLETION: ☐ TIME: ____

NUMBER SEQUENCE 2

Can you uncover the logic behind this sequence and fill in the next number in the series?

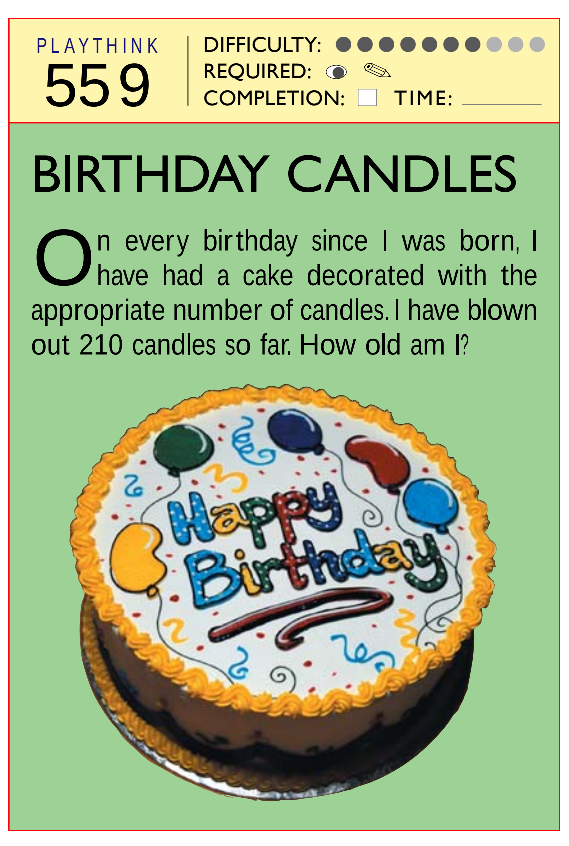




PLAYTHINK 558 DIFFICULTY: ●●●●●●○○○○ REQUIRED: 👁 COMPLETION: ☐ TIME: ____

PERSISTENCE SEQUENCE

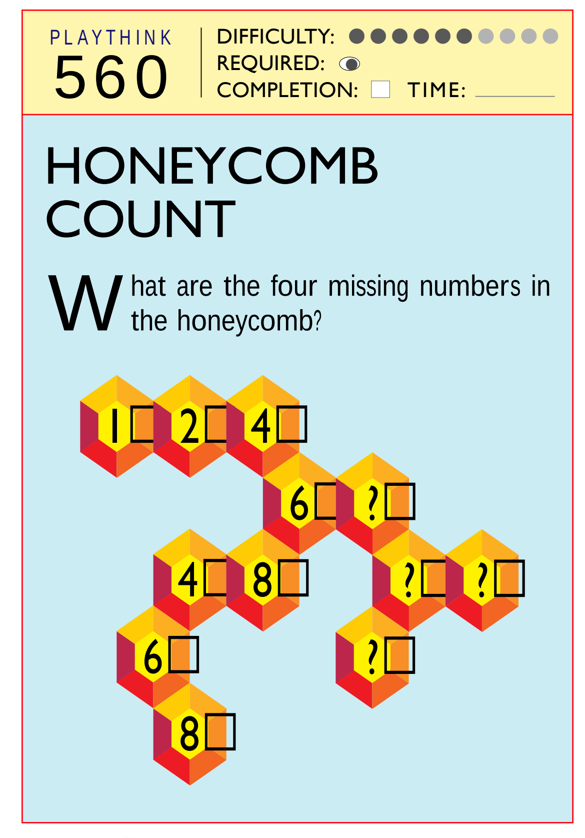
Examine the sequence. Can you discover the underlying logic and fill in the final number?



PLAYTHINK 559 DIFFICULTY: ●●●●●●●○○○ REQUIRED: 👁 ✎ COMPLETION: ☐ TIME: ____

BIRTHDAY CANDLES

On every birthday since I was born, I have had a cake decorated with the appropriate number of candles. I have blown out 210 candles so far. How old am I?



PLAYTHINK 560 DIFFICULTY: ●●●●●●○○○○ REQUIRED: 👁 COMPLETION: ☐ TIME: ____

HONEYCOMB COUNT

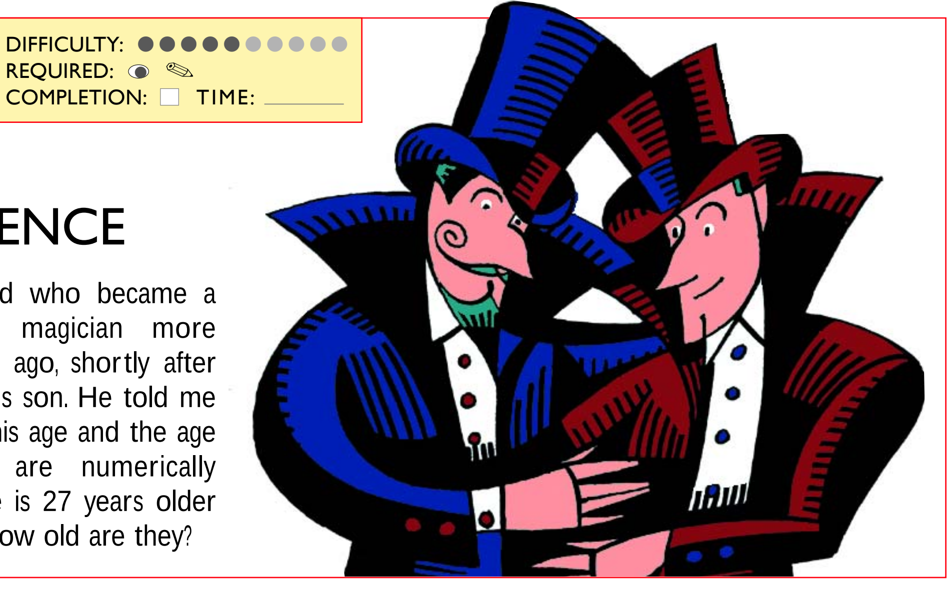
What are the four missing numbers in the honeycomb?

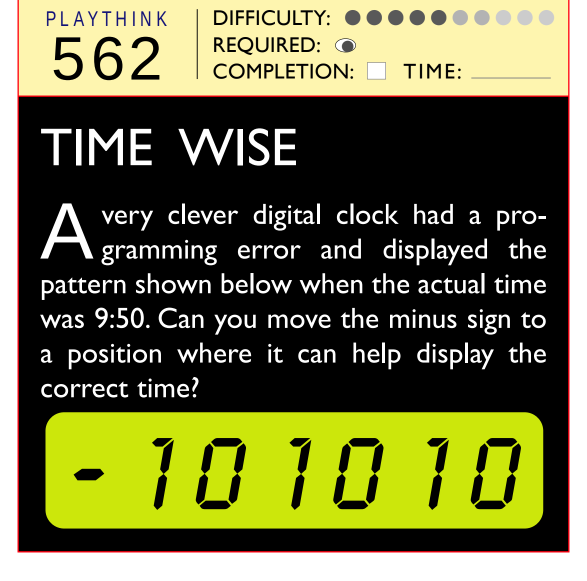


PLAYTHINK 561 DIFFICULTY: ●●●●●○○○○○ REQUIRED: 👁 ✎ COMPLETION: ☐ TIME: ____

AGE DIFFERENCE

I have a friend who became a professional magician more than 45 years ago, shortly after the birth of his son. He told me recently that his age and the age of his son are numerically reversed. If he is 27 years older than his son, how old are they?

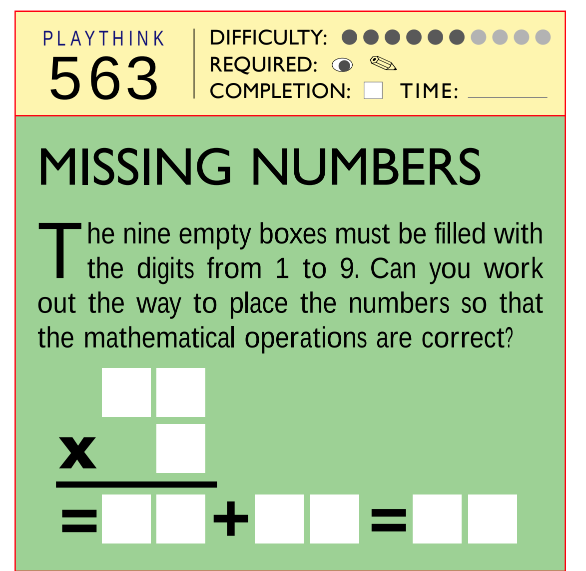




PLAYTHINK 562 DIFFICULTY: ●●●●●○○○○○ REQUIRED: 👁 COMPLETION: ☐ TIME: ____

TIME WISE

A very clever digital clock had a programming error and displayed the pattern shown below when the actual time was 9:50. Can you move the minus sign to a position where it can help display the correct time?



PLAYTHINK 563 DIFFICULTY: ●●●●●●○○○○ REQUIRED: 👁 ✎ COMPLETION: ☐ TIME: ____

MISSING NUMBERS

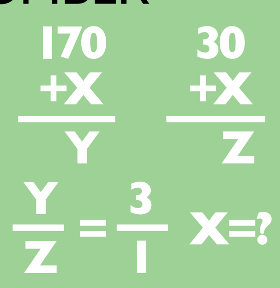
The nine empty boxes must be filled with the digits from 1 to 9. Can you work out the way to place the numbers so that the mathematical operations are correct?

PLAYTHINK 564

DIFFICULTY:
REQUIRED:
COMPLETION: TIME:

ADD A NUMBER

Can you work out what number can be added to both 170 and 30 so that the resultant sums have a ratio of 3:1?



PLAYTHINK 565

DIFFICULTY:
REQUIRED:
COMPLETION: TIME:

RIGHT EQUATION

Can you move one digit to a new position so that the equation below is correct? (Moving signs is not allowed.)

62−63=1

PLAYTHINK 566

DIFFICULTY:
REQUIRED:
COMPLETION: TIME:

JIGSAW

A jigsaw puzzle has 100 pieces. A move involves joining two clusters of pieces, or joining one piece to a cluster. Work out the fewest moves needed to complete the puzzle.

PLAYTHINK 567

DIFFICULTY:
REQUIRED:
COMPLETION: TIME:

MONASTERY PROBLEM

Place digits from 0 to 9 in the outer squares of the grid. Every red square must contain the same number; every yellow square should contain the same number; the sum of the numbers on each side should equal nine. How many different solutions can you find, not counting the one shown?

3	3	3
3		3
3	3	3

PLAYTHINK 568

DIFFICULTY:
REQUIRED:
COMPLETION: TIME:

WINE DIVISION

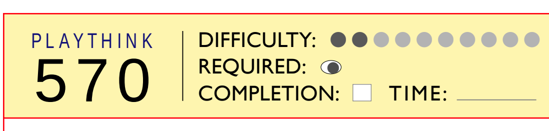
There are fourteen wineglasses on a table: seven are full, seven are half full. Without changing the amount of wine in any glass, can you divide the glasses into three groups so that each has the same total amount of wine?

PLAYTHINK 569

DIFFICULTY:
REQUIRED:
COMPLETION: TIME:

SOCCER ELIMINATION

Fifty-eight teams are entered in a single-elimination soccer tournament. How many matches must be scheduled?

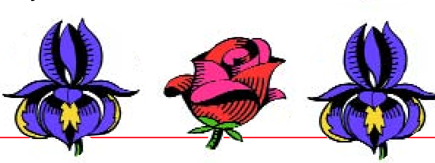


PLAYTHINK 570 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

FLOWERS PURPLE AND RED



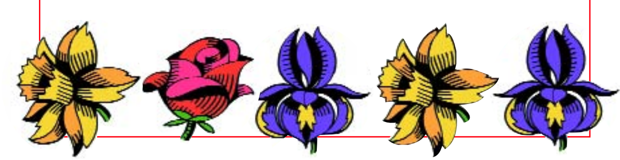
There are exactly forty flowers, red and purple, in a garden. And no matter which two flowers you pick, at least one will be purple. Can you work out how many red flowers there are?



PLAYTHINK 571 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

FLOWERS PURPLE, RED AND YELLOW

There are purple, red and yellow flowers in a garden. Anytime you pick three flowers, at least one will be red and at least one will be purple. From that information, can you work out how many flowers there are?



PLAYTHINK 572 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

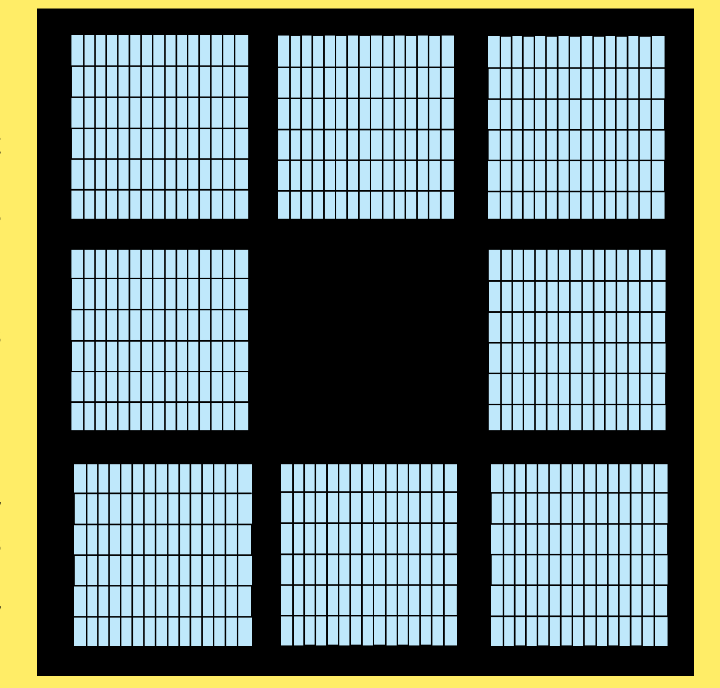
PRISON ESCAPE

A prison warden runs a two-story prison that has eight cells on each floor. To provide extra security, he has the cells occupied according to very specific rules:

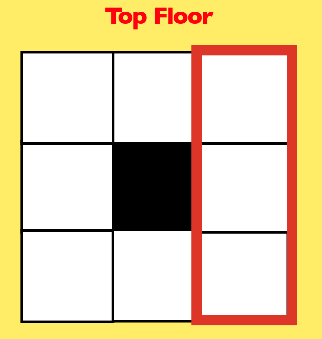
1. There must always be twice as many prisoners on the top floor as on the ground floor.
2. No cell may be unoccupied.
3. There must always be exactly eleven prisoners in the six cells running along any given exterior wall (as marked by heavy red lines on diagrams of top and ground floors.)

One night nine prisoners escape. Yet the next morning when the warden makes his rounds, all the cells are occupied according to his rules. Can you work out how many prisoners there were to begin with and how they rearranged themselves to conceal their escape?

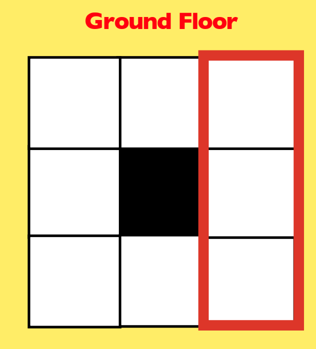
Prison Layout



Top Floor



Ground Floor



PLAYTHINK 573 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

COUNTING ANIMALS

I went to the zoo and saw the camels and the emus. If, all told, I saw thirty-five heads and ninety-four feet, how many camels and emus did I see?



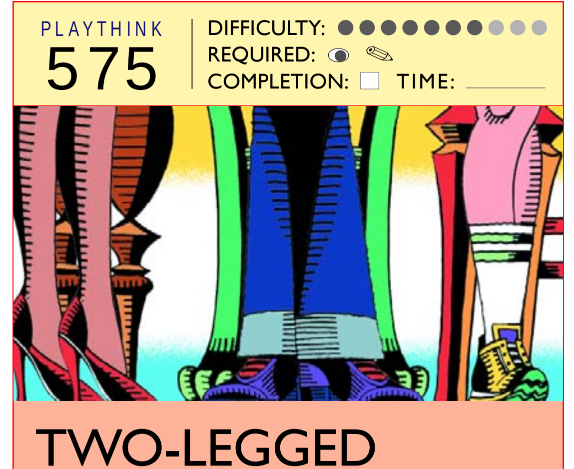
PLAYTHINK 574 | DIFFICULTY: ●●●○○○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

ZOO MIX

On another trip to the zoo, I counted thirty-six heads and one hundred feet. Can you work out how many birds and how many beasts I saw?

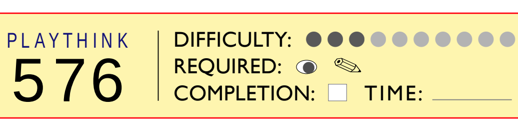


PLAYTHINK 575 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____



TWO-LEGGED THREE-LEGGED

In a reading room at a library, there are several three-legged stools and four-legged chairs, and they are all occupied. If you count thirty-nine legs in the room, is it possible to figure out how many stools, chairs and people there are?

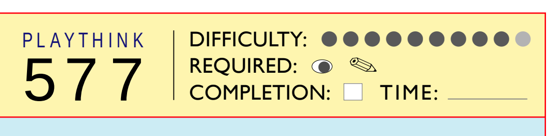


PLAYTHINK 576 | DIFFICULTY: ●●●○○○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

PUPPIES GALORE

A woman owns ten female dogs. Every one of the dogs has had a puppy, and none has had as many as ten. Does that mean that at least two of the dogs have had the same number of puppies?

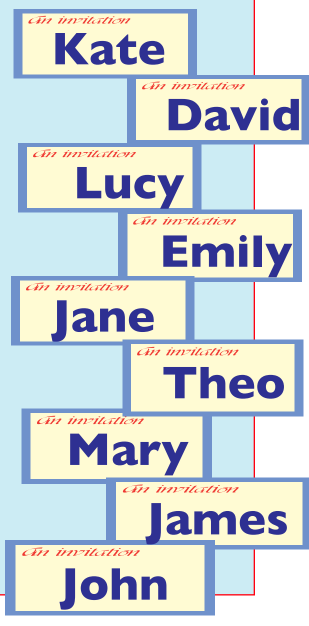


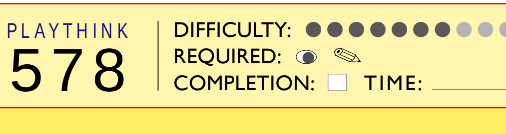


PLAYTHINK 577 | DIFFICULTY: ●●●●●●●●●○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

THREE'S COMPANY

There are nine people in your circle of friends, and you want to invite them to dinner, three at a time, over the next twelve Saturdays. Is there a way to arrange the invitations in such a way that pairs of friends meet each other at your dinners just once?





PLAYTHINK 578 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

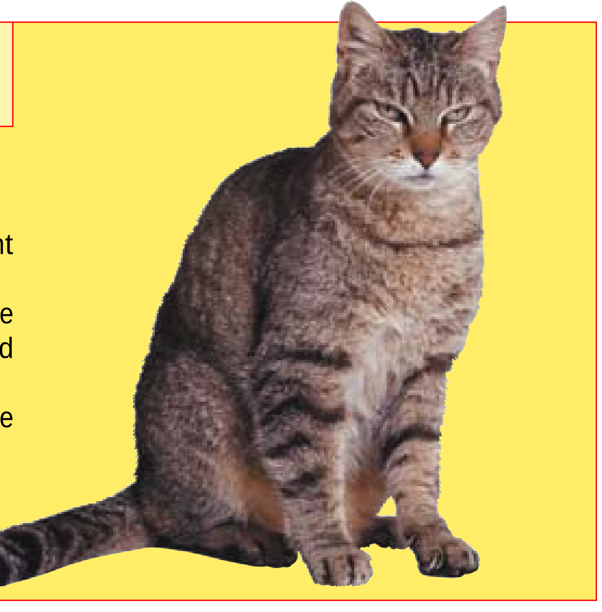
CAT LIVES

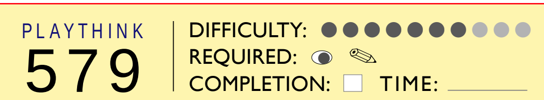
The following is derived from an ancient Egyptian puzzle.

A mother cat has spent seven of her nine lives. Some of her kittens have spent six, and some have spent only four.

Together, the mother and her kittens have a total of twenty-five lives left.

Can you tell with certainty how many kittens there are?

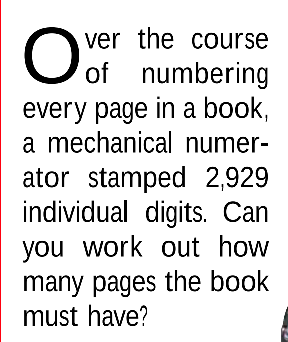


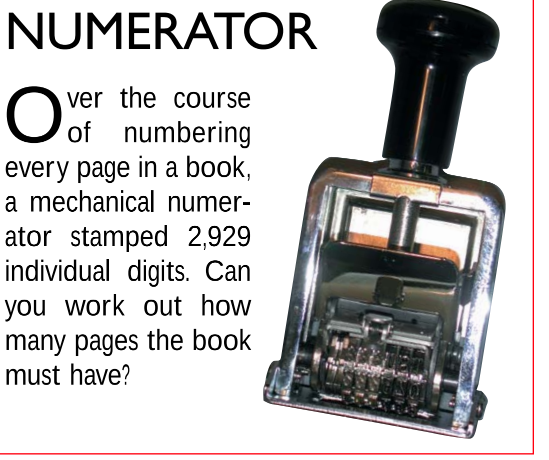


PLAYTHINK 579 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

NUMERATOR

Over the course of numbering every page in a book, a mechanical numerator stamped 2,929 individual digits. Can you work out how many pages the book must have?

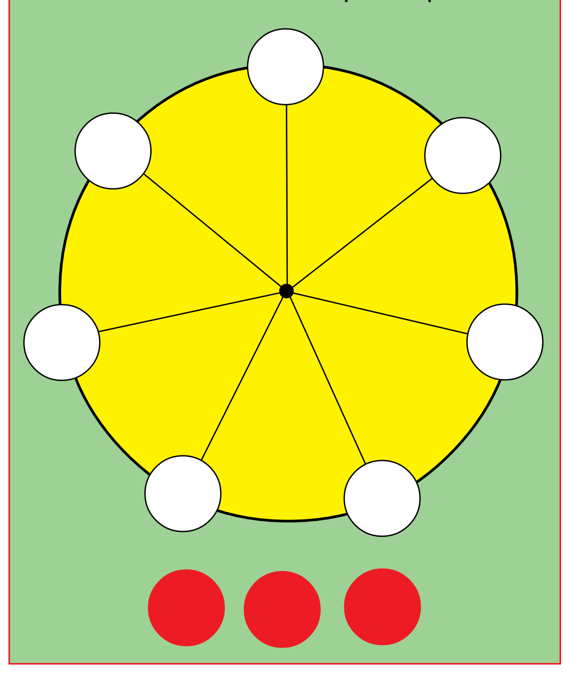




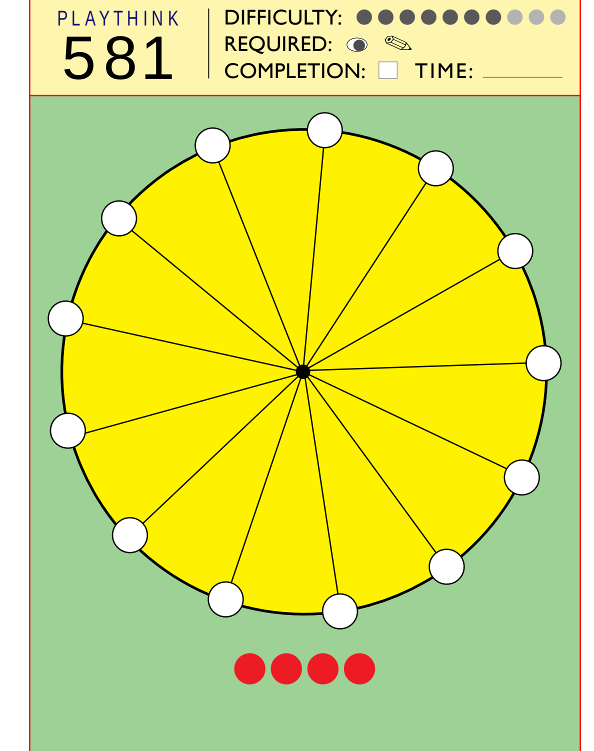
PLAYTHINK 580 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ____

MINIMAL LENGTH CIRCLE 1

The circumference of a circle, shown below, is divided into seven equal distances. Can you place three points on the circumference so that every number from 1 to 6 corresponds to an arc distance between two of the three placed points?



PLAYTHINK 581 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ____



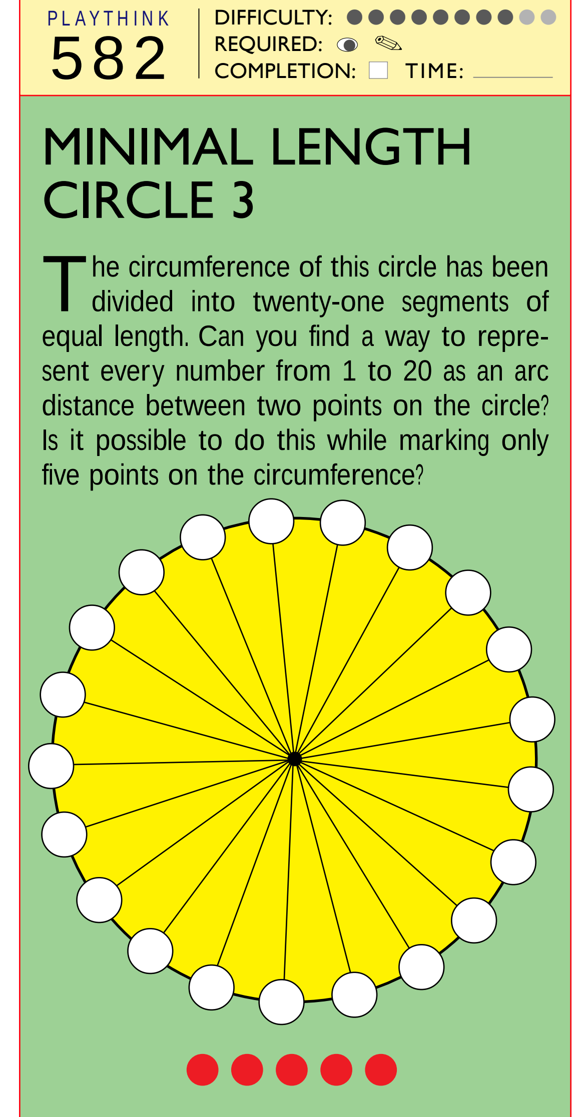
MINIMAL LENGTH CIRCLE 2

The circumference of this circle is divided into thirteen segments of equal length. Can you place four points along the circumference so that every number from 1 to 12 will correspond to an arc distance between two of the four points?

PLAYTHINK 582 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ____

MINIMAL LENGTH CIRCLE 3

The circumference of this circle has been divided into twenty-one segments of equal length. Can you find a way to represent every number from 1 to 20 as an arc distance between two points on the circle? Is it possible to do this while marking only five points on the circumference?



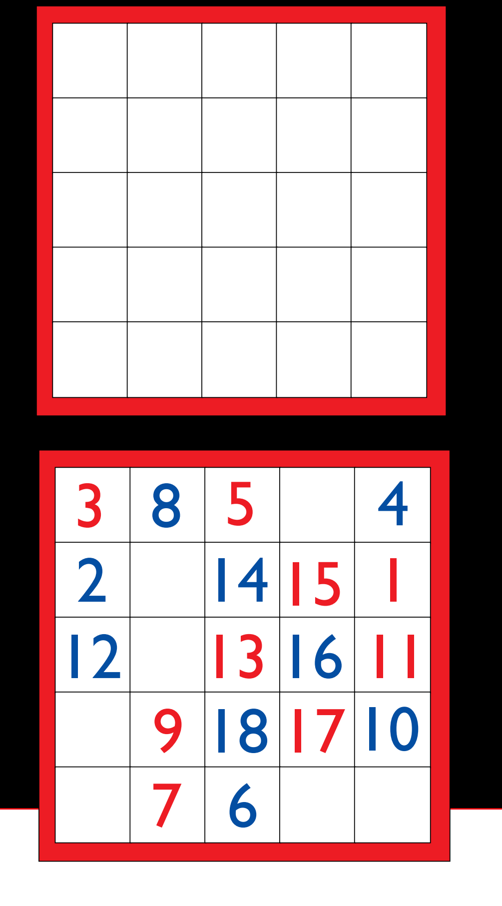
PLAYTHINK 583 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ____

PERSISTO

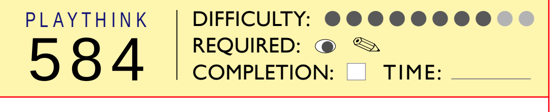
Here is a challenging paper-and-pencil number game. Two players take turns entering numbers in the cells of a square grid. The first player may enter a 1 anywhere on the grid. Subsequent numbers must be entered in the same column or row as the previously played number, with the restriction that the new number must have a clear "line of sight" with the old number. In other words, no player may "jump" a previously played number.

Whoever plays the last number scores that number of points. Play continues until one side tops 100.

A sample round with a score of 18 is shown at bottom.



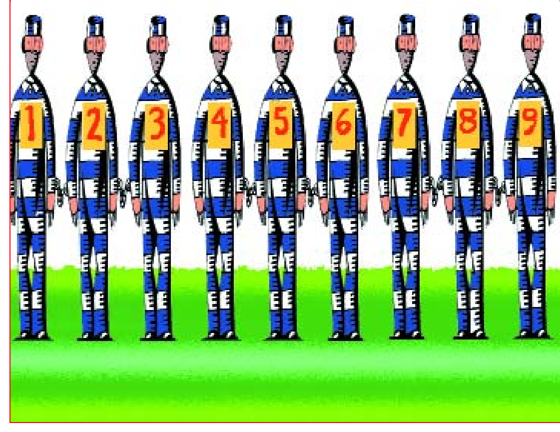
3	8	5		4
2		14	15	1
12		13	16	11
	9	18	17	10
	7	6		



PLAYTHINK 584 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ____

JAILHOUSE WALK

Nine prisoners are handcuffed in groups of three for their daily exercise. If the warden wants to arrange the men so that no two individuals are chained side by side more than once over the course of a six-day period, how might he handcuff them?



PLAYTHINK 585 DIFFICULTY: ●●●●●●●●●● REQUIRED: 👁 COMPLETION: ☐ TIME: ____

JEKYLL AND HYDE

Sixteen coins are distributed randomly on a four-by-four game board. On one side of each coin is Jekyll; on the other, Hyde.

The object of the game is to turn over the coins until all show Jekylls or all show Hydes. The coins are flipped according to one simple rule: At each turn you must flip all the coins in a single row, column or diagonal. (A diagonal may be a short one—even the corner counts as a diagonal of just one coin.)

Two random starting configurations are shown below. Can you work out whether every starting configuration will lead to an all-Jekyll or all-Hyde outcome?

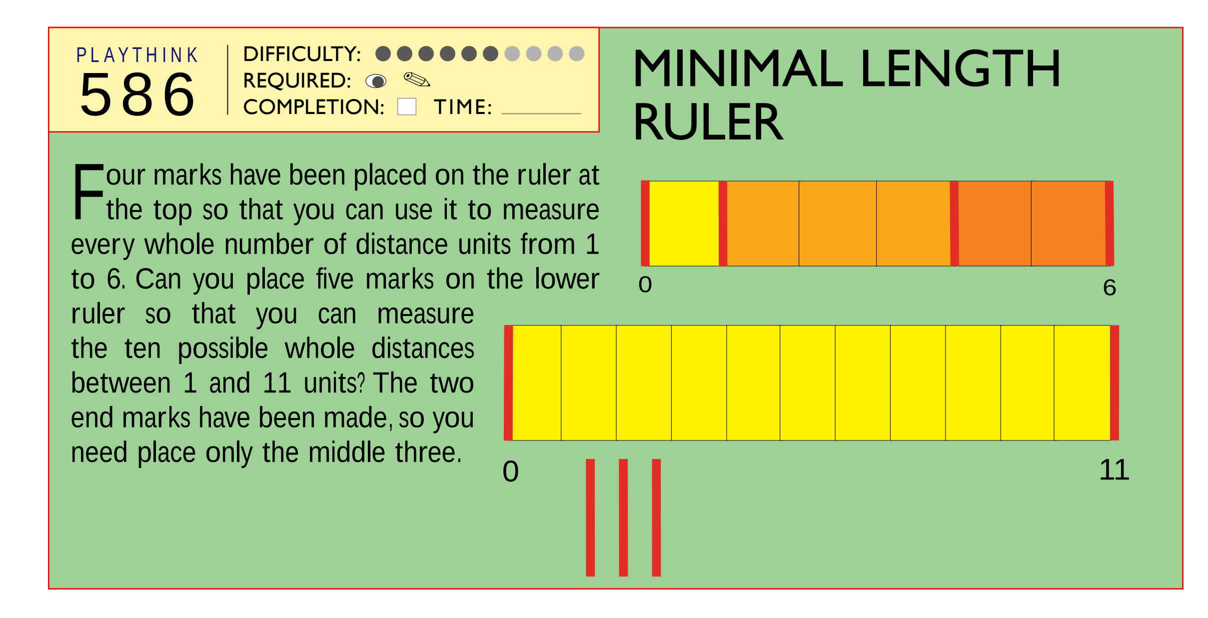




PLAYTHINK 586 DIFFICULTY: ●●●●●●●●●● REQUIRED: 👁 ✎ COMPLETION: ☐ TIME: ____

MINIMAL LENGTH RULER

Four marks have been placed on the ruler at the top so that you can use it to measure every whole number of distance units from 1 to 6. Can you place five marks on the lower ruler so that you can measure the ten possible whole distances between 1 and 11 units? The two end marks have been made, so you need place only the middle three.



PLAYTHINK 587 DIFFICULTY: ●●●●●●●●●● REQUIRED: 👁 ✎ COMPLETION: ☐ TIME: ____

LADYBUG FAMILY

One-fifth of the ladybug family flew to the garden with the yellow roses. One-third of the family flew to the violets, and three times the difference between these two numbers flew to the red poppies far away. And the mother of the ladybug family went to the river to do laundry. When all the ladybugs met up back home, how many were there?

PLAYTHINK 588 DIFFICULTY: ●●●●●●●●●● REQUIRED: 👁 ✎ COMPLETION: ☐ TIME: ____

HINGED RULER 1

Five unmarked rulers have been hinged at two points, as shown. What lengths should each of the rulers have so that one or a combination of rulers can measure every distance from 1 to 15 units?

PLAYTHINK 589 DIFFICULTY: ●●●●●●●●●● REQUIRED: 👁 ✎ COMPLETION: ☐ TIME: ____

HINGED RULER 2

Three unmarked rulers are hinged at one point, as shown. What three lengths should the rulers have so that, singly or in combination, they can measure every length from 1 to 8 units?

It can't be done? Try including measurements in which the rulers are folded back against one another.

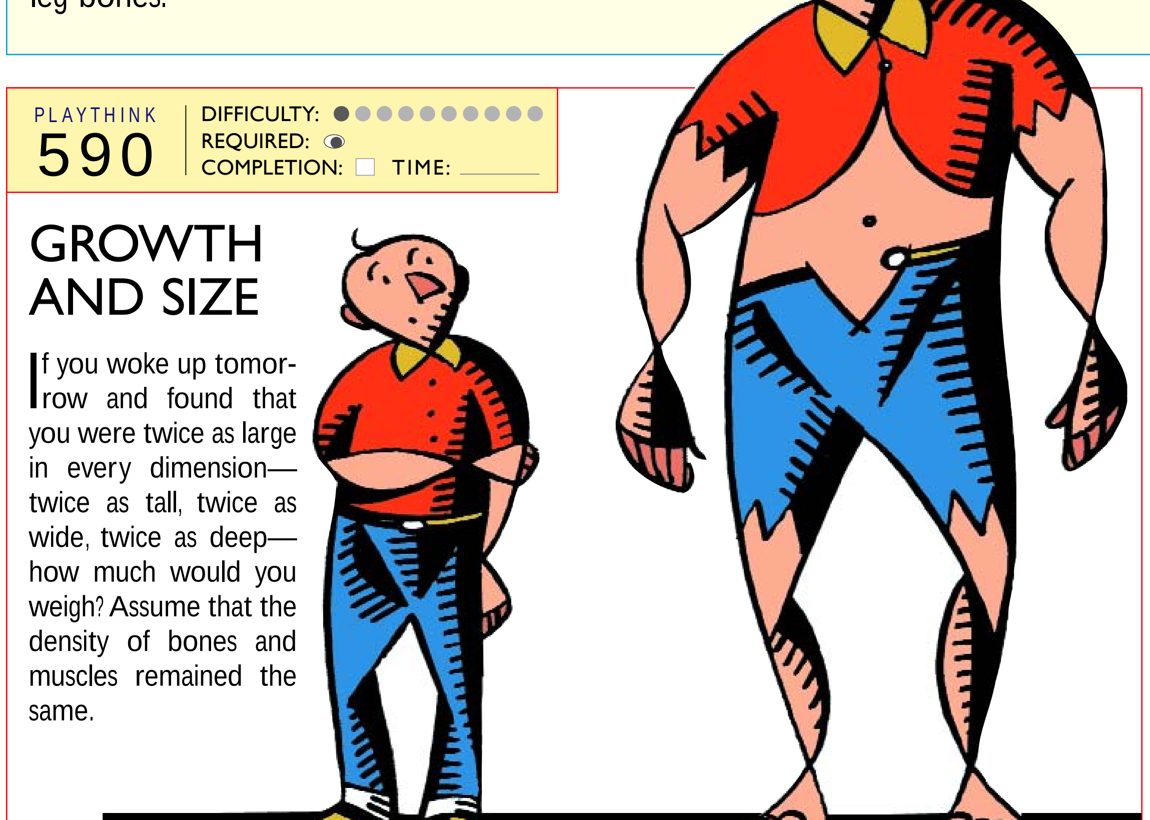
The Principle of Similitude

In *Gulliver's Travels*, Jonathan Swift described Brobdingnag, a land of giants where every person was twelve times taller than normal. But could a 70-foot-tall man even support his own weight? Actually, no: such people are physical impossibilities. When scaling objects linearly, you have to keep in mind that their cross-sectional area goes up by the square of the linear factor, and the volume goes up by the cube. A person who was twelve times larger in every dimension would weigh 12^3, or 1,728 times more than a normal human. What's more, because the strength of a bone scales according to its cross-sectional area, the bones would be only 144 times as strong. Any Brobdingnagian who tried to stand would snap his leg bones.

This sort of problem faces anyone who tries to scale an object up or down. A thirty-story office building cannot be constructed in the same manner as a three-story house. A model airplane and a modern jetliner are built with different types of materials. Many would-be inventors have been disappointed through ignorance of the effects of change in scale.

Galileo, with his law of similitude, explained why large bodies suffer relatively greater deformation from the force of their own weight than do smaller objects. The stability of objects of identical shape, the law states, decreases directly with increased height because the distorting force of gravity increases with volume while the magnitude of supporting ability, because it depends upon cross-sectional area, cannot exhibit a comparable increase. A body that increases in linear dimensions by a factor of 10 increases in volume by a factor of 1,000.

Also, surface area per unit of volume is greater for the smaller object than for the larger one; small animals are particularly vulnerable to the loss of water by evaporation due to their comparatively large surface area. The law of similitude, then, helps explain why elephants and mice not only look different but act different. And why you are unlikely ever to meet a Brobdingnagian.

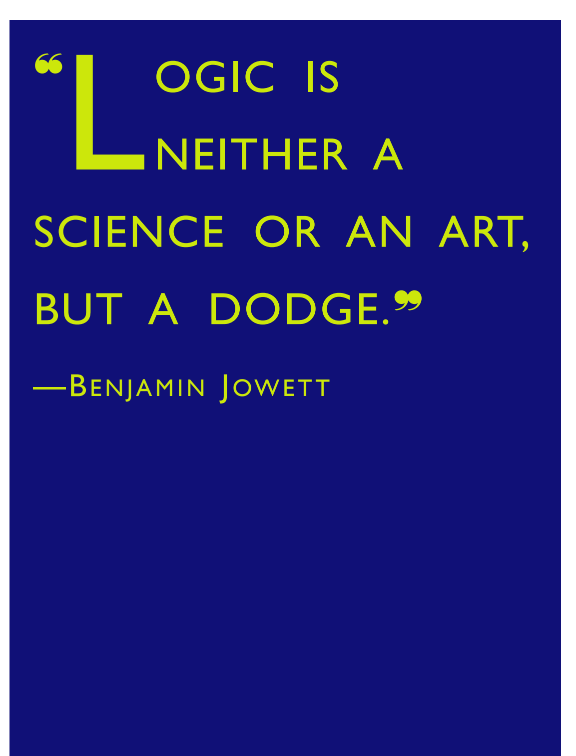


PLAYTHINK 590

DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

GROWTH AND SIZE

If you woke up tomorrow and found that you were twice as large in every dimension—twice as tall, twice as wide, twice as deep—how much would you weigh? Assume that the density of bones and muscles remained the same.



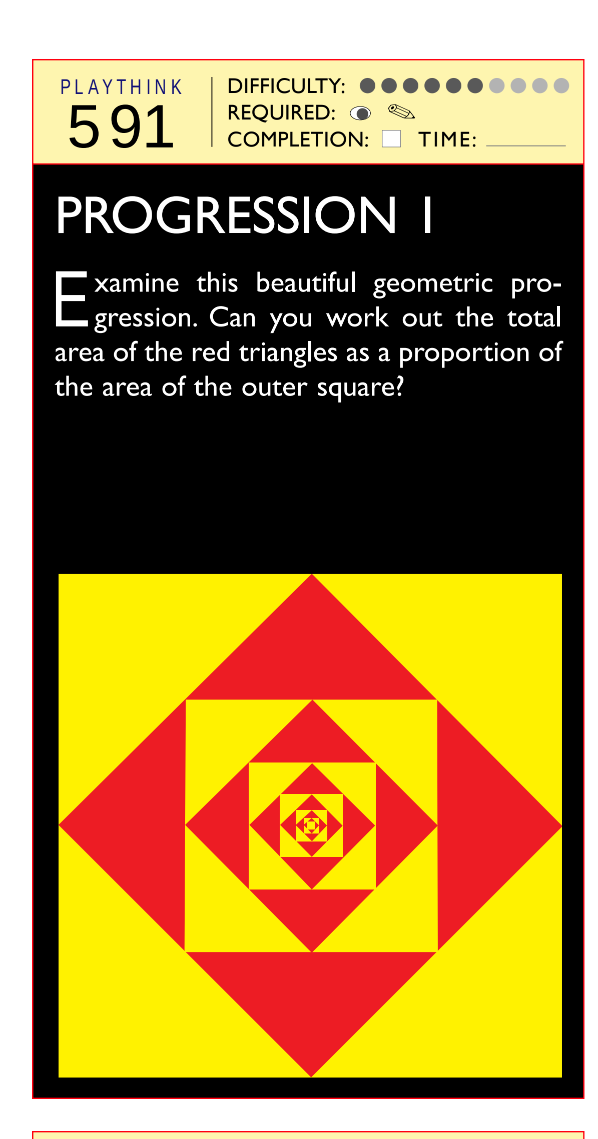
"Logic is neither a science or an art, but a dodge."

—Benjamin Jowett

PLAYTHINK 591 | DIFFICULTY: ●●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

PROGRESSION 1

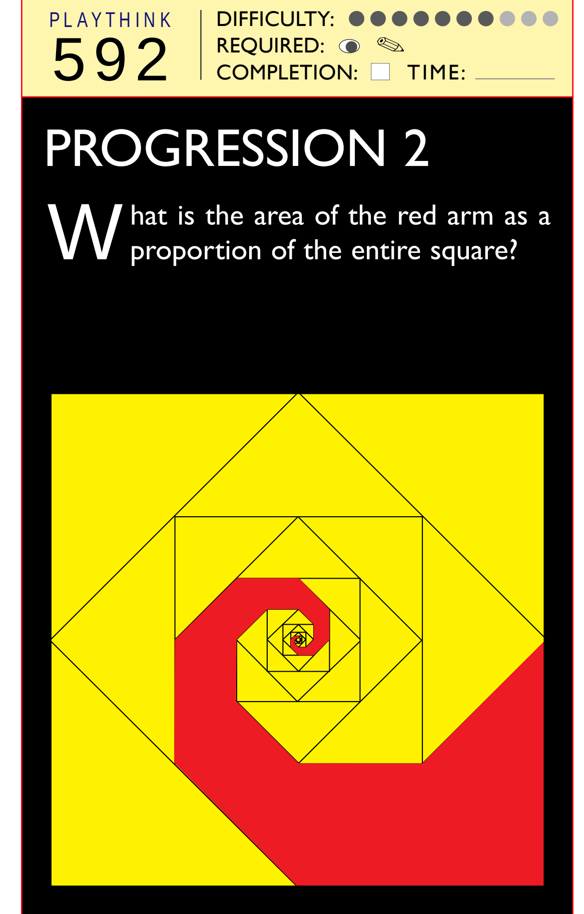
Examine this beautiful geometric progression. Can you work out the total area of the red triangles as a proportion of the area of the outer square?



PLAYTHINK 592 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

PROGRESSION 2

What is the area of the red arm as a proportion of the entire square?



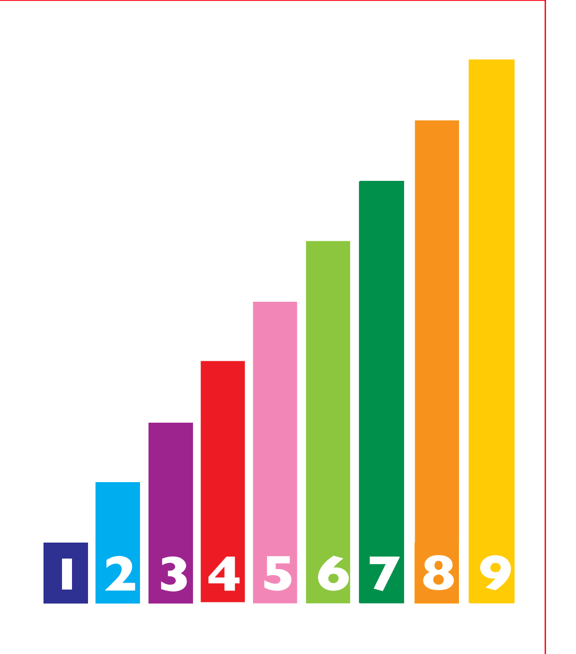
PLAYTHINK 593 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

ASCENT–DESCENT

Can you arrange the nine strips in a row so that you will be unable to find four that are either in ascending or descending order?

The strips in an ascending or descending sequence need not be next to each other. For example, 7, 5, 8, 1, 9, 4, 6, 2, 3 fails because 7, 5, 4, 2 is a descending sequence, in spite of the fact that other numbers come between them.

Can you find at least one sequence that follows the rule?

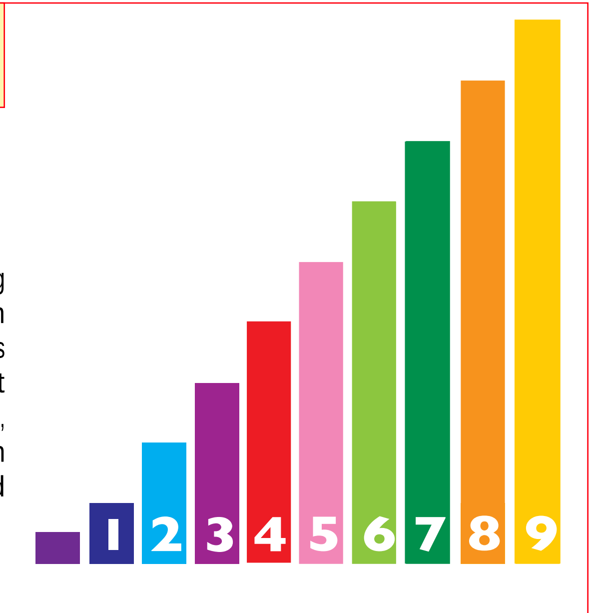


PLAYTHINK 594 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

INCREASING–DECREASING

Is it possible to arrange ten strips of differing lengths so that there is no set of four in ascending or descending order? The four strips do not have to be next to each other to count as a sequence. For example, in the sequence 1, 2, 8, 0, 3, 6, 9, 4, 5, 7, the set 1, 2, 8, 9 is an ascending sequence, even though the 8 and the 9 are not side by side.

Can you find a sequence that works?

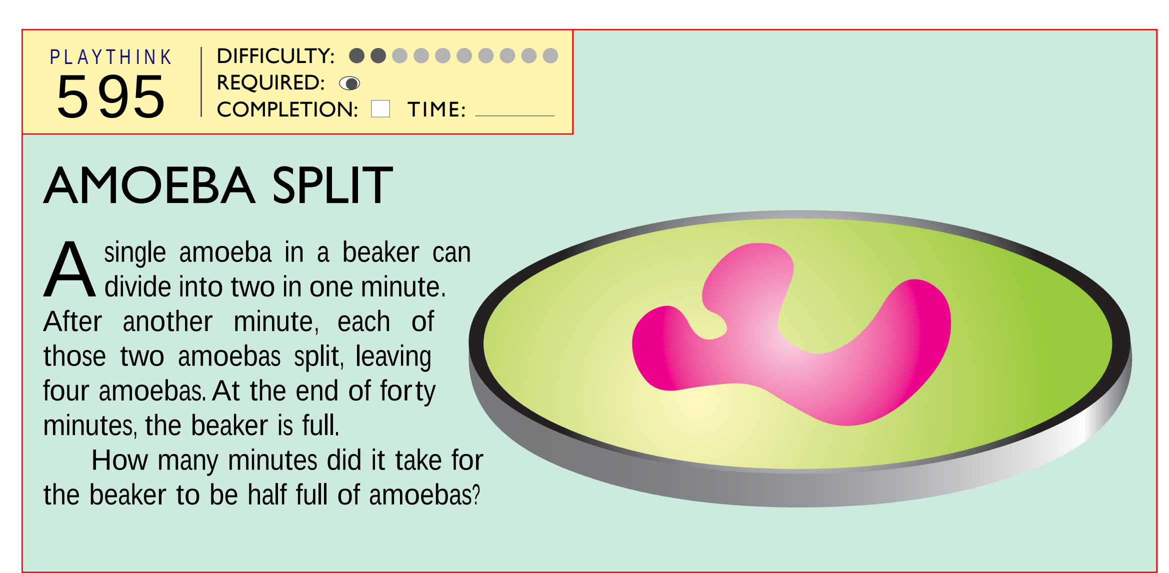


PLAYTHINK 595 | DIFFICULTY: ●●○○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

AMOEBA SPLIT

A single amoeba in a beaker can divide into two in one minute. After another minute, each of those two amoebas split, leaving four amoebas. At the end of forty minutes, the beaker is full.

How many minutes did it take for the beaker to be half full of amoebas?

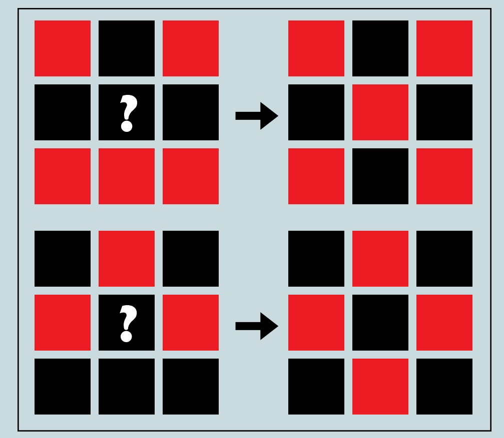


PLAYTHINK 596

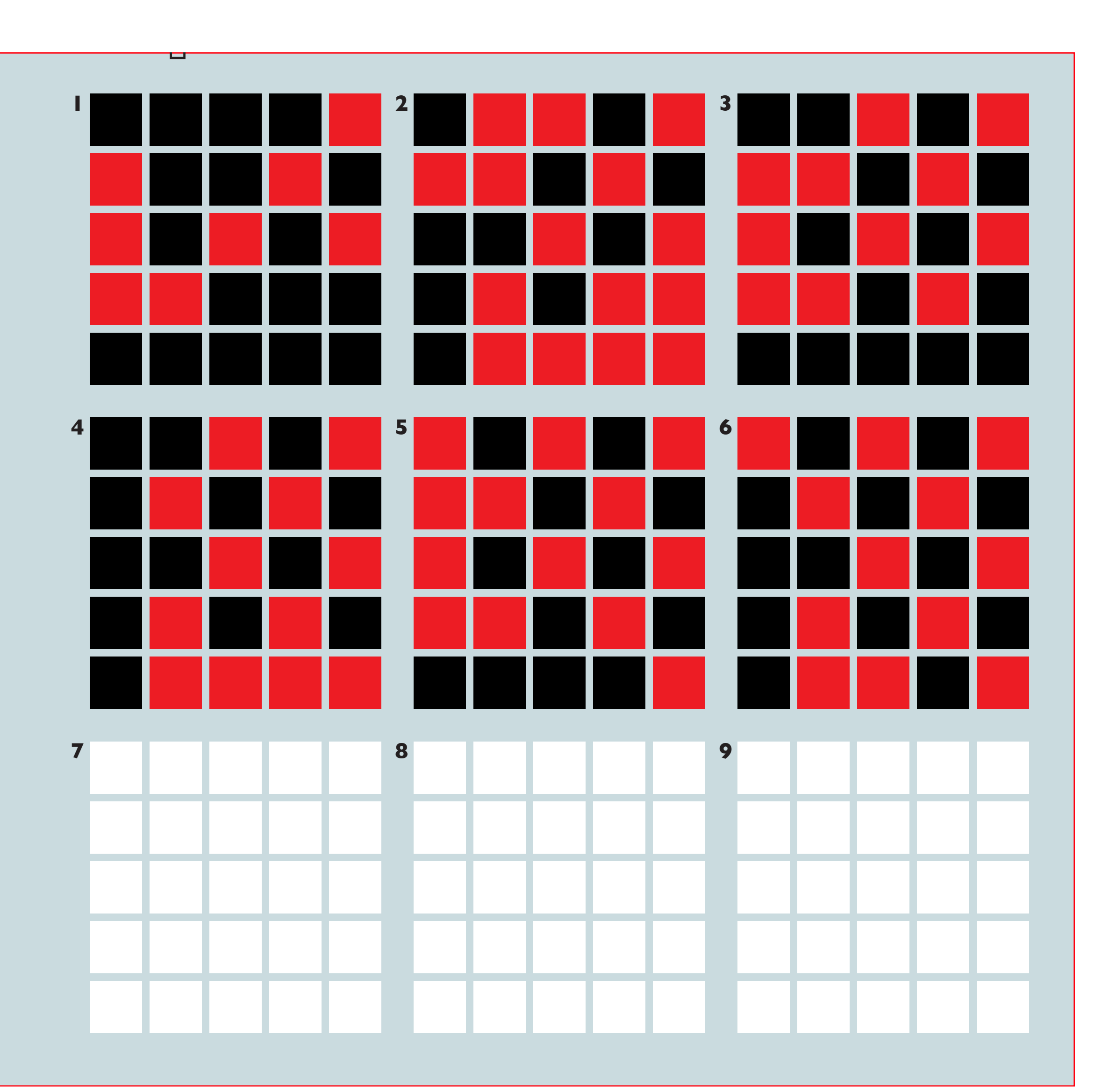
DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

CELLULAR AUTOMATON

The squares in grid 1 are randomly distributed between red and black. In every grid after that, the color of each square is determined by that of its neighbors in the generation before. For example, if a black square is surrounded by a majority of black squares, it will flip from black to red. A majority of red squares changes the color to black. (In case of a tie, the color remains the same.)



Six generations of the puzzle are shown. Can you complete the next three?



PLAYTHINK 597

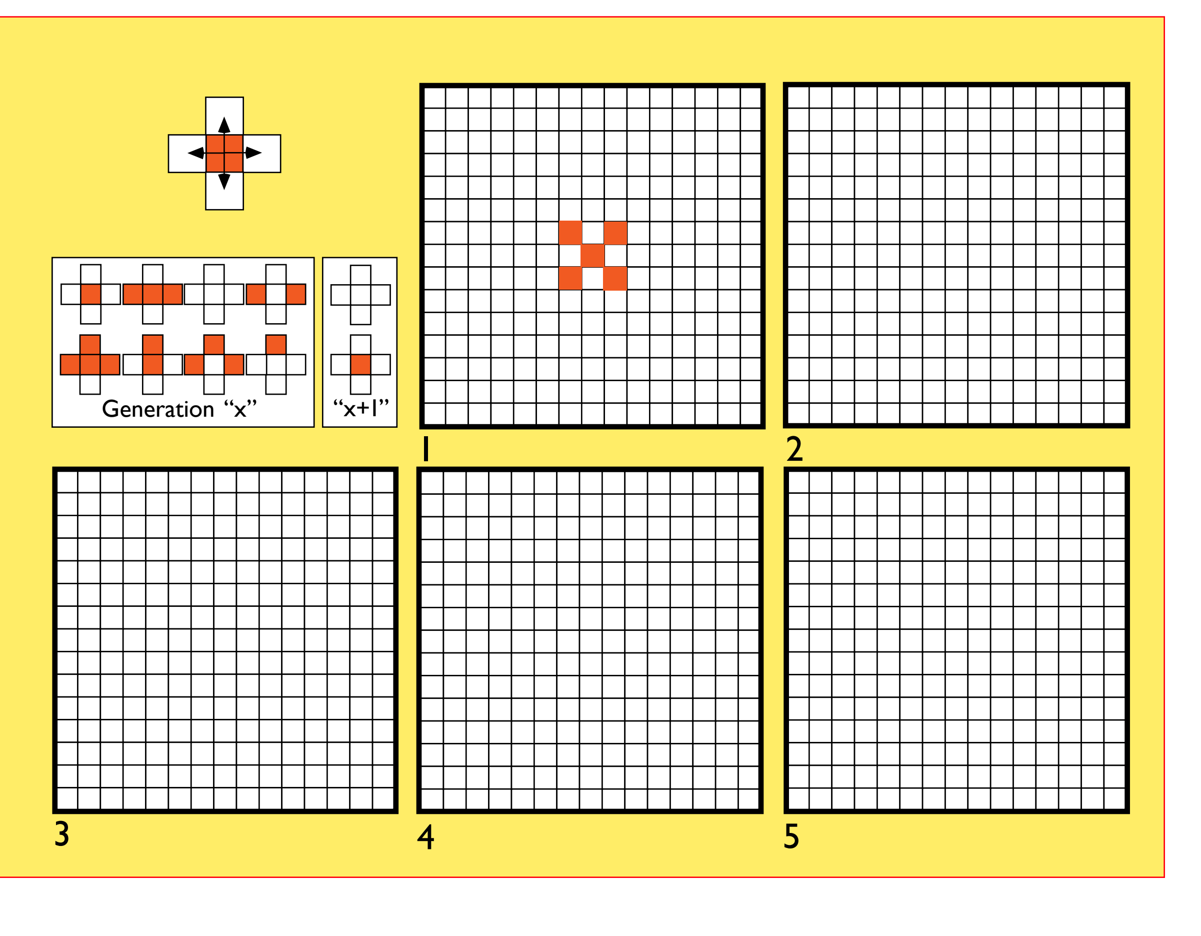
DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

FREDKIN'S CELLULAR AUTOMATON

■ *A Two-Dimensional Self-Generating Mechanism*

Five red cells sit in the middle of grid 1. Each successive grid holds a new generation of cells that have been added or subtracted according to a simple rule: If the number of red cells horizontally or vertically adjacent to the cell is even, then the cell is white in the next generation; if the number of adjacent red cells is odd, the cell is red in the next generation. (See the inset for a demonstration of the growth pattern.)

Can you carry out the growth pattern over five generations? If you do, you will see a surprising result.



PLAYTHINK 598

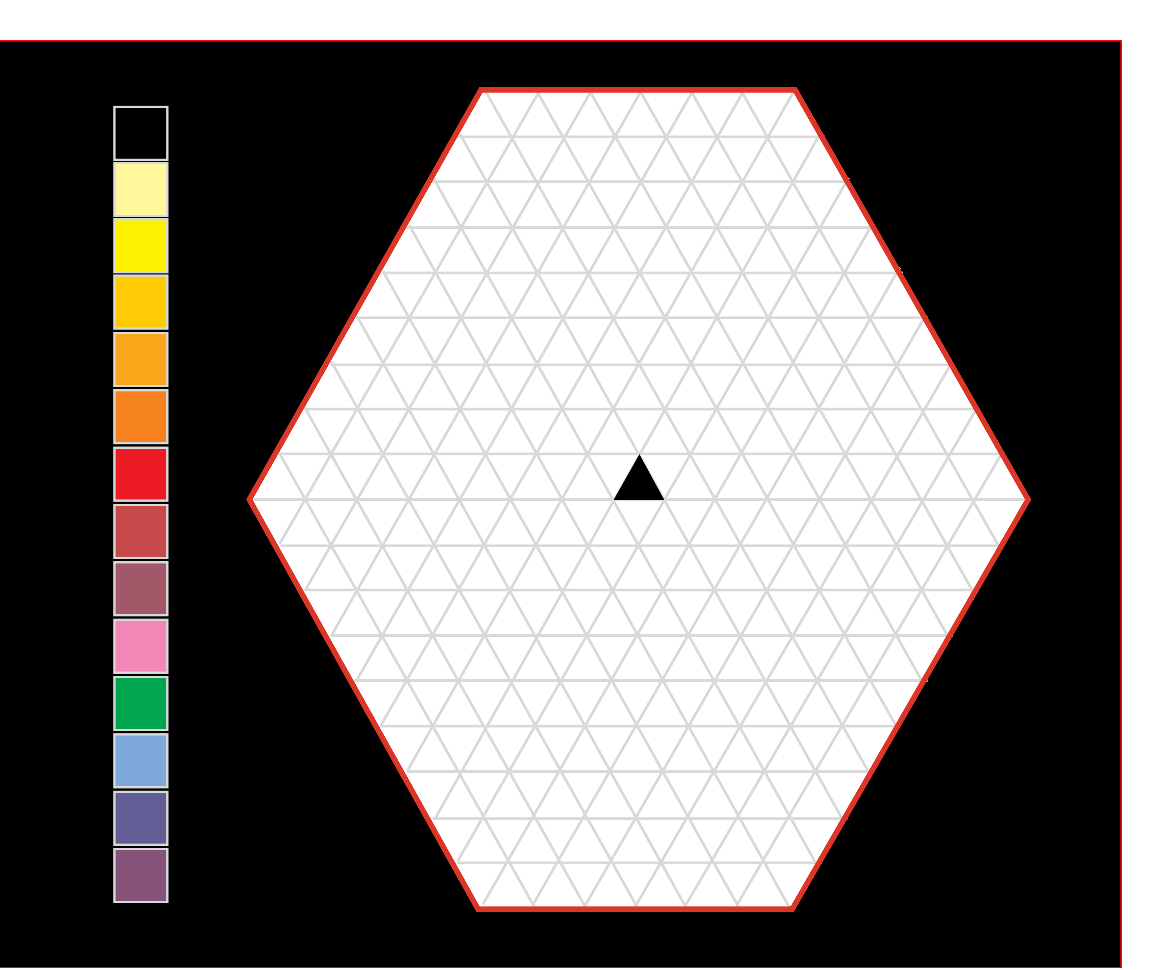
DIFFICULTY: ●●●●○○○○○○
REQUIRED:
COMPLETION: TIME:

GROWTH PATTERN TRIANGLES

Many items in nature—crystals, colonies of bacteria, even the clouds that form stars—show highly geometric patterns in their growth. This puzzle helps harness such a pattern for the purpose of art.

Begin with a single triangle in the center of a grid, as shown. Add triangles one generation at a time, following one simple rule: Each new triangle should touch one—and only one—side of a triangle from the previous generation. To make each wave of growth distinct, use the palette for the color of each generation of triangles. After fourteen generations you can recycle the colors.

How many triangles are there in each generation? Is there any regularity in the sequence of numbers?



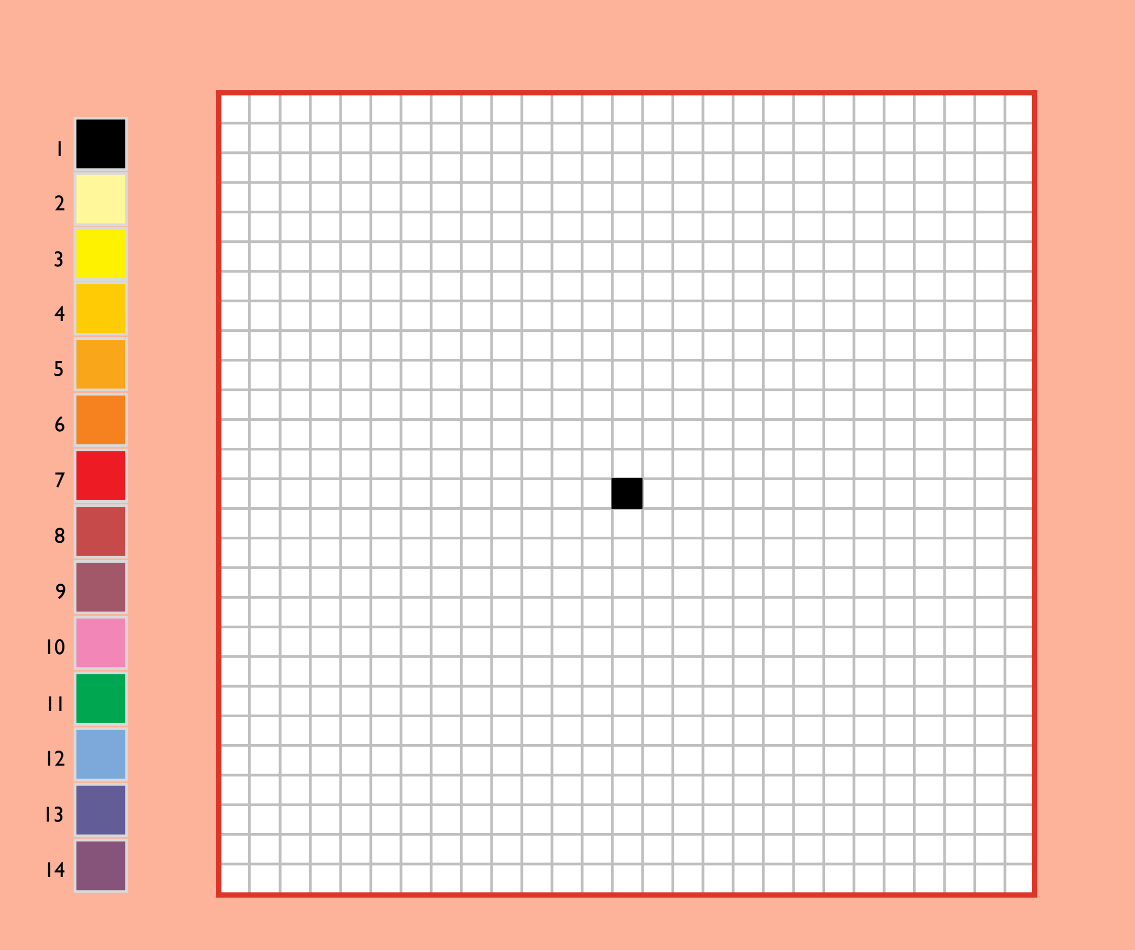
PLAYTHINK 599

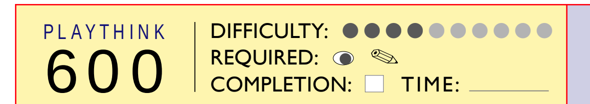
DIFFICULTY: ●●●●○○○○○○
REQUIRED:
COMPLETION: TIME:

GRADIENT PATTERN SQUARES

There is a single dark square at the center of the grid. Additional squares can be added to the grid following a simple growth rule: Squares are added one generation at a time so that each new square touches one—and only one—square from the previous generation.

To help show the patterns of growth, color each generation according to the palette at right. Can you work out how many squares will be added for each generation? Is there a pattern to the number of new squares in each generation?





LADYBUG WALKS

These five games build on a regular series of walks and turns. Imagine that five ladybugs follow the circuits described below. Will any of them return to their starting places?

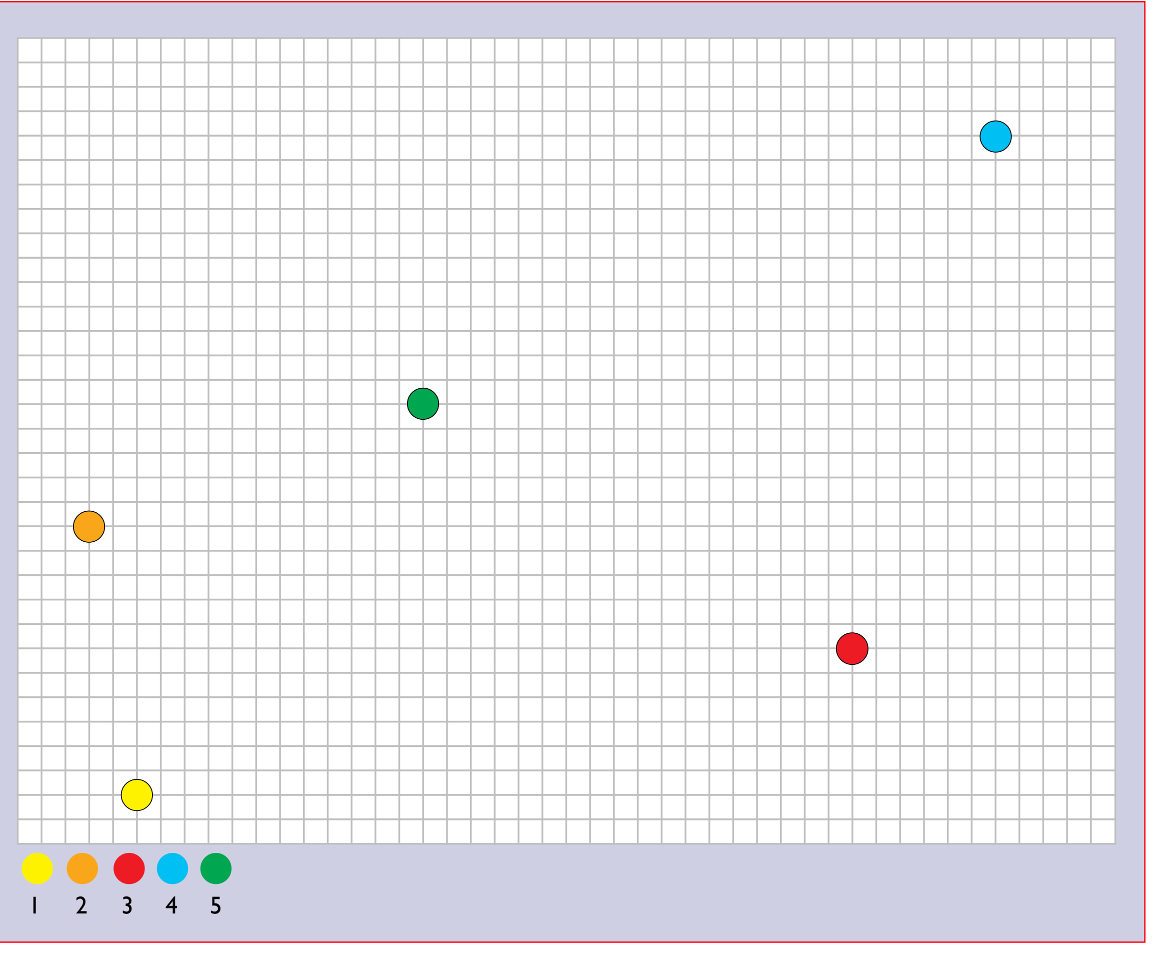
Game 1—Starting at the yellow point, crawl a distance of 1 unit up, then turn right. Crawl 2 units, then turn right again. Crawl 3 units and so forth, up to a 5-unit crawl. After 5 units, turn right and start the sequence over again with a 1-unit crawl.

Game 2—The same as Game 1, except that the sequence builds to a 6-unit crawl before returning to 1 unit.

Game 3—As above, except that it is extended to 7 units.

Game 4—As above, except extended to 8 units.

Game 5—As above, except extended to 9 units.



PLAYTHINK 601 DIFFICULTY: REQUIRED: COMPLETION: TIME:

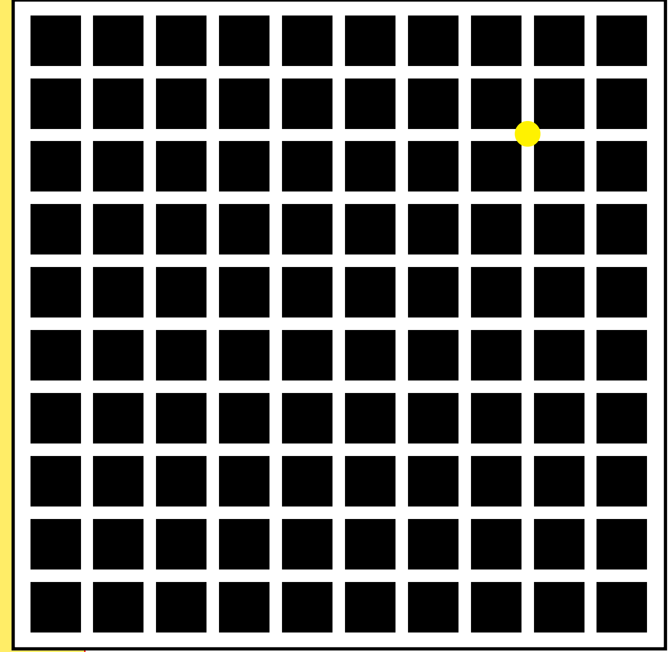
GOLYGONS

■ *A Walk in a Square Matrix*

The mathematician Lee Sallows of the University of Nijmegen in The Netherlands conceived of the following problem.

Start at the yellow point on the grid. Pick a direction and "walk" one block. At the end of the block, turn left or right and walk two more blocks; turn left or right and then walk three blocks. Continue this way, walking one more block in each segment than before. If after a number of turns you return to the starting point, then the path you have traced is the boundary of a golygon.

The simplest golygon has eight sides, meaning it can be traced in eight segments. Can you find it?

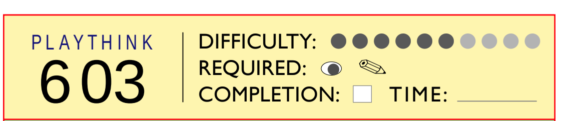


PLAYTHINK 602 DIFFICULTY: REQUIRED: COMPLETION: TIME:

PRIME DOUBLES

Can you always find a prime number somewhere between any number and its double (excluding 1, of course)?





PRIME CHECK

There are exactly 9!, or 362,880, different nine-digit numbers in which all the digits from 1 to 9 appear. The number below is an obvious example. Of those 362,880 numbers, can you work out how many will be prime—divisible only by 1 and themselves?

123,456,789

Snowflake Curve

What kind of shape has an infinite length yet only a finite area?

It sounds impossible, but, surprisingly, such figures exist. One of them is the beautiful snowflake curve. This curve is essentially a growth pattern created as a sequence of polygons. The snowflake curve is built on the sides of an equilateral triangle according to a very simple progression principle. On the central third of each side, another equilateral triangle is added, and that progression is carried out generation after generation forever.

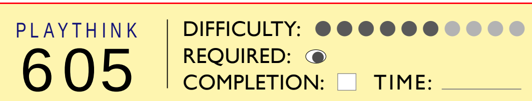
The snowflake curve is a good first introduction to the idea of limit and the concept of fractals. It is not possible to draw the limiting curve. We can create the polygons only for the next sequence, and the ultimate curve must be left to the imagination.

PLAYTHINK 604

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

SNOWFLAKE AND ANTI-SNOWFLAKE CURVES

The red figures at the bottom of this illustration show the first four stages of the famous snowflake pattern. As the fractal pattern continues indefinitely, can you work out the limit on the length of its perimeter and the area it will eventually enclose?



INFINITY AND LIMIT

Each picture is half the height of the image it is set in. If this pattern continued, there would be an infinite number of pictures. Rather than setting them one inside another, imagine stacking them atop each other. How tall would the tower of pictures grow?



Fractal Geometry

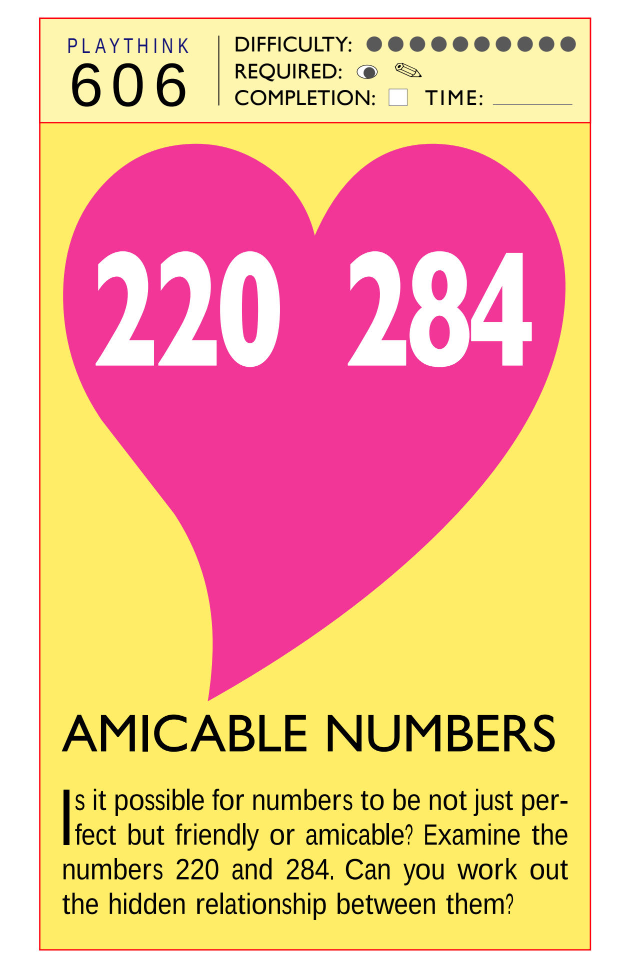
Twentieth-century mathematicians revolted against the classical mathematics of previous centuries when they discovered that mathematical structures and curves did not fit the patterns laid down by Euclid. The new structures and curves were at first regarded as "pathological," because they seemed to upset established standards of the time. How ironic that term is, since the bizarre, abstract structures invented to break free of the Euclidean mold turned out to be present in many familiar objects.

Fractals are an example. Forests, coastlines, star clusters and atomic tracks may not seem to have much in common, but they are all linked by this extraordinary geometrical notion.

Fractals begin with the simple. Take the shortest distance between two points, a straight line. Add a few kinks and bumps, and it gets longer. The more convoluted it becomes, the longer it grows. If the line becomes irregular enough, it will become infinitely long. Then you have a fractal. A coastline is a perfect example. No matter how much you magnify the scale of a map, a coastline presents the same sort of jagged shape.

A set discovered by Polish mathematician Benoit Mandelbrot in 1977 is another example of the confluence of simplicity and complexity found in fractal geometry. His fractal set can be generated by just a few lines of computer code, but an infinite amount of information would be needed to create a full description of the shape of its outline. Fractals derived from Mandelbrot's work have been used by computer graphic artists to create imaginary landscapes that seem as natural as any found on earth.

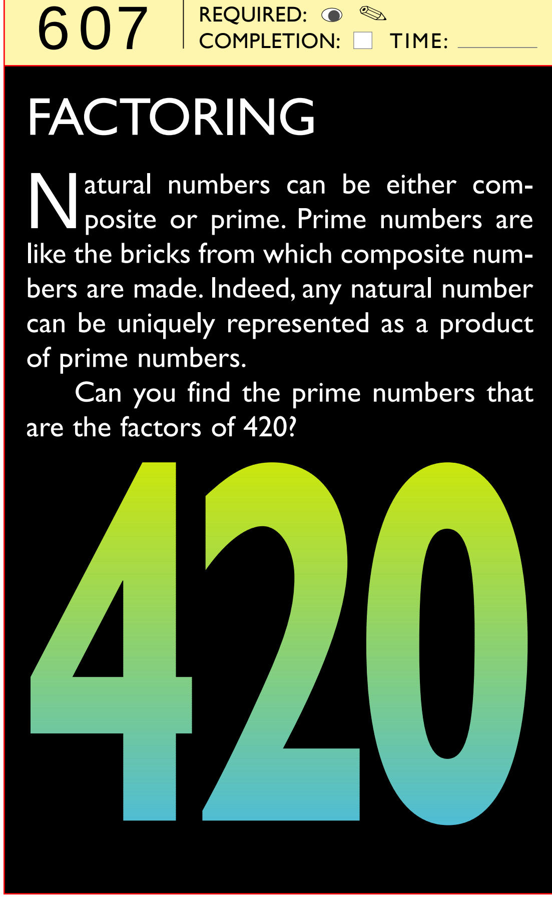
Fractals delineate a whole new way of thinking about structure and form. They show that the world of pure mathematics contains a richness of possibilities that go far beyond the simple structures that earlier mathematicians saw in nature.



PLAYTHINK 606 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: TIME:

AMICABLE NUMBERS

Is it possible for numbers to be not just perfect but friendly or amicable? Examine the numbers 220 and 284. Can you work out the hidden relationship between them?

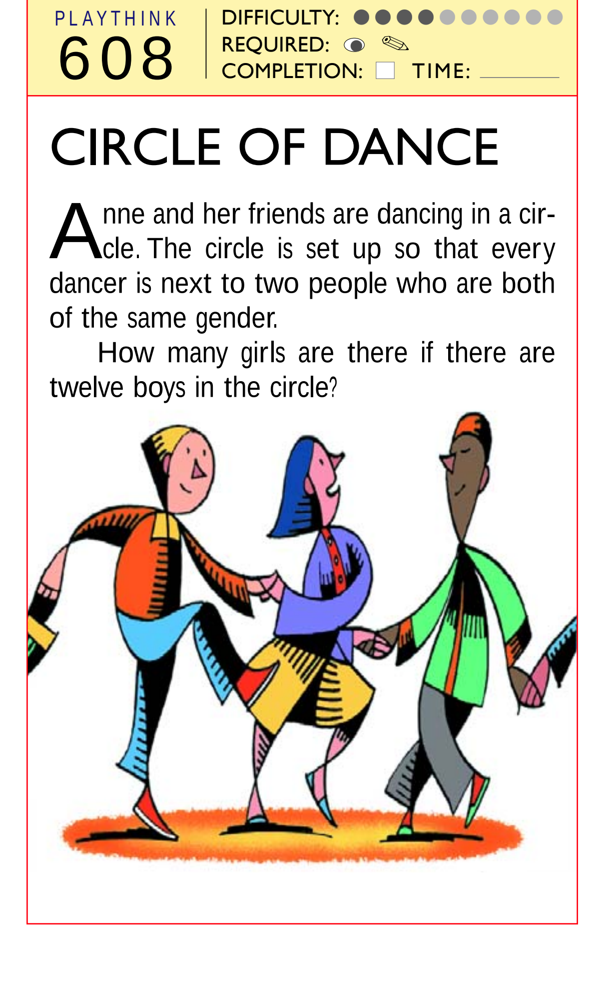


PLAYTHINK 607 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: TIME:

FACTORING

Natural numbers can be either composite or prime. Prime numbers are like the bricks from which composite numbers are made. Indeed, any natural number can be uniquely represented as a product of prime numbers.

Can you find the prime numbers that are the factors of 420?



PLAYTHINK 608 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: TIME:

CIRCLE OF DANCE

Anne and her friends are dancing in a circle. The circle is set up so that every dancer is next to two people who are both of the same gender.

How many girls are there if there are twelve boys in the circle?

Big Numbers

What counts as a really large number? One Indian legend recounts a gift granted by King Shirhan to his vizier, who had just invented the game of chess. The vizier, thinking about the most he could ask for without being presumptuous, said, "Give me a gram of wheat to put on the first square of the chessboard, and two grams on the second square. Continue this doubling for each successive square for all sixty-four squares of the chessboard." The king agreed to the request at once, which was a big mistake. Although the first few squares could be covered easily, the power of doubling soon made the vizier's request impossible. The sequence, called a geometrical progression, runs:

$$1 + 2 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 \ldots$$
$$2^{62} + 2^{63} = 2^{64} - 1$$

The amount requested by his vizier, totaling more than 10 billion billion grams, turned out to be equal to the world's wheat production for a period of some 2,000 years. Although this is an almost unbelievably large number, it is still finite and, theoretically at least, given enough time, one could count it down to the last decimal.

Infinite numbers, on the other hand, are larger than any number you can possibly write down no matter how long you work. Many ideas related to infinite numbers are surprising and counterintuitive; for example, it is possible to compare two infinite sets and determine which set is larger.

The German mathematician Georg Cantor, known as the founder of the arithmetic of infinity, found the answer. Cantor concluded that if one can pair two objects of two infinite groups so that each object of one infinite collection pairs with each object of another infinite collection and no object in either group is left alone, the two infinities are equal. Otherwise, one infinite set is larger than the other.

Applying this rule leads to some surprising results. Compare, for example, the infinity of all even numbers to the infinity of all odd numbers. No problem here: your intuition tells you that there are as many even numbers as there are odd. But what about the infinite set of all whole numbers versus the set of just the even numbers? Surely the set of whole numbers is greater than the set of just the even numbers—after all, the even numbers are contained within the whole numbers. But when one begins to compare the two sets, one finds:

1-2, 2-4, 3-6, 4-8, 5-10,
6-12, 7-14, 8-16 . . .

For every whole number there is an even number. The infinity of even numbers, therefore, is exactly as large as the infinity of all numbers. It is a paradox, but one of the bizarre things about dealing with infinities is that *a part may be equal to the whole*.

Not every infinity is the same. There are many more geometrical points on a line than there are integers or fractional numbers because it is impossible to establish a one-to-one correspondence between the points on a line and the integer numbers. But it also follows that the same number of points are in lines 1 inch, 1 foot, or 1 mile long. But the number of all geometrical points, though larger than the number of all integers and fractional numbers, is not the largest infinity known to mathematicians; the number of geometrical curves is greater than the collection of all geometrical points on a line.

Cantor denoted the different infinities by the Hebrew letter aleph (ℵ), and the complete sequence of all numbers today looks like:

$\aleph_1$ integers and fractional numbers

$\aleph_2$ points on a line

$\aleph_3$ different geometric curves

The Tower of Hanoi

The Babylon puzzle (PlayThink 609) is a variation of one of the most beautiful puzzles ever created: the Tower of Hanoi. Created by the French mathematician Edouard Lucas in 1883, the puzzle is framed within a legend. At a great temple at Benares, there is a brass plate into which three vertical pins are fixed. At the beginning of time, sixty-four golden disks were stacked on one pin in decreasing order of size, with the largest resting at the bottom of the brass plate. Day and night, so the legend goes, a priest transfers the disks from one pin to another at a constant rate, never allowing any disk to be placed on top of a smaller one. Once the tower is rebuilt on one of the other two pins, the universe will end.

Even if the legend were true, there would be no reason to worry. Allowing one second per move of a disk, the task would take about 600 billion years, or about sixty times longer than the lifetime of the sun.

The number of moves necessary to complete a Tower of Hanoi of a given number of disks can be calculated as $2^n - 1$. So two disks require three moves, three disks require seven and so on.

PLAYTHINK 609 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: | COMPLETION: ☐ | TIME: ____

BABYLON

This puzzle is a variation of the classic Tower of Hanoi. You can play it on several different levels of difficulty and with variant sets of rules.

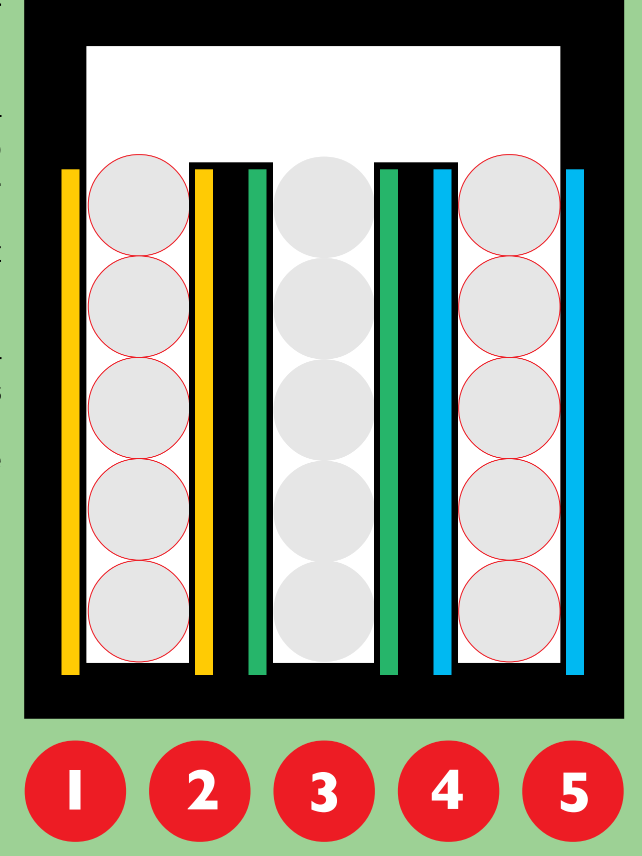
The puzzle begins with a stack of disks in the left-hand column, as shown in the insets below. Your objective in each puzzle is to transfer the disks to the right-hand column, keeping the same numerical order.

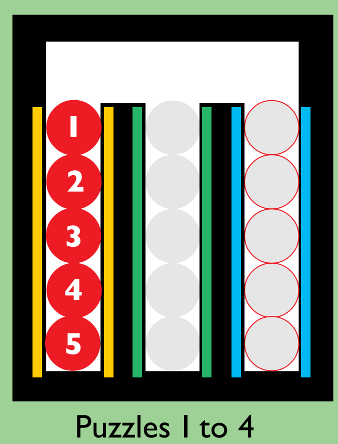
The basic rule is, don't place a disk on another disk of smaller value. Otherwise, shuttle the disks, one at a time, among the three columns until you have the proper arrangement in the right-hand column.

Puzzles 1, 2, 3 and 4 (see first diagram below left)—Find the minimum number of moves to transfer 2, 3, 4 and 5 disks, respectively, to the right-hand column.

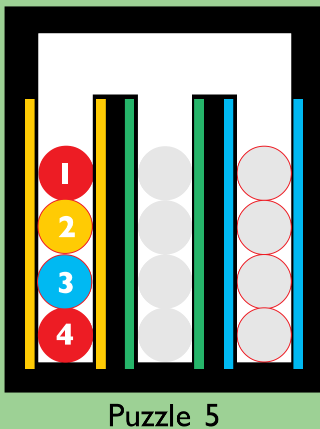
Puzzle 5 (second diagram below)—Find the minimum number of moves to transfer the four disks, observing an additional rule that a disk cannot be placed on another disk of the same color. That means that disk 1 cannot be placed on disk 4.

Puzzle 6 (third diagram below)—Find the minimum number of moves to transfer the four disks, observing an additional rule that a disk cannot be placed on another disk of the same color. That means that disk 1 cannot be placed on disk 3, and disk 2 cannot be placed on disk 4.

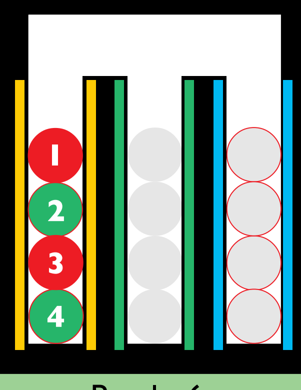




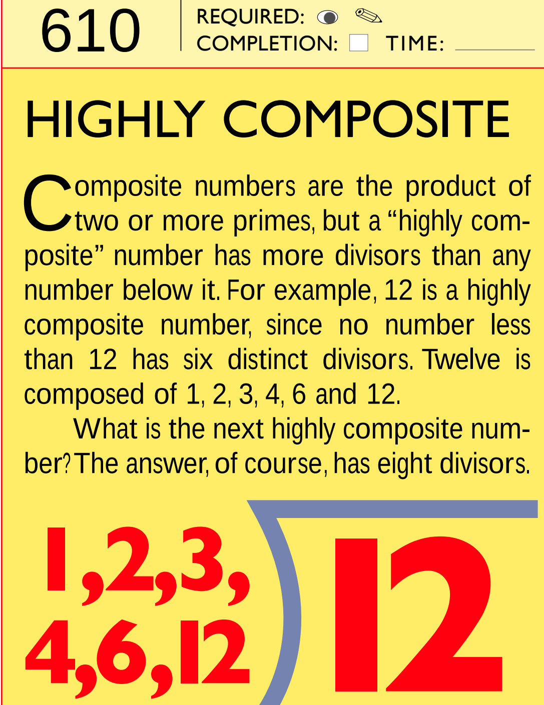
Puzzles 1 to 4



Puzzle 5



Puzzle 6

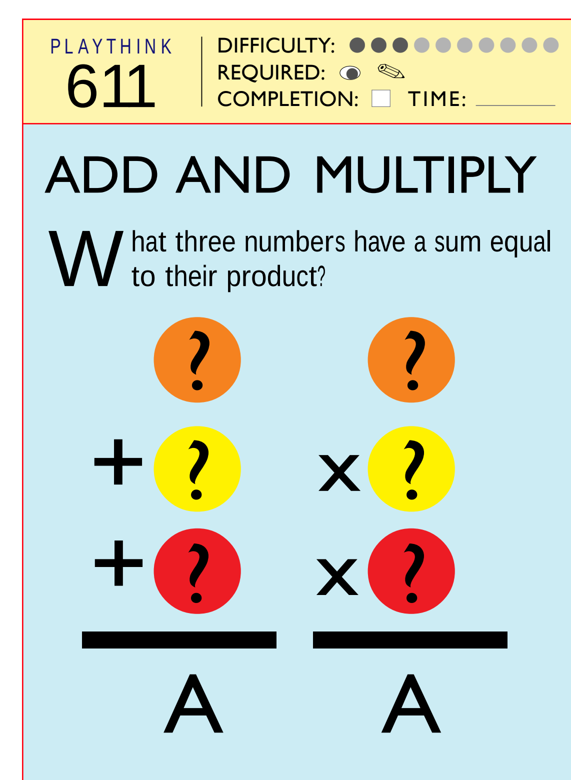


PLAYTHINK 610 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

HIGHLY COMPOSITE

Composite numbers are the product of two or more primes, but a "highly composite" number has more divisors than any number below it. For example, 12 is a highly composite number, since no number less than 12 has six distinct divisors. Twelve is composed of 1, 2, 3, 4, 6 and 12.

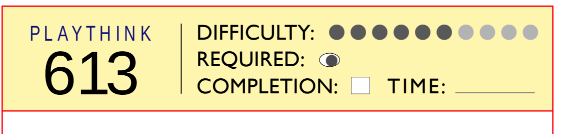
What is the next highly composite number? The answer, of course, has eight divisors.



PLAYTHINK 611 | DIFFICULTY: ●●●○○○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

ADD AND MULTIPLY

What three numbers have a sum equal to their product?



PLAYTHINK 613 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

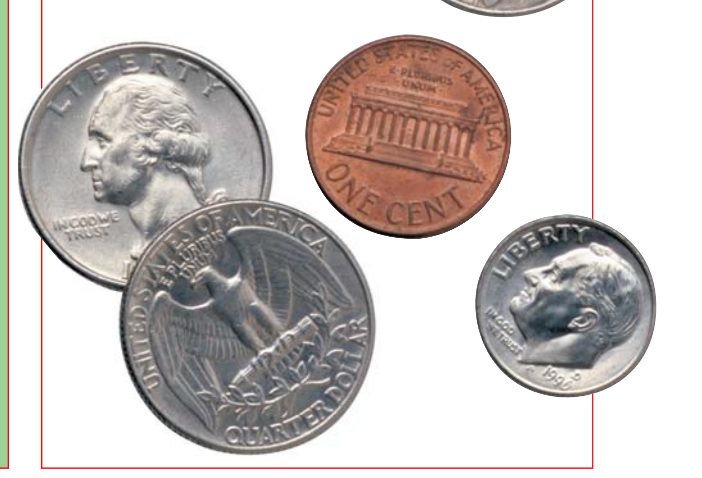
HIDDEN MAGIC COIN

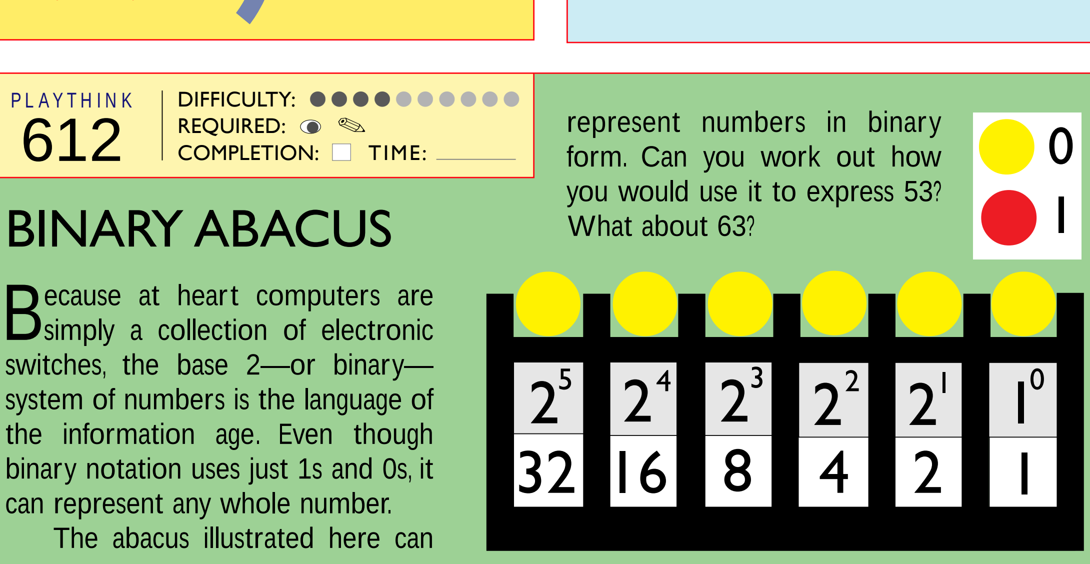
One of the most beautiful coin tricks is often explained as a feat of extrasensory perception. But it is really an example of the mathematical concept of parity.

Ask someone to toss a handful of coins on a table. After a quick peek at the result, turn your back and ask the person to turn over pairs of coins at random—as many pairs as he or she likes. Then ask the person to cover up one coin.

When you turn around, you can tell immediately whether the covered coin is showing heads or tails.

Can you work out the mathematical secret at the heart of this trick?





PLAYTHINK 612 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

BINARY ABACUS

Because at heart computers are simply a collection of electronic switches, the base 2—or binary—system of numbers is the language of the information age. Even though binary notation uses just 1s and 0s, it can represent any whole number.

The abacus illustrated here can represent numbers in binary form. Can you work out how you would use it to express 53? What about 63?

Bits and Computers

For all their prowess at making calculations and controlling machinery, computers are essentially little more than a collection of switches. Each of the thousands of electronic circuits in a computer can switch on and off incredibly fast. When a pulse of electricity flows through a circuit, it is on; when no electricity flows, it is off. Circuits that are on have a value of 1; circuits that are off have a value of 0.

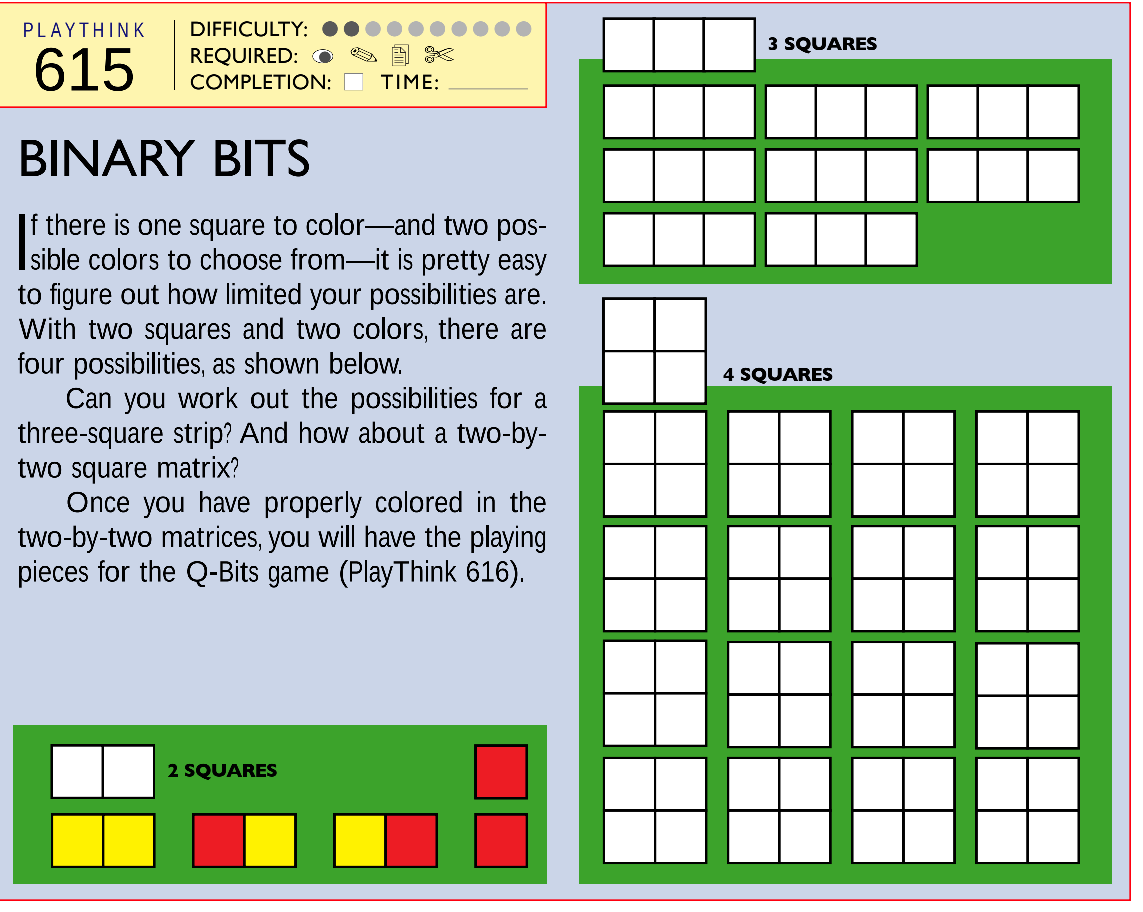
The digits 1 and 0 are the basis of the binary number system used in computers. Each number is called a bit, short for binary digit. Computers usually deal with strings of eight or sixteen bits at a time. (A group of eight bits is a byte.)

Four switches can be set in 2^4, or 16, different ways. Those four switches can be represented by the cells of a two-by-two square, with the "on" switches colored red and the "off" switches colored yellow. Running through all the binary possibilities will give you a set of sixteen tiles with which you can play games and puzzles.



BINARY GRIDS

An important message is hidden in the four square grids above. Can you use the clues in the sample (right) and other information on the page to find the hidden message?

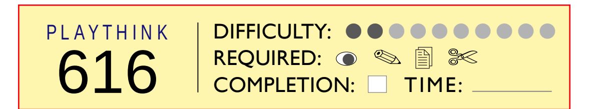


BINARY BITS

If there is one square to color—and two possible colors to choose from—it is pretty easy to figure out how limited your possibilities are. With two squares and two colors, there are four possibilities, as shown below.

Can you work out the possibilities for a three-square strip? And how about a two-by-two square matrix?

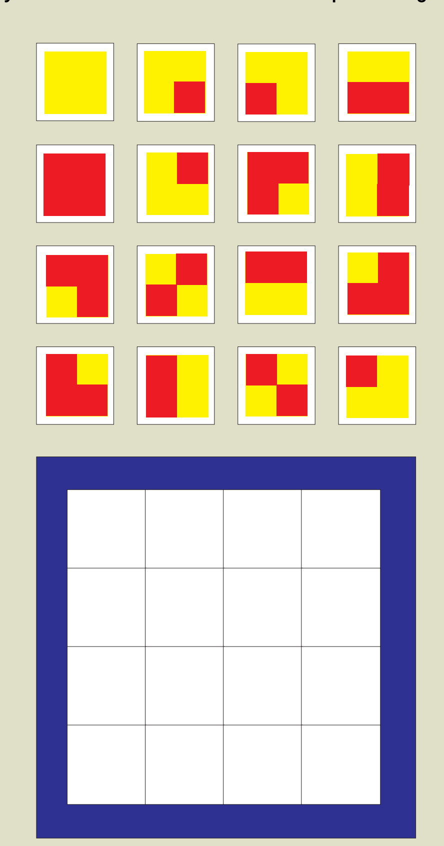
Once you have properly colored in the two-by-two matrices, you will have the playing pieces for the Q-Bits game (PlayThink 616).



Q-BITS

There are many different ways to arrange sixteen tiles on a four-by-four grid. But is it possible to do it in such a way that the colors of adjacent tiles will match along every edge?

That is the goal of this puzzle, which can be played as either a solitaire or competitive game.



To play as a solitaire game, cover the board with all sixteen tiles from PlayThinks 615 according to the domino principle: with all touching edges matching. How many different solutions can you find? If you copy your solutions onto grid paper, you will see that some of them have a strong aesthetic appeal.

To play as a two-person game, start by mixing the tiles face down. The players take turns selecting a tile and placing it on the board. As in the solitaire game, any tiles that touch must possess matching colors along their edges. The last player who can place a tile according to the rules wins.

The longest game is sixteen moves and will fill up the board. Can you find the shortest possible game, that is, the fewest tiles needed to block further moves?

PLAYTHINK 617

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

HEXABITS 1

If you divide a hexagon with lines drawn between its vertices, you can fill in the alternating regions with two different colors, as shown. Discounting rotations but accepting reflections as different, there are nineteen unique patterns that can be created in this manner. You have been given seventeen—can you find the other two?

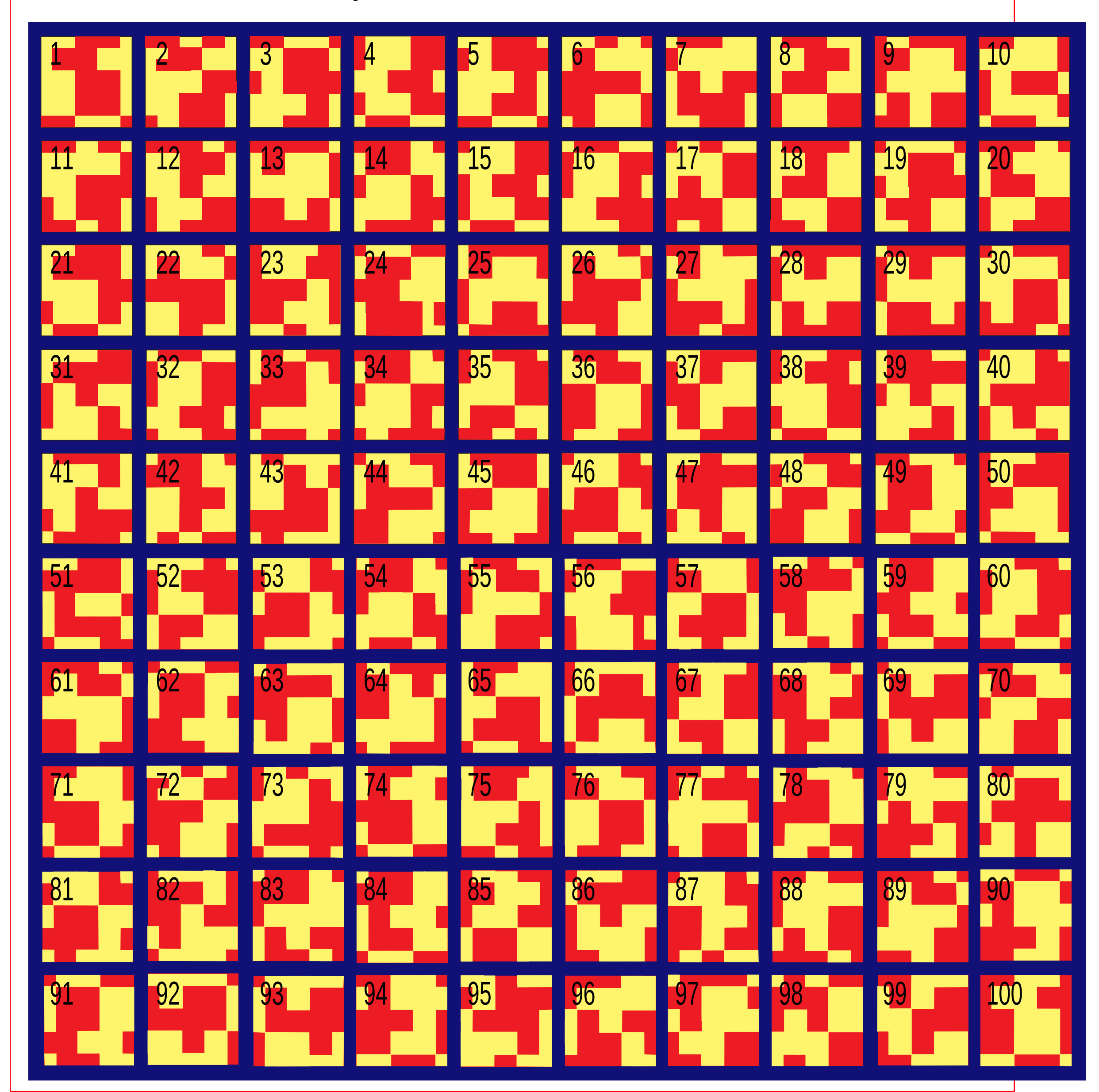
PLAYTHINK 618

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

POSI-NEGA Q-BITS

The fifty solutions to the Q-Bits puzzle are shown below, along with their color-reversed duplicates. The tiles are numbered 1 to 10 on the first row, 11 to 20 on the second row and so on. As you can see, tiles 1 and 100 are a color-reversed pair.

How long will it take you to match all fifty more pairs?



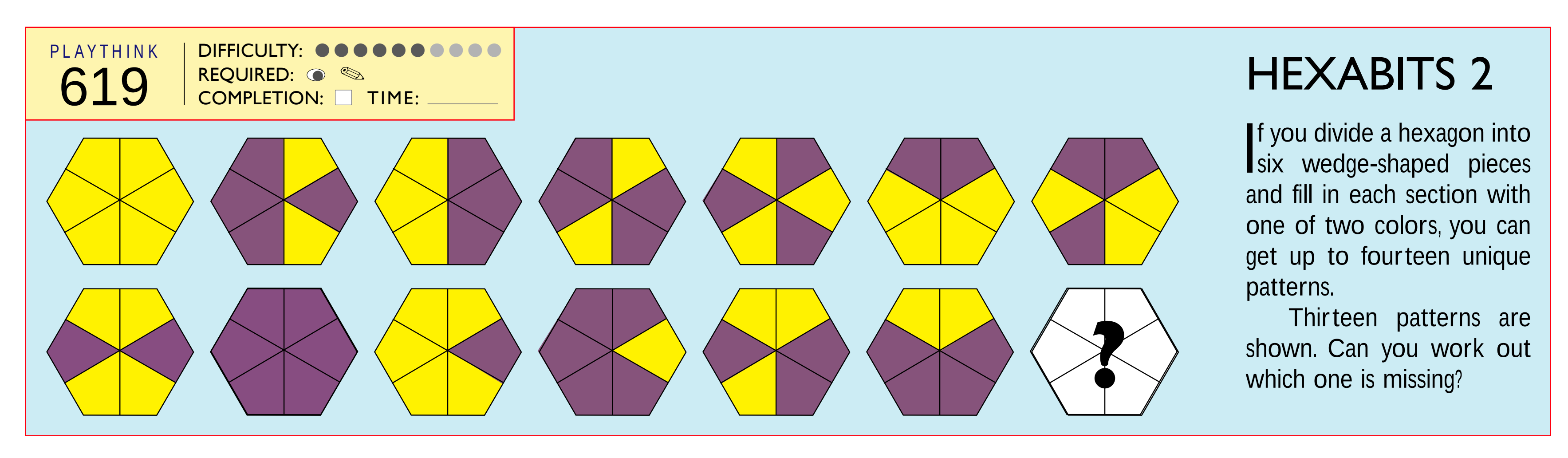
PLAYTHINK 619

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

HEXABITS 2

If you divide a hexagon into six wedge-shaped pieces and fill in each section with one of two colors, you can get up to fourteen unique patterns.

Thirteen patterns are shown. Can you work out which one is missing?



PLAYTHINK 620

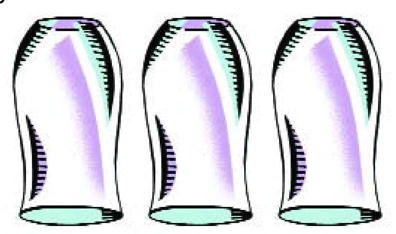
DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

THREE GLASSES TRICK



Place three glasses on a table, as shown above. Your goal is to bring all three glasses to the upright position in exactly three moves, turning over two glasses at a time. A quick examination will reveal that this is easy to do—in fact, it can be done after any number of moves.

Once you succeed, turn all three glasses over to the inverted position, as shown below. Then challenge your friends to duplicate your feat.



PLAYTHINK 621

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

SIX GLASSES PROBLEM

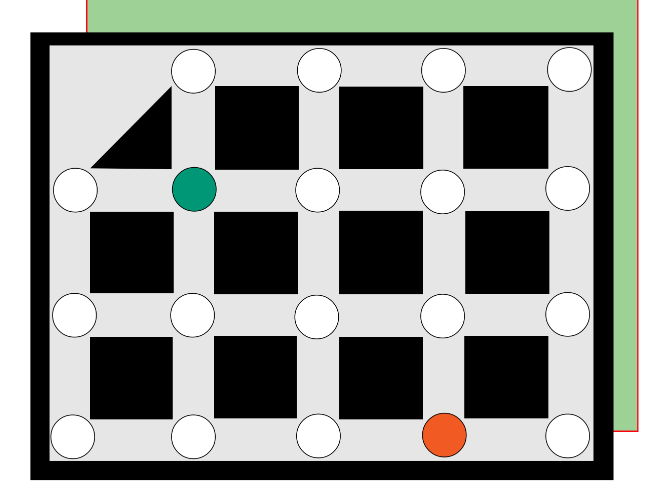
Place six glasses on a table, as shown. Take any pair and invert them. If you continue to invert pairs for as long as you like, will you ever end up with all six glasses upright? How about all six glasses upside-down?

PLAYTHINK 622

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

POLICE CHASE

In this game the policeman (the green dot) chases the thief (the red dot). They alternate moves, going from circle to adjacent circle. The policeman catches the thief if, in his move, he can place his green dot on the red dot. Can the policeman catch the thief in fewer than ten moves?

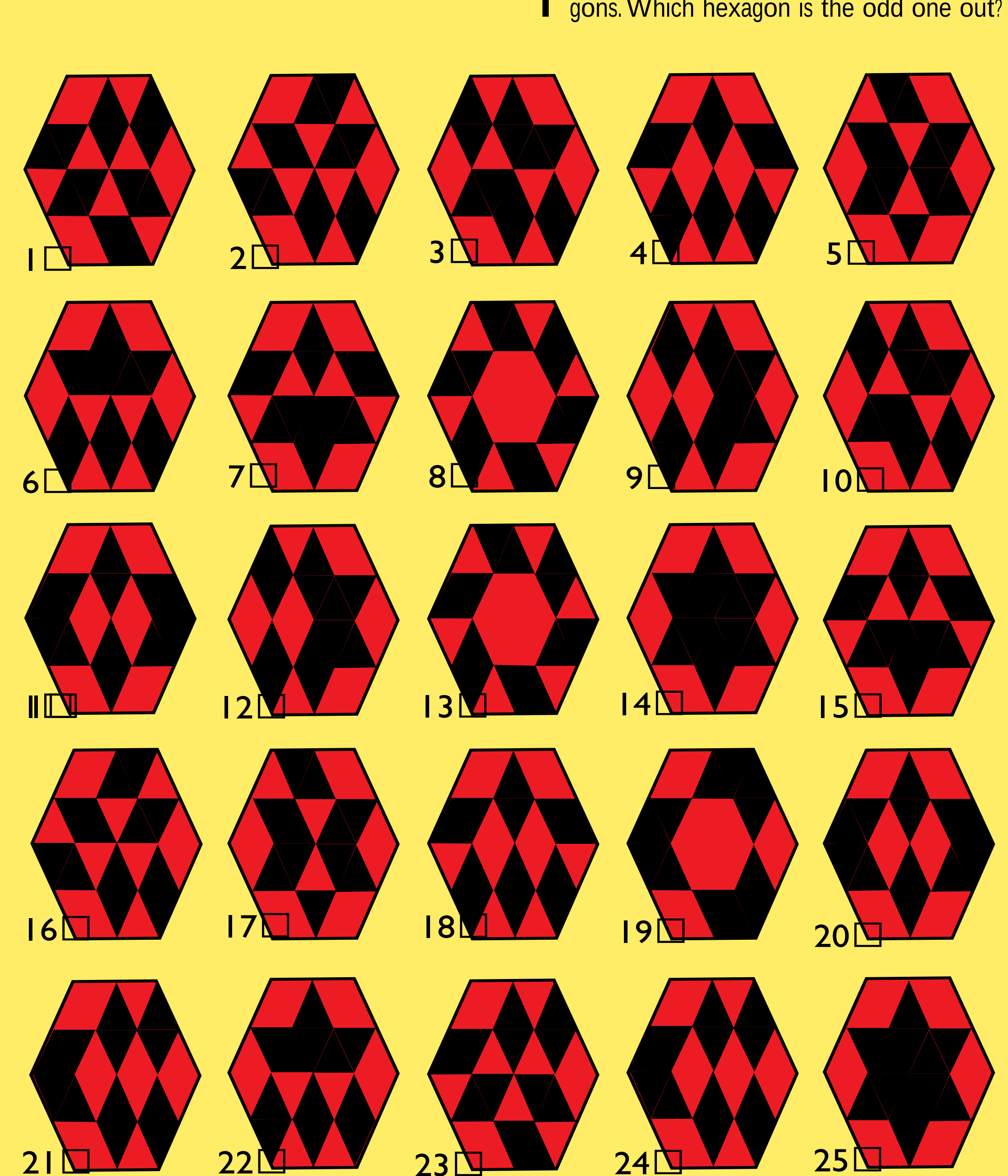


PLAYTHINK 623

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

PAIRING HEXAGONS

There are twelve pairs of identical hexagons. Which hexagon is the odd one out?

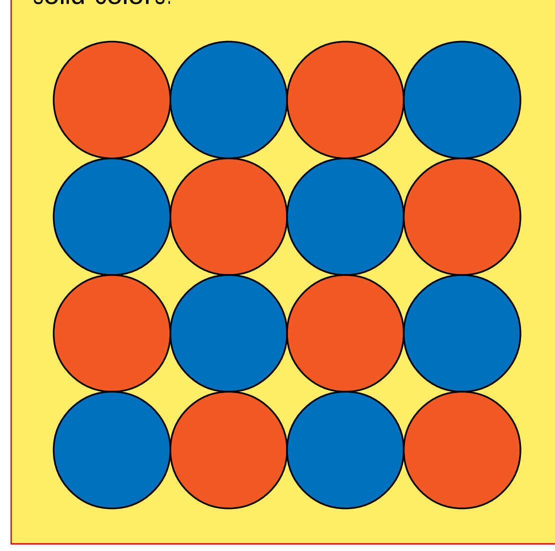


PLAYTHINK 624 DIFFICULTY: ●●●●●○○○○○ REQUIRED: 👁 COMPLETION: ☐ TIME: ____

POKER CHIPS PATTERN

Sixteen chips lie on a table in an alternating pattern, as shown.

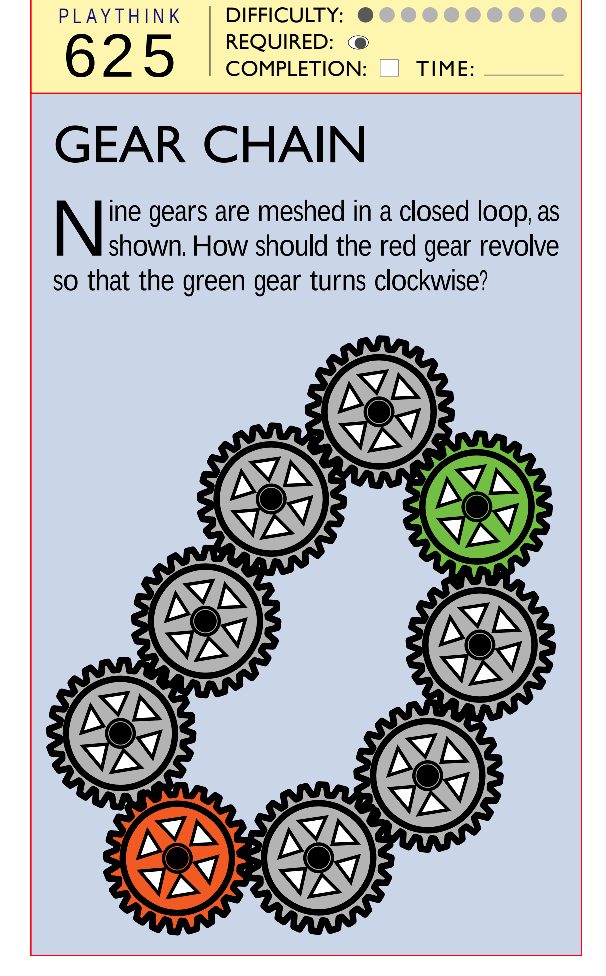
If you are allowed to slide only two chips into new positions, can you find a way to turn the pattern into horizontal rows of solid colors?



PLAYTHINK 625 DIFFICULTY: ●○○○○○○○○○ REQUIRED: 👁 COMPLETION: ☐ TIME: ____

GEAR CHAIN

Nine gears are meshed in a closed loop, as shown. How should the red gear revolve so that the green gear turns clockwise?

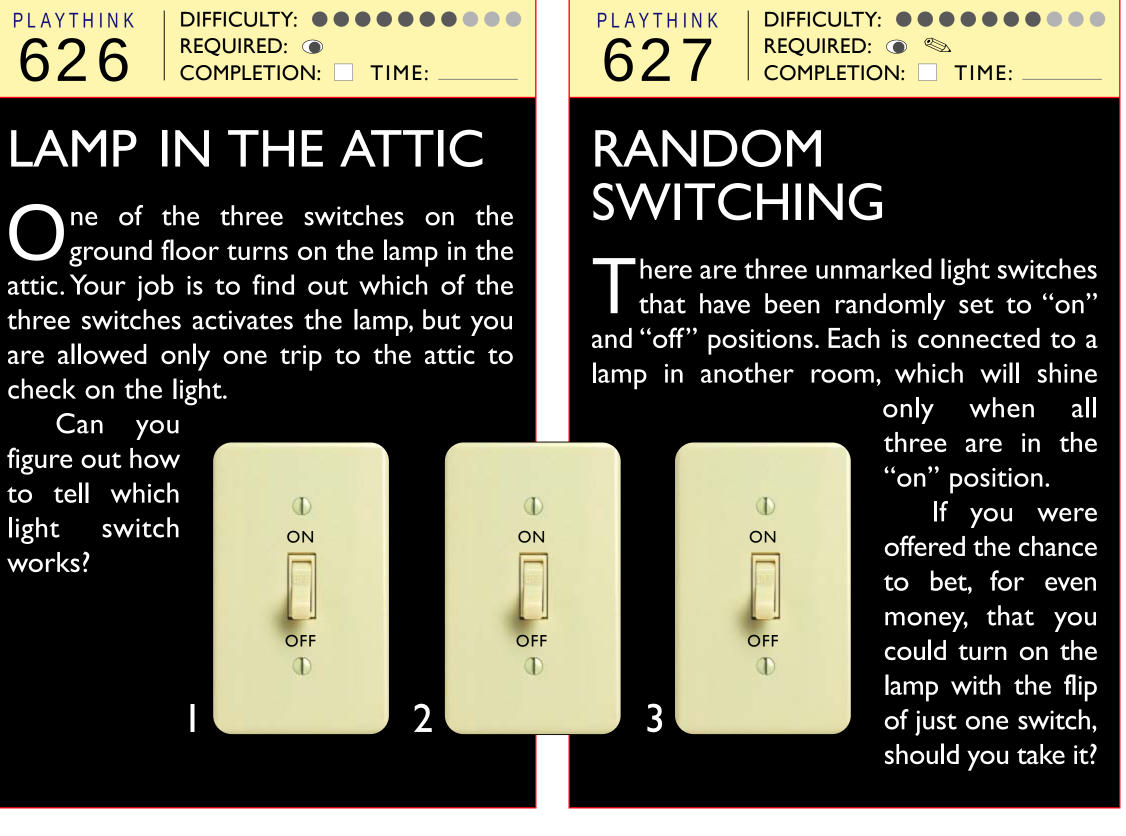


PLAYTHINK 626 DIFFICULTY: ●●●●●●●○○○ REQUIRED: 👁 COMPLETION: ☐ TIME: ____

LAMP IN THE ATTIC

One of the three switches on the ground floor turns on the lamp in the attic. Your job is to find out which of the three switches activates the lamp, but you are allowed only one trip to the attic to check on the light.

Can you figure out how to tell which light switch works?



PLAYTHINK 627 DIFFICULTY: ●●●●●●●○○○ REQUIRED: 👁 ✎ COMPLETION: ☐ TIME: ____

RANDOM SWITCHING

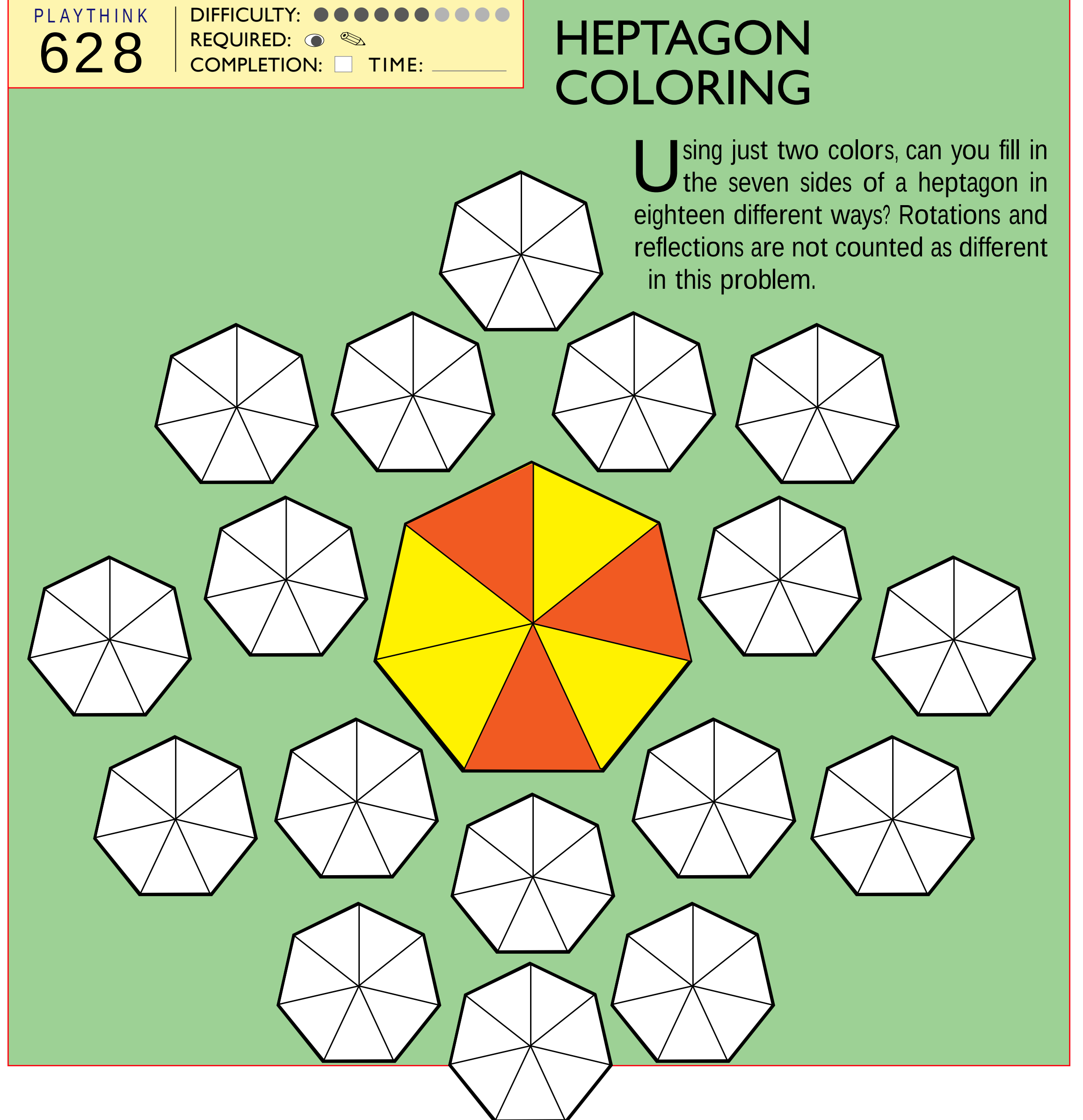
There are three unmarked light switches that have been randomly set to "on" and "off" positions. Each is connected to a lamp in another room, which will shine only when all three are in the "on" position.

If you were offered the chance to bet, for even money, that you could turn on the lamp with the flip of just one switch, should you take it?

PLAYTHINK 628 DIFFICULTY: ●●●●●●○○○○ REQUIRED: 👁 ✎ COMPLETION: ☐ TIME: ____

HEPTAGON COLORING

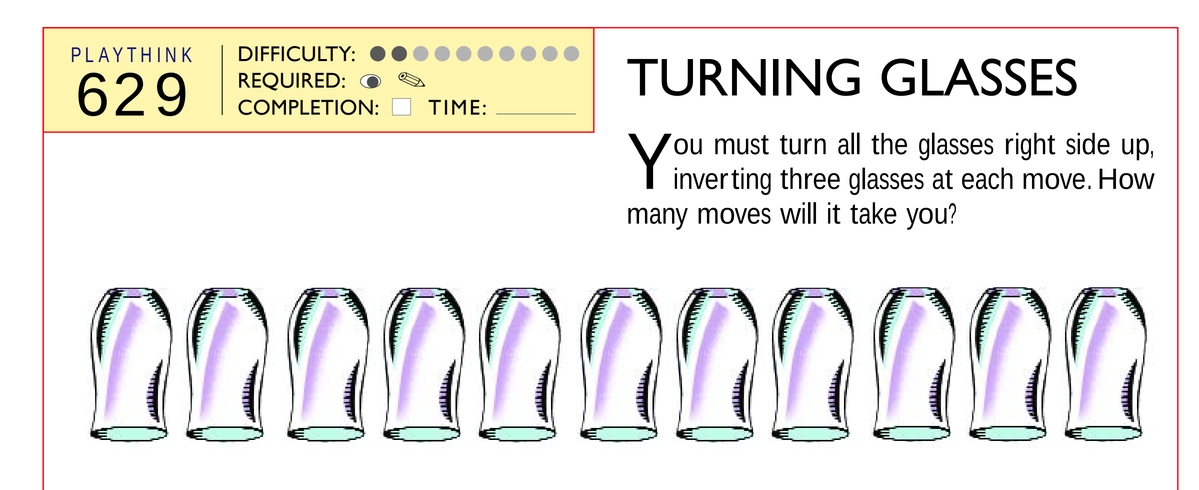
Using just two colors, can you fill in the seven sides of a heptagon in eighteen different ways? Rotations and reflections are not counted as different in this problem.



PLAYTHINK 629
DIFFICULTY: ●●○○○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

TURNING GLASSES

You must turn all the glasses right side up, inverting three glasses at each move. How many moves will it take you?

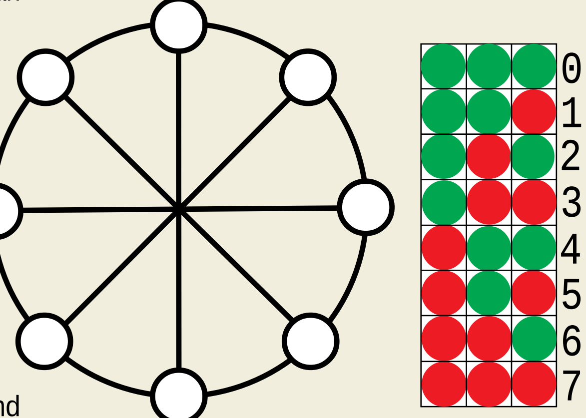


PLAYTHINK 630
DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

BINARY OR MEMORY WHEEL 1

All the possible triplets of digits 1 and 0 can be embodied in three switches, which may be in either the "on" or "off" position. These triplets represent the first eight numbers (including 0) of the binary numbering system. It is interesting to note that, altogether, twenty-four switches are needed to express the first eight digits simultaneously, as shown at right.

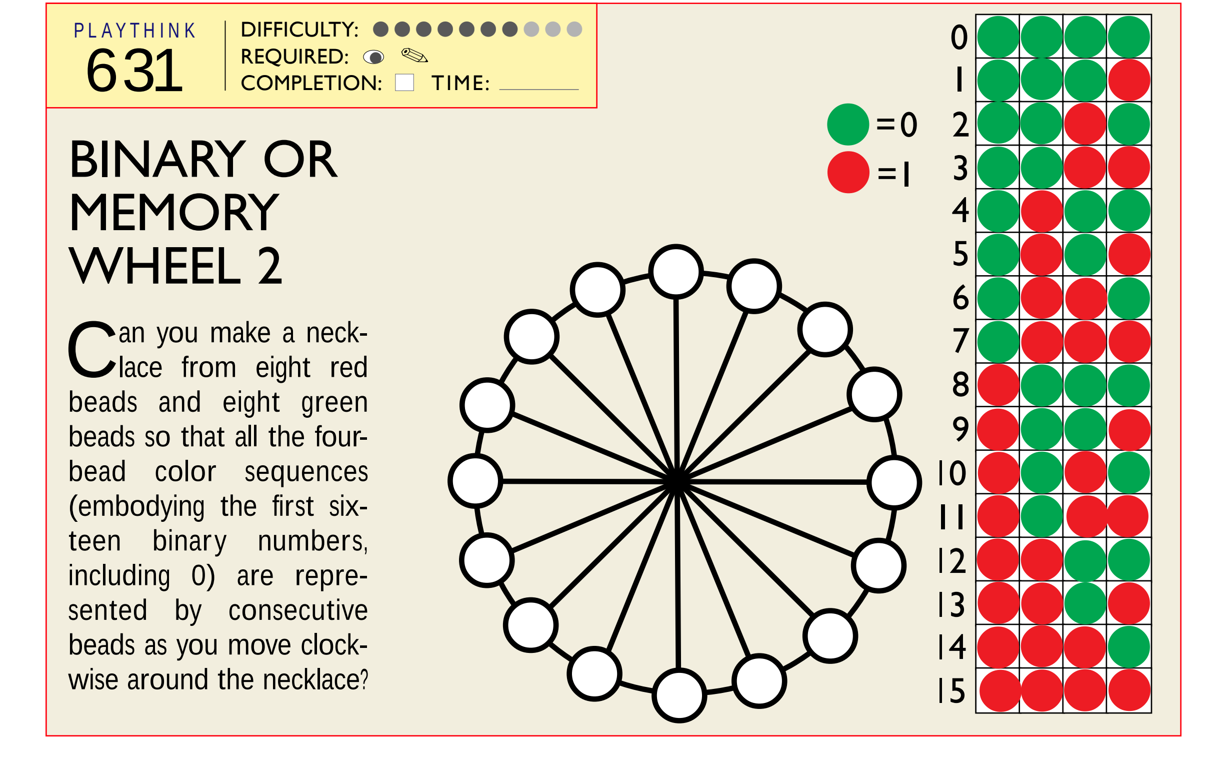
In the "binary" or "memory" wheel, the same amount of information can be condensed to just eight switches. To show how, examine the necklace outline. Can you find a way to use four red and four green beads in such a way that all eight triplets will be represented by consecutive beads as you go around the necklace clockwise? Although the beads in the triplet must be consecutive, each triplet need not be next to the other.



PLAYTHINK 631
DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

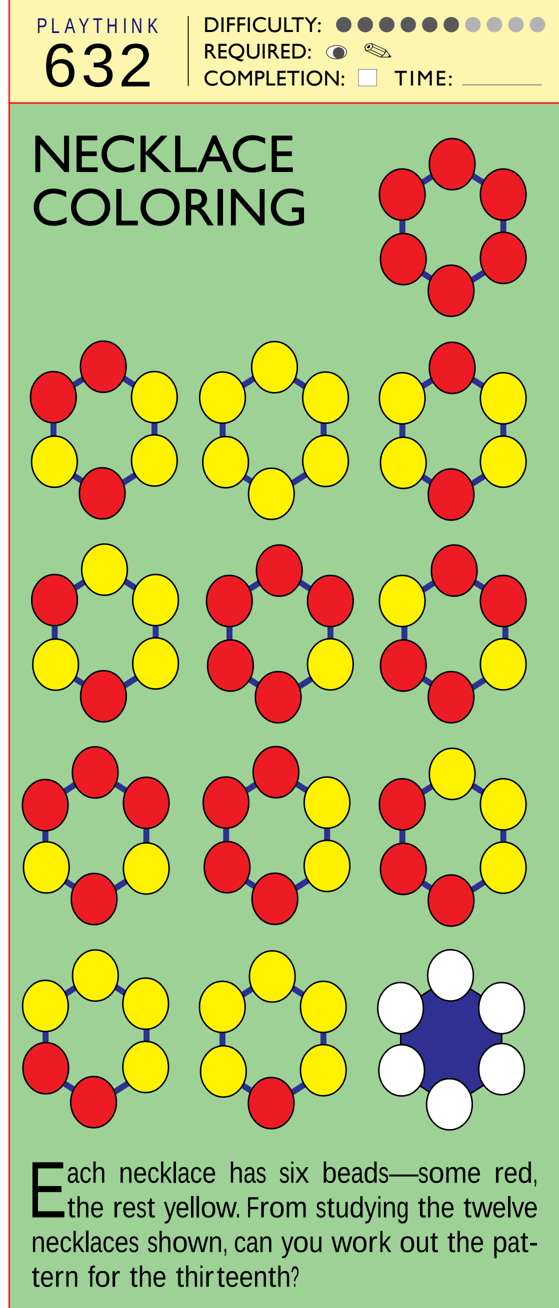
BINARY OR MEMORY WHEEL 2

Can you make a necklace from eight red beads and eight green beads so that all the four-bead color sequences (embodying the first sixteen binary numbers, including 0) are represented by consecutive beads as you move clockwise around the necklace?



PLAYTHINK 632
DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

NECKLACE COLORING

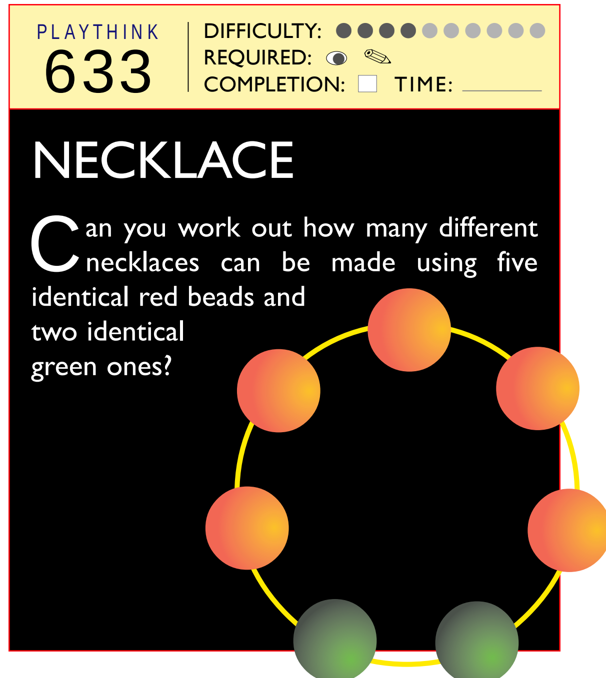


Each necklace has six beads—some red, the rest yellow. From studying the twelve necklaces shown, can you work out the pattern for the thirteenth?

PLAYTHINK 633
DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

NECKLACE

Can you work out how many different necklaces can be made using five identical red beads and two identical green ones?





10

Logic and Probability

PLAYTHINK 634 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

HIERARCHY

In logic the basic form of reasoning is deduction, in which a specific conclusion is reached based on one or more premises. The conclusion must be true if all the premises are true.

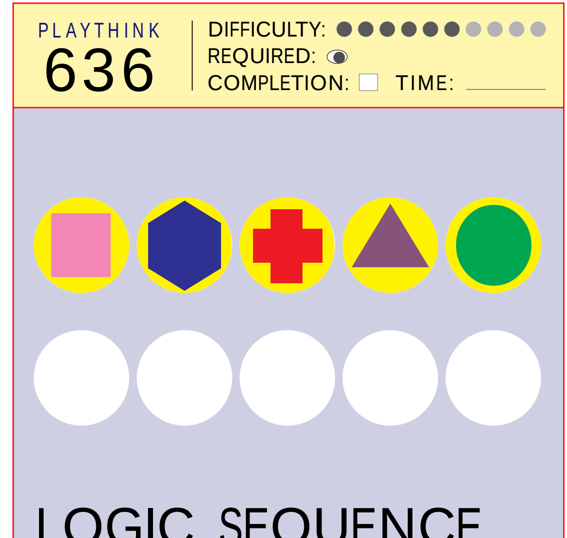
Here is a classic deduction problem that will show you how this works.

In a certain company the positions of chairman, director and secretary are held by Gerry, Anita and Rose—but not necessarily in that order. The secretary, who is an only child, earns the least. And Rose, who is married to Gerry's brother, earns more than the director.

From that information, can you work out who does what?



PLAYTHINK 636 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____



LOGIC SEQUENCE

The lower row of shapes, which is hidden, is in a different sequence than the top row. The hidden row does, however, conform to the following rules:

- Neither the cross nor the circle is next to the hexagon.
- Neither the cross nor the circle is next to the triangle.
- Neither the circle nor the hexagon is next to the square.
- The triangle is just to the right of the square.

Can you work out the hidden sequence?

PLAYTHINK 635 | DIFFICULTY: ●●○○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

PARROT

Madame M. decided to buy a parrot to keep her company. But she wanted one that talked. "Does this parrot speak?" Madame M. asked the pet store clerk .

The clerk was unequivocal. "This parrot," he said, "repeats every word he hears."

That convinced Madame M. to buy the bird. But after months of trying to teach the parrot to speak, she never heard a word out of him.

Was the clerk lying? Or is there a piece of important information he left out?



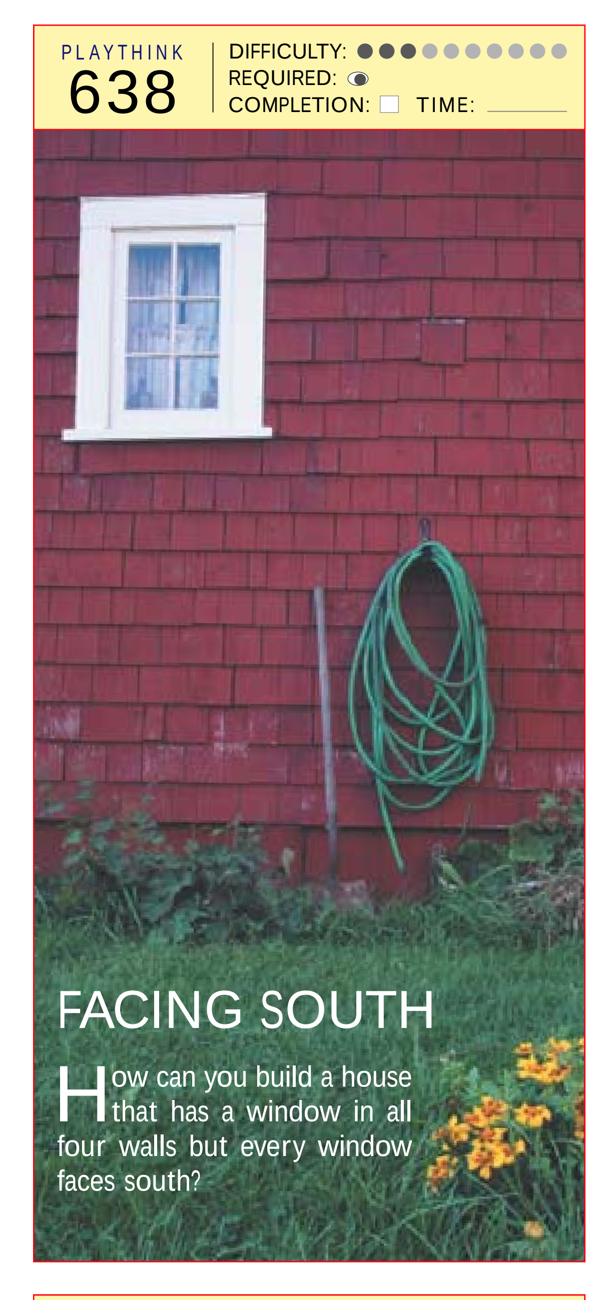
PLAYTHINK 637 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

GIRL-GIRL

Mr. and Mrs. Smith have two children, and they tell you that at least one of them is a girl. Assuming that boys and girls are equally likely, what is the probability that their other child is a girl?



PLAYTHINK 638 | DIFFICULTY: ●●●○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____



FACING SOUTH

How can you build a house that has a window in all four walls but every window faces south?

PLAYTHINK 639 | DIFFICULTY: ●●○○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

GHOTI

The word below may seem odd, but it is pronounced just like a common English word. Pronounce the *gh* as in "tough," the *o* as in "women" and the *ti* as in "emotion."

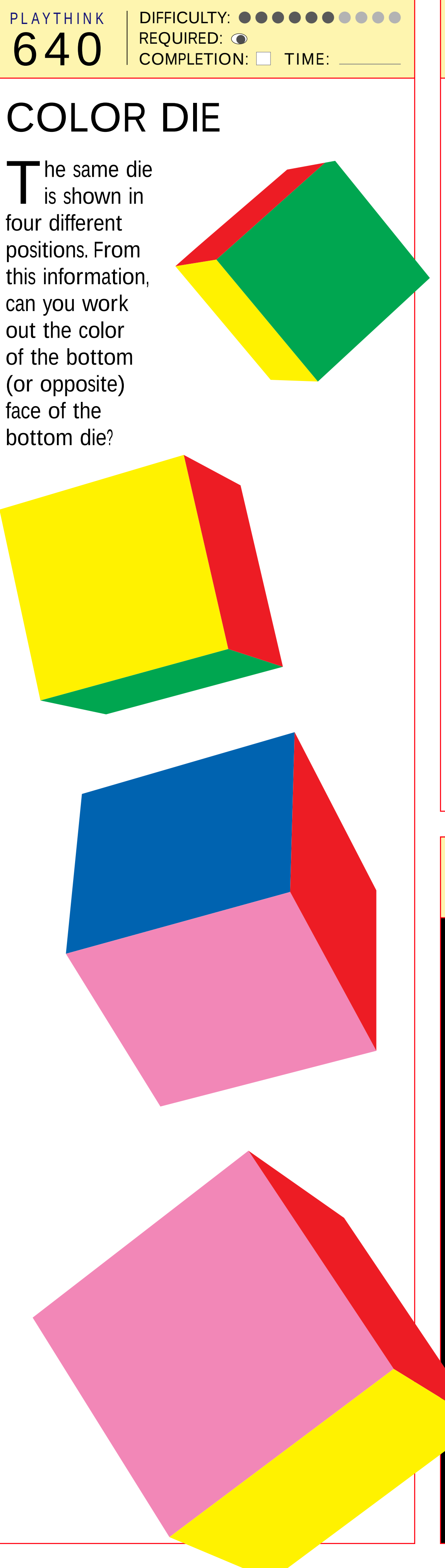
What, then, is the common word that "ghoti" sounds like?

GHOTI

PLAYTHINK 640 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

COLOR DIE

The same die is shown in four different positions. From this information, can you work out the color of the bottom (or opposite) face of the bottom die?



PLAYTHINK 641 | DIFFICULTY: ●●●○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

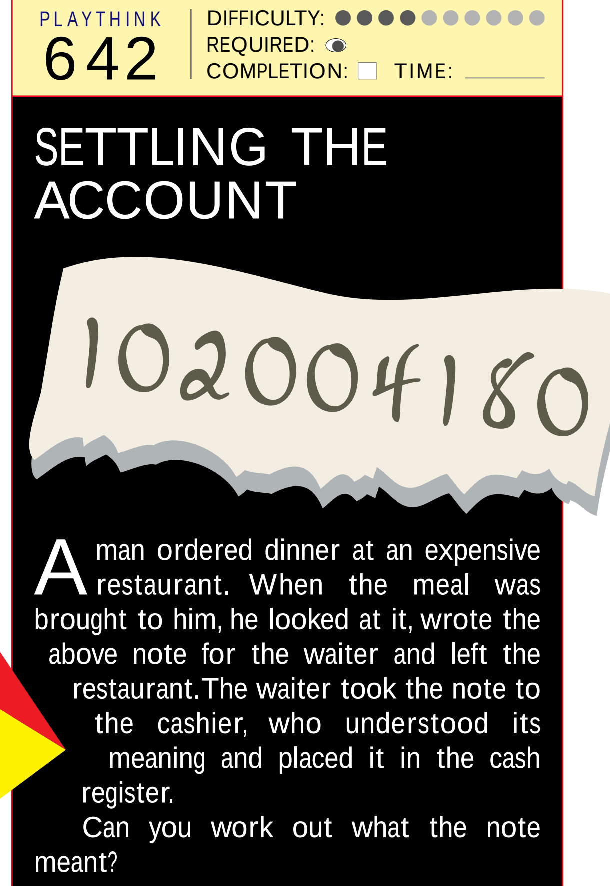
MARRIAGE

Many years ago a man married the sister of his widow. How did he do it?



PLAYTHINK 642 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

SETTLING THE ACCOUNT



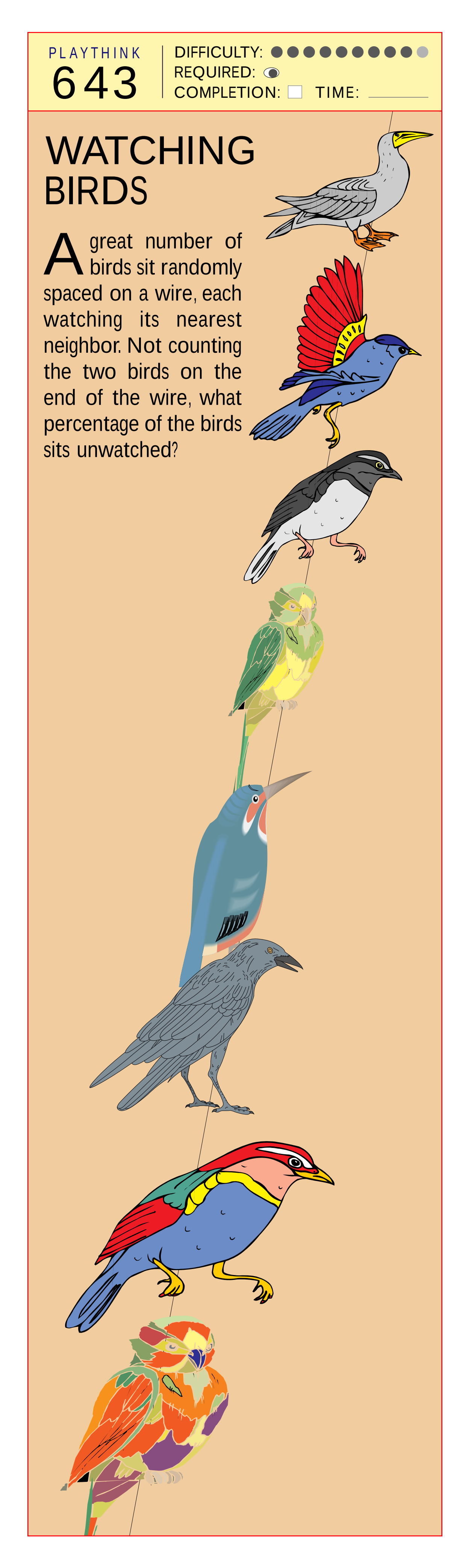
A man ordered dinner at an expensive restaurant. When the meal was brought to him, he looked at it, wrote the above note for the waiter and left the restaurant. The waiter took the note to the cashier, who understood its meaning and placed it in the cash register.

Can you work out what the note meant?

PLAYTHINK 643

DIFFICULTY: ●●●●●●●●●○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

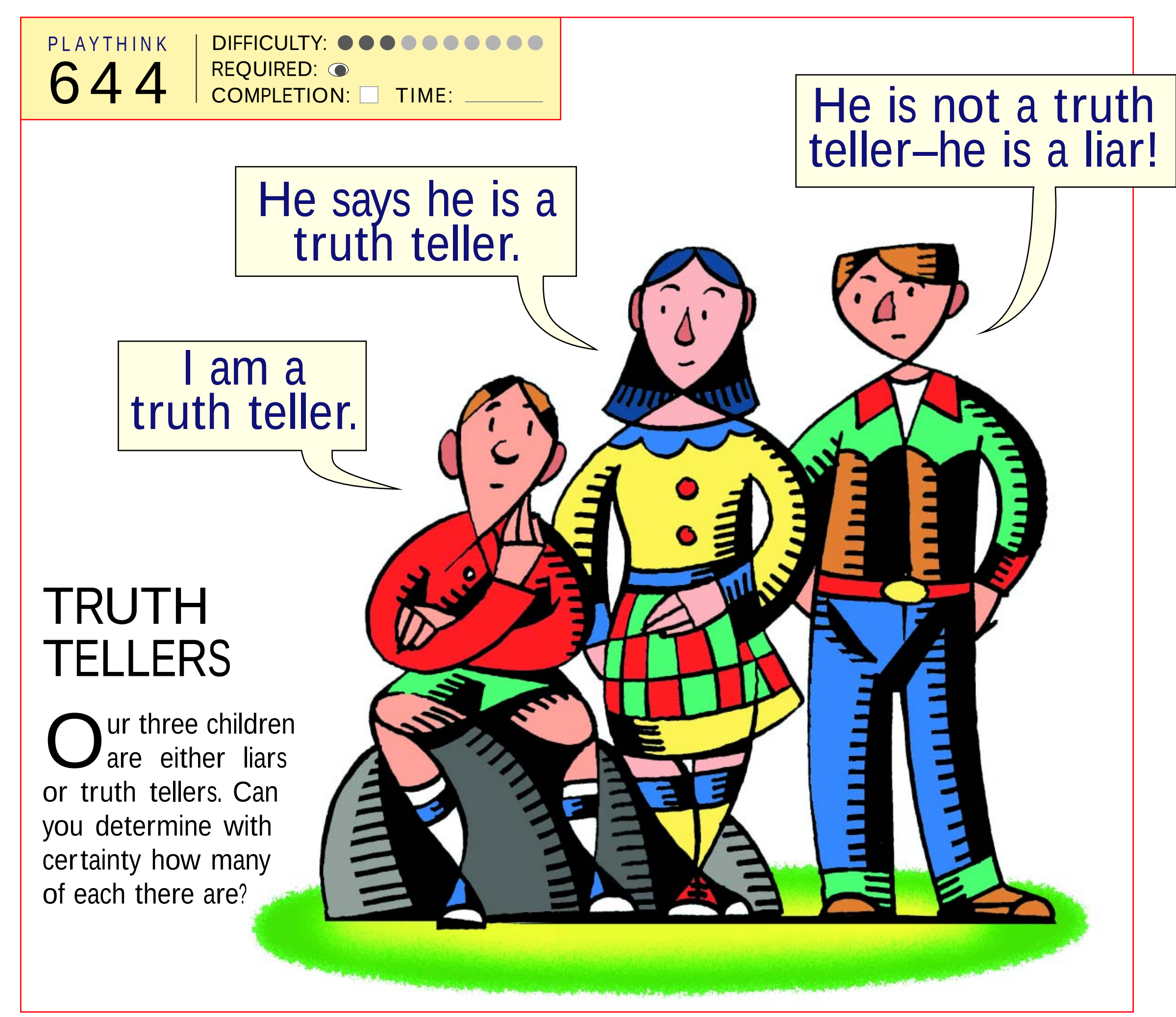
WATCHING BIRDS



A great number of birds sit randomly spaced on a wire, each watching its nearest neighbor. Not counting the two birds on the end of the wire, what percentage of the birds sits unwatched?

PLAYTHINK 644

DIFFICULTY: ●●●○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____



TRUTH TELLERS

Our three children are either liars or truth tellers. Can you determine with certainty how many of each there are?

PLAYTHINK 645

DIFFICULTY: ●○○○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

HORSE RACE



The will of an eccentric old man stipulated that his two heirs were to stage a horse race and the owner of the losing horse would receive the entire inheritance. At the appointed hour, the race took place, but both heirs kept their horses from crossing the finish line. To break the stalemate, the executor of the will thought up a slight change to the race. Following the executor's idea, the two heirs raced again, and the one who finished first won the inheritance.

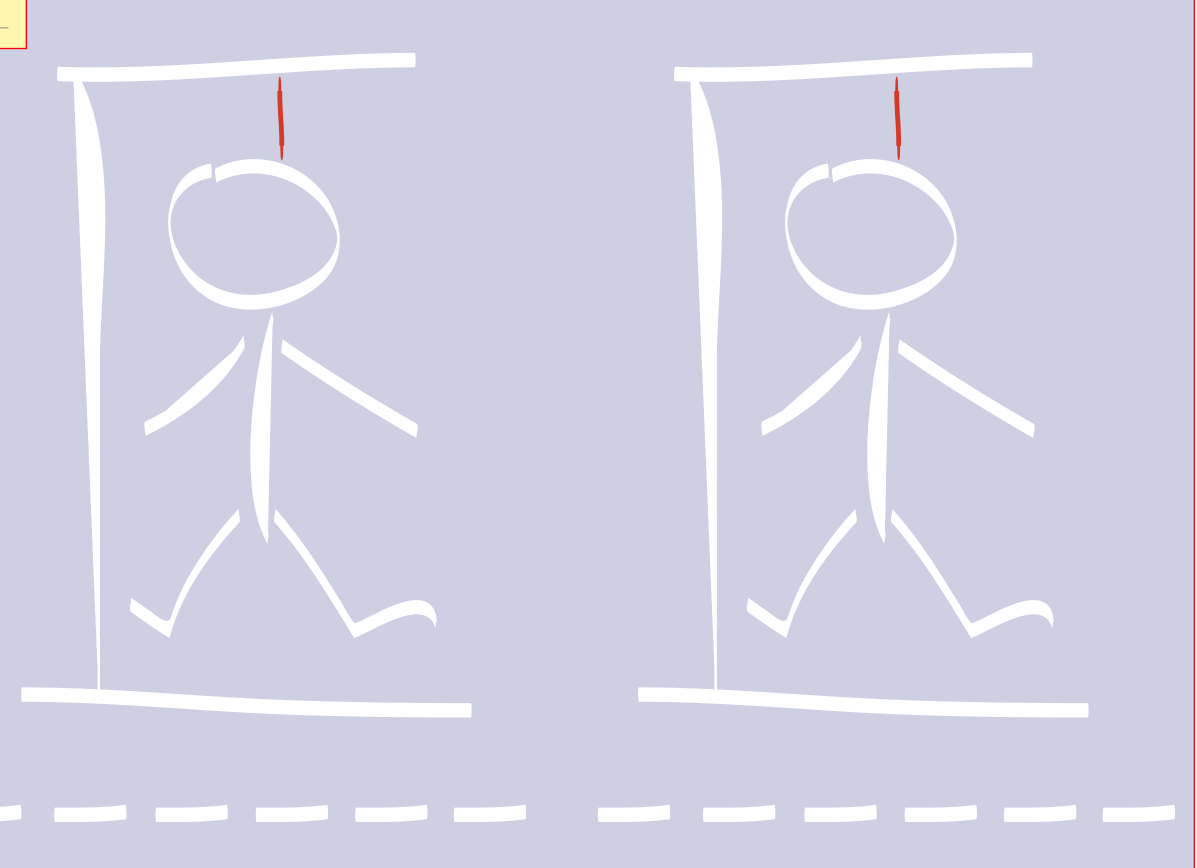
How could that be the case if everyone stayed true to the letter of the will?

PLAYTHINK 646 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

HANGMAN

In this version of the classic word game, both players get a hangman. They both think of a word of up to six letters and enter a number of dashes on the opponent's board equal to the letters in the secret word.

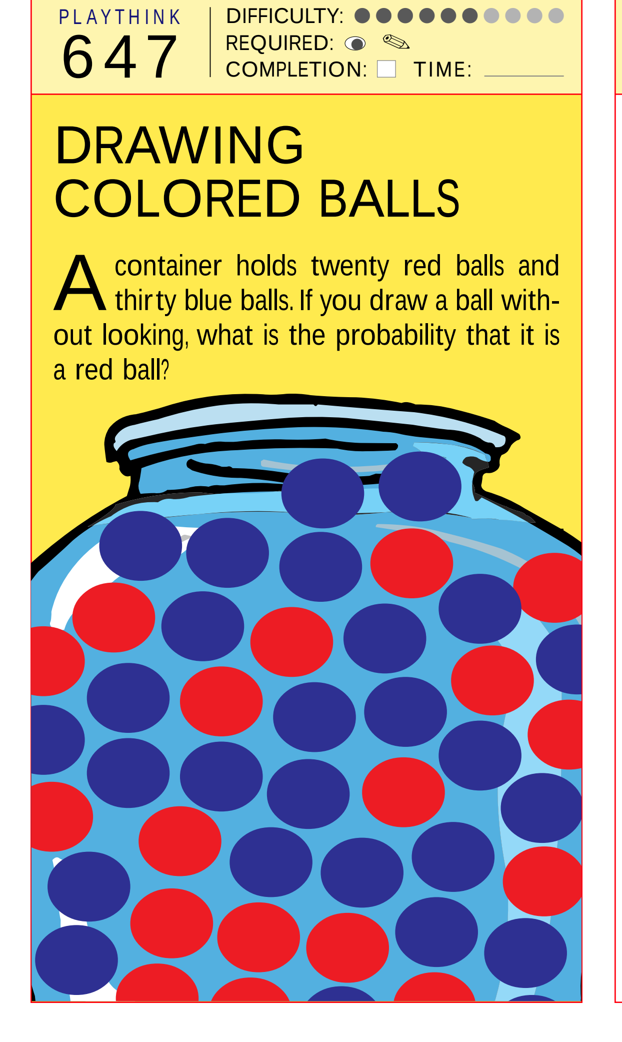
Players alternate calling out one letter at a time. If the letter is part of the secret word, it is entered above the appropriate dash (and if the letter occurs more than once, it must be entered as often as it appears). If the guess is incorrect, the opponent starts to draw in the pieces of the illustration one at a time—first the gallows, then the six parts of the condemned man. If a player calls out seven incorrect letters, his man is hanged.



PLAYTHINK 647 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

DRAWING COLORED BALLS

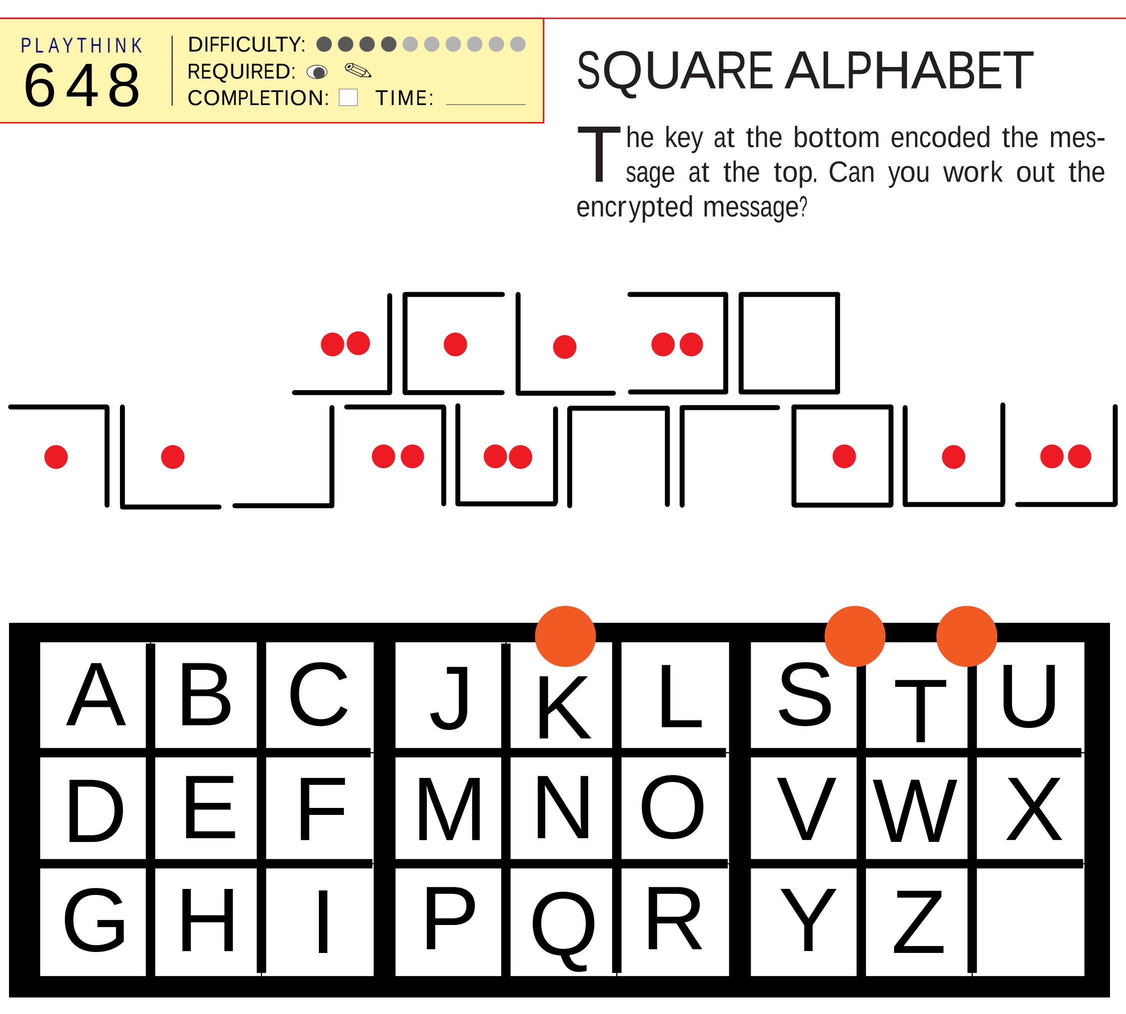
A container holds twenty red balls and thirty blue balls. If you draw a ball without looking, what is the probability that it is a red ball?



PLAYTHINK 648 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

SQUARE ALPHABET

The key at the bottom encoded the message at the top. Can you work out the encrypted message?

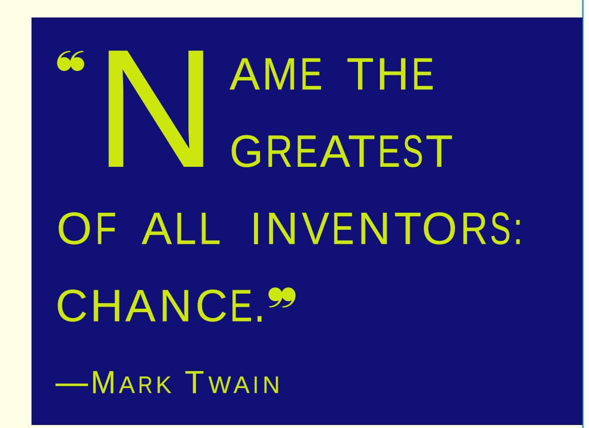


Chance

Classical logic and high school mathematics tend to operate in an unreal world of utter certainty. Every question can be answered by "yes" or "no," and every decision is either "right" or "wrong."

But the real world is quite a different place. Few answers and few decisions are wholly right or wholly wrong. The whole physical universe obeys the laws of chance. The seeming order of large-scale phenomena is sometimes simply the average outcome of millions of elementary random events.

That doesn't mean that any answer or decision is just as good as another. Most events follow the laws of probability, and if we know those laws, our chances of finding the most likely answers and the most promising decisions are greatly enhanced. There are varying degrees of plausibility or probability for every alternative. They can be compared, their reliability fixed, and useful estimates can be made of the comparative possibilities. This is the kind of logic that is developed in the theory of probability.



PLAYTHINK 649 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

HATCHECK

Six men check their hats at the theater. An inattentive attendant mixes up the claim checks, so when the men return after the show, the hats are essentially handed out at random.

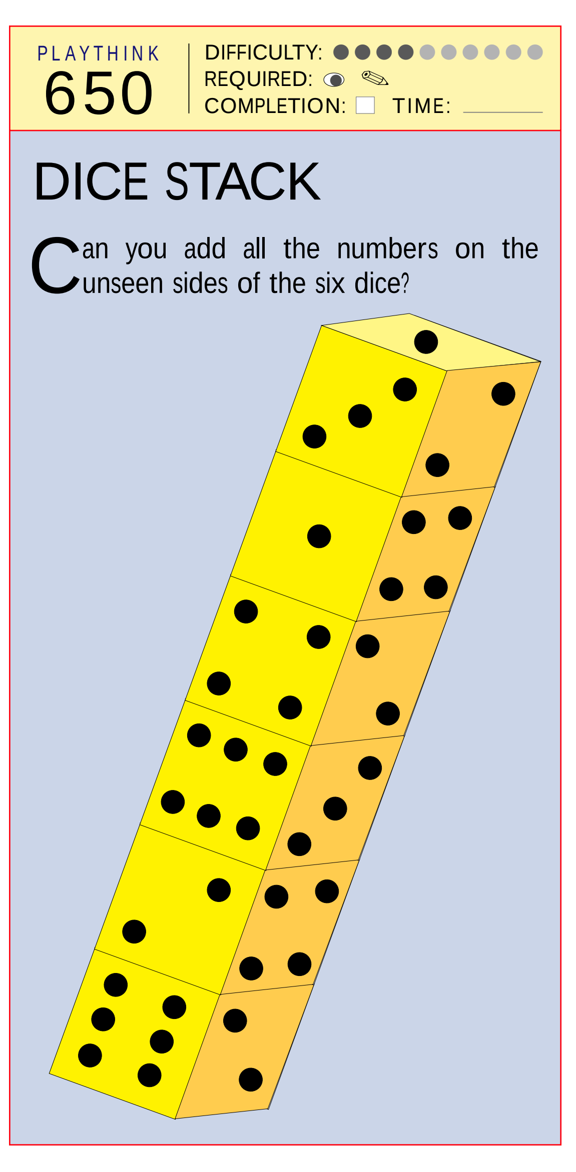
If someone offered you even money to bet that at least one of the men got his own hat back, would you take the bet? In other words, do you believe the probability of one of the six men getting his own hat back is greater than 0.5?



PLAYTHINK 650 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

DICE STACK

Can you add all the numbers on the unseen sides of the six dice?



Probability

Probability is the likelihood that an event will occur. The study of probability deals with questions that are answered, in colloquial terms, with "possibly," "sometimes," "often" or "almost always."

Unlike the fuzziness of "maybe," however, probabilities can be measured, calculated or—when calculation is impossible—estimated. The result is a numerical value. A probability of 1 corresponds to absolute certainty; a value of 0 means the outcome is impossible. Values that fall in between give a sense of likelihood: 0.7 for something that is fairly likely, 0.1 for something that is rather rare, and 0.5 for an event that is purely random, such as the toss of a coin.

Like all numbers, probabilities can be compared. Researchers use past events to calculate the probability of similar events occurring in the future. Such calculations have an important role to play in preparations for natural disasters. In locations where the probability of a hurricane is high but that of an earthquake is low, local safety workers can be trained in rescue techniques that are different from those in areas where the dangers are reversed.

In general, the probability of an event is defined by the equation:

$$P = n/N$$

in which N is the total number of equally probable outcomes and n is the number of specified outcomes whose probability is being calculated.

In many games it is customary to talk about the odds for (or against) an outcome, rather than its probability. Odds are calculated as n to N – n, so for an event that has a $^1/_5$ probability of occurring, the odds are 1 in 4.

PLAYTHINK 651

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

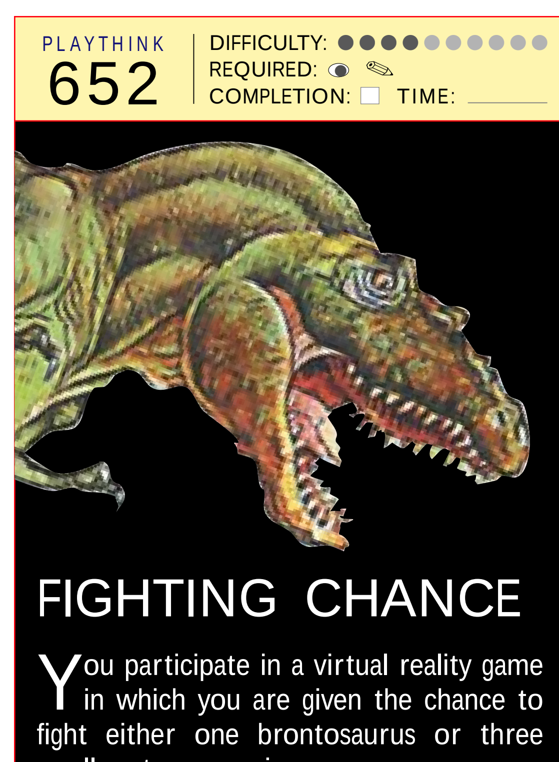
PROBABILITY MACHINE

If sixteen balls are released from the top of the hopper, how many on average will end up in each of the five compartments, according to the laws of probability?

This puzzle is based on the famous probability machine designed in the nineteenth century by Francis Galton. And though you won't be able to say how any individual ball may fall, you will be able to predict how a great many balls will be distributed. Although one random event is unpredictable, a great number of random events generally adhere to the laws of probability. Even the relatively few balls in this demonstration should give you a feel for how such a device works.



PLAYTHINK 652 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ______



FIGHTING CHANCE

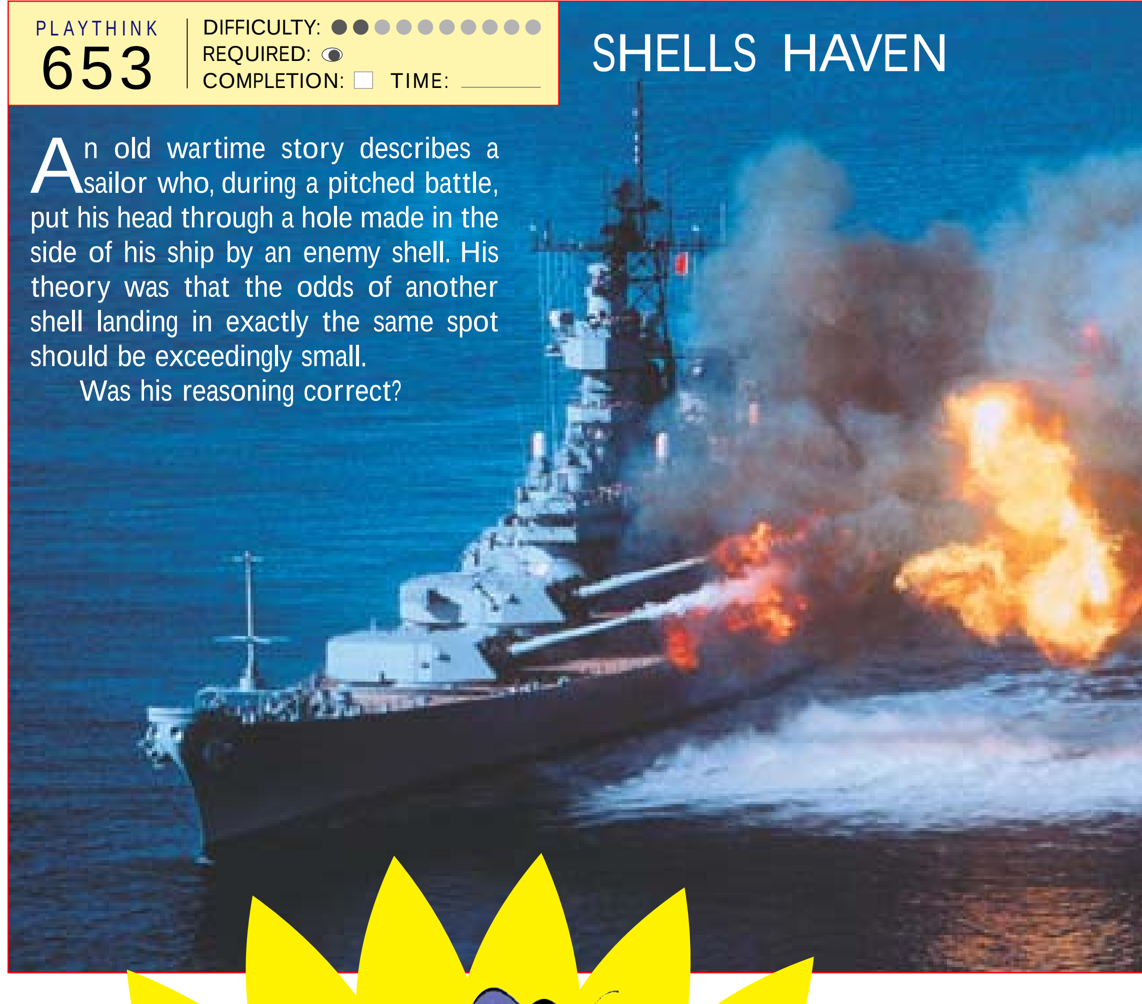
You participate in a virtual reality game in which you are given the chance to fight either one brontosaurus or three smaller stegosaurs in a row.

You know in advance that your chances of defeating the brontosaurus are one in seven, while the probability of defeating one of the stegosaurs is $^1/_2$.

Which alternative should you choose?

PLAYTHINK 653 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ______

SHELLS HAVEN



An old wartime story describes a sailor who, during a pitched battle, put his head through a hole made in the side of his ship by an enemy shell. His theory was that the odds of another shell landing in exactly the same spot should be exceedingly small.

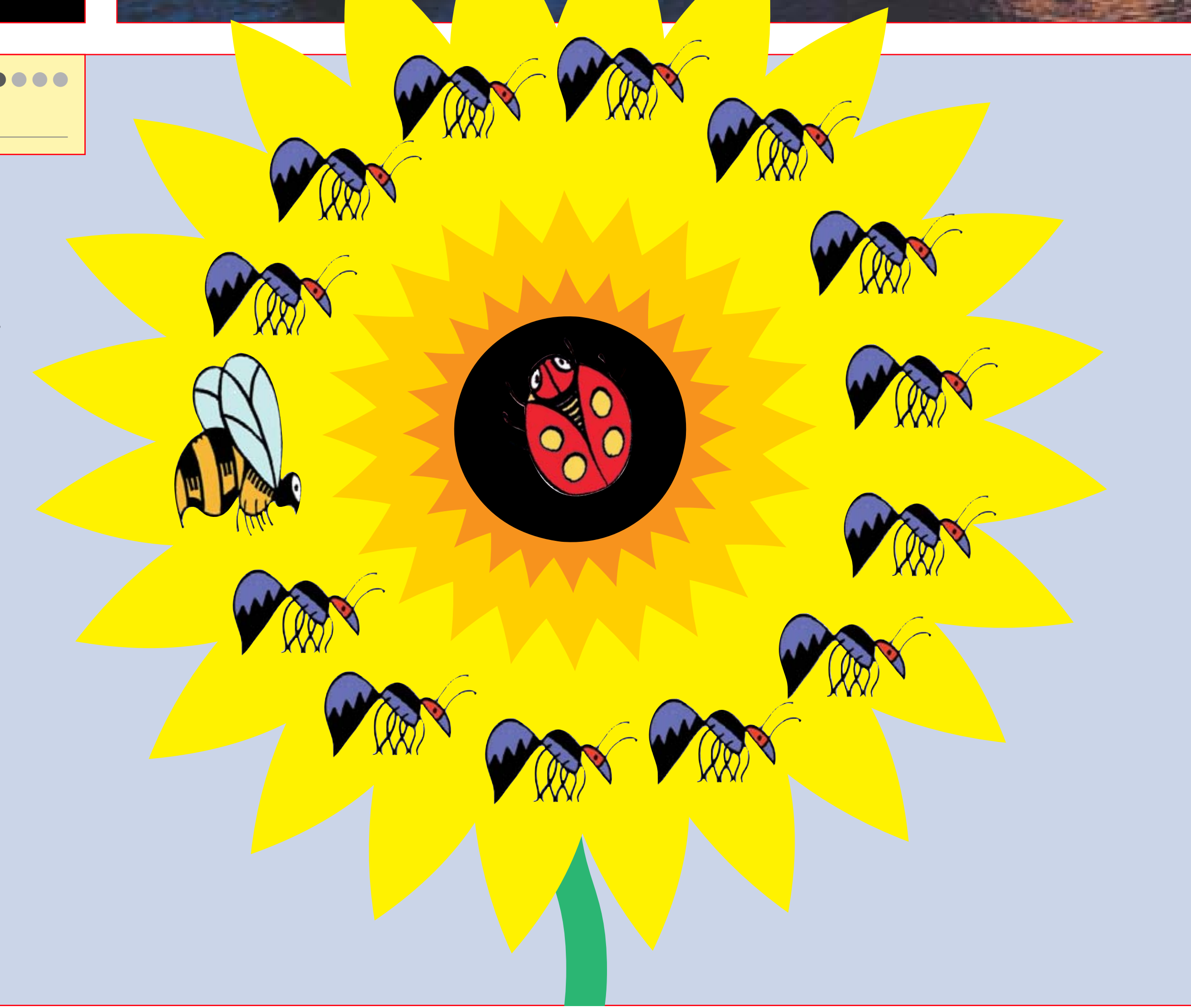
Was his reasoning correct?

PLAYTHINK 654 DIFFICULTY: ●●●●●●●●●● REQUIRED: COMPLETION: ☐ TIME: ______

VORACIOUS LADYBUGS

A hungry ladybug will try to eat every thirteenth bug she comes across as she goes around the flower. If it is an aphid, she will be happy; if it is a bumblebee, she will be stung.

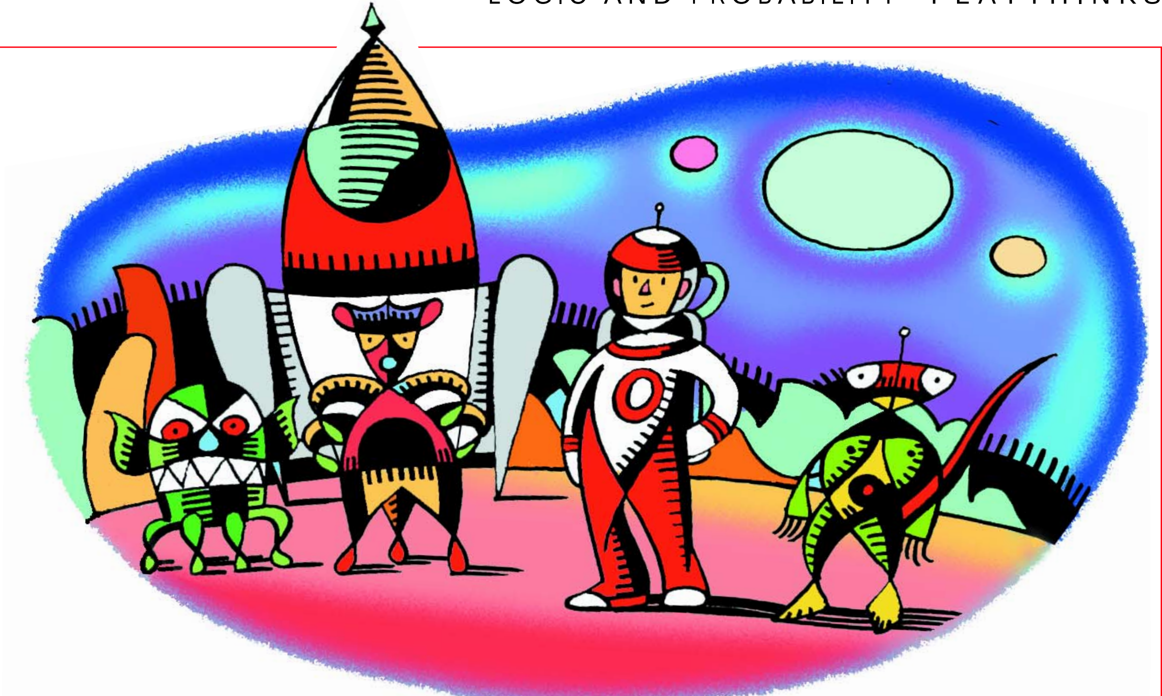
Can you work out which aphid she should start with so that she will eat all thirteen aphids and avoid the bumblebee?



PLAYTHINK 655

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

INTERPLANETARY COURIER



I have a job (in my dream) as an interplanetary courier at the Alpha Centauri spaceport, which means I am responsible for transporting passengers from the spaceport to the spaceliner in orbit many zerks above us. My shuttlecraft can hold just two people at a time—a passenger and me. Also, all the passengers must wait in the spaceliner's airlock until the last one has arrived.

Generally, the job is hassle-free, but on one recent occasion it was a real nightmare. There were three passengers waiting to be transported: a Rigellian, a Denebian and a weird-looking quadripedal creature called a Terrestrial. This caused all sorts of problems. First, the Denebians and the Rigellians were at war, so leaving them alone at the airlock could have caused an intergalactic incident. And unlike the vegetarian Rigellian, the Denebian was a voracious carnivore and, if left alone with the Terrestrial, would have devoured the hapless creature in a second.

It took me a minute, but I found a way to shuttle the passengers up to the spaceliner without any "accidents." One passenger may have had to accompany me more than once, but at the end all three were able to emerge safely from the airlock. Can you work out how I did it?

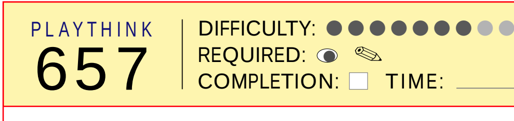
PLAYTHINK 656

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

LIKES AND DISLIKES

The picture below shows members of a group I belong to discussing their favorite food. Can you work out who is who and who likes what?





PLAYTHINK 657

DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

THREE COINS PARADOX

Suppose you have three coins—one with a head and a tail, one with two heads and one with two tails—that are dropped in a hat. If you draw one coin from the hat and lay it flat on a table without looking at it, what are the chances that the hidden side is the same as the visible side?



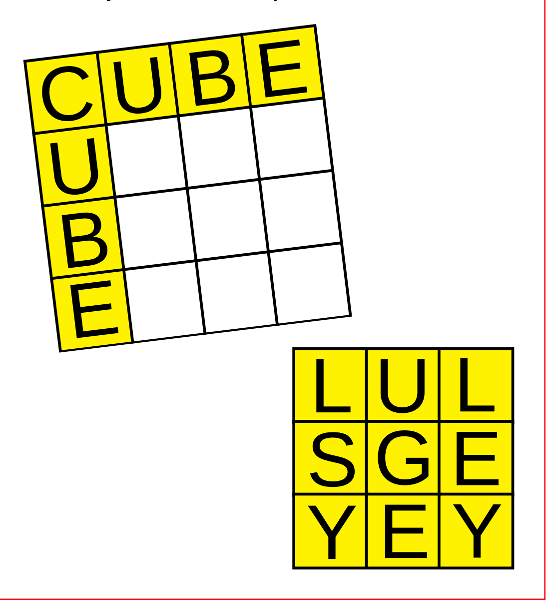
PLAYTHINK 658

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

WORD SQUARE

Word squares are matrices in which the same set of words appears both horizontally and vertically.

Can you fit in the extra letters to form a four-by-four word square?



PLAYTHINK 659

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

SPLIT GREETINGS

The two transparent disks are each one-half of a special greeting. If you superimpose one disk on the other, can you work out the hidden message?



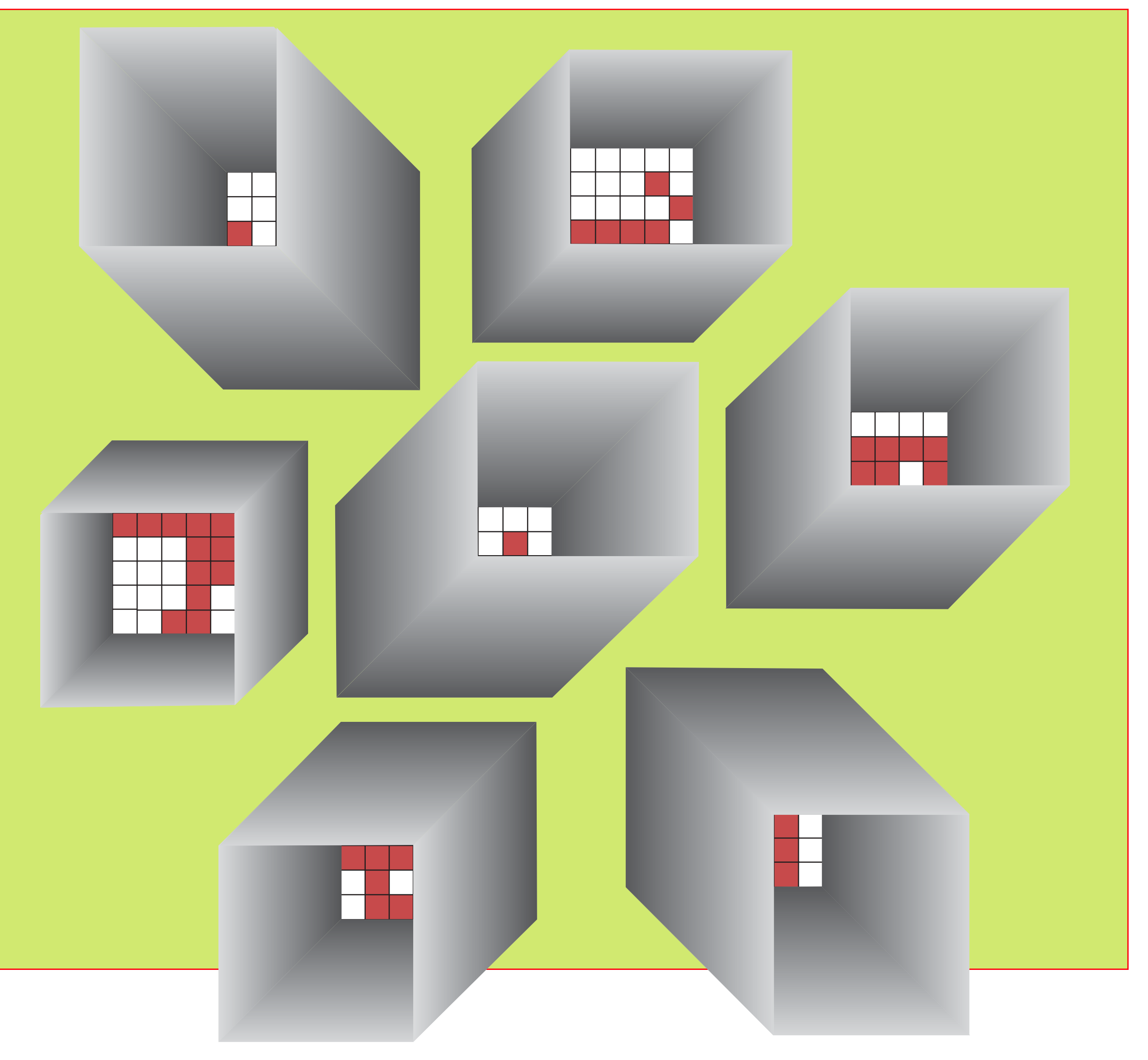
PLAYTHINK 660

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

HOLLOW CUBE 1

Imagine you can peer into a hollow cube that has an eight-by-eight mosaic on the bottom. At any one time, however, only parts of the mosaic can be seen. The pattern involves a bit of bilateral symmetry, so it is possible to deduce the answer from the visual information given.

Can you construct or deduce the whole mosaic from the bits you see here?



PLAYTHINK 661

DIFFICULTY: ●○○○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

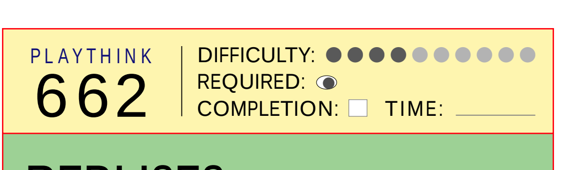


ROULETTE

What is the only sure way to win at roulette?

PLAYTHINK 662

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____



REBUSES

Can you solve the two rebus word problems illustrated below?

ME JUST YOU

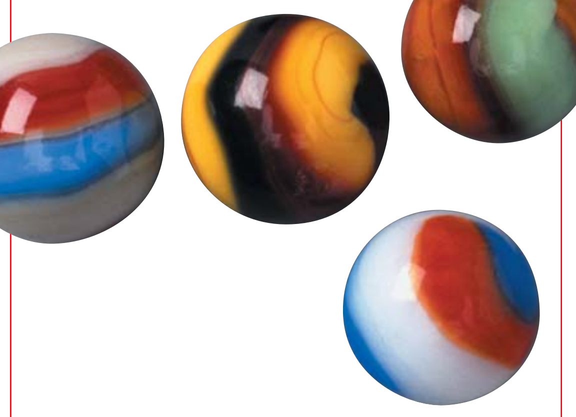
TIMING TIM ING

PLAYTHINK 664

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

ROLLING MARBLES

Peter and Paul are equally good at marbles. If Peter has two marbles and Paul has one, can you work out the probability of Peter winning? To win, a marble must land closest to a fixed point.

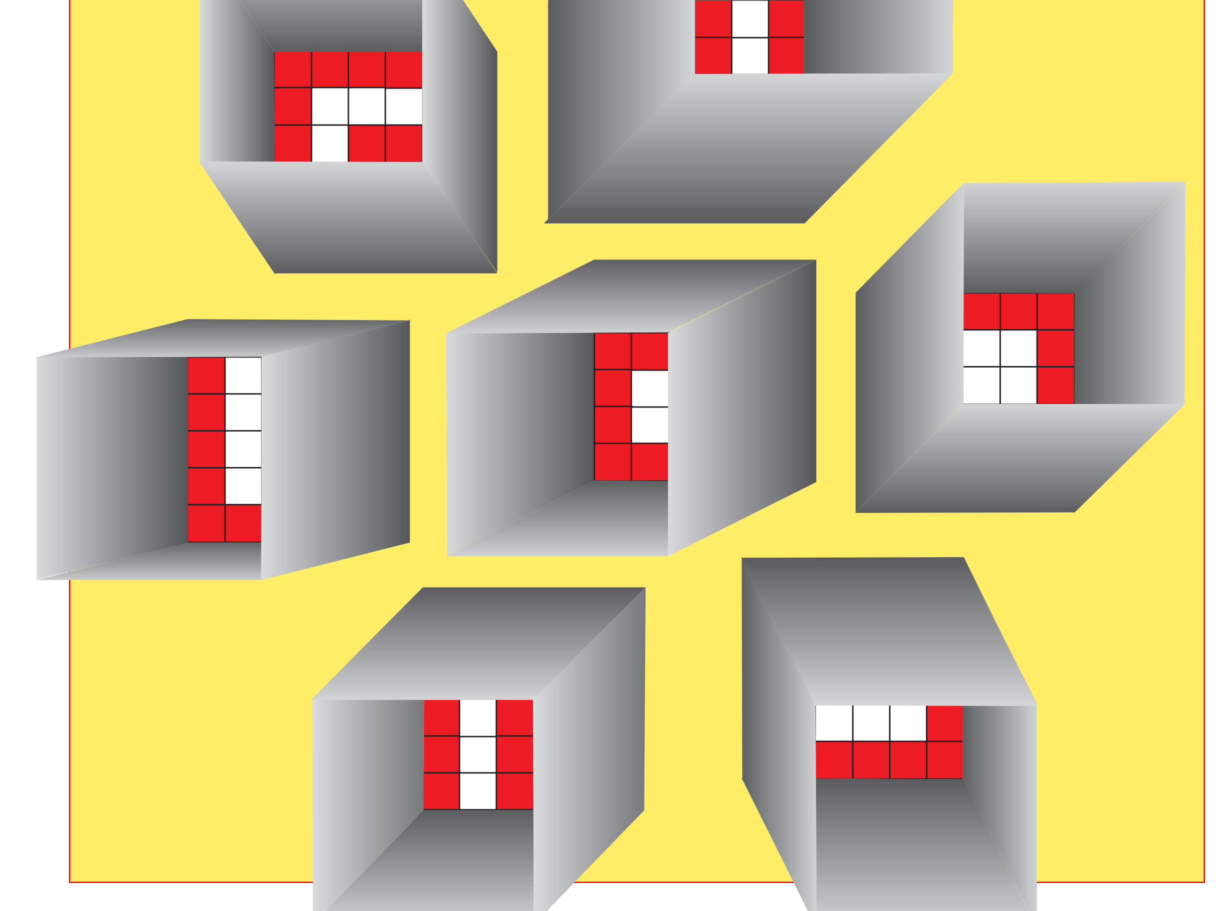


PLAYTHINK 663

DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

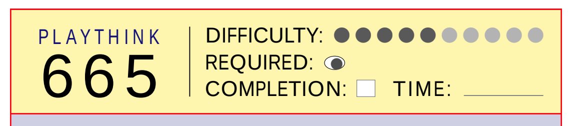
HOLLOW CUBE 2

Imagine you are peering into the bottom of a hollow cube that contains a six-by-six mosaic at its base. Only bits of the pattern can be seen at any given time. Can you piece together your observations to construct or deduce the pattern of the mosaic?



PLAYTHINK 665

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____



THREE MISTAKES

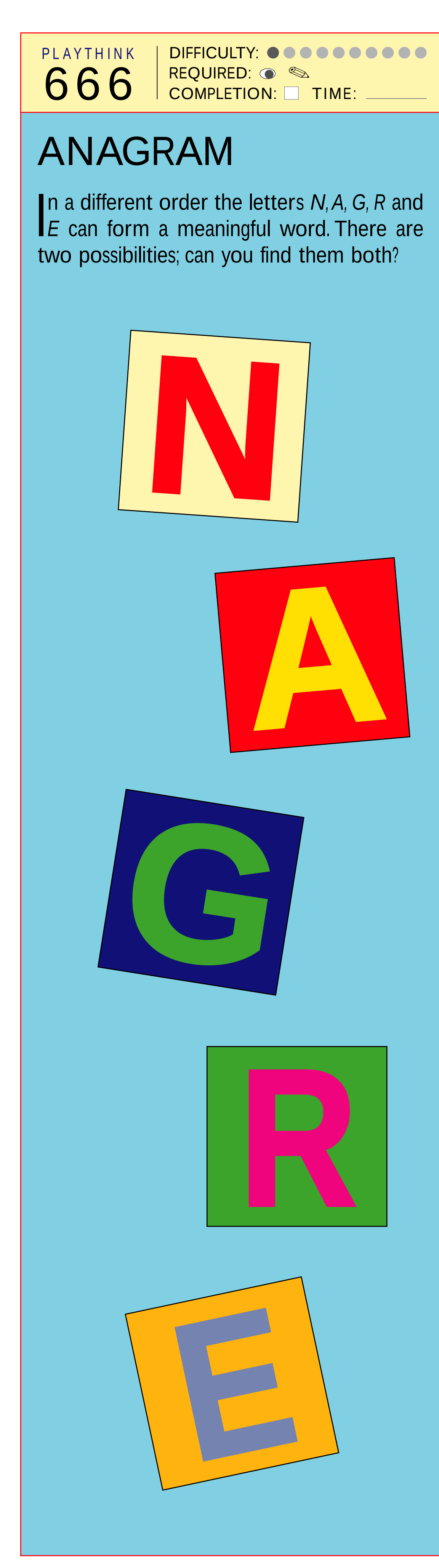
There are three mistakes in the message below. Can you spot them all?

What are the tree mistake
in this sentence?

PLAYTHINK 666 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

ANAGRAM

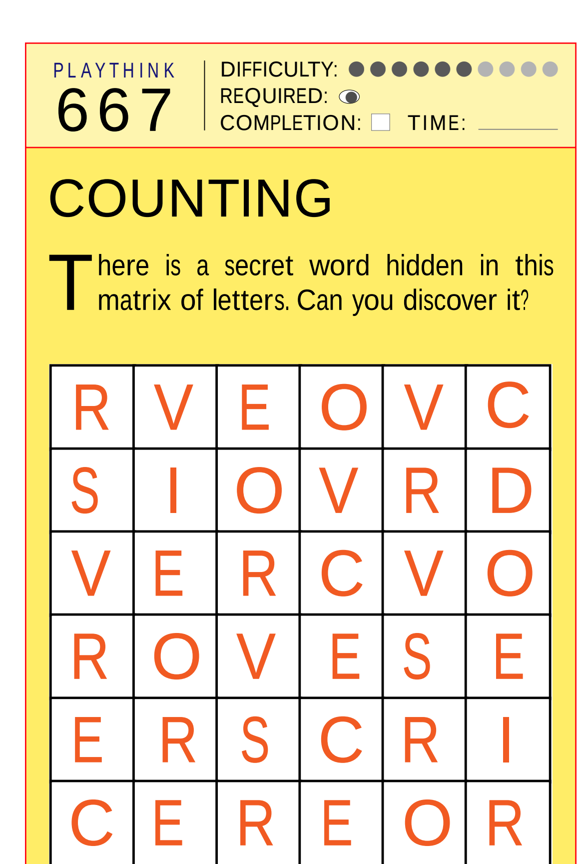
In a different order the letters *N, A, G, R* and *E* can form a meaningful word. There are two possibilities; can you find them both?



PLAYTHINK 667 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

COUNTING

There is a secret word hidden in this matrix of letters. Can you discover it?



R	V	E	O	V	C
S	I	O	V	R	D
V	E	R	C	V	O
R	O	V	E	S	E
E	R	S	C	R	I
C	E	R	E	O	R

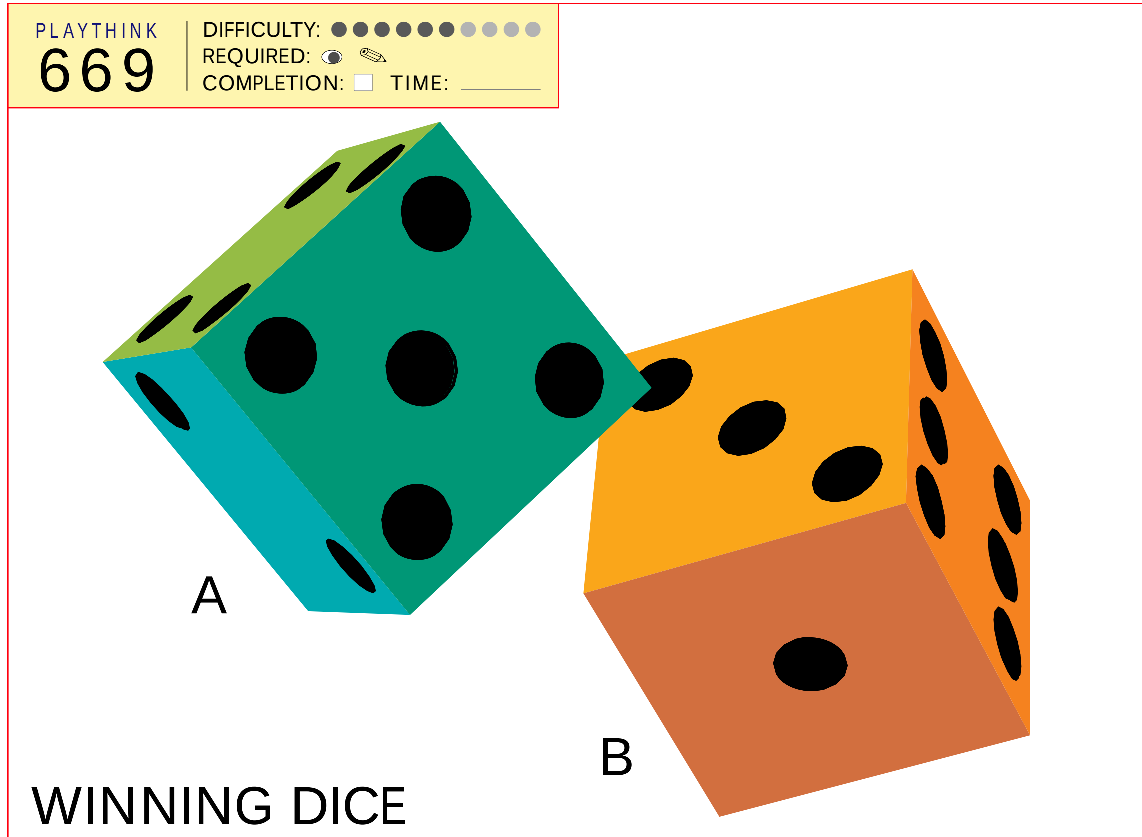
PLAYTHINK 668 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

SMALL WORLD

Pick any two of the 284 million people living in the United States. If you wanted to link those two by a chain of acquaintances (a friend of a friend of a friend . . .) how many people (or "links") would you need on average?



PLAYTHINK 669 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____



WINNING DICE

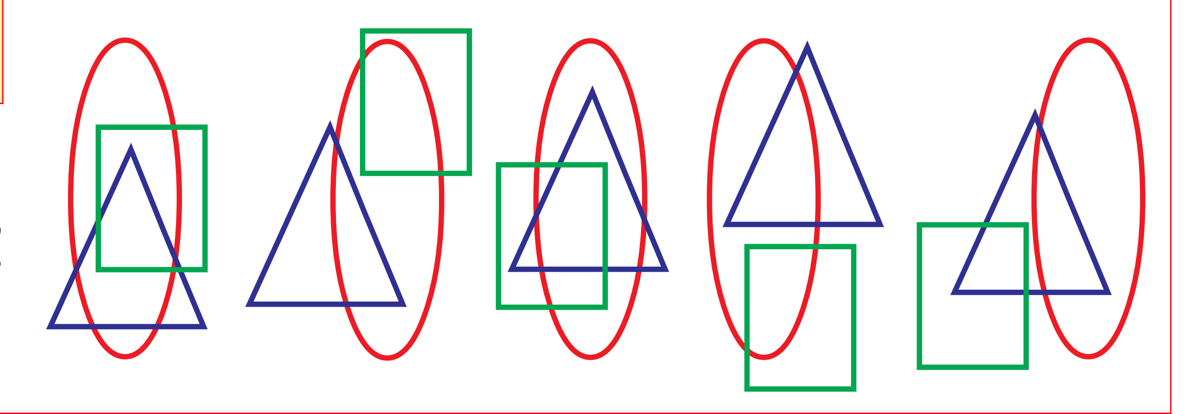
Two prisoners are spending their life sentences throwing dice. Each of them has just one old die so worn out that only three sides are legible. The three legible sides for each are shown above.

If their game awards the player who rolls the highest number, which player will win most often over the long run? (A game is not counted if either die lands with an illegible side face up.)

PLAYTHINK 670 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: eye, pencil | COMPLETION: ☐ TIME: ____

BASIC SHAPES

Five overlapping compositions of a triangle, a rectangle and an oval are illustrated here. Can you find the odd one out?



PLAYTHINK 671 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: eye | COMPLETION: ☐ TIME: ____

ALIEN LANDING

Twenty-four aliens just landed on the earth. They look identical except for the color of their hair, eyes, noses and mouths, which come in four colors in different combinations.

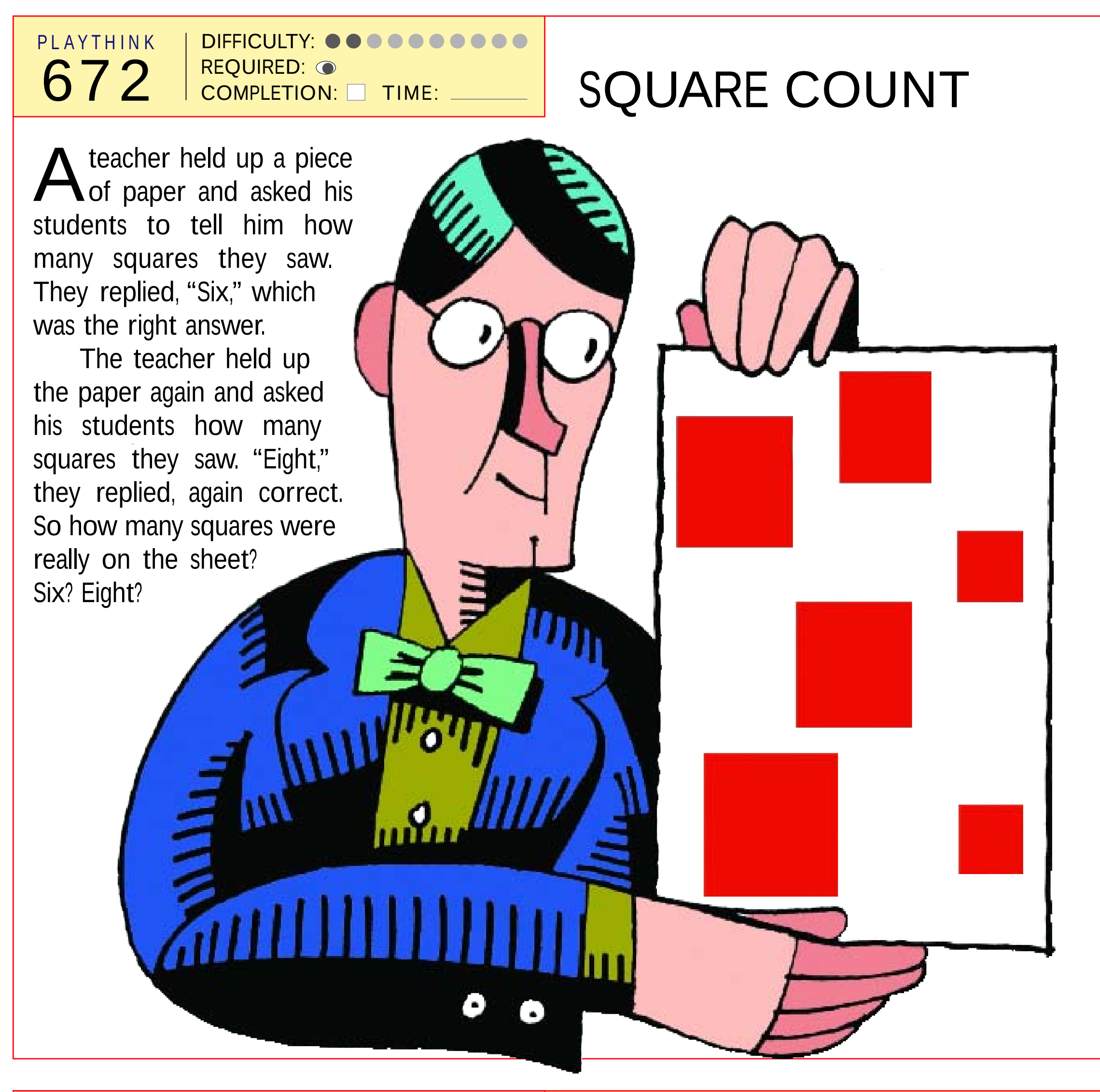
The aliens all carry a letter or a space for a message in English, but they are not in the right order. Can you use the color key below to put the aliens in the correct order and spell out their important message?

O	-	U	A	-	A
E	T	E	S	K	U
-	S	T	-	O	E
R	-	R	D	L	Y

	HAIR	EYES	NOSE	MOUTH

PLAYTHINK 672 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

SQUARE COUNT



A teacher held up a piece of paper and asked his students to tell him how many squares they saw. They replied, "Six," which was the right answer.

The teacher held up the paper again and asked his students how many squares they saw. "Eight," they replied, again correct. So how many squares were really on the sheet? Six? Eight?

PLAYTHINK 673 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

TRUE STATEMENT

Which of the three statements is true?

1) One statement here is false.
2) Two statements here are false.
3) Three statements here are false.

PLAYTHINK 674 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

TUNNEL PASSAGE

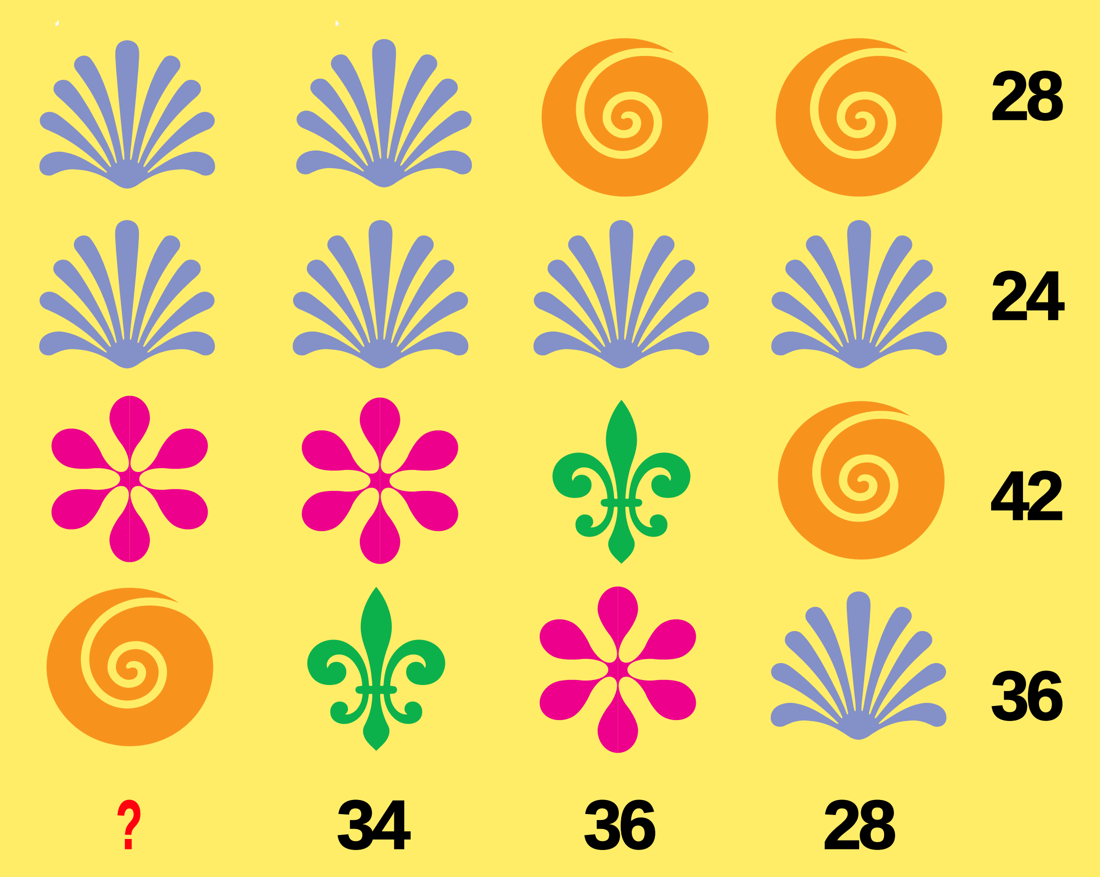
Three men sat by open windows on a steam train that passed through a tunnel. All three of their faces became covered with soot. When the three passengers saw this, they started laughing at one another. Then one of them suddenly stopped because he realized that his face was also soiled.

What was his reasoning?

PLAYTHINK 675 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

LOGIC PATTERN

Each of the symbols in this matrix represents a number. The sum for each row and for three of the four columns is given. From that information, can you find the value of each symbol?



Coincidence

The improbable, Aristotle once said, is extremely probable. But when one looks at the bizarre coincidences that occur on a weekly or daily basis, it is easy to conclude that many coincidences are too improbable to be explained by known laws.

The parapsychologist Lewis Vaughn warned in *Skeptics* magazine of the dangers of trying to predict synchronistic events by telling the following tale.

A man was the seventh child of parents who were each the seventh child in their families. The man was born on the Sabbath—the seventh day—of the seventh month of 1907. Over the course of his life, a number of odd things occurred, all related to the number seven, which he soon took as his lucky number. On his twenty-seventh birthday, the man went to the racetrack and saw that a horse named Seventh Heaven was listed to run from the seventh gate in the seventh race. The odds were seven to one. The man borrowed all the money he could and bet on the horse. The horse, of course, came in seventh.

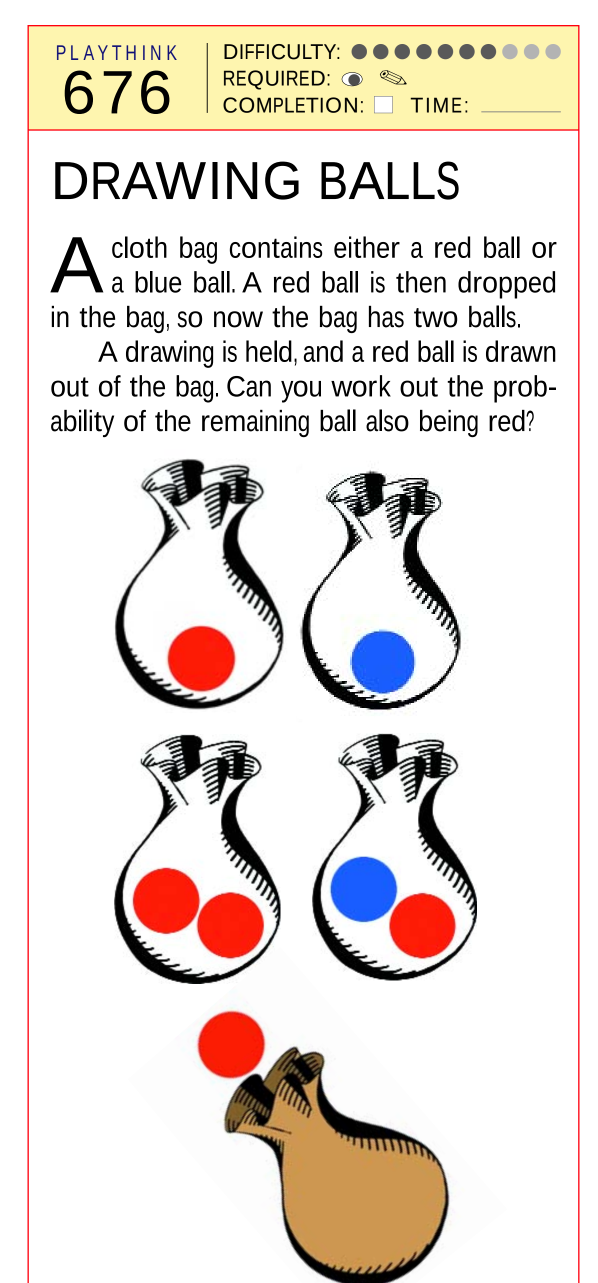
Like the man in the story, we can develop subjective concepts about probability that may lead us to wrong conclusions. In order to relate to chance intelligently, we must learn to understand the laws of chance, which often have a strongly counterintuitive aspect.

PLAYTHINK 676 — DIFFICULTY: ●●●●●●●○○○ REQUIRED: 👁 ✎ COMPLETION: ☐ TIME: ____

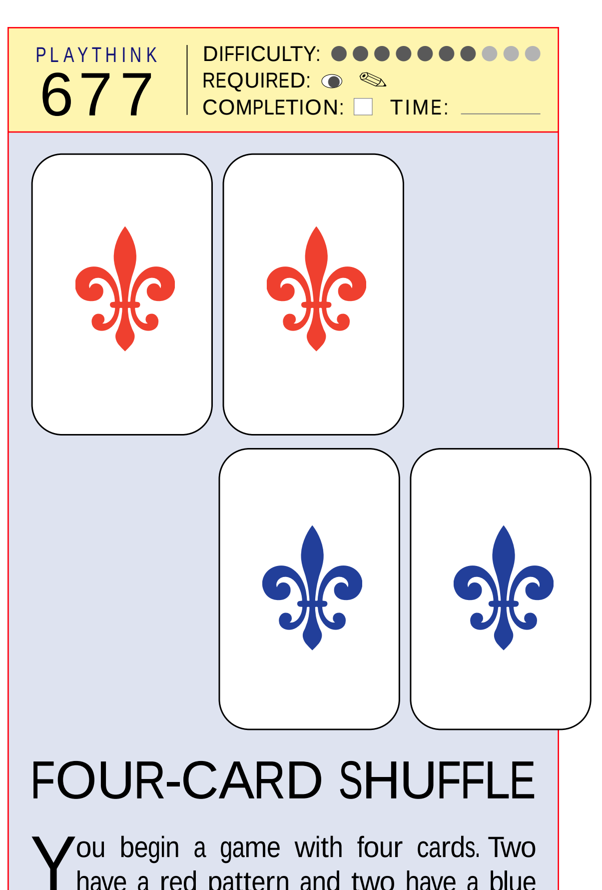
DRAWING BALLS

A cloth bag contains either a red ball or a blue ball. A red ball is then dropped in the bag, so now the bag has two balls.

A drawing is held, and a red ball is drawn out of the bag. Can you work out the probability of the remaining ball also being red?



PLAYTHINK 677 — DIFFICULTY: ●●●●●●●○○○ REQUIRED: 👁 ✎ COMPLETION: ☐ TIME: ____



FOUR-CARD SHUFFLE

You begin a game with four cards. Two have a red pattern and two have a blue pattern, and all are blank on one side.

You shuffle the four cards and place them face down. If you pick two cards at random, what is the probability that the two cards will be the same color?

Your friend tries to convince you that the chances are $^2/_3$ with this reasoning: there are three possibilities—two red, two blue or one of each—and since two of those are of the same color, the chances are two out of three.

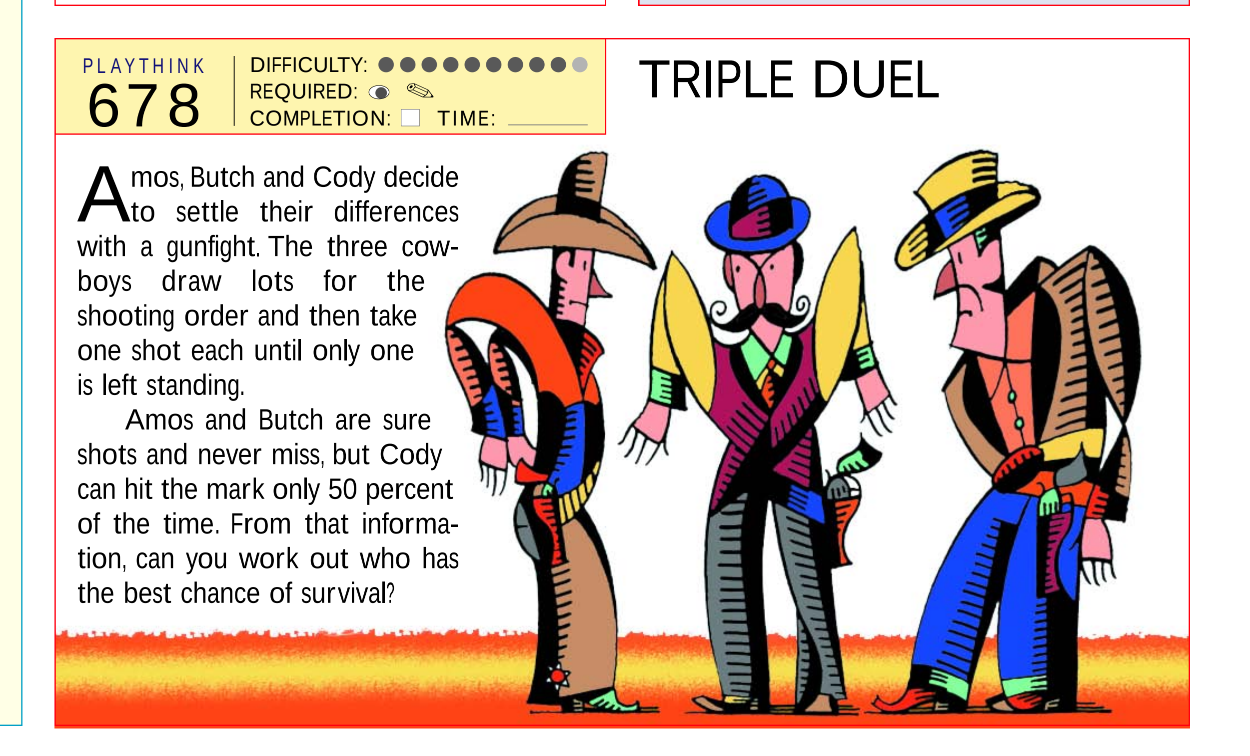
Are you convinced?

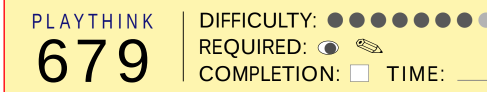
PLAYTHINK 678 — DIFFICULTY: ●●●●●●●●●○ REQUIRED: 👁 ✎ COMPLETION: ☐ TIME: ____

TRIPLE DUEL

Amos, Butch and Cody decide to settle their differences with a gunfight. The three cowboys draw lots for the shooting order and then take one shot each until only one is left standing.

Amos and Butch are sure shots and never miss, but Cody can hit the mark only 50 percent of the time. From that information, can you work out who has the best chance of survival?





LAST ALIVE

Imagine you have just become the emperor of Rome. One of your first duties is to condemn thirty-six prisoners to be eaten by lions in the arena. The lions can eat only six victims a day, and there are six hated enemies you would like to dispatch right away, but you also want to appear impartial.



The traditional Roman way to select prisoners for execution is decimation—picking every tenth person. If you have the prisoners stand in a circle, is there a way to plant your enemies at specific positions so they will be the first six selected to die?

Coin Tossing

Although no one can say with certainty the outcome of a single toss of a coin, the result of a million tosses is easy to predict: half a million heads and half a million tails, or within a percent or two of each. This, in essence, is the basis of the theory of probability.

Two laws underlie probability: the "both-and" law, employed to calculate the probability of two events both happening, and the "either-or" law, used to calculate the probability of one or the other of two events happening. The both-and law states that the chance of two independent events both happening is equal to the probability of one happening multiplied by the probability of the other happening. For example, the chance of one flip of a coin turning up heads is $\frac{1}{2}$. The chance of the coin turning up heads on both the first and second flips is $\frac{1}{2} \times \frac{1}{2}$, or only $\frac{1}{4}$.

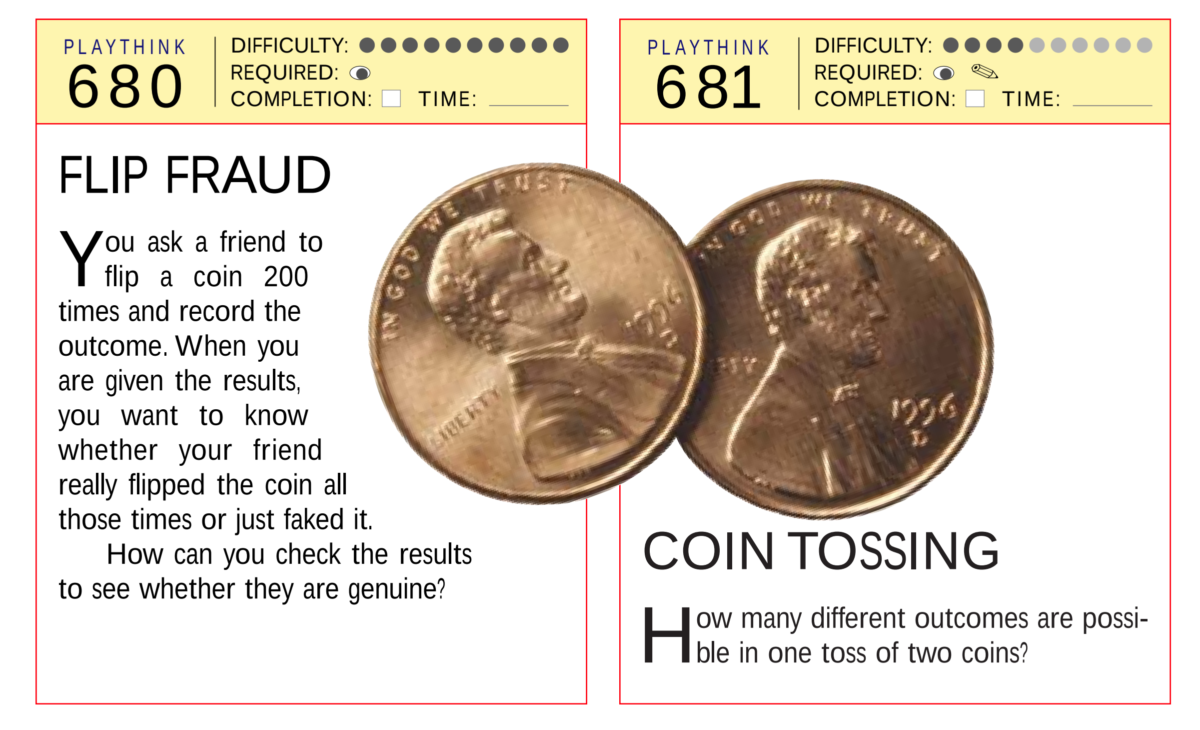
The either-or law states that the chance of either one or the other of two mutually exclusive probabilities coming true equals the sum of the separate chance that each would occur individually. The chance of one flip of a coin turning up either heads or tails is equal to the chance of throwing heads plus the chance of throwing tails: $\frac{1}{2} + \frac{1}{2}$, or 1— absolute certainty.

PLAYTHINK 680 DIFFICULTY: REQUIRED: COMPLETION: TIME:

FLIP FRAUD

You ask a friend to flip a coin 200 times and record the outcome. When you are given the results, you want to know whether your friend really flipped the coin all those times or just faked it.

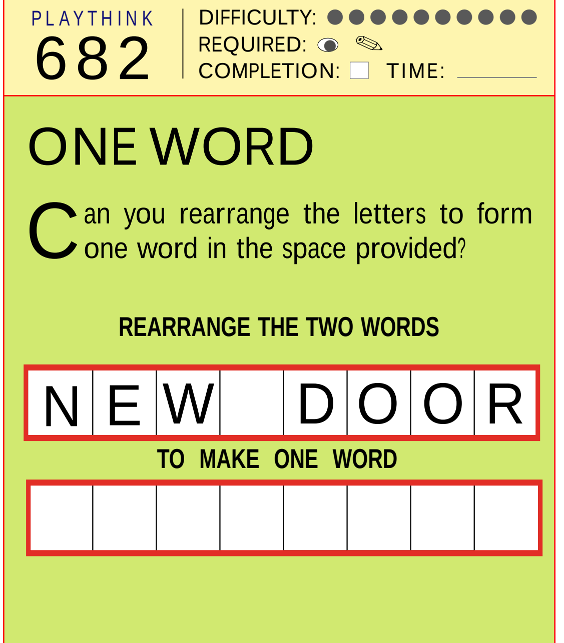
How can you check the results to see whether they are genuine?



PLAYTHINK 681 DIFFICULTY: REQUIRED: COMPLETION: TIME:

COIN TOSSING

How many different outcomes are possible in one toss of two coins?



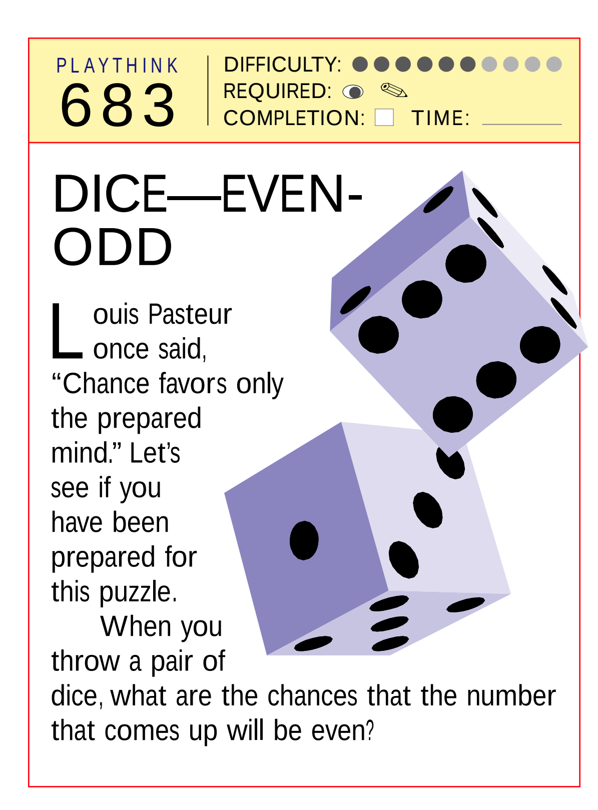
PLAYTHINK 682 DIFFICULTY: REQUIRED: COMPLETION: TIME:

ONE WORD

Can you rearrange the letters to form one word in the space provided?

PLAYTHINK 683 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ______

DICE—EVEN-ODD



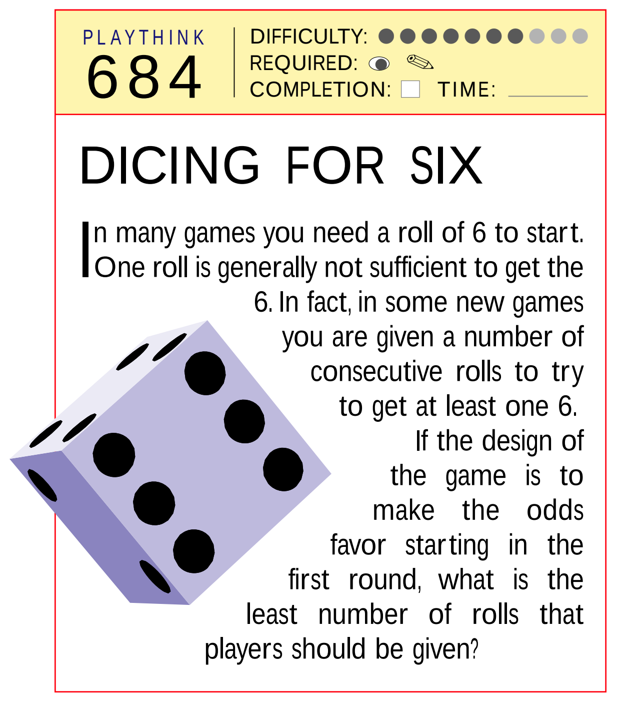
Louis Pasteur once said, "Chance favors only the prepared mind." Let's see if you have been prepared for this puzzle.

When you throw a pair of dice, what are the chances that the number that comes up will be even?

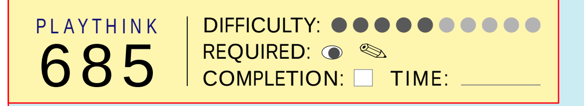
PLAYTHINK 684 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ______

DICING FOR SIX

In many games you need a roll of 6 to start. One roll is generally not sufficient to get the 6. In fact, in some new games you are given a number of consecutive rolls to try to get at least one 6.



If the design of the game is to make the odds favor starting in the first round, what is the least number of rolls that players should be given?



PLAYTHINK 685 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ______

BIRTHDAY PARADOX

You want to have a party at which at least two people share the same birthday—same month and day but not necessarily the same year. If you don't know the birthdays of any of your guests, how many people do you have to invite so that the probability of two people sharing a birthday is more than 0.5? How many people do you need to invite for birthday sharing to be a practical certainty?



PLAYTHINK 686 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ______

COLOR WORDS

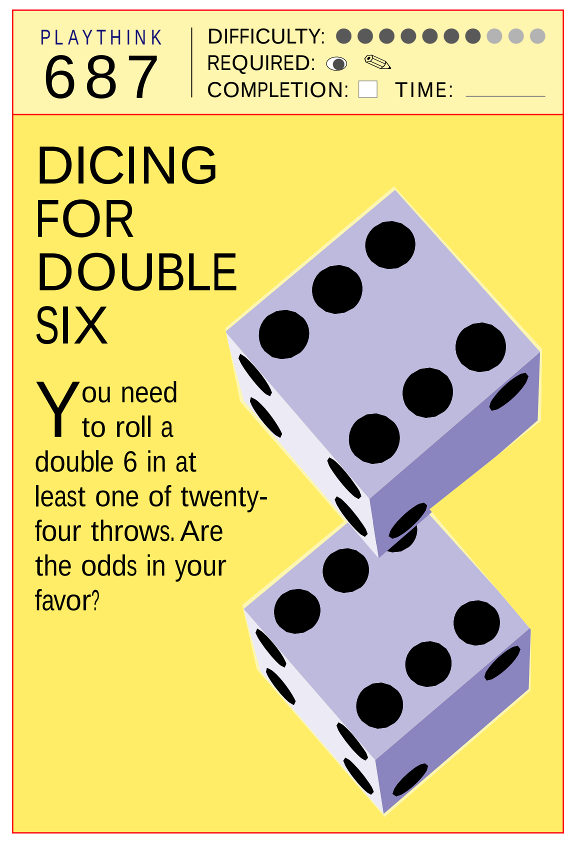
How strongly do words affect perception? Try reading the four lines of colored words at right—but instead of saying the words, say the color of each word.

Can you say more than five in a row without making a mistake?

RED YELLOW BLUE GREEN
YELLOW BLUE GREEN RED
GREEN RED YELLOW BLUE
BLUE GREEN RED YELLOW

PLAYTHINK 687
DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

DICING FOR DOUBLE SIX



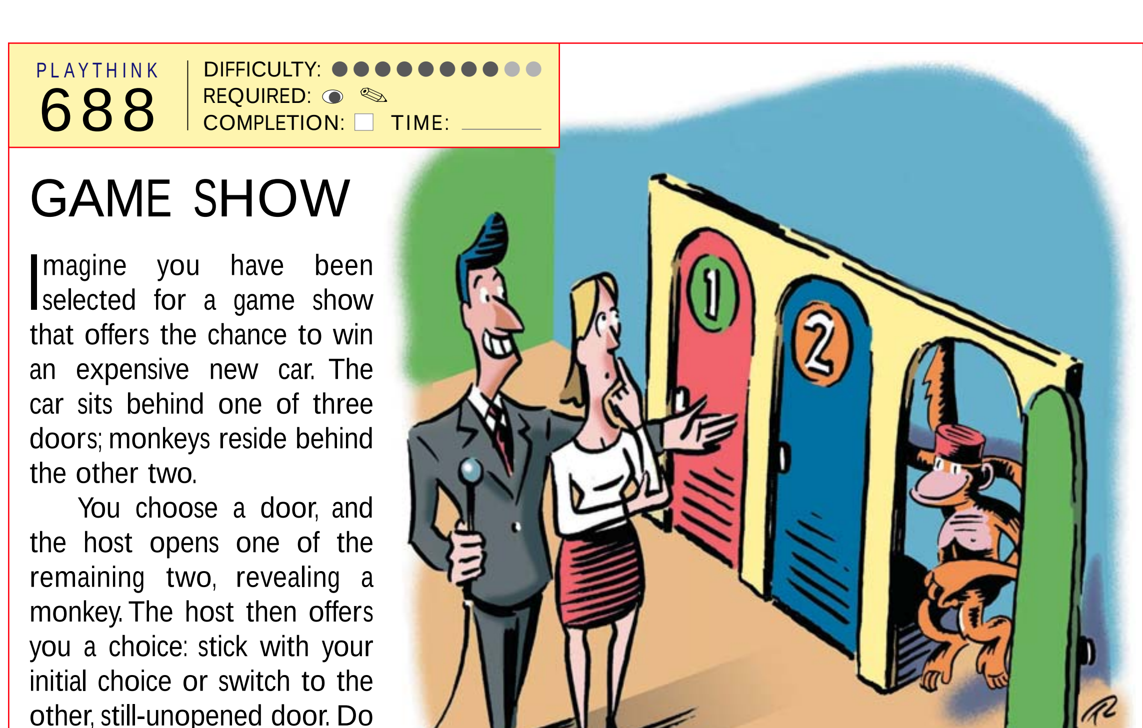
You need to roll a double 6 in at least one of twenty-four throws. Are the odds in your favor?

PLAYTHINK 688
DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

GAME SHOW

Imagine you have been selected for a game show that offers the chance to win an expensive new car. The car sits behind one of three doors; monkeys reside behind the other two.

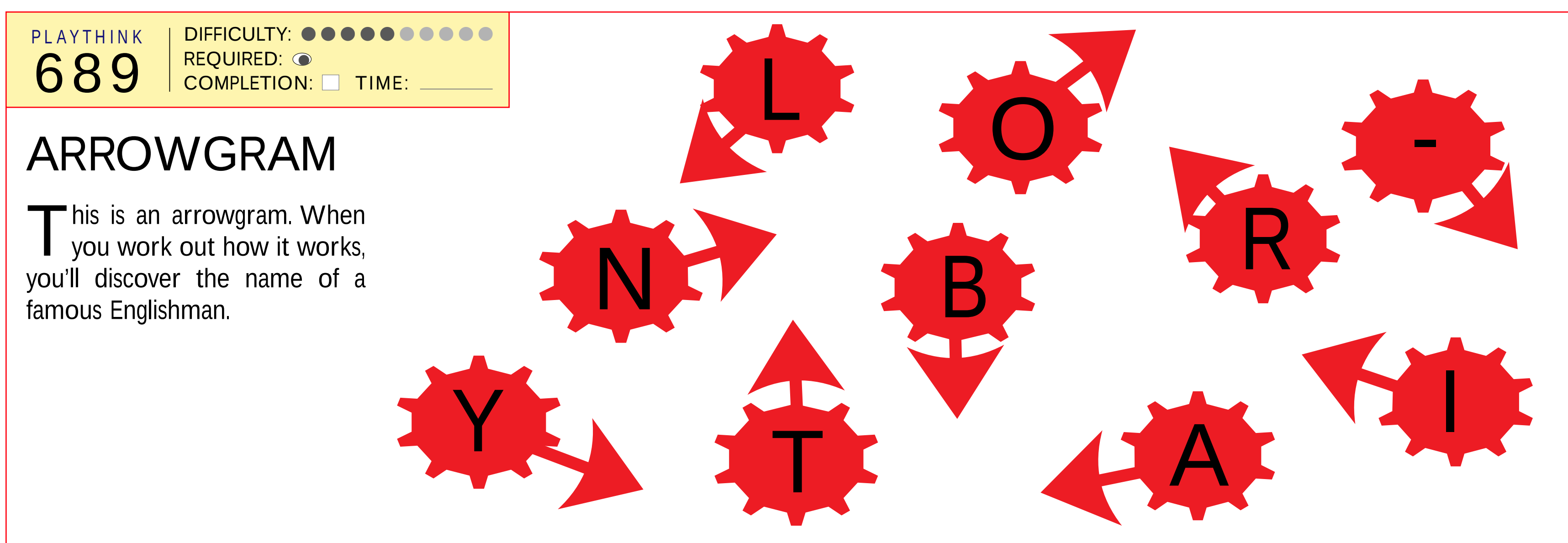
You choose a door, and the host opens one of the remaining two, revealing a monkey. The host then offers you a choice: stick with your initial choice or switch to the other, still-unopened door. Do you stick with your door, or do you take the host up on the offer?



PLAYTHINK 689
DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

ARROWGRAM

This is an arrowgram. When you work out how it works, you'll discover the name of a famous Englishman.



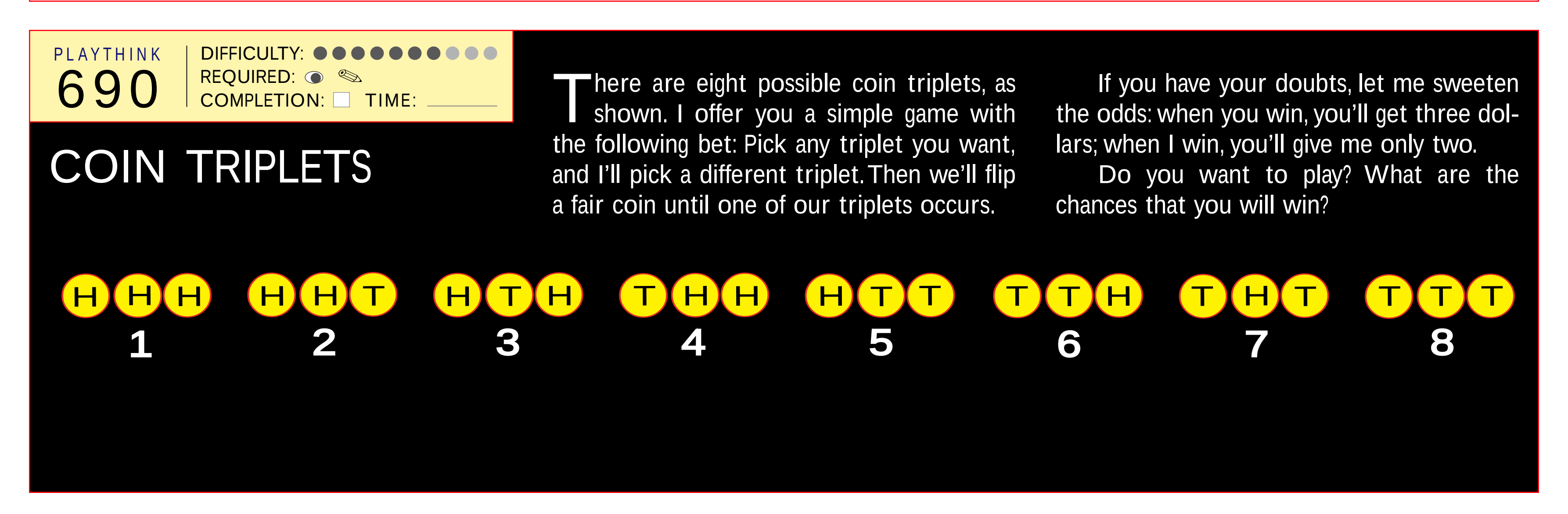
PLAYTHINK 690
DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

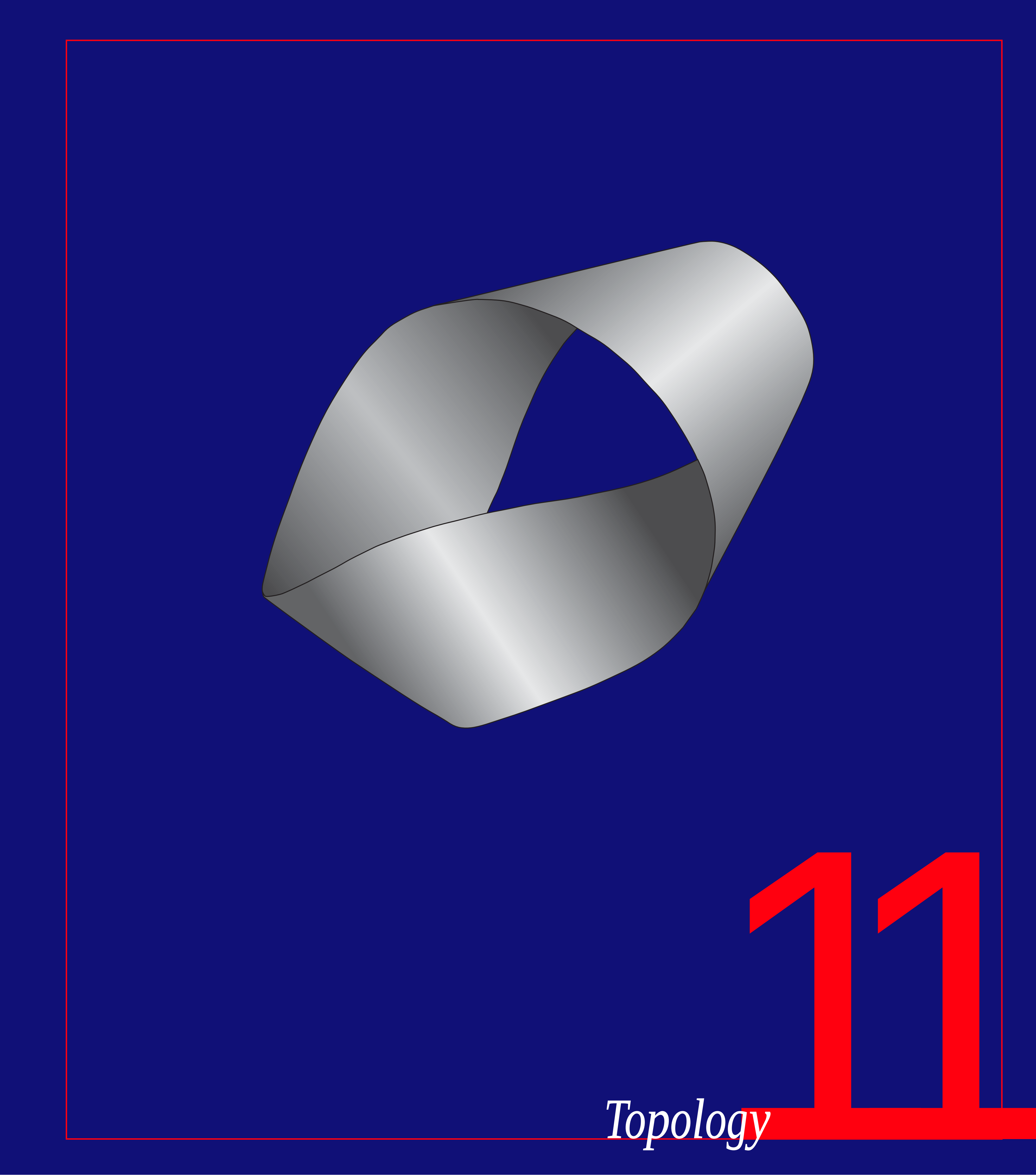
COIN TRIPLETS

There are eight possible coin triplets, as shown. I offer you a simple game with the following bet: Pick any triplet you want, and I'll pick a different triplet. Then we'll flip a fair coin until one of our triplets occurs.

If you have your doubts, let me sweeten the odds: when you win, you'll get three dollars; when I win, you'll give me only two.

Do you want to play? What are the chances that you will win?





11

Topology

What is Topology?

Euclidean geometry is clear: A triangle is utterly different from a circle, which has little in common with a trapezoid. But not every type of mathematics ascribes to those boundaries. Take topology: the emphasis in topology isn't on angles or curves but on surfaces. It studies those properties of a figure that remain unchanged under deformation.

Little traditional geometry survives from the topological perspective. From the point of view of a topologist, the number of sides and angles of a triangle are unimportant. One can easily deform a triangle to make its angles change. The lengths of the sides are similarly uninteresting. Indeed, even being a triangle is not a topological property—by introducing a bend in one side of a triangle, the shape can be continuously deformed into a rectangle. In fact, to topologists, a triangle is the same as a square, a parallelogram—even a circle.

The topologist studies surfaces, and topology looks at the continuity from one surface to another. The fact that a triangle has an inside and an outside and that it is impossible to pass from one to the other without crossing an edge of the triangle—those are topological properties. The fact that a car inner tube has a hole in the middle is a topological property. Whether a loop or string is knotted is a topological property.

Two figures are topologically equivalent if one can be continuously deformed into another. (Continuous deformation means a shape is bent, twisted, stretched or compressed.) So a sphere and a cube are topologically equivalent, as are the figure *8* and letter *B*. A fundamental problem in topology is to group objects into classes of things that are topologically equivalent.

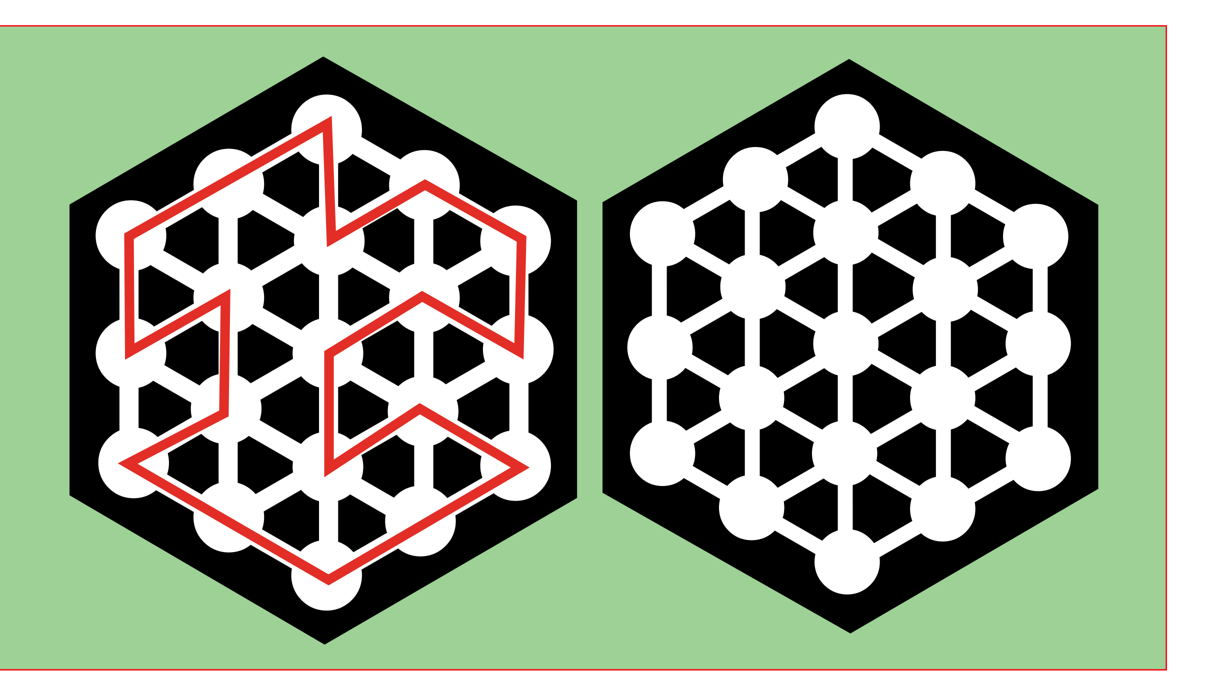
The basic concepts of topology include many ideas we learn in childhood: insideness and outsideness, right- and left-handedness, linking, knotting, connectedness and disconnectedness. Indeed, some of the concepts are so basic that topologists have been called mathematicians who don't know the difference between a coffee cup and a doughnut. But topology has become a cornerstone of modern mathematics. During the last forty years it has been applied to problems in practically all fields of science.

Because topology deals with space, surfaces, solids, regions and networks and because it is full of impossibilities and paradoxes, it is a rich topic for fun, games, puzzles and problem solving.

PLAYTHINK 691 | DIFFICULTY: ●●●○○○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

DOT WIGGLING 1

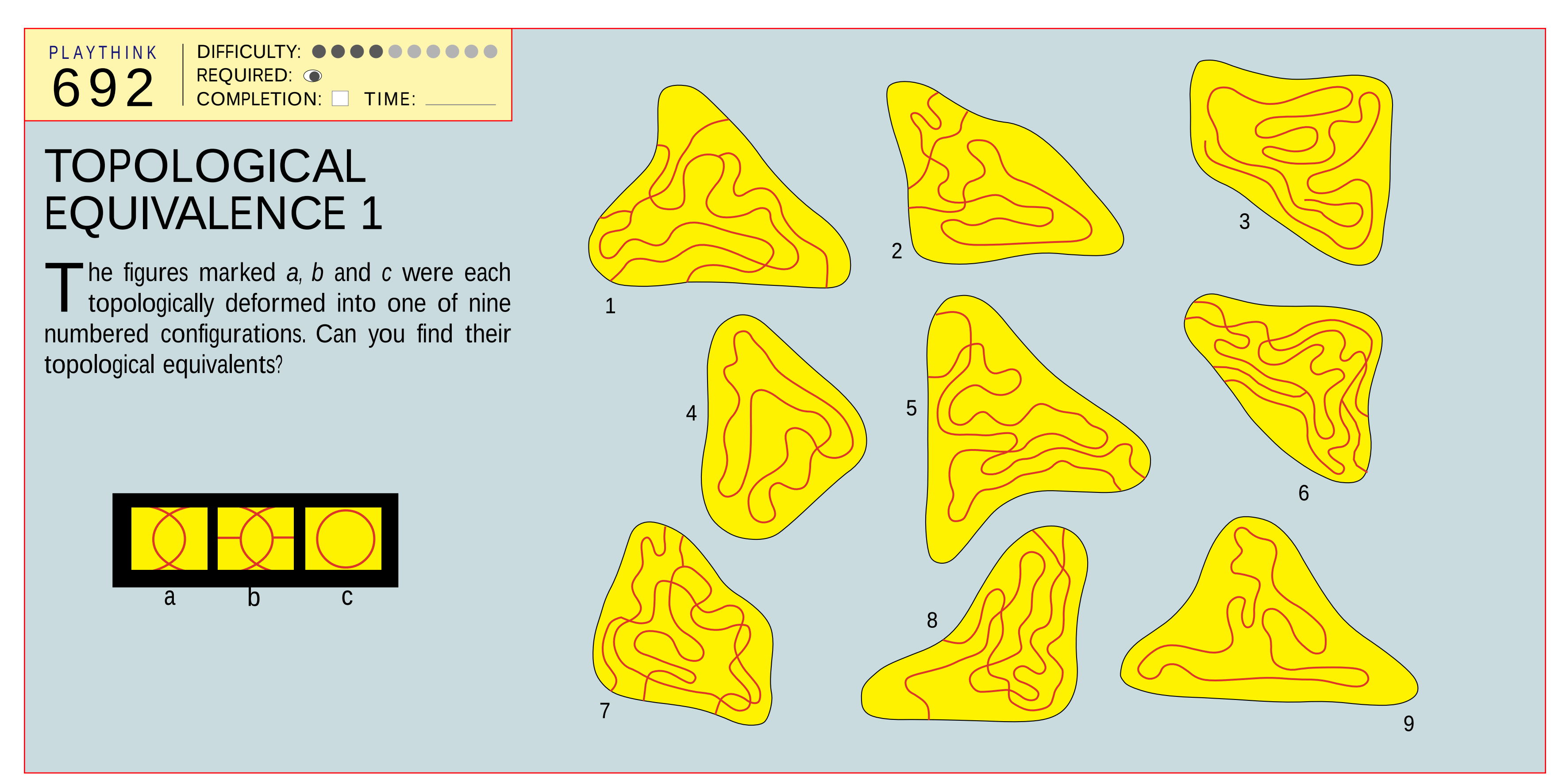
Anyone can connect all nineteen points in a closed, continuous path. But can you find the path that possesses the most wiggles? The path shown in the diagram at the left has seventeen angles. Can you find another path that has seventeen angles?



PLAYTHINK 692 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

TOPOLOGICAL EQUIVALENCE 1

The figures marked *a*, *b* and *c* were each topologically deformed into one of nine numbered configurations. Can you find their topological equivalents?



PLAYTHINK 693 | DIFFICULTY: ●●●○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

PICK-UP STICKS 2

In this puzzle each stick can be picked up only when no other stick lies on top of it. Can you work out the sequence in which to pick up all twenty sticks? Also, how many different lengths are represented?

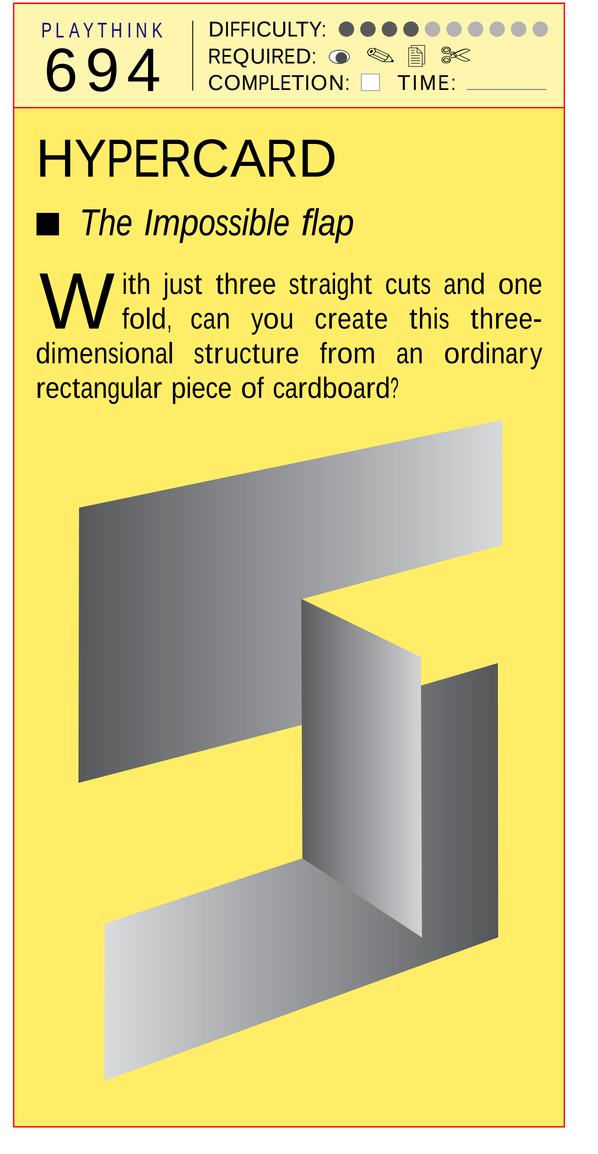


PLAYTHINK 694 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 ✎ 📄 ✂ | COMPLETION: ☐ | TIME: ____

HYPERCARD

■ *The Impossible flap*

With just three straight cuts and one fold, can you create this three-dimensional structure from an ordinary rectangular piece of cardboard?



The Four-Color Theorem

Until recently a long-standing problem in topology dealt with the coloring of maps. In the mid-nineteenth century an Englishman named Francis Guthrie was filling in a map of England—coloring in the counties so that no two adjacent counties had the same color—and wondered how many colors were necessary to complete the job. That bit of puzzling set off a mathematical problem that stayed alive for more than a century.

Mathematicians streamlined the question considerably to make it more general. How many colors, they asked, are needed so that *any* map can be colored in such a way that no adjacent regions (which must touch along an edge, not just at a point) have the same color? It is easy to show that at least four colors are needed. In 1879, a few years after Guthrie posed the four-color problem, an English mathematician named Alfred Bray Kempe published a proof that no map needed five colors, but in 1890 a subtle but crucial mistake was found in his proof: it actually showed that no map requires *six* colors.

Mathematicians wrestled with the problem for almost a century. No one could find a map that actually needed five colors, but then no one could prove that no such map existed. The four-color problem became notorious as one of the simplest remaining unsolved mathematical problems. To make matters worse, in the efforts to answer such a simple question, analogous problems dealing with more complicated surfaces were solved conclusively. For example, a map on a doughnut can always be colored with seven colors. A strange, one-sided surface called the Klein bottle requires six or more colors to fill in all possible regions.

It took a pair of mathematicians using a supercomputer to finally crack the four-color problem. Wolfgang Haken and Kenneth Appel of the University of Illinois broke the problem into a set of subproblems, each of which could be solved by computer. By 1976 they had found a solution, and now the former "problem" is called the four-color theorem.

PLAYTHINK 695



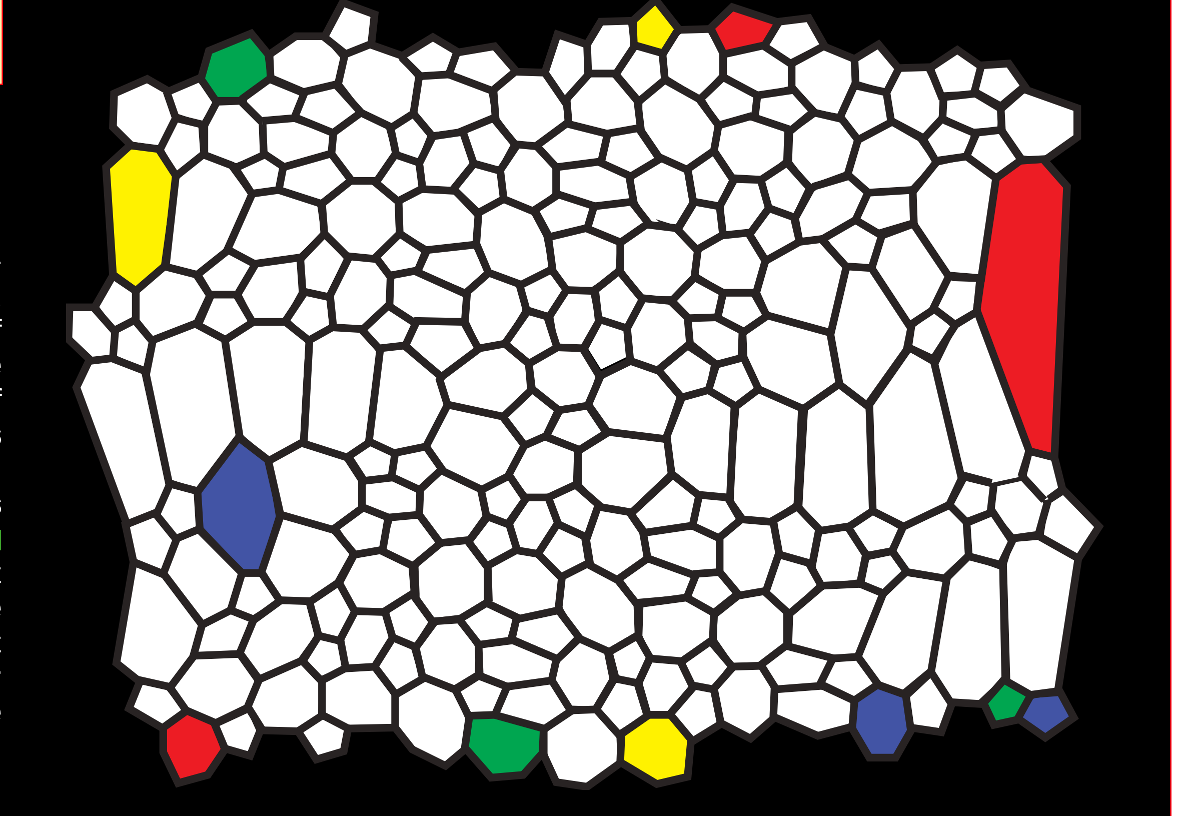
DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

COLOR CUL-DE-SAC

■ *Coloring Map of 210 Countries*

Can you fill in this map with just four colors? If you start filling in the regions, you may soon run into problems. The difficulty of avoiding color cul-de-sacs—regions that have been filled in to create areas where none of the four colors can be used—is what makes this two-player game so much fun.

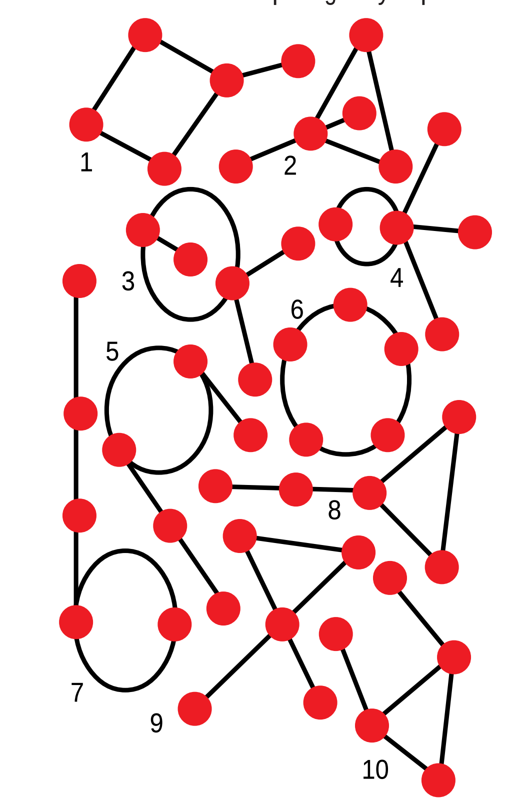
The first player selects a region and fills it in with one of the four colors: ■ ■ ■ ■ The second player colors in an adjacent region, following the exact opposite of the domino principle—no two regions that touch can be the same color. The last player who can fill in a region while following the rules wins the game.



PLAYTHINK 696 DIFFICULTY: ●●●●●○○○○○ REQUIRED: 👁 COMPLETION: ☐ TIME: ______

TOPOLOGICAL EQUIVALENCE 2

Consider that these structures are made from rubber bands and beads. Can you work out which are topologically equivalent?



> "A CHILD'S . . . FIRST GEOMETRICAL DISCOVERIES ARE TOPOLOGICAL. . . . IF YOU ASK HIM TO COPY A SQUARE OR A TRIANGLE, HE DRAWS A CLOSED CIRCLE."
>
> —JEAN PIAGET

PLAYTHINK 697 DIFFICULTY: ●●●○○○○○○○ REQUIRED: 👁 ✎ 📄 ✂ COMPLETION: ☐ TIME: ______

FOUR-COLOR HONEYCOMB

■ *A Topological Game*

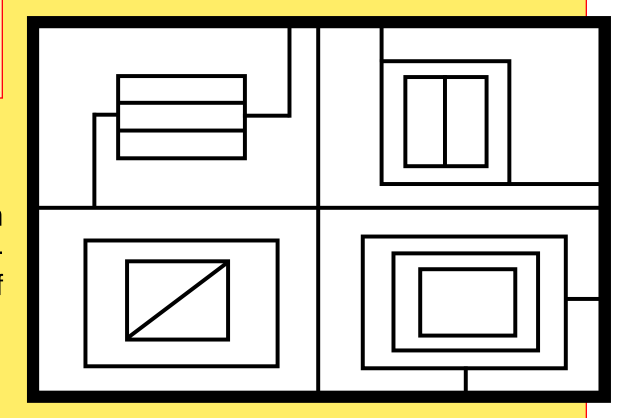
This two-person game tests your ability to anticipate so-called color cul-de-sacs.

Copy and cut out the sixteen colored hexagons and lay them face down on the table. The first player selects a hexagon and places it on any space on the board that does not share an edge with a boundary region of the same color. Players then alternate selecting and placing hexagons on spaces that do not border a boundary region or hexagon of the same color. The last player who can place a hexagon wins.

PLAYTHINK 698 DIFFICULTY: ●●○○○○○○○○ REQUIRED: 👁 ✎ COMPLETION: ☐ TIME: ______

COLORING PATTERN

Say you wanted to fill in this outlined pattern without using the same color in two adjacent areas. What is the minimum number of colors you would need?

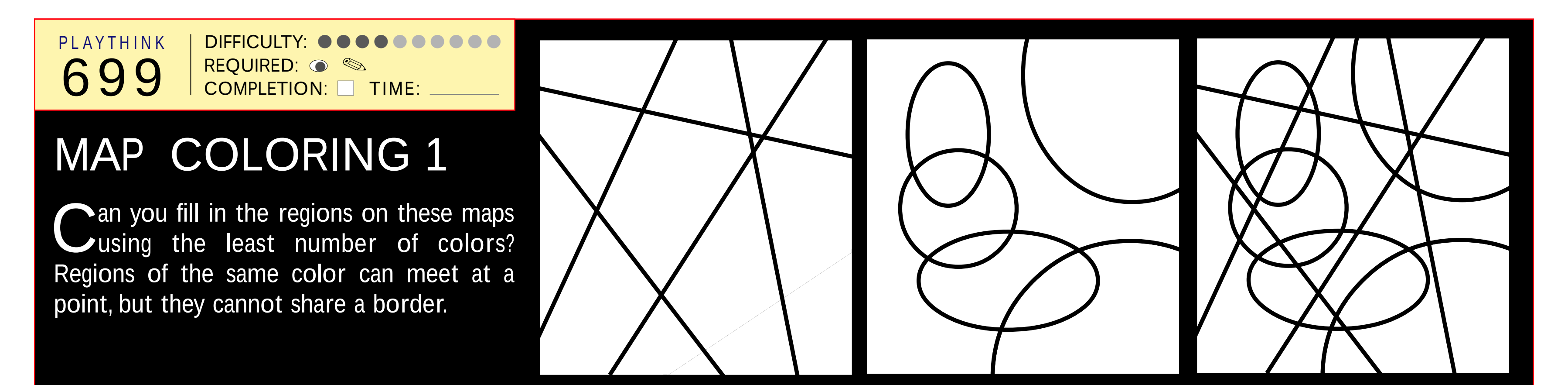


The Two-Color Theorem

Although four colors are needed for regular maps, maps drawn in a special way may not need even that many. One extreme case involves maps drawn using only straight lines. A little scratch-paper experimenting suggests that two colors might be sufficient. Is this true?

The proof that it is requires little effort to grasp. Simply add lines one by one to a map; as you add each line, interchange the two colors on all regions that lie on one side of the new line. The colored regions that remain the same still differ across old boundaries, while they differ across the new borders thanks to the interchange of colors.

The same proof can be generalized to apply to maps on which the boundaries are either single curves that run right across the whole plane or closed loops. Each of those two-color maps possesses an even number of edges that meet at any junction. That must be true of any map that can be colored with just two colors, because the regions around a junction or corner must be of alternate colors. Moreover, it can be proved that any map on a plane can be colored with just two colors if and only if all its junctions have an even number of edges. That's the two-color theorem.

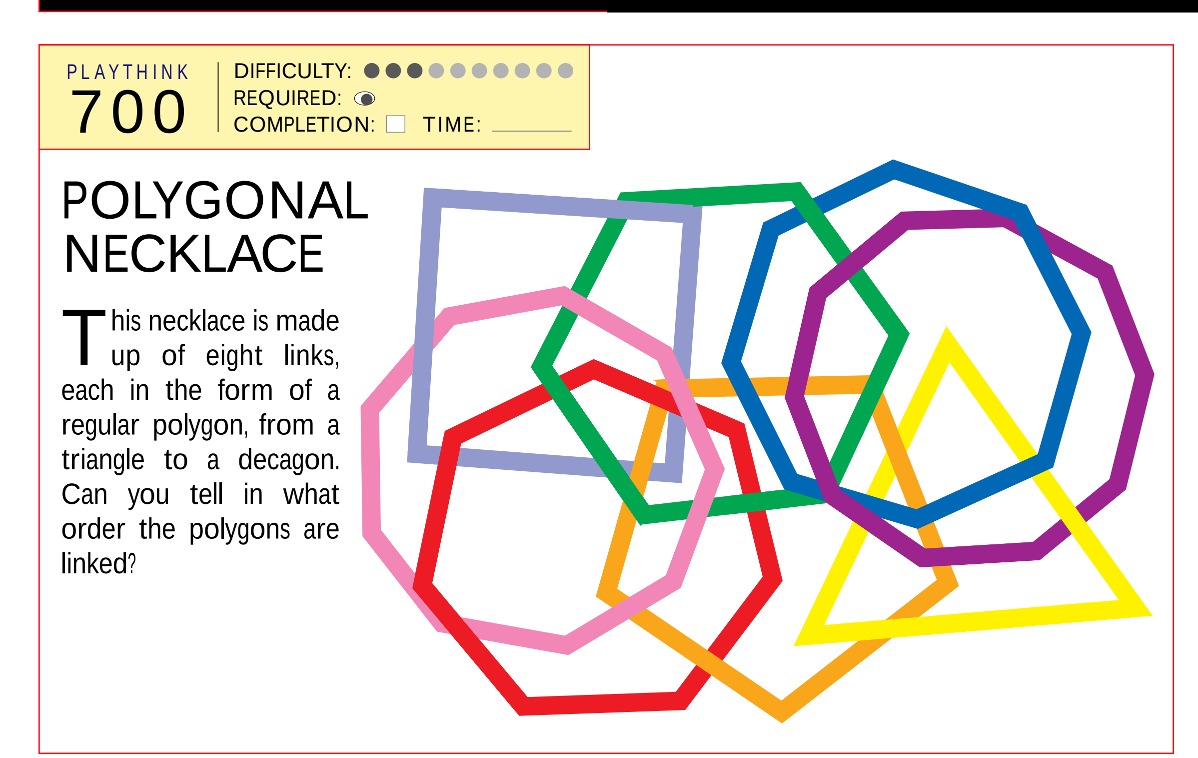


PLAYTHINK 699

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

MAP COLORING 1

Can you fill in the regions on these maps using the least number of colors? Regions of the same color can meet at a point, but they cannot share a border.



PLAYTHINK 700

DIFFICULTY: ●●●○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

POLYGONAL NECKLACE

This necklace is made up of eight links, each in the form of a regular polygon, from a triangle to a decagon. Can you tell in what order the polygons are linked?

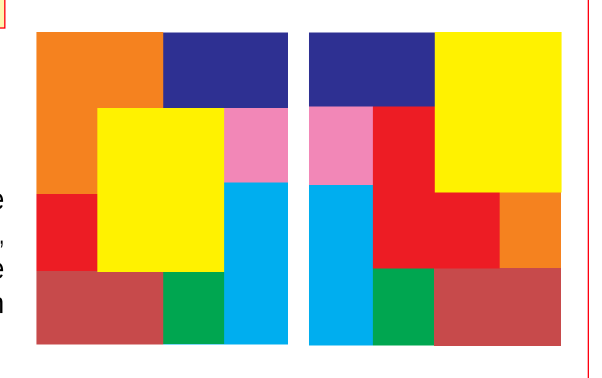
"A TOPOLOGIST IS ONE WHO DOESN'T KNOW THE DIFFERENCE BETWEEN A DOUGHNUT AND A COFFEE CUP."

—John L. Kelley

PLAYTHINK 701 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ______

OVERLAPPING CARDS

Eight playing cards of different colors are stacked in two overlapping patterns, shown here. Can you work out the order the cards were laid down—from 8 for the bottom card to 1 for the top—for each pile?

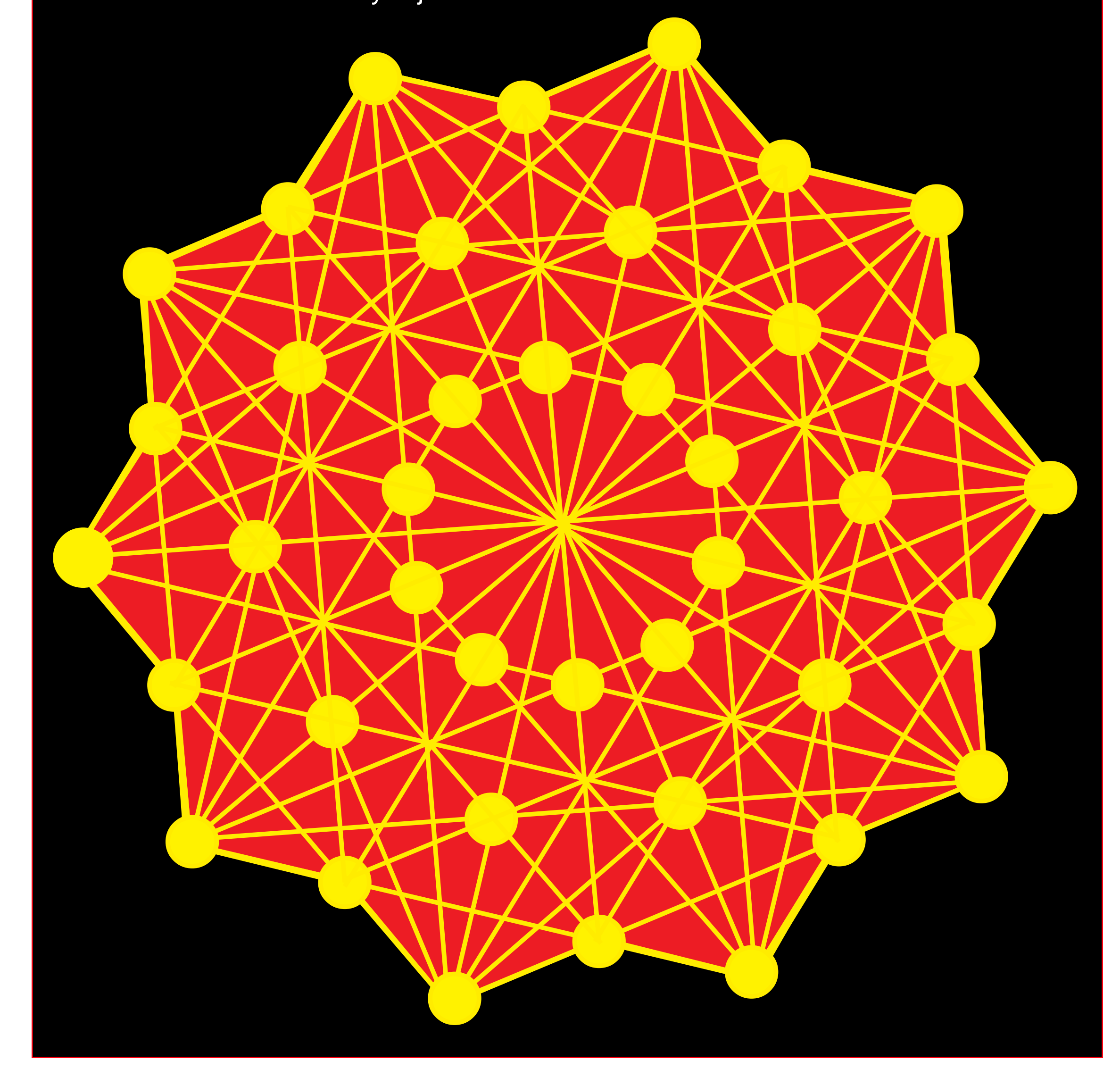


PLAYTHINK 702 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ______

FOUR-IN-A-ROW GAME

Up to four can play this strategy game. Play begins with each player making ten counters in a color of his or her choice. Then players take turns placing one counter at a time on the board. After all the counters are on the board, each person in turn moves one counter from its circle to any adjacent circle connected by a line. If a move places a counter in a row in which the player has more counters than an opponent, the player may remove that opponent's piece.

Play continues until a player wins by moving four counters to the same line.



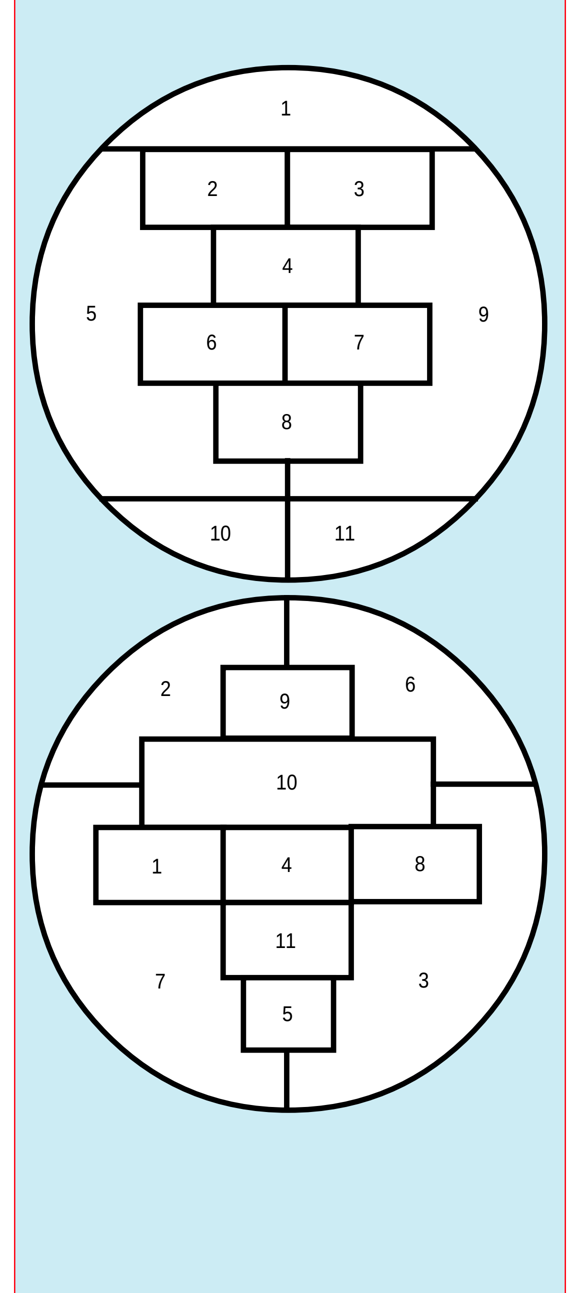
PLAYTHINK 703 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ______

MARS COLONY

German mathematician Gerhard Ringel proposed this map problem in 1950.

Imagine that the eleven major nations on Earth have staked out territory on Mars for colonization. There is one region for each nation. To help keep the political distinctions clear, the nations insist that maps of Mars depict colonies in the same color used for mother countries on Earth maps.

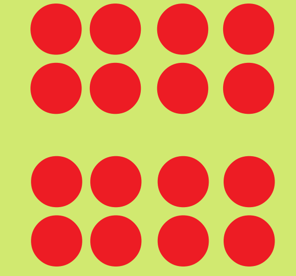
Using the same color for regions that have the same number, can you fill in both maps so that no neighboring regions share a color? How many colors will you need?

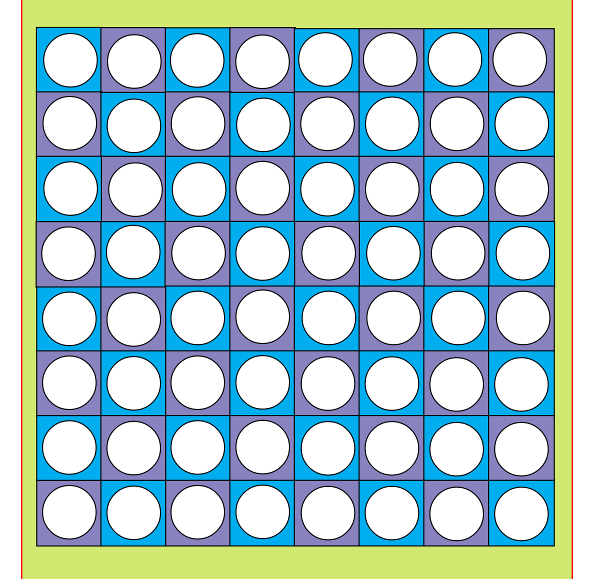


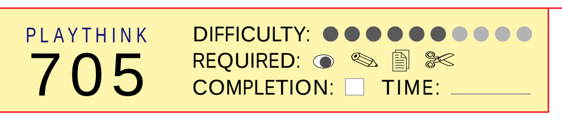
PLAYTHINK 704
DIFFICULTY: ●●●●●●●●○○
REQUIRED:
COMPLETION: ☐ TIME: ______

QUEENS' STANDOFF

1. Can you place ten queens on a standard chessboard so that each queen can attack only one other queen?
2. Can you place fourteen queens so that each can attack exactly two other queens?
3. Can you place sixteen queens on a standard chessboard so that each can attack only three other queens?





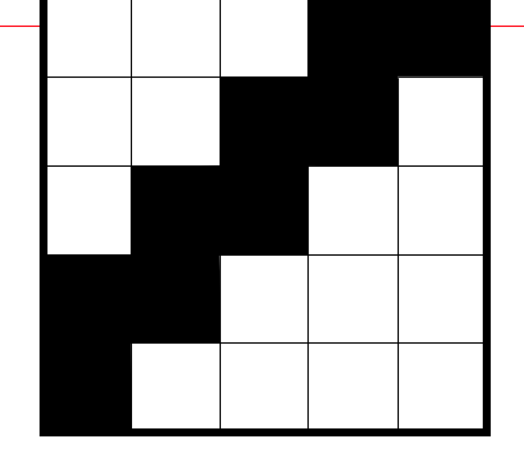


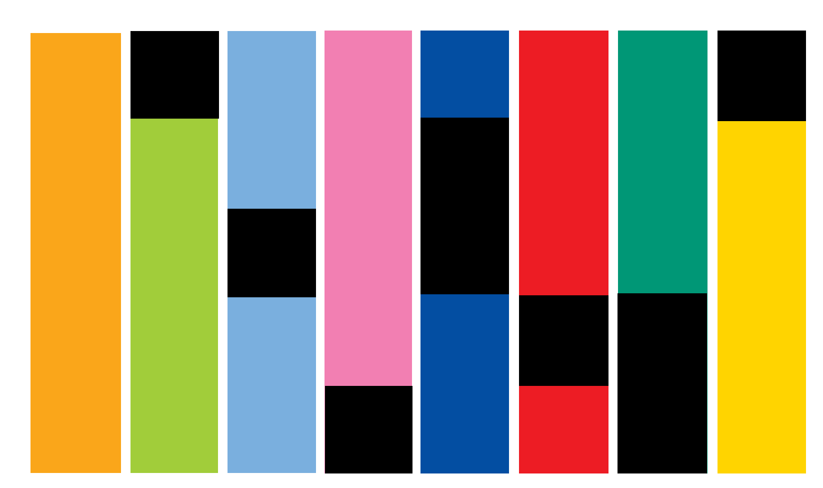
PLAYTHINK 705
DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

ZIGZAG OVERLAP

Can you work out how to fit the eight strips into the five-by-five grid so that you will see the continuous black band running diagonally across the board, as shown?

What is the sequence in which the strips must be laid?



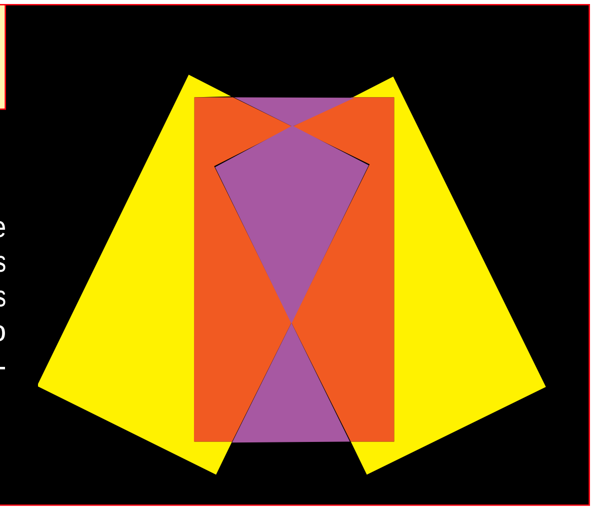




PLAYTHINK 706
DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

OVERLAP

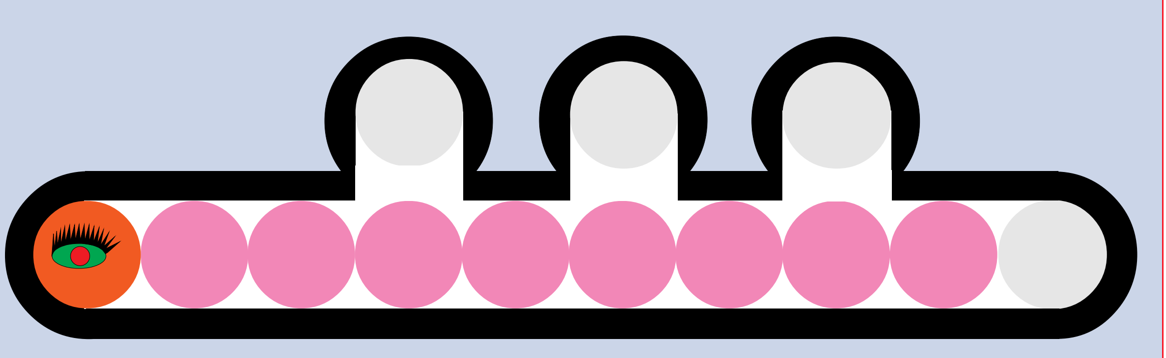
Three identical rectangular frames are placed one on top of the other, as shown. The result of their intersections is seven regions. Can you work out a way to obtain twenty-five regions from the intersection of the same overlapping rectangles?



PLAYTHINK 707
DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

SNAKE

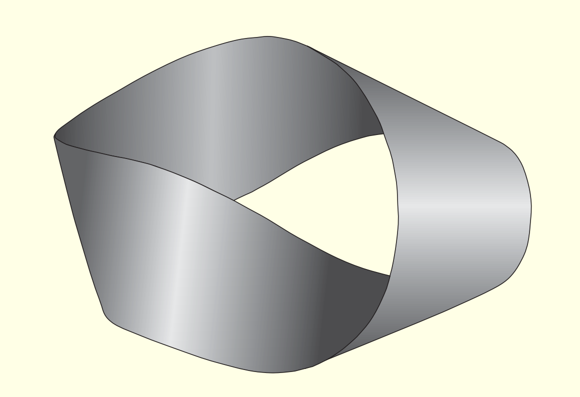
Nine disks are arranged as shown, with the eye of the snake on the left. The object of this puzzle is to transfer the eye to the other end in the fewest possible number of moves. (In this puzzle a move counts as an instance in which you place a disk in one of the three spaces in the side of the snake.)



Möbius Strip

Bizarre shapes and strange connections make math interesting, and nothing is more strangely fascinating than the simplicity and topology of the Möbius strip. The nineteenth-century German mathematician A. F. Möbius discovered that it was possible to make a surface that has only one side and one edge.

Although such an object seems impossible to imagine, making a Möbius strip is very simple: take a strip of ordinary paper and give one end a twist, then glue the two ends together. And there it is. If you begin drawing a line lengthwise down the strip, after one full revolution you will be at the point where you started—but on the opposite "side" of the strip! Drawing the line through another full revolution will find you back at the beginning.

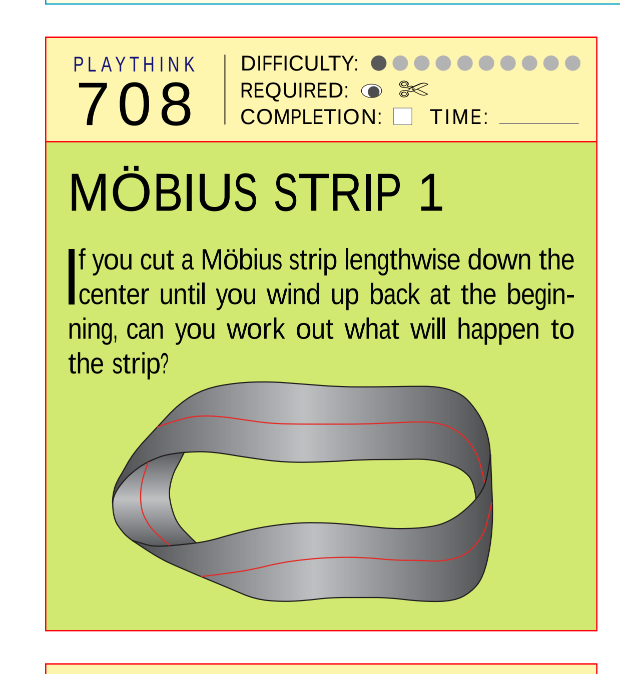


Möbius strips are fun to play with, but industrial engineers have made good use of the shape as well. Conveyor belts are often designed as Möbius strips so that the surface wears out half as fast.

PLAYTHINK 708 | DIFFICULTY: ●○○○○○○○○○ | REQUIRED: eye, scissors | COMPLETION: ☐ TIME: ____

MÖBIUS STRIP 1

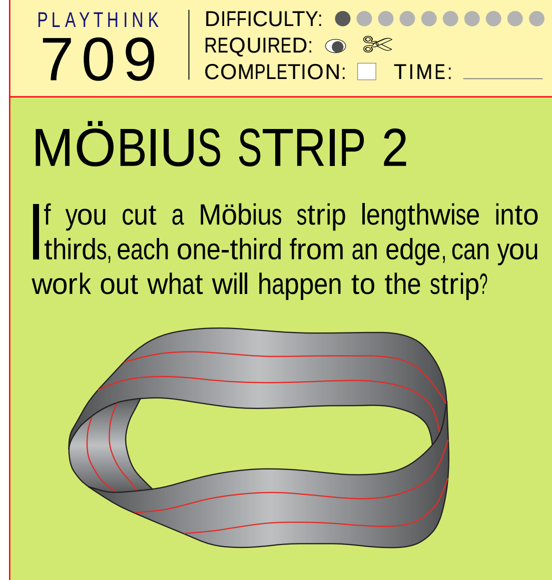
If you cut a Möbius strip lengthwise down the center until you wind up back at the beginning, can you work out what will happen to the strip?



PLAYTHINK 709 | DIFFICULTY: ●○○○○○○○○○ | REQUIRED: eye, scissors | COMPLETION: ☐ TIME: ____

MÖBIUS STRIP 2

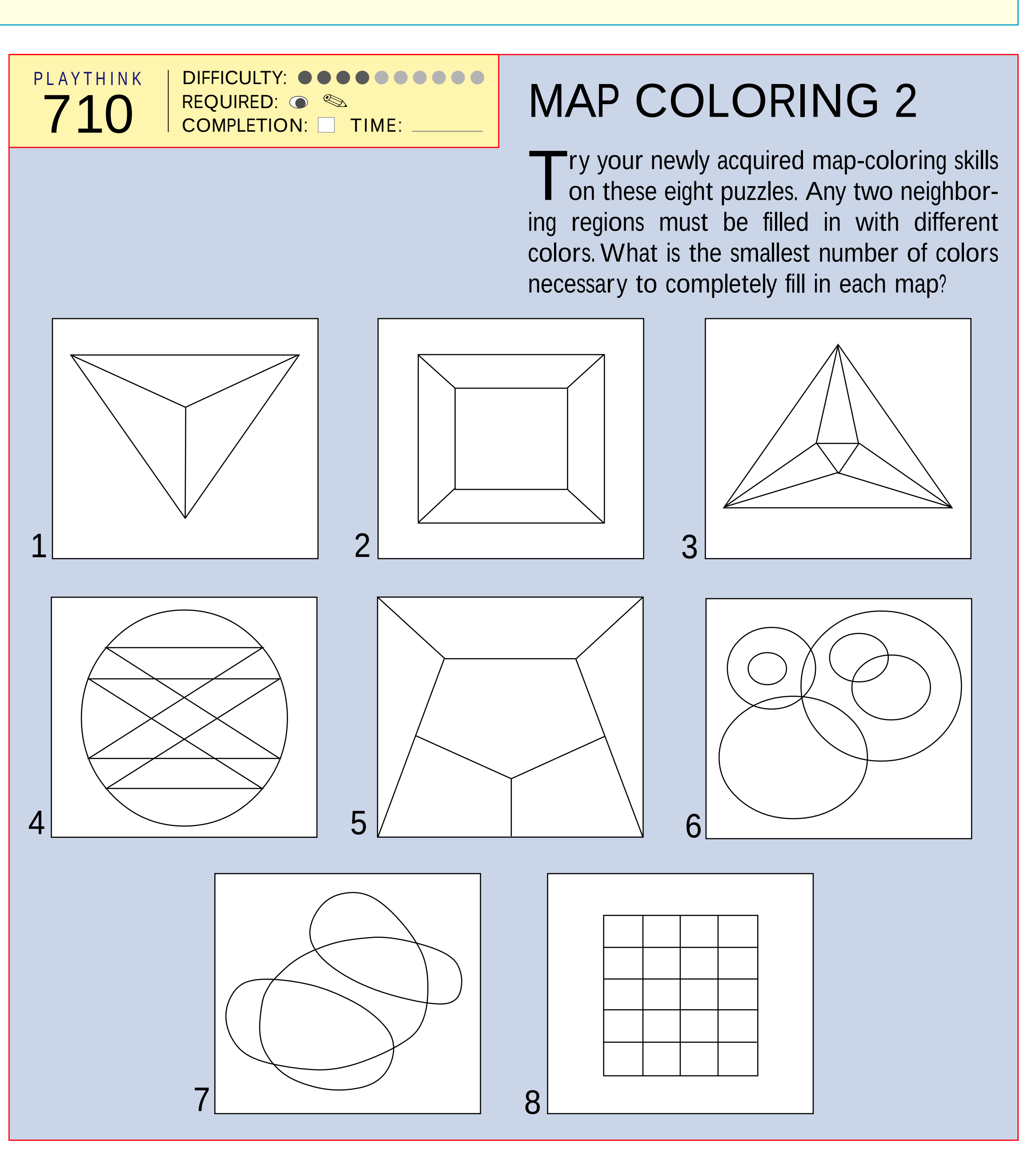
If you cut a Möbius strip lengthwise into thirds, each one-third from an edge, can you work out what will happen to the strip?



PLAYTHINK 710 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: eye, pencil | COMPLETION: ☐ TIME: ____

MAP COLORING 2

Try your newly acquired map-coloring skills on these eight puzzles. Any two neighboring regions must be filled in with different colors. What is the smallest number of colors necessary to completely fill in each map?

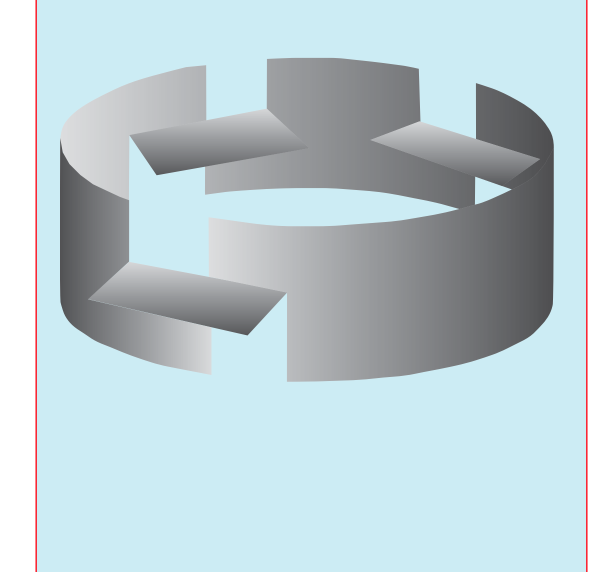


PLAYTHINK 711 DIFFICULTY: ●●●○○○○○○○ REQUIRED: COMPLETION: ☐ TIME: ____

HYPERCARD RING

This diagram shows a curious piece of furniture made from a single piece of bent plywood. As you can see, it is circular, with two benches on the inside of the ring and one bench on the outside.

Can you construct a model of the structure from a single strip of paper? Once you have made the model, can you see how to use it to show an opposite configuration for the benches—two on the outside and one on the inside?



PLAYTHINK 712 DIFFICULTY: ●●●●●○○○○○ REQUIRED: COMPLETION: ☐ TIME: ____

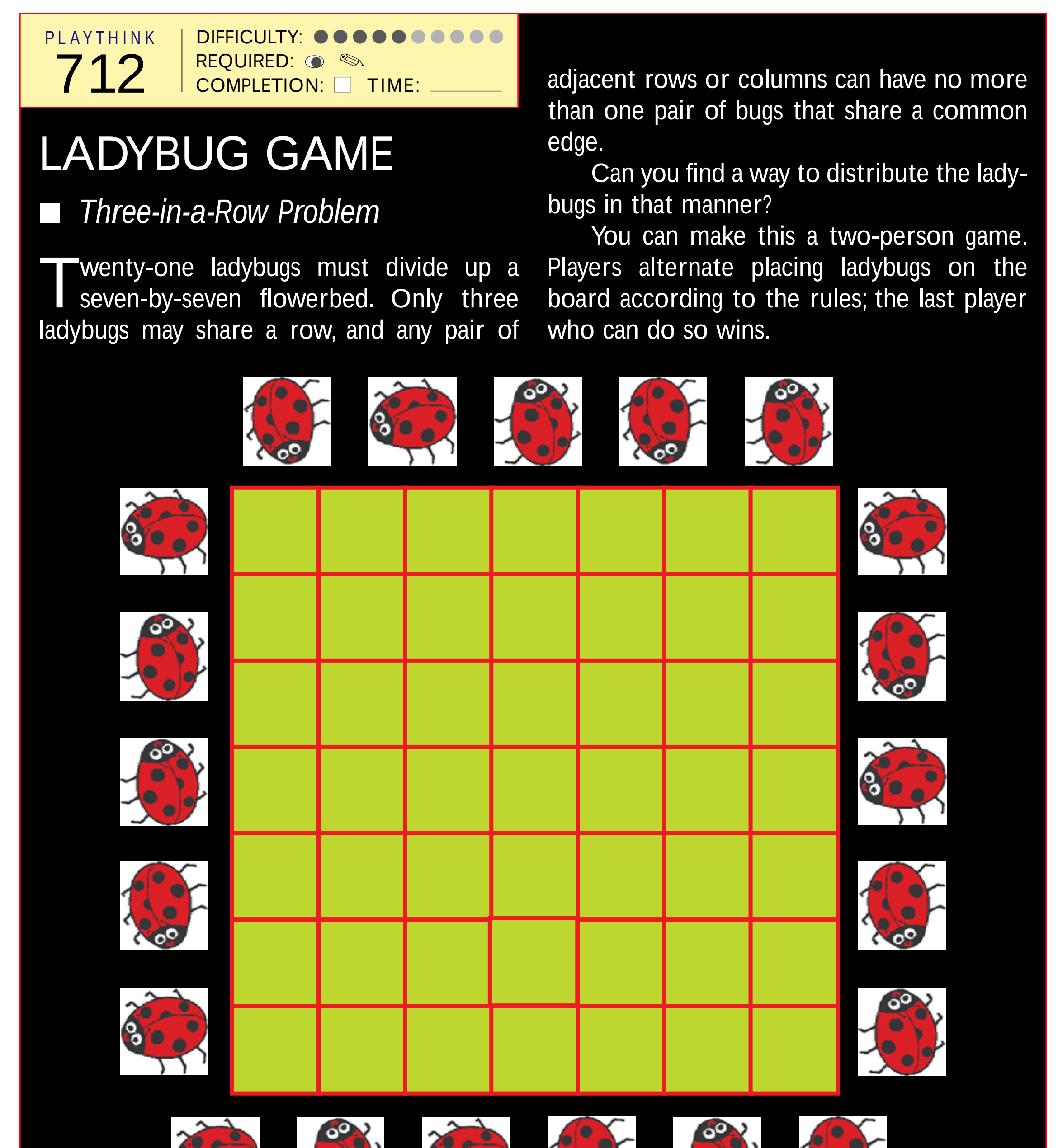
LADYBUG GAME

■ *Three-in-a-Row Problem*

Twenty-one ladybugs must divide up a seven-by-seven flowerbed. Only three ladybugs may share a row, and any pair of adjacent rows or columns can have no more than one pair of bugs that share a common edge.

Can you find a way to distribute the ladybugs in that manner?

You can make this a two-person game. Players alternate placing ladybugs on the board according to the rules; the last player who can do so wins.

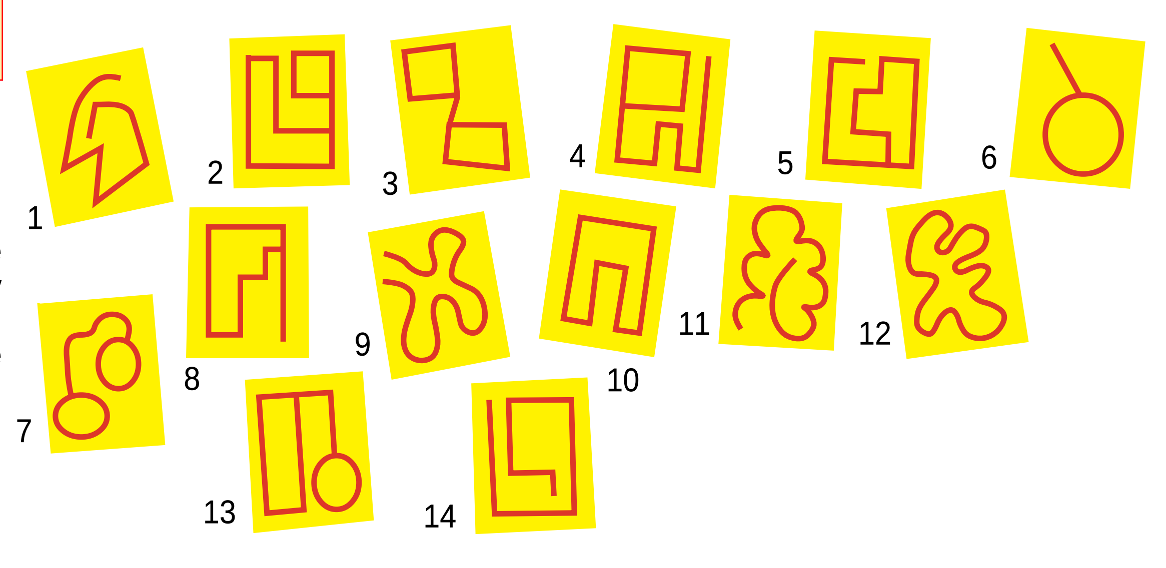


PLAYTHINK 713 DIFFICULTY: ●●●●●●○○○○ REQUIRED: COMPLETION: ☐ TIME: ____

TOPOLOGICAL EQUIVALENCE 3

These fourteen drawings include three quartets and one pair of topologically equivalent figures.

Can you identify the solitary pair amid the quartets?



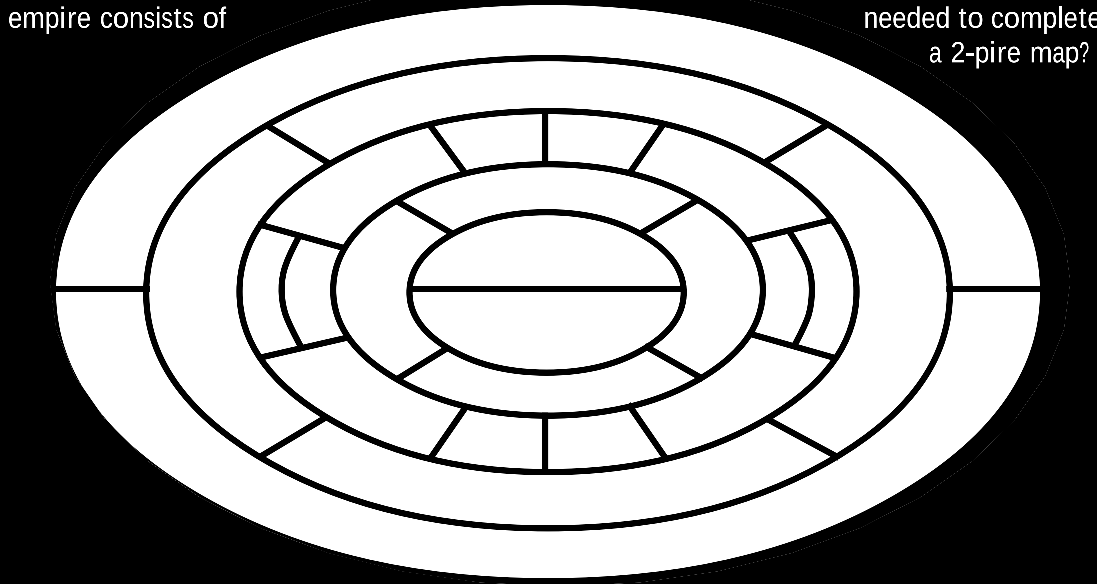
PLAYTHINK 714 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

M-PIRE COLORING GAME

The four-color problem is the most famous map puzzle, but there's another, equally challenging puzzle that involves allowing different regions of the map to belong to the same "empire," which requires that they receive the same color. If each empire consists of exactly *M* regions this is called the "M-pire" problem.

The usual map-coloring question is a 1-pire problem. A 2-pire problem considers maps in which pairs of allied regions are considered—and the map must use the same color for both halves of the 2-pire, though different 2-pires can be different colors. And, of course, no two adjacent areas can be the same color.

Given that, can you work out the minimum number of colors needed to complete a 2-pire map?



PLAYTHINK 715 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

TOPOLOGY OF THE ALPHABET

Two figures are topologically equivalent if one can be continuously deformed into the other. A triangle, to the topologist's eye, is no different than a square or even a circle.

The letter *E*, in the font shown below, is topologically equivalent to five other letters. Can you work out which ones?

ABCDE
FGHIJ
KLMNO
PQRST
UVWXYZ

PLAYTHINK 716 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

COLORING POLYHEDRONS

There are five regular solids, or polyhedrons: the tetrahedron (four faces), the cube (six faces), the octahedron (eight faces), the dodecahedron (twelve faces) and the icosahedron (twenty faces). To help you color in each face, you can think of each polyhedron as a map on a sphere, although it's a rather bent and bumpy sphere.

To aid in coloring, the five regular solids shown at right are covered with a rubber sheet. This allows us to "skin" the figures to create plane graphs that can easily be colored.

Using these graphs as a guide, can you work out how many colors are needed to fill in the faces of the five regular polyhedrons? Remember, the areas beyond the edges of the graph count as an additional side.

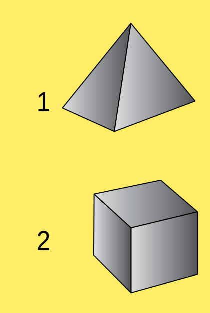
1

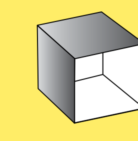
2

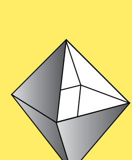
3

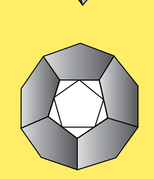
4

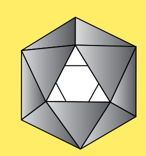
5

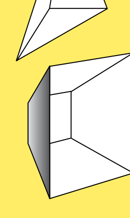


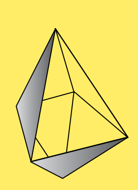


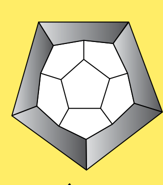


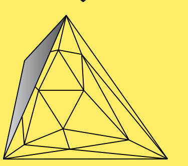


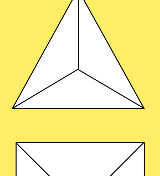


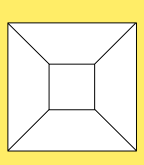


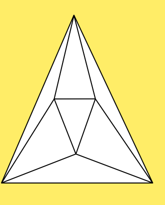


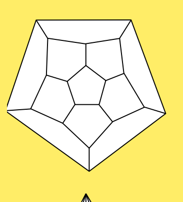


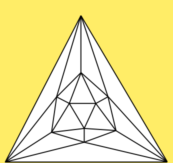








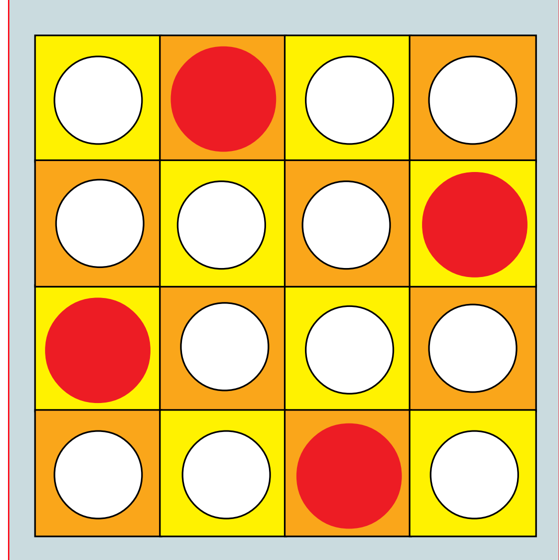




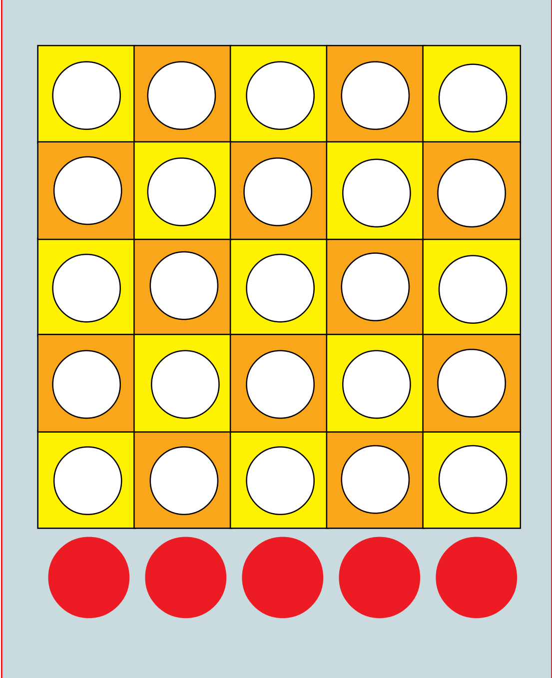
PLAYTHINK 717 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

NO-TWO-IN-A-LINE 1

■ *Queens' Standoff*



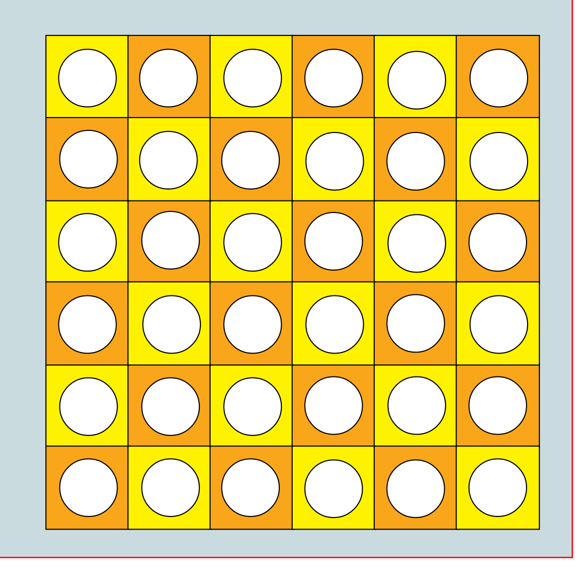
The four red counters in the top diagram are placed in a four-by-four matrix so that no two lie on the same vertical, horizontal or diagonal line. Can you place five counters on a five-by-five board under the same restrictions?



PLAYTHINK 718 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

NO-TWO-IN-A-LINE 2

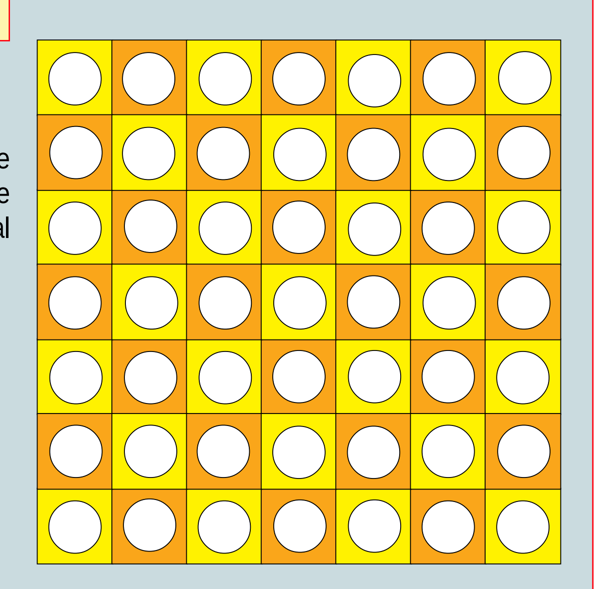
Can you place six counters on this six-by-six board so that no two lie on the same vertical, horizontal or diagonal line?



PLAYTHINK 719 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

NO-TWO-IN-A-LINE 3

Can you place seven counters on the seven-by-seven board so that no two lie on the same vertical, horizontal or diagonal line?



PLAYTHINK 720 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

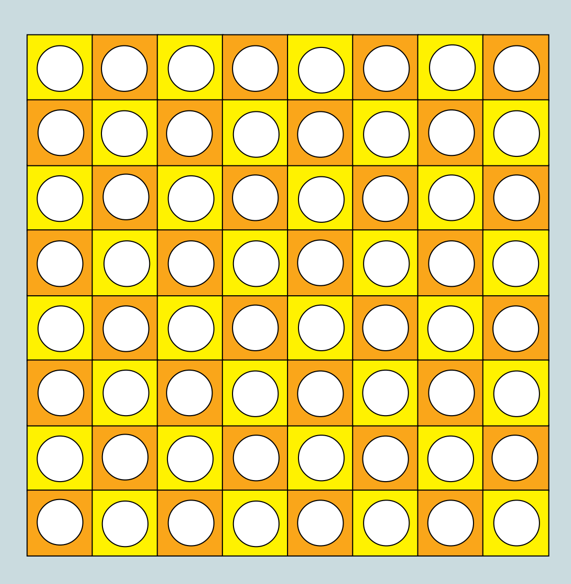
NO-TWO-IN-A-LINE 4

■ *The Eight Queens Puzzle*

Can you place eight counters on the eight-by-eight board so that no two are on the same vertical, horizontal or diagonal line?

This is essentially the same as asking how to place eight queens on a chessboard so that none can attack another. Can you find the twelve different solutions?



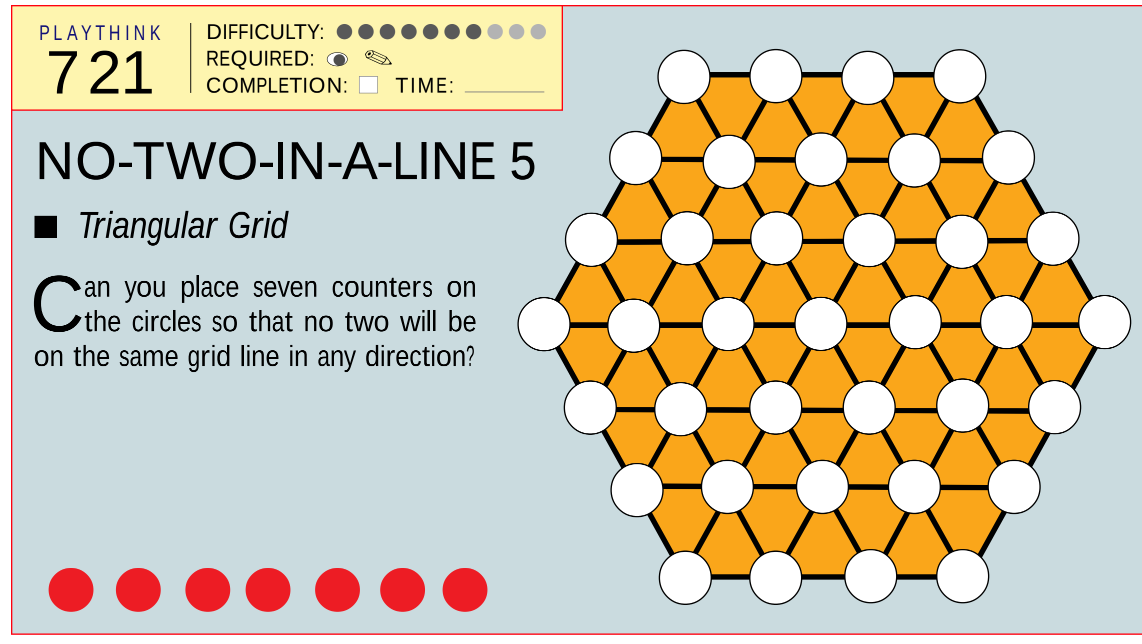


PLAYTHINK 721 DIFFICULTY: ●●●●●●●●○○ REQUIRED: COMPLETION: ☐ TIME: ____

NO-TWO-IN-A-LINE 5

■ *Triangular Grid*

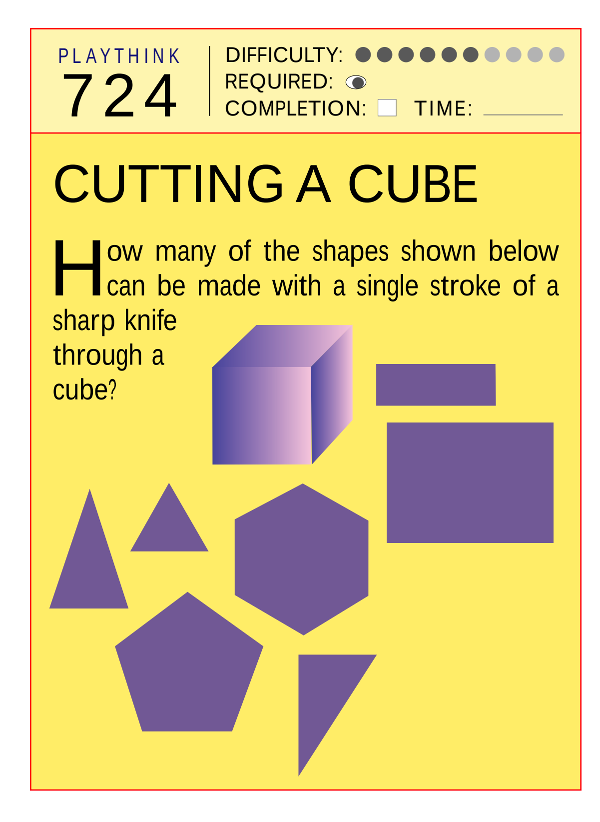
Can you place seven counters on the circles so that no two will be on the same grid line in any direction?



PLAYTHINK 724 DIFFICULTY: ●●●●●●○○○○ REQUIRED: COMPLETION: ☐ TIME: ____

CUTTING A CUBE

How many of the shapes shown below can be made with a single stroke of a sharp knife through a cube?

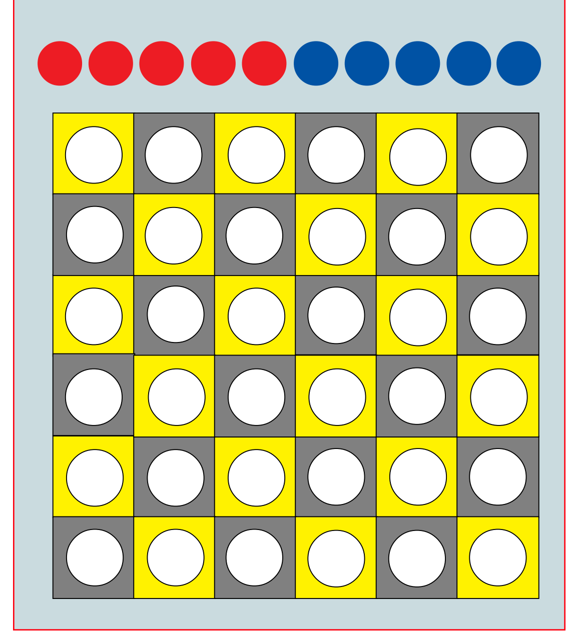


PLAYTHINK 722 DIFFICULTY: ●●●●●●●○○○ REQUIRED: COMPLETION: ☐ TIME: ____

QUEENS' COLOR STANDOFF 1

An interesting variation of the classic eight queens problem involves the placement of differently colored queens. The general question is, how many queens of two (or even more) colors can be placed on a given board so that no queen can be attacked by a piece of another color? That is, no two queens of differing colors may lie on the same vertical, horizontal or diagonal line.

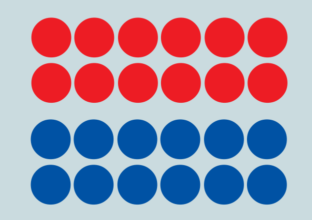
Can you work out how to place five red queens and five blue queens on a six-by-six chessboard so that no queen can be attacked?

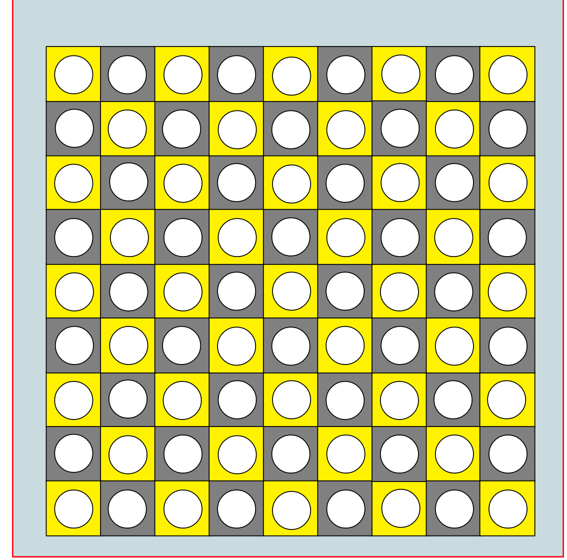


PLAYTHINK 723 DIFFICULTY: ●●●●●●●○○○ REQUIRED: COMPLETION: ☐ TIME: ____

QUEENS' COLOR STANDOFF 2

Can you put all twelve red queens and all twelve blue queens on a nine-by-nine chessboard so that no queen can attack a queen of another color?



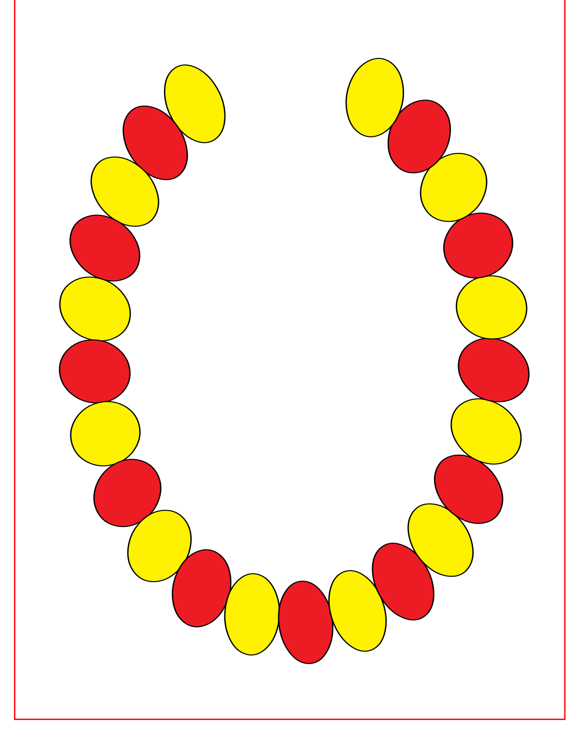


PLAYTHINK 725 DIFFICULTY: ●●●●●○○○○○ REQUIRED: COMPLETION: ☐ TIME: ____

MINIMAL NECKLACE

A twenty-three-bead necklace is shown here. You want to disconnect individual beads to break the necklace into smaller lengths that can then be rejoined to form every possible length from one to twenty-three beads.

Can you work out how many beads must be disconnected to accomplish this?



Knots

Everyone who can tie his shoes understands a little about knots. But mathematicians have turned knots into a field for deep topological study. Don't expect to untie a mathematical knot: both ends of the mathematician's knot are joined to form an endless loop. Such linear structures extending into three dimensions are the simplest representations of curves in three-dimensional space. (The more advanced topological concepts are surfaces and the multidimensional structures known as manifolds.)

This is the first question in knot theory: Can two closed strings made of extensible but impenetrable material be changed by continuous transformation into strings of congruent form? Although knots are one-dimensional, they are trickier than surfaces; untying them poses great problems, many of which are still unanswered. Even in the simplest cases, establishing the proof is a daunting task.

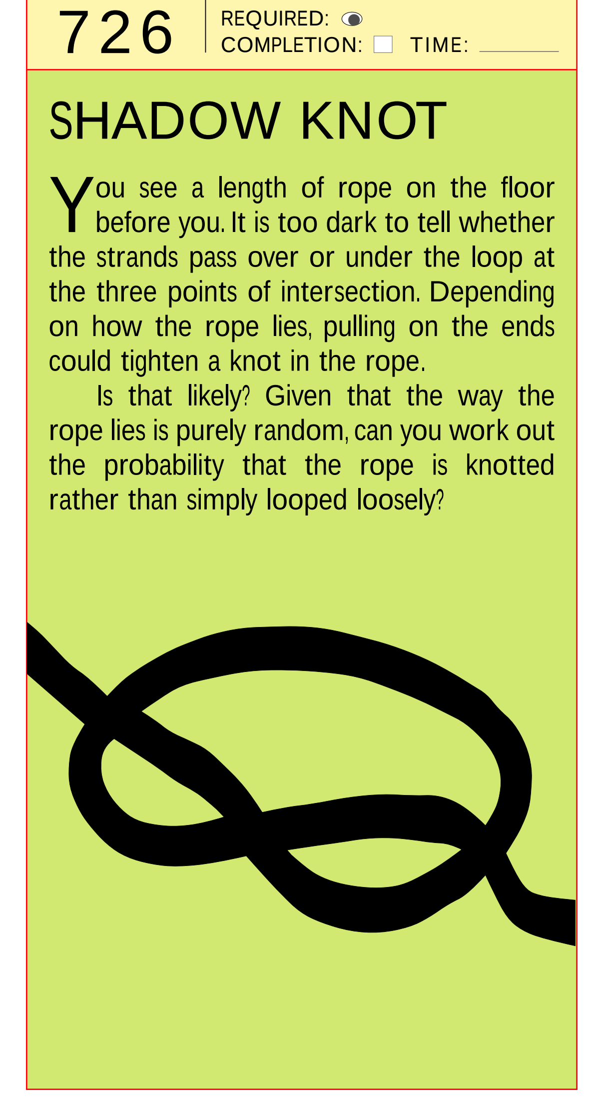
The topology of knots is not merely of interest to recreational and professional mathematicians. It has enormous importance in several other branches of science, particularly molecular biology. The structure of the DNA molecule and those of complexly folded proteins have been elucidated with the help of the mathematical answer to the question, How does one untangle very long three-dimensional knots?

PLAYTHINK 726 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

SHADOW KNOT

You see a length of rope on the floor before you. It is too dark to tell whether the strands pass over or under the loop at the three points of intersection. Depending on how the rope lies, pulling on the ends could tighten a knot in the rope.

Is that likely? Given that the way the rope lies is purely random, can you work out the probability that the rope is knotted rather than simply looped loosely?



PLAYTHINK 727 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

WATER HOSE

This garden hose is a mess. If you pull it tight at both ends, how many knots will be formed?



PLAYTHINK 728

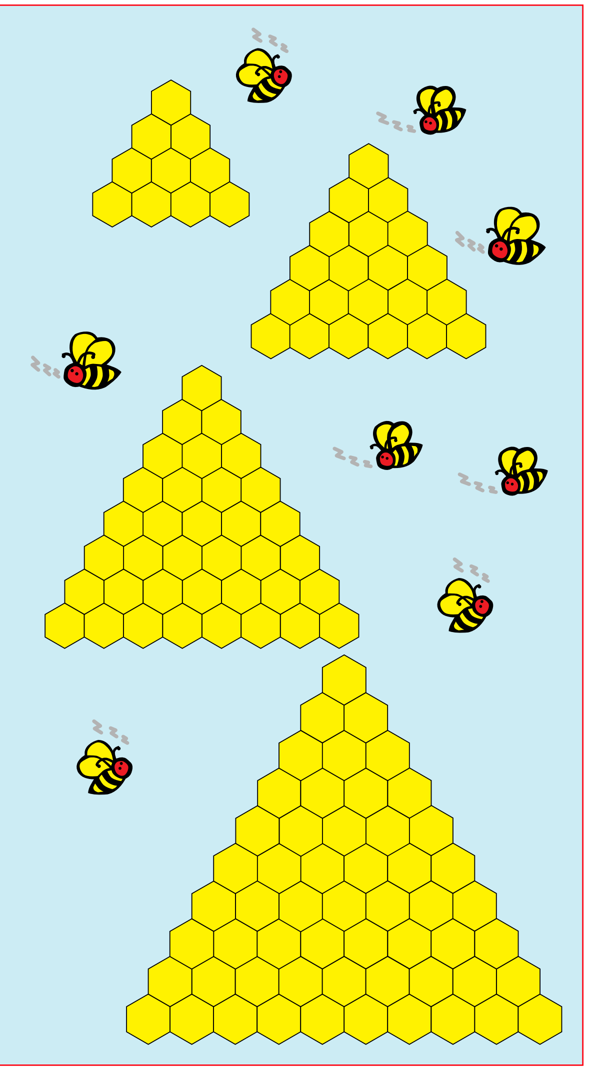
DIFFICULTY: ●●●●●○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

BEE ROOKS

The mathematician Herbert Taylor investigated the nonattacking principle found in the queens' standoff puzzles on hexagonal and triangular matrices. Here's a puzzle based on his findings.

Bees will attack one another if they share the same triangular row or column in the hexagonal grid. With that in mind, can you work out the greatest number of bees that can be placed on each of the four grids shown here?

Can you work out the minimum number of bees needed to guard the four grids, placed so that the addition of one more bee would trigger an attack?



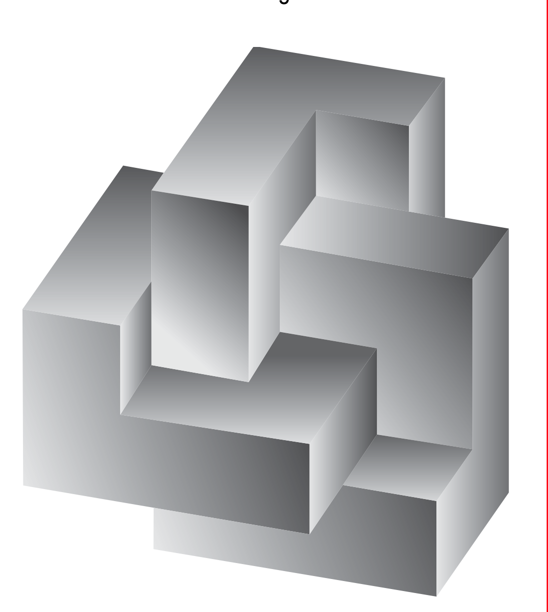
PLAYTHINK 729

DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

3-D KNOT

This figure shows a three-dimensional knot composed of the least possible number of unit cubes. Each cube is the same size, there are no loose ends, and the cubes are connected across their full faces.

Can you work out the number of cubes needed to make this figure?



PLAYTHINK 730

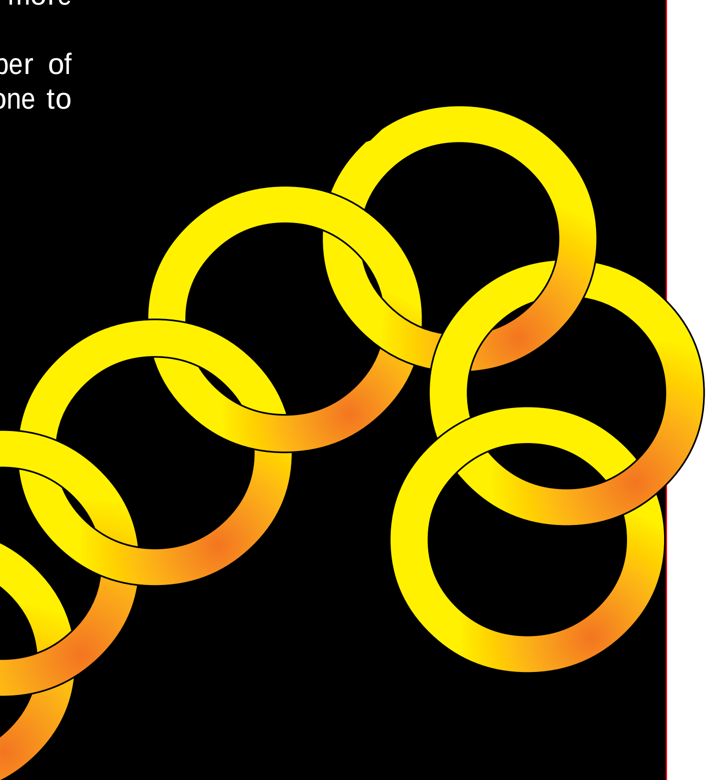
DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

DIVORCEE'S BELT

A faded movie queen, recently divorced (for the third time) and down to her last mink coat, is stranded in a fashionable hotel in Cannes with no immediately available liquid assets. The hotel management has cut her credit line and put her on a cash-in-advance status. The ex-star is willing to make payments only one day at a time. She is expecting an alimony payment in just eleven days, but until then she has no cash on hand.

She has persuaded the hotel to accept a link of her gold belt for a day's rent, with the understanding that when the check arrives, she can buy it back. Her problem is that because she expects to have belt back soon, she does not want to cut it up any more than required.

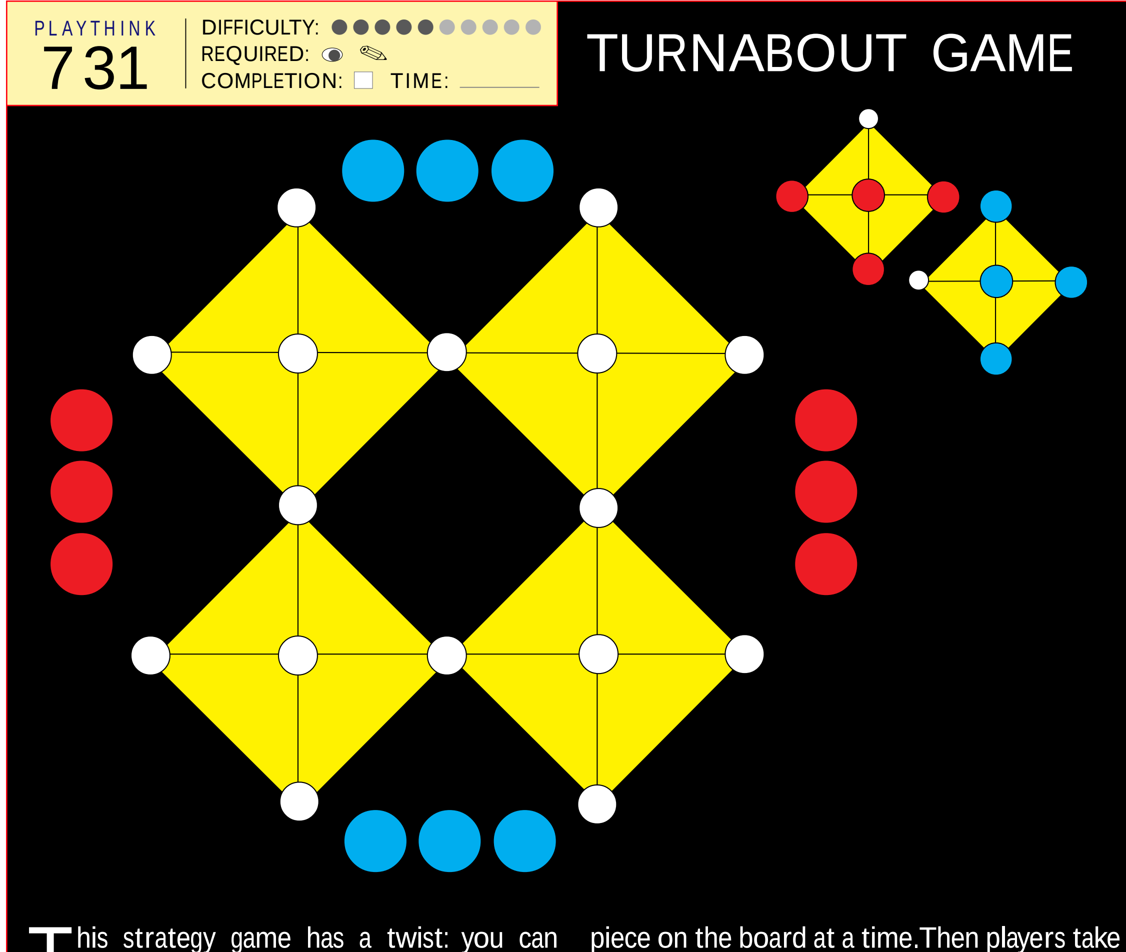
Can you find the minimum number of cuts needed so that she can pay for one to eleven days, one day at a time?



PLAYTHINK 731

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

TURNABOUT GAME



This strategy game has a twist: you can neutralize and play your opponent's pieces.

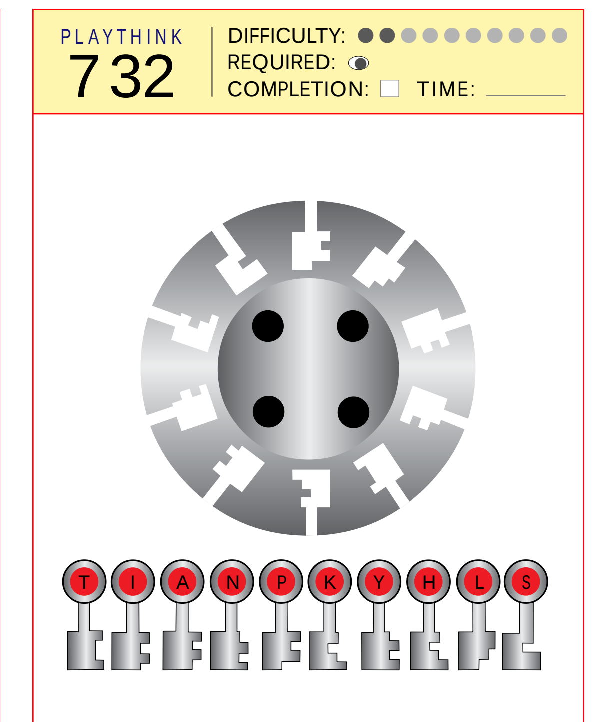
Each player gets six counters—either red or blue on one side and black on the other. When a piece is flipped over to its black side, it can be moved by either player.

Play begins with each person placing one piece on the board at a time. Then players take alternate turns by making one of three types of moves: sliding a counter to an empty adjacent space, jumping over another piece into an empty space or turning a counter over.

The first player who can maneuver four of his or her non-neutral pieces into the shape of a triangle, as shown in the inset diagram, wins.

PLAYTHINK 732

DIFFICULTY: ●●○○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____



COMBINATION LOCK

A safe has ten locks requiring ten keys, each bearing a different letter. The safe opens only after all ten keys have been inserted in the locks.

There are 3.6 million possible combinations, but fortunately, you have a diagram of the interior of the locks that shows the shapes of the appropriate keys.

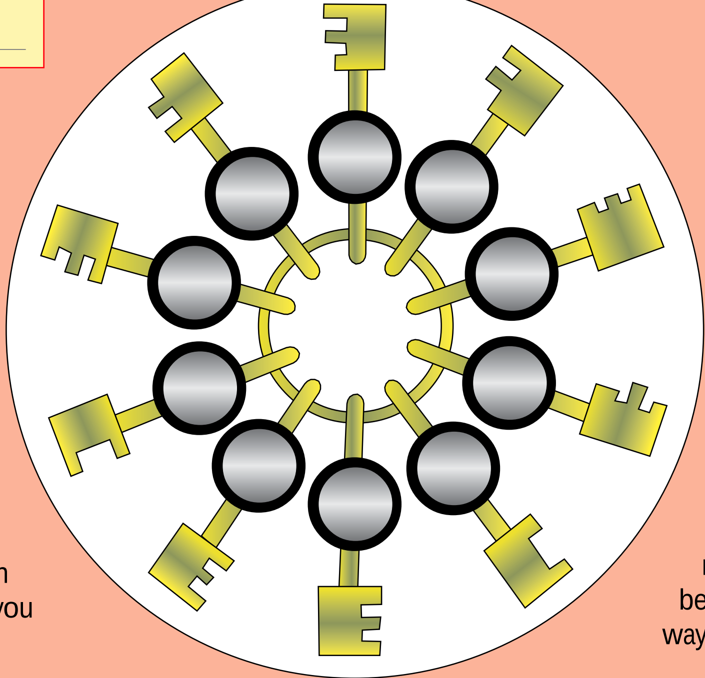
Can you work out the correct order for the keys? What word do the keys spell out?

PLAYTHINK 733

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

KEYS TO THE KEYS

A circular ring holds ten keys, each with a round handle. The keys are in an order that you have memorized, and each fits one of ten different locks. The trouble is that you work in the dark and you can't see the key ring—you have to feel the keys with your fingers. If you had a way to tell by touch which key was which, it wouldn't take you long to unlock the doors.



One solution is to give the keys differently shaped handles. But do you have to give all ten different handles?

Can you work out the smallest number of keys you need to mark so that you can identify where you are on the ring by touch alone? Is there a specific arrangement that the marked keys must have along the ring?

One clue: Remember that any symmetrical arrangement of keys will fail. That's because in the dark you won't know which way you are holding the ring.

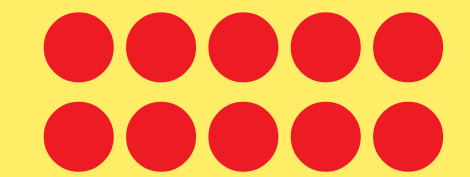
PLAYTHINK 734 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

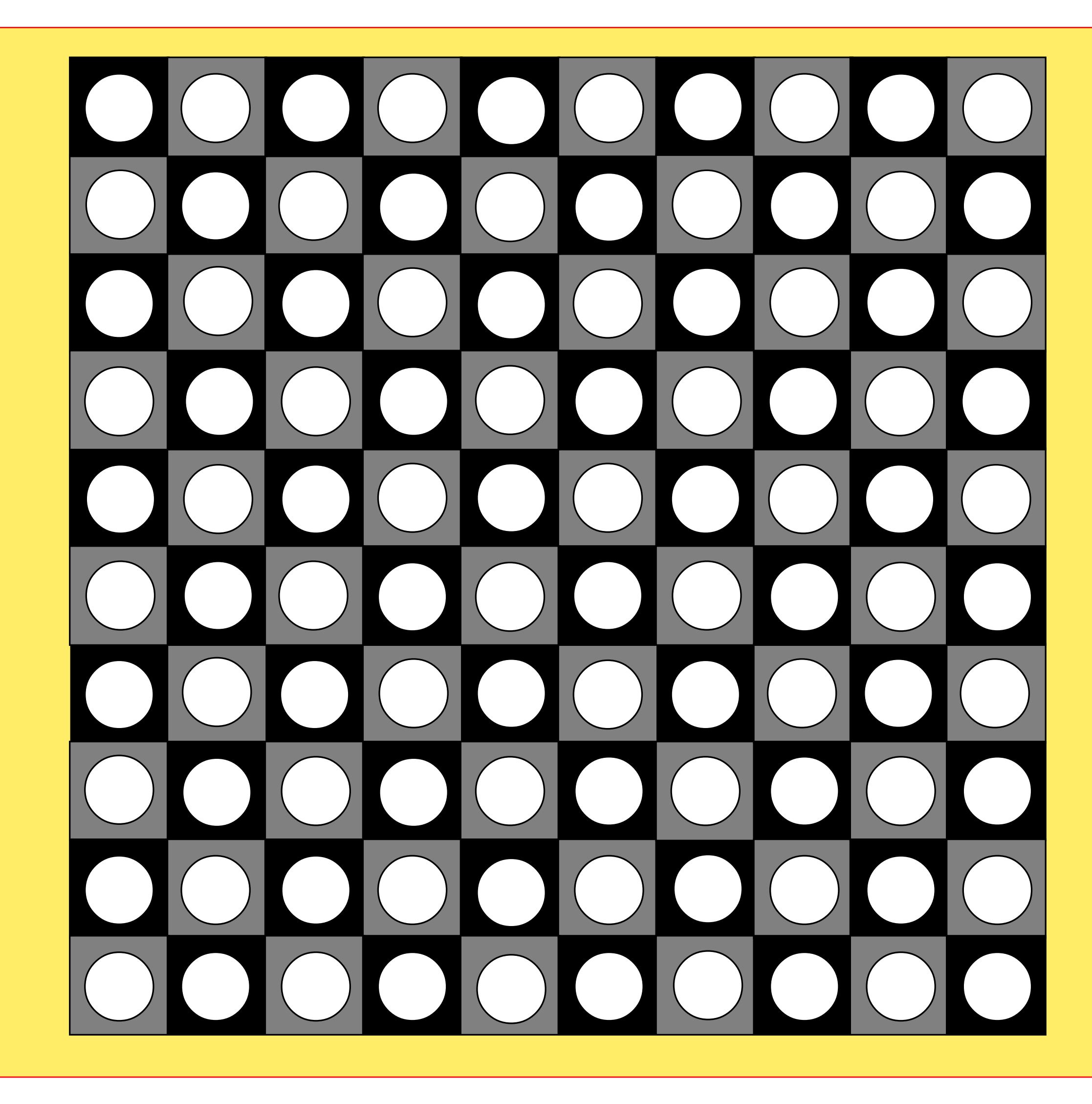
SUPERQUEENS

A superqueen is an imaginary chess piece that has the attacking range of a queen and a knight combined.

More than sixty years ago the mathematician George Polya discovered that with less than ten, it was impossible to place *n* superqueens on an *n*-by-*n* chessboard so that none would come under attack.

But what about ten superqueens? Can you work out how to place ten superqueens on a ten-by-ten board so that none of them can attack another?



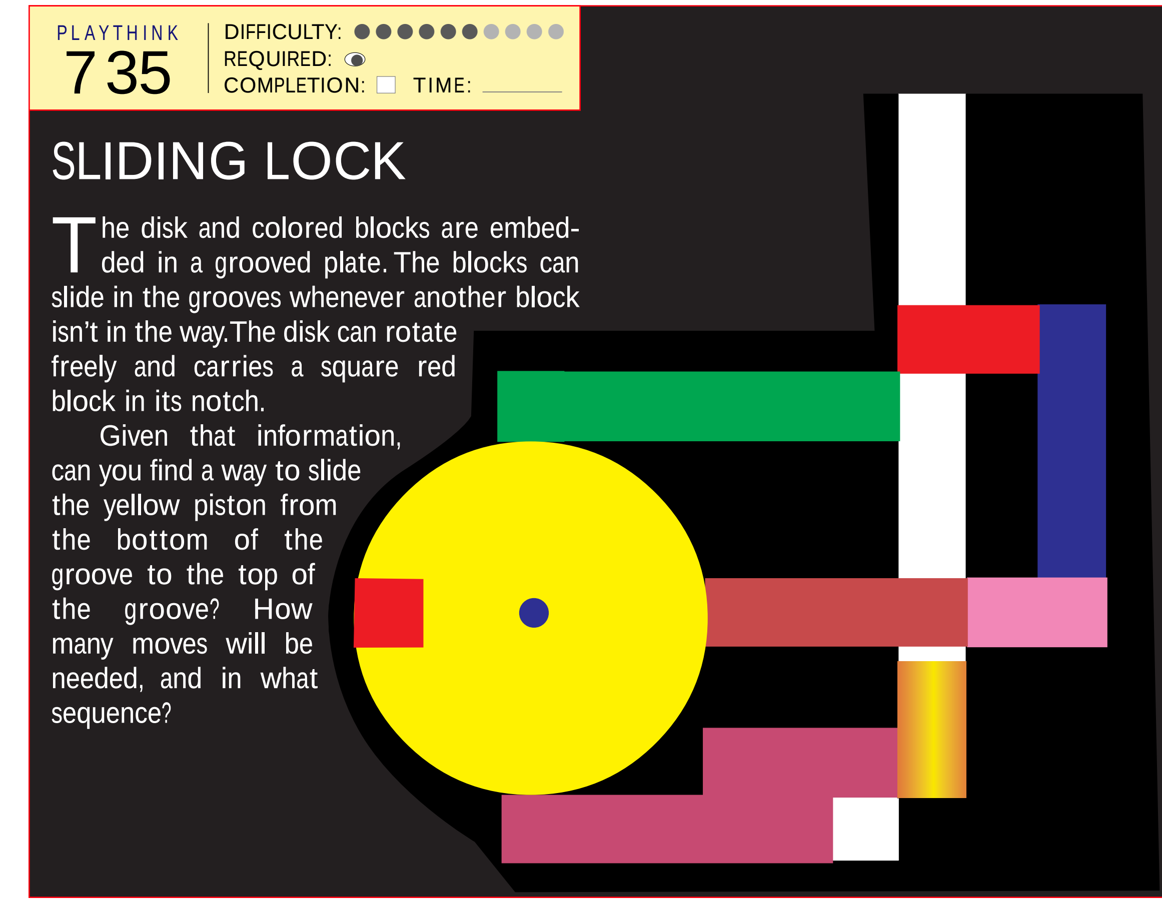


PLAYTHINK 735 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

SLIDING LOCK

The disk and colored blocks are embedded in a grooved plate. The blocks can slide in the grooves whenever another block isn't in the way. The disk can rotate freely and carries a square red block in its notch.

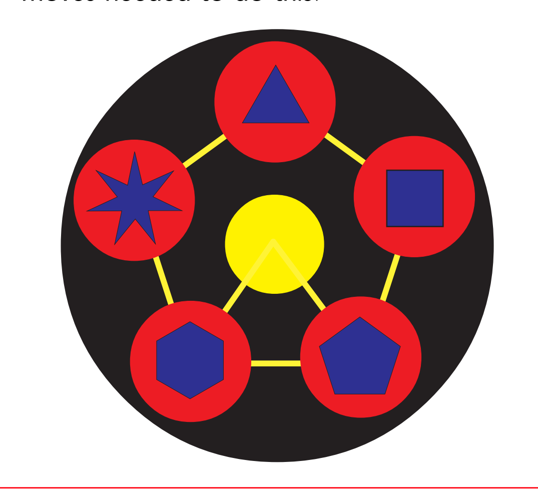
Given that information, can you find a way to slide the yellow piston from the bottom of the groove to the top of the groove? How many moves will be needed, and in what sequence?



PLAYTHINK 736 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

POLYGON CYCLE

Set up the five disks with polygons as shown. Moving one disk at a time into an empty adjacent circle along the connecting lines, can you exchange the star with the hexagon? What is the smallest number of moves needed to do this?



PLAYTHINK 737

DIFFICULTY: ●●●●●○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

SLIDING ZOO

Ten animals have been distributed randomly into ten cages. Each cage has three doorsize—one to the cage on either side and one to the center enclosure.

Without putting two animals together in the same cage or in the central enclosure at the same time, can you work out a plan for moving all the animals to their proper cages (by matching the colors in the circles to the colors of the cages)? How many moves will it take?

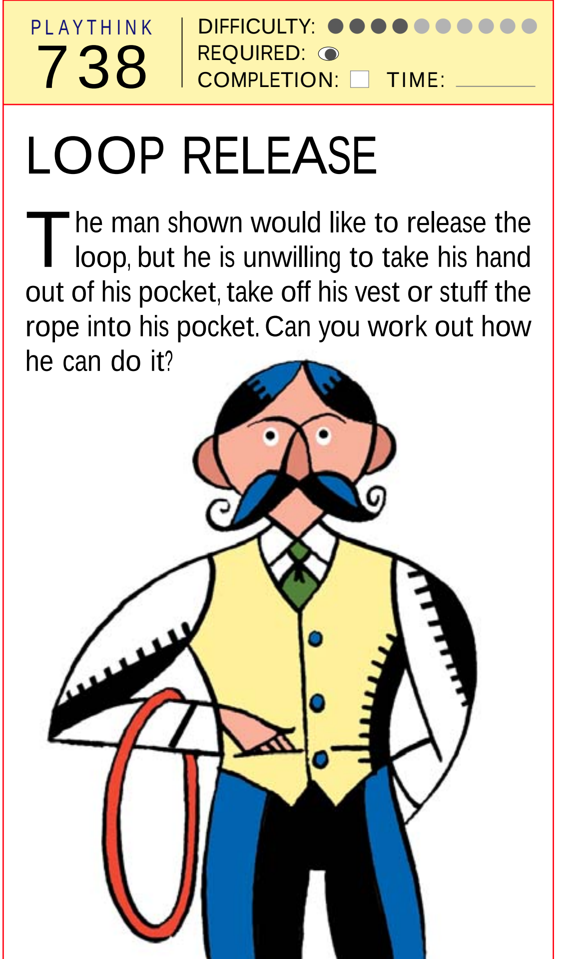


PLAYTHINK 738

DIFFICULTY: ●●●●○○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

LOOP RELEASE

The man shown would like to release the loop, but he is unwilling to take his hand out of his pocket, take off his vest or stuff the rope into his pocket. Can you work out how he can do it?



PLAYTHINK 739

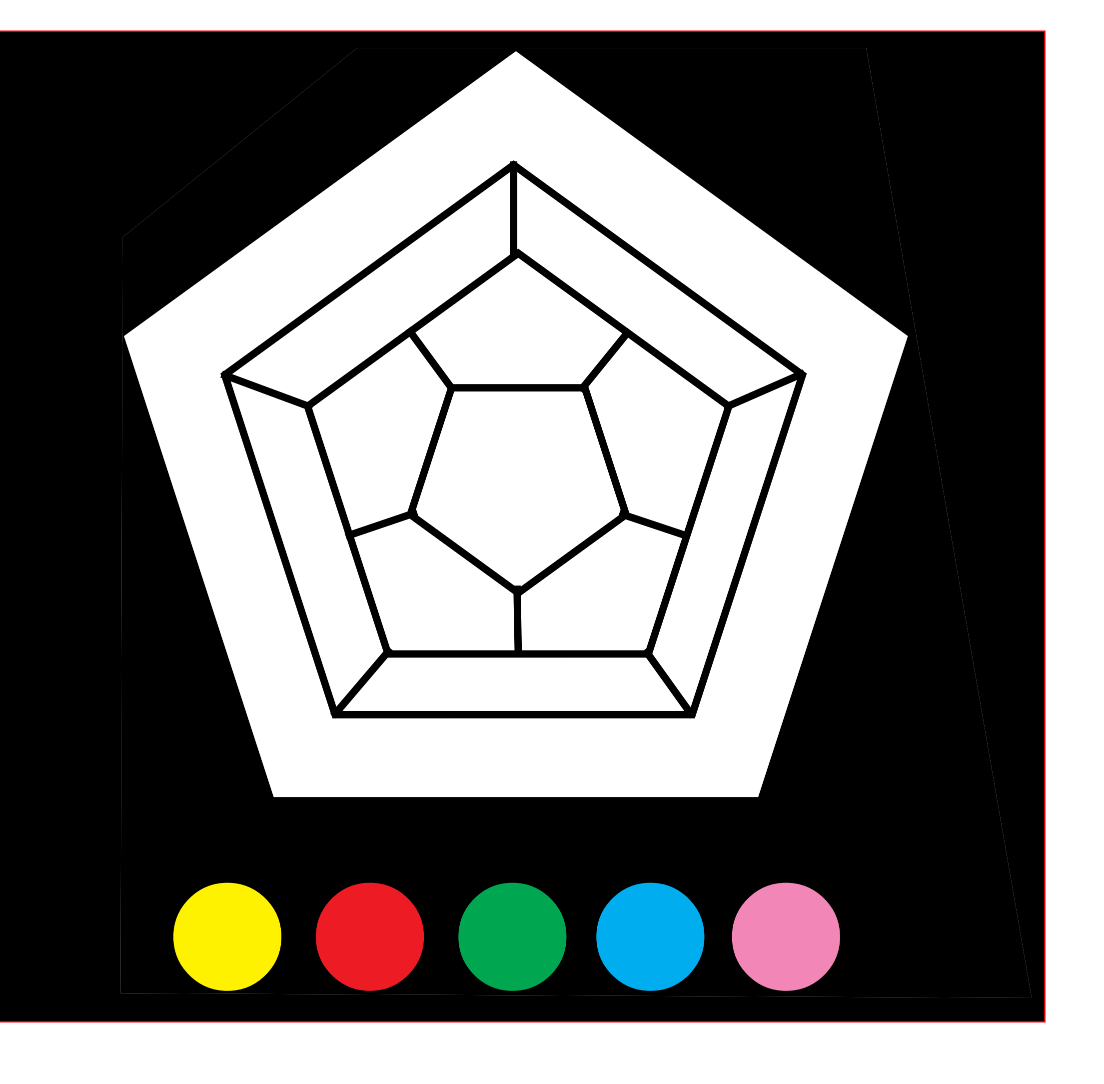
DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ____

BRAMS'S COLORING GAME

This is a more advanced map-coloring game devised by New York University political scientist Steven J. Brams. Two players take turns filling in the map one region at a time so that no two adjacent regions have the same color. Each player has a selection of five colors to choose from.

This may sound like other map-coloring games, but here's where it differs: The two players have different roles. Player 1 is the minimizer, whose aim is to play so that by the end of the game the entire map has been filled in using five or fewer colors. Player 2 is the maximizer, whose aim is to play so that at some point none of the five colors will be sufficient to fill in an empty region. Whoever achieves his or her aim wins the game.

Can you work out a strategy for the maximizer so that he or she will always win?

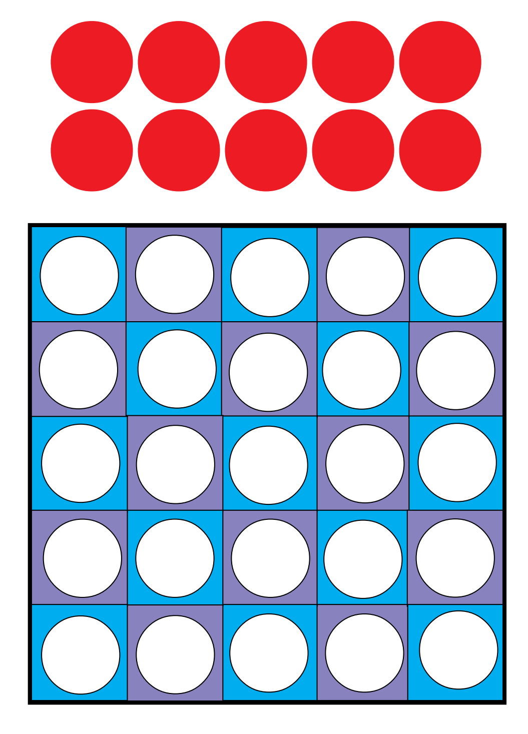


PLAYTHINK 740 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

NO-THREE-IN-A-LINE 1

Minimal Problem
Can you place six counters on the five-by-five board so that placing a seventh counter on any vacant circle will make a vertical, horizontal or diagonal line contain three counters?

Maximal Problem
Can you place ten counters on the five-by-five board so that placing an eleventh counter on any vacant circle will make a vertical, horizontal *and* diagonal line contain three counters?

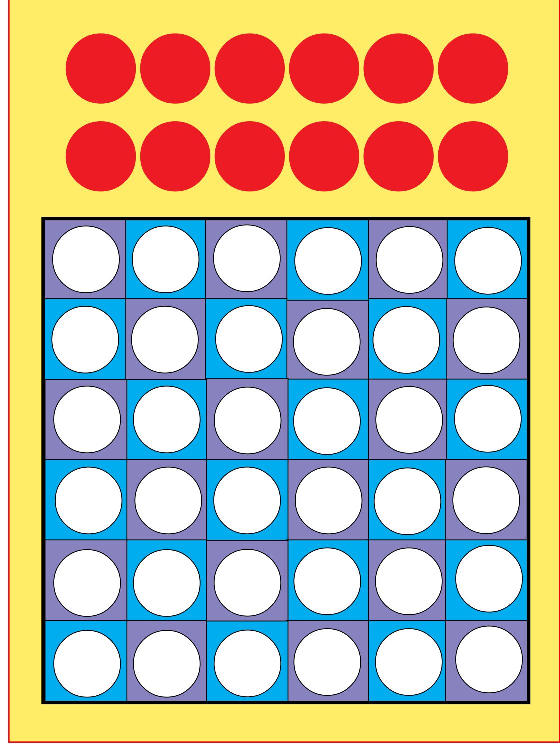


PLAYTHINK 741 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

NO-THREE-IN-A-LINE 2

Minimal Problem
Can you place six counters on the six-by-six board so that placing a seventh counter on any vacant circle will make a vertical, horizontal or diagonal line contain three counters?

Maximal Problem
Can you place twelve counters on the six-by-six board so that placing a thirteenth counter on any open circle will make a vertical, horizontal *and* diagonal line contain three counters?

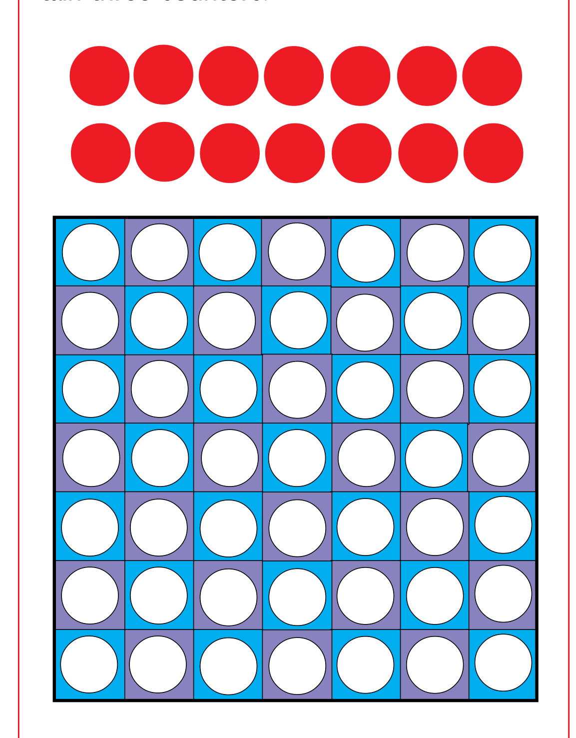


PLAYTHINK 742 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

NO-THREE-IN-A-LINE 3

Minimal Problem
Can you place eight counters on the seven-by-seven board so that placing a ninth counter on any open circle will make a vertical, horizontal or diagonal line contain three counters?

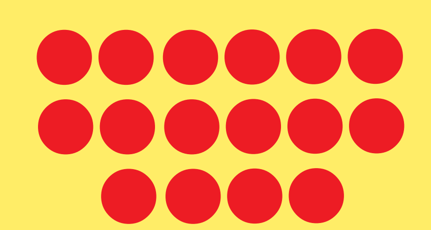
Maximal Problem
Can you place fourteen counters on the seven-by-seven board so that placing a fifteenth counter on any open circle will make a vertical, horizontal *and* diagonal line contain three counters?

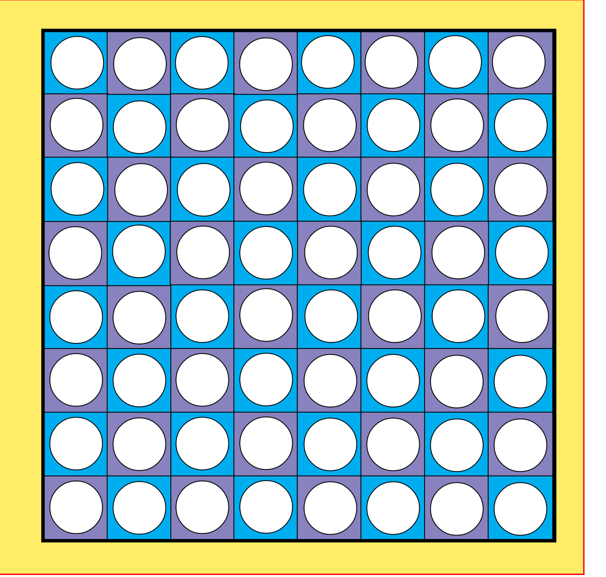


PLAYTHINK 743 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

NO-THREE-IN-A-LINE 4

Can you place sixteen counters on the eight-by-eight board so that placing a seventeenth counter on any open circle will make a vertical, horizontal or diagonal line contain three counters?





Map Folding

Ever since the twentieth-century Polish mathematician Stanislaw Ulam first posed the question of how many different ways a map can be folded, the problem has frustrated researchers in the field of modern combinatorial theory. Indeed, the general problem is still unsolved.

The difficulty arises from the fact that even the simplest map—or any rectangular piece of paper—has many possible ways of being folded. There is an old saying that you have no doubt come across: "The easiest way to fold a map is differently."

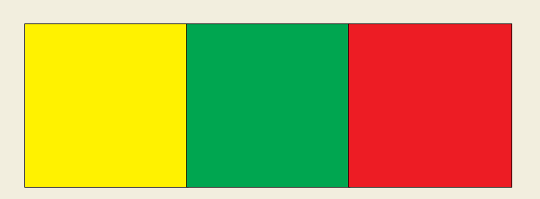
PLAYTHINK 744
DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁 ✂
COMPLETION: ☐ TIME: ____

FOLDING A THREE-SQUARE STRIP

How many different ways can you find to fold a three-square paper strip?

The folds must be confined to creases between the squares, and the final product must be a stack with each square neatly under another.

The squares are the same color on both sides, so it doesn't matter which side is up on the final stack.



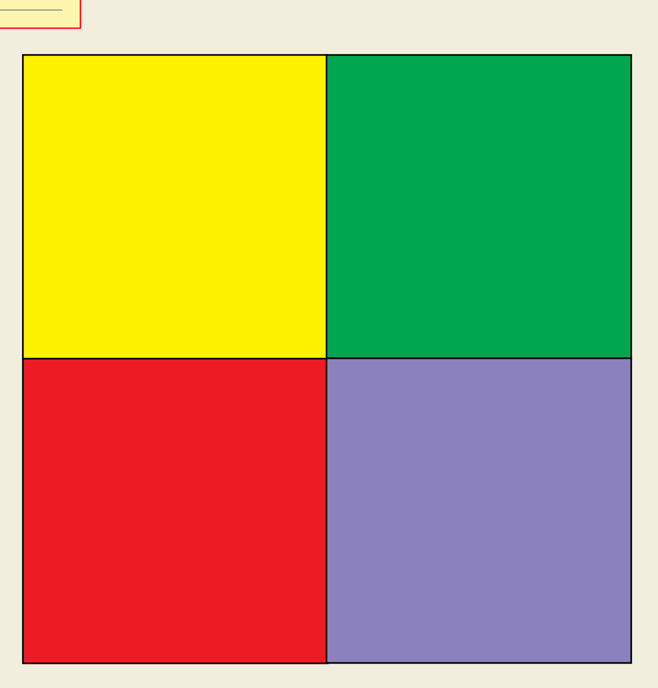
PLAYTHINK 745
DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✂
COMPLETION: ☐ TIME: ____

FOLDING A FOUR-SQUARE SQUARE

How many different ways can you find to fold a four-square paper square?

The folds must be confined to creases between the squares, and the final product must be a stack with each square neatly under another.

The squares are the same color on both sides, so it doesn't matter which side is up on the final stack.

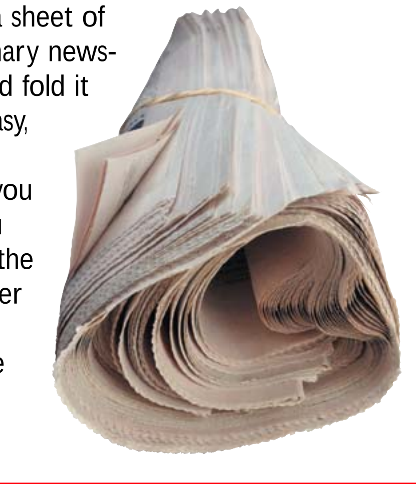


PLAYTHINK 746
DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

FOLDING A NEWSPAPER

Take a sheet of ordinary newspaper and fold it in half. Easy, right?

Do you think you can fold the newspaper on itself ten more times?



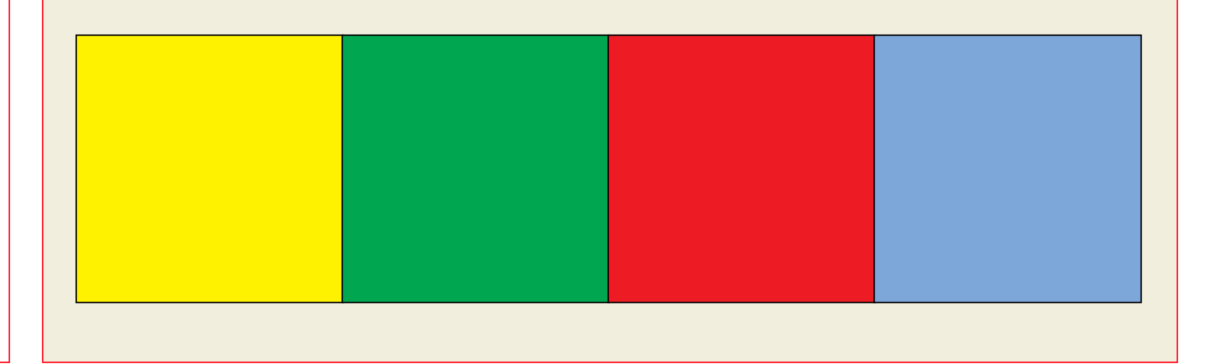
PLAYTHINK 747
DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✂
COMPLETION: ☐ TIME: ____

FOLDING A FOUR-SQUARE STRIP

How many different ways can you find to fold a four-square paper strip?

The folds must be confined to creases between the squares, and the final product must be a stack with each square neatly under another.

The squares are the same color on both sides, so it doesn't matter which side is up on the final stack.



PLAYTHINK 748

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

WORD CHAINS

This diagram shows a collection of letters connected by moving linkages, all of which can be bent and manipulated over the green board. The only fixed point is the letter *Y*; the other linkages revolve and bend around that point.

When properly aligned on the five-by-five game board, the letters spell out an important message. Can you work out what it is?

PLAYTHINK 749

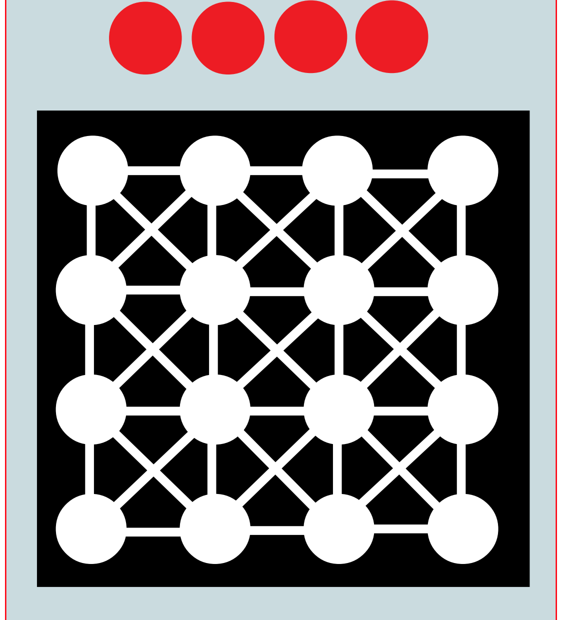
DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

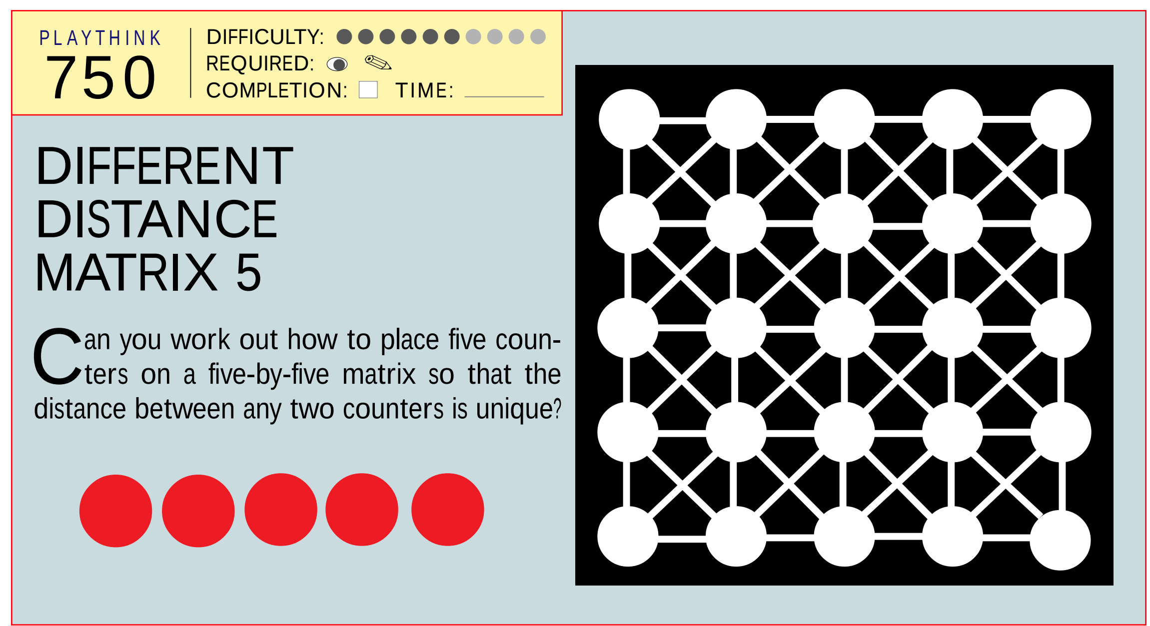
DIFFERENT DISTANCE MATRIX 4

The class of problems known as different distance matrices asks how to place counters on a square grid so that the distance between every two counters is unique. On a straight line the problem is very simple—three counters on a number line could be at points 0, 1 and 3 to ensure that the distance between every pair of counters is unique. In two dimensions the problem becomes much more complicated.

For the purposes of these puzzles, assume that each counter marks the center of the circle and that the distances are measured along a straight line joining the two centers.

Can you work out how to place four counters on a four-by-four matrix so that every pair of counters has a unique distance?





PLAYTHINK 750

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

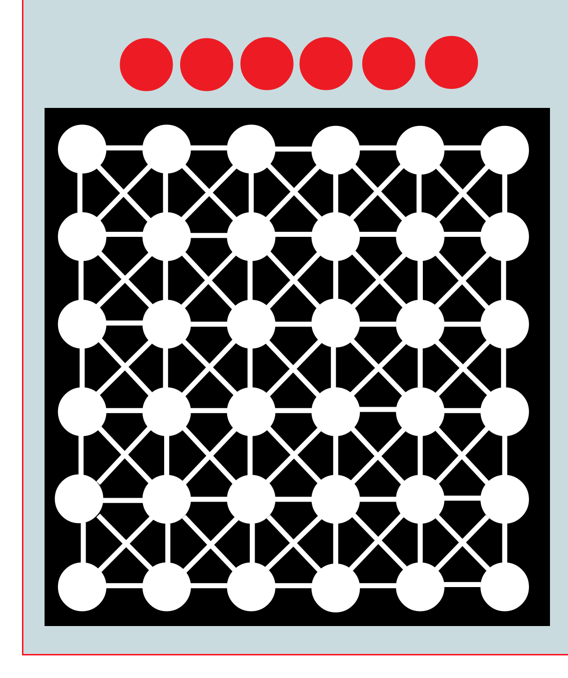
DIFFERENT DISTANCE MATRIX 5

Can you work out how to place five counters on a five-by-five matrix so that the distance between any two counters is unique?

PLAYTHINK 751 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

DIFFERENT DISTANCE MATRIX 6

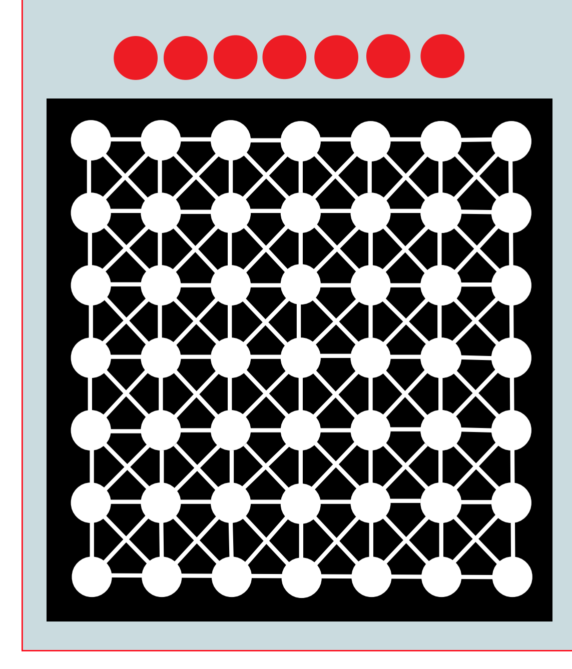
Can you work out how to place six counters on a six-by-six matrix so that the distance between any pair of counters is unique?



PLAYTHINK 752 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

DIFFERENT DISTANCE MATRIX 7

Can you work out how to place seven counters on a seven-by-seven matrix so that the distance between any pair of counters is unique?

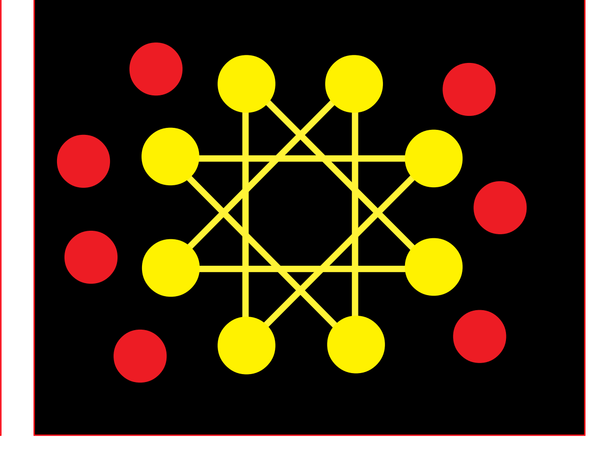


PLAYTHINK 753 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

CROSSROADS

The object of this puzzle is to place seven coins or counters on the eight points of the octagonal star. Coins are placed one at a time on any unoccupied circle. But every coin that is placed must be immediately transferred to one of two other points that are connected by a straight line to the initial circle. Once moved, a coin cannot be moved again.

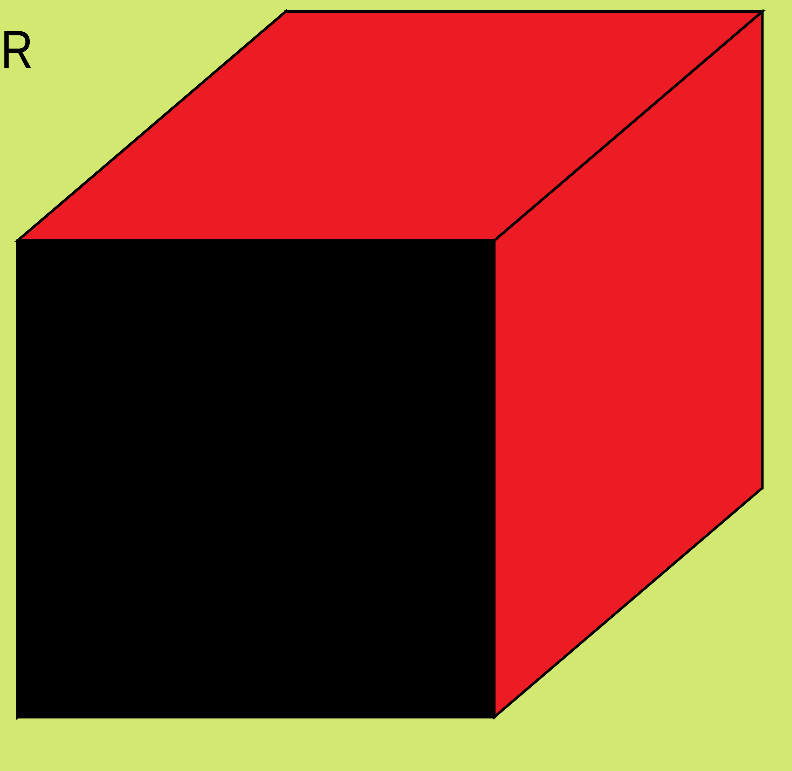
Although the puzzle is complicated, a simple strategy will enable you to solve it every time. Can you work it out?



PLAYTHINK 754 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

TWO-COLOR CUBES

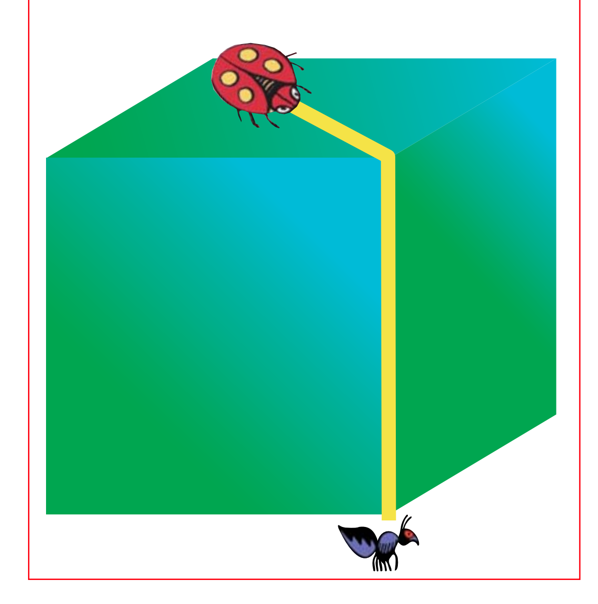
Can you work out all the distinct ways the faces of a cube may be filled in using just two colors?



PLAYTHINK 755 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

SHORTEST CATCH

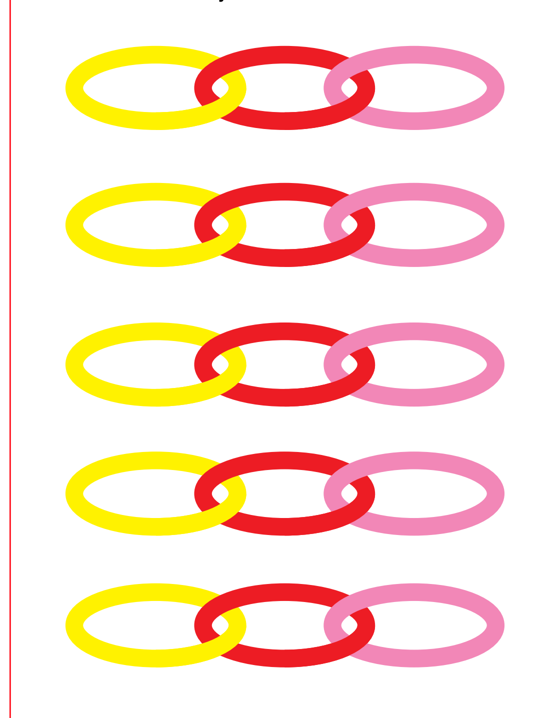
The ladybug wants to reach the aphid as quickly as possible. Is the path marked the shortest route possible?



PLAYTHINK 756 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

LINK RINGS

A blacksmith has been asked to make one long chain from five three-link bits of chain. Can you find a way to do it so that he has to make just three welds?

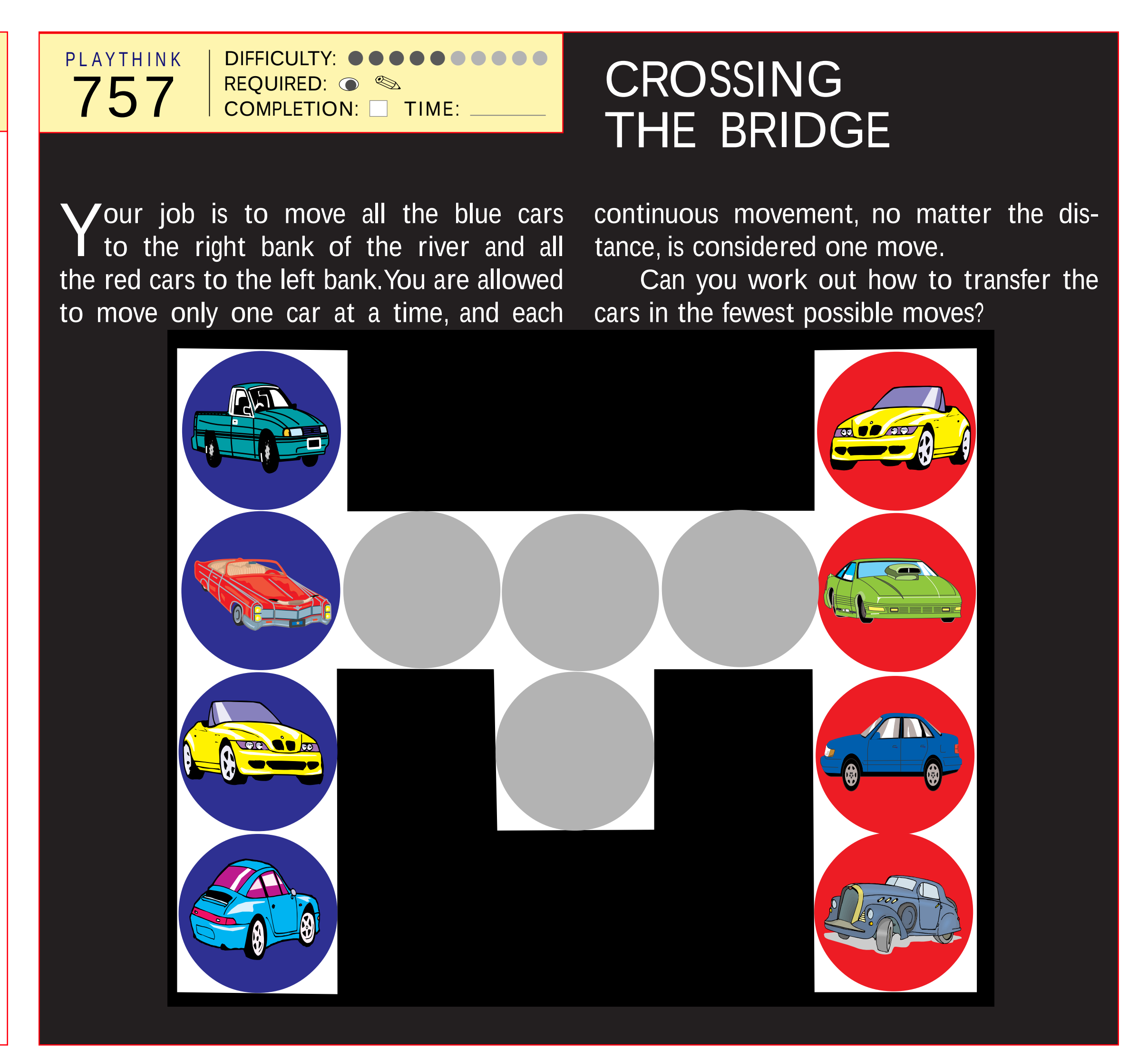


PLAYTHINK 757 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

CROSSING THE BRIDGE

Your job is to move all the blue cars to the right bank of the river and all the red cars to the left bank. You are allowed to move only one car at a time, and each continuous movement, no matter the distance, is considered one move.

Can you work out how to transfer the cars in the fewest possible moves?



PLAYTHINK 758 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

JUMPING DISKS

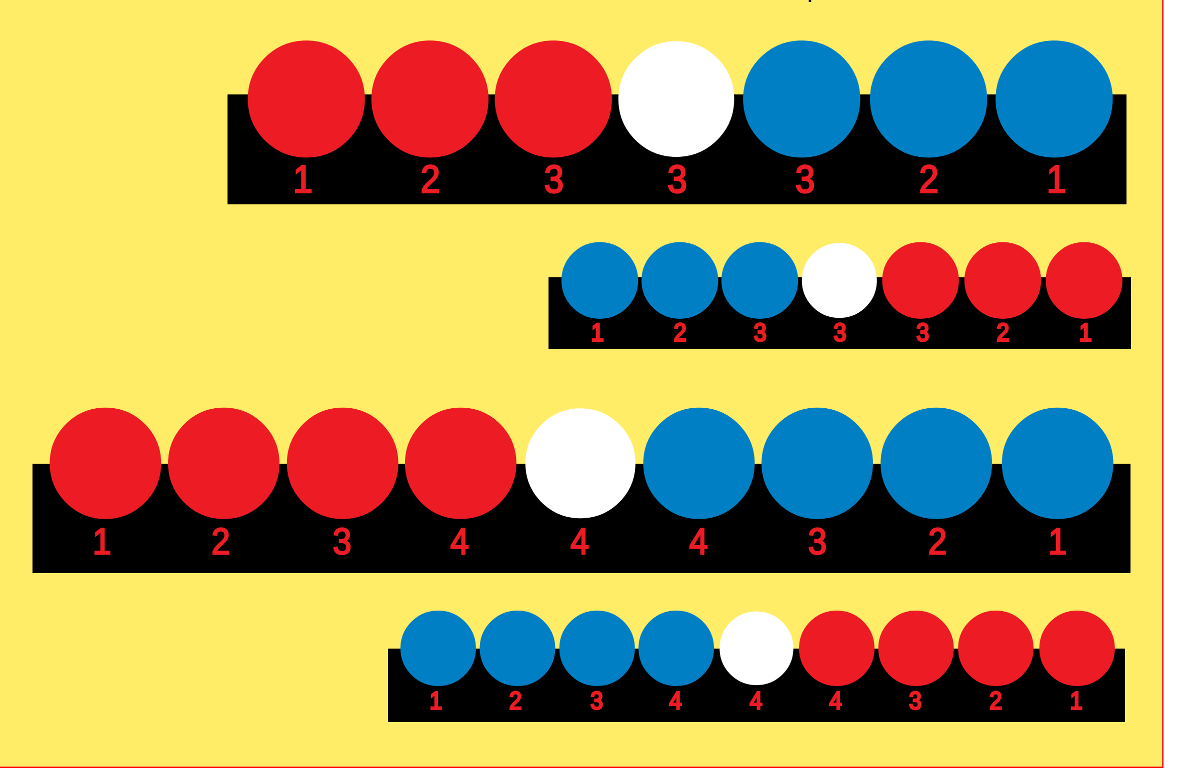
The object of these two puzzles is to reverse the pattern by exchanging the two sets of disks. To do this, five rules must be observed:

1. Only one disk may be moved at a time.
2. A disk may move into an adjacent empty space.
3. A disk may jump over a disk of the opposite color into the space immediately beyond it.
4. A disk may not jump over a disk of the same color.
5. No backward moves are permitted.

Also, each move must involve landing in a numbered space (no jumping off the ends!), and no move may disturb another disk in the process (no pushing).

Can you solve the puzzles in fifteen and twenty-four moves, respectively?

For a hint, look at the numbers below the disks. If you find the key there, you can solve not only these puzzles but others that are much more complicated.

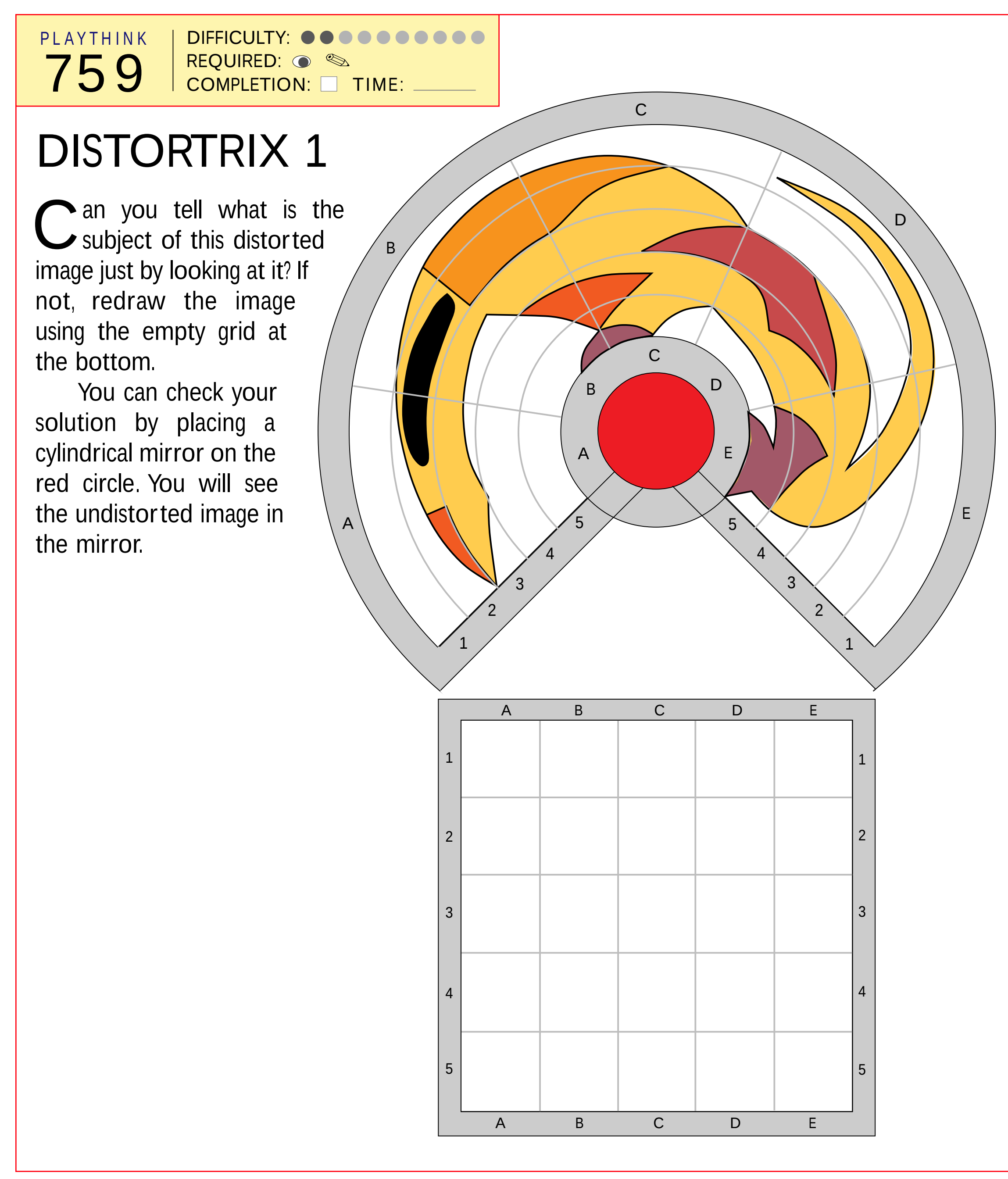


PLAYTHINK 759 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

DISTORTRIX 1

Can you tell what is the subject of this distorted image just by looking at it? If not, redraw the image using the empty grid at the bottom.

You can check your solution by placing a cylindrical mirror on the red circle. You will see the undistorted image in the mirror.

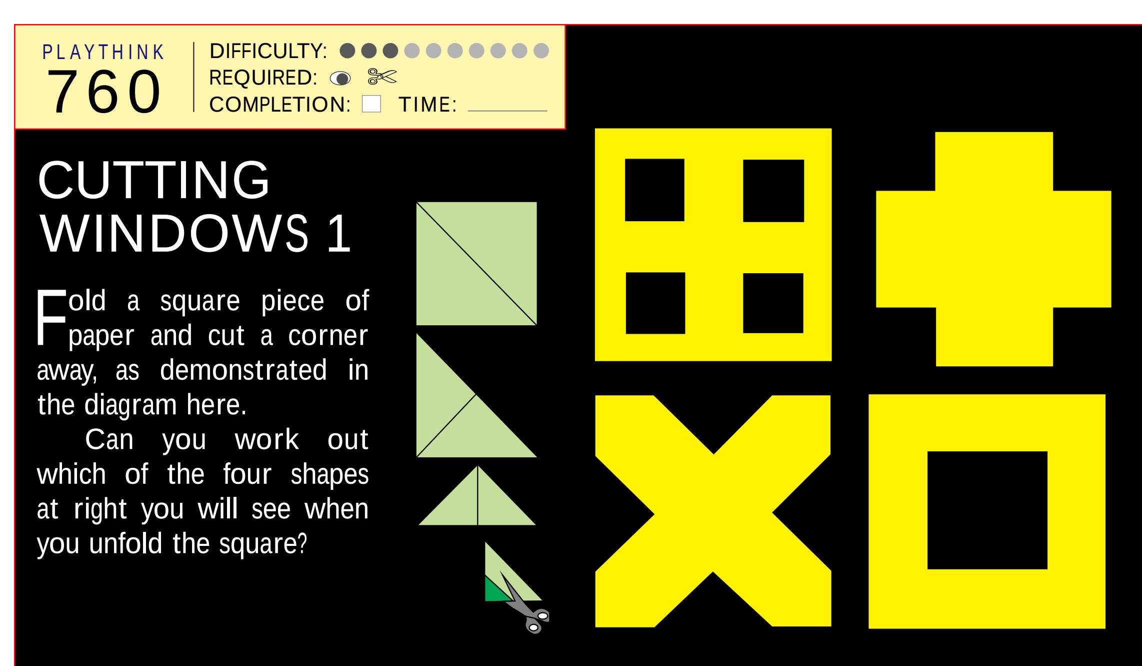


PLAYTHINK 760 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 ✂ | COMPLETION: ☐ TIME: ____

CUTTING WINDOWS 1

Fold a square piece of paper and cut a corner away, as demonstrated in the diagram here.

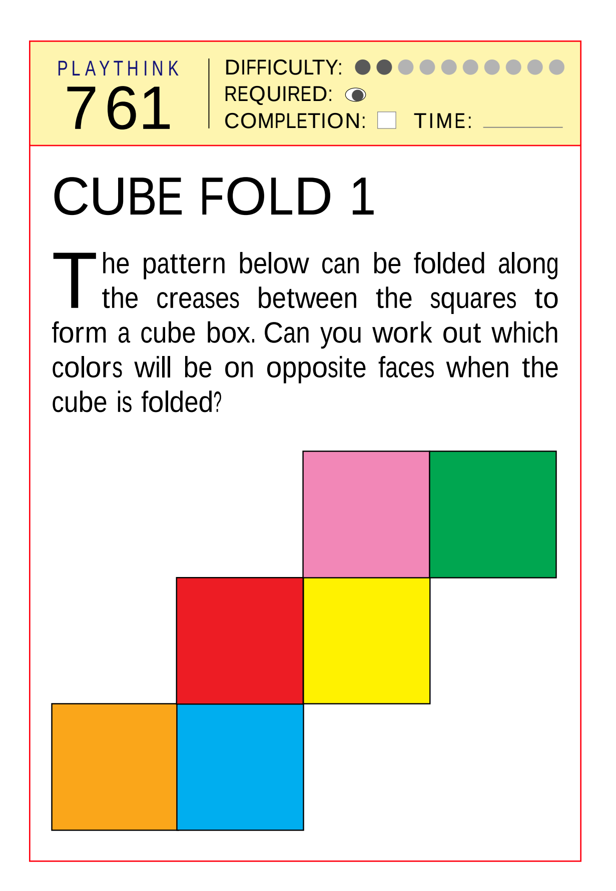
Can you work out which of the four shapes at right you will see when you unfold the square?



PLAYTHINK 761 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

CUBE FOLD 1

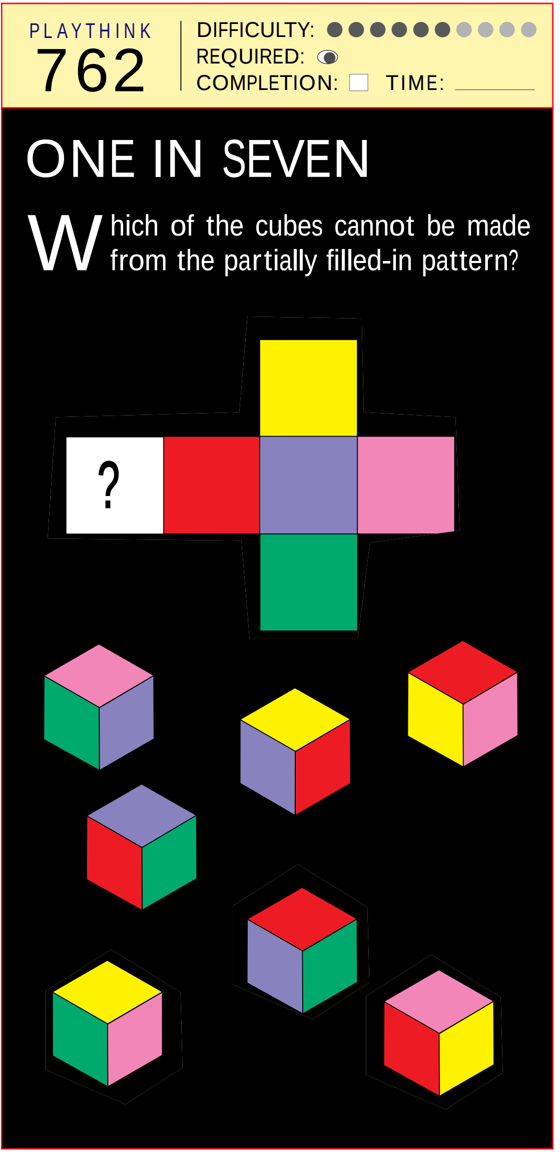
The pattern below can be folded along the creases between the squares to form a cube box. Can you work out which colors will be on opposite faces when the cube is folded?



PLAYTHINK 762 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

ONE IN SEVEN

Which of the cubes cannot be made from the partially filled-in pattern?



Distortions and Impossibilities

Toward the end of the nineteenth century, biologists noticed that in many cases evolution proceeded in a way that corresponded to a distorted framework—that is, it seemed as if the original plan of the creature had been distorted to form the more modern one. Then, in 1917, D'Arcy Thompson published his classic work, *Growth and Form*, in which he illustrated animal species that differed from one another only by anamorphic distortions—the animals shared a body plan, but certain parts had been stretched or shrunk in a mathematically predictable way.

This was all very intriguing until it was discovered that there are many, many cases in which two creatures that bear a close resemblance in form are not even closely related.

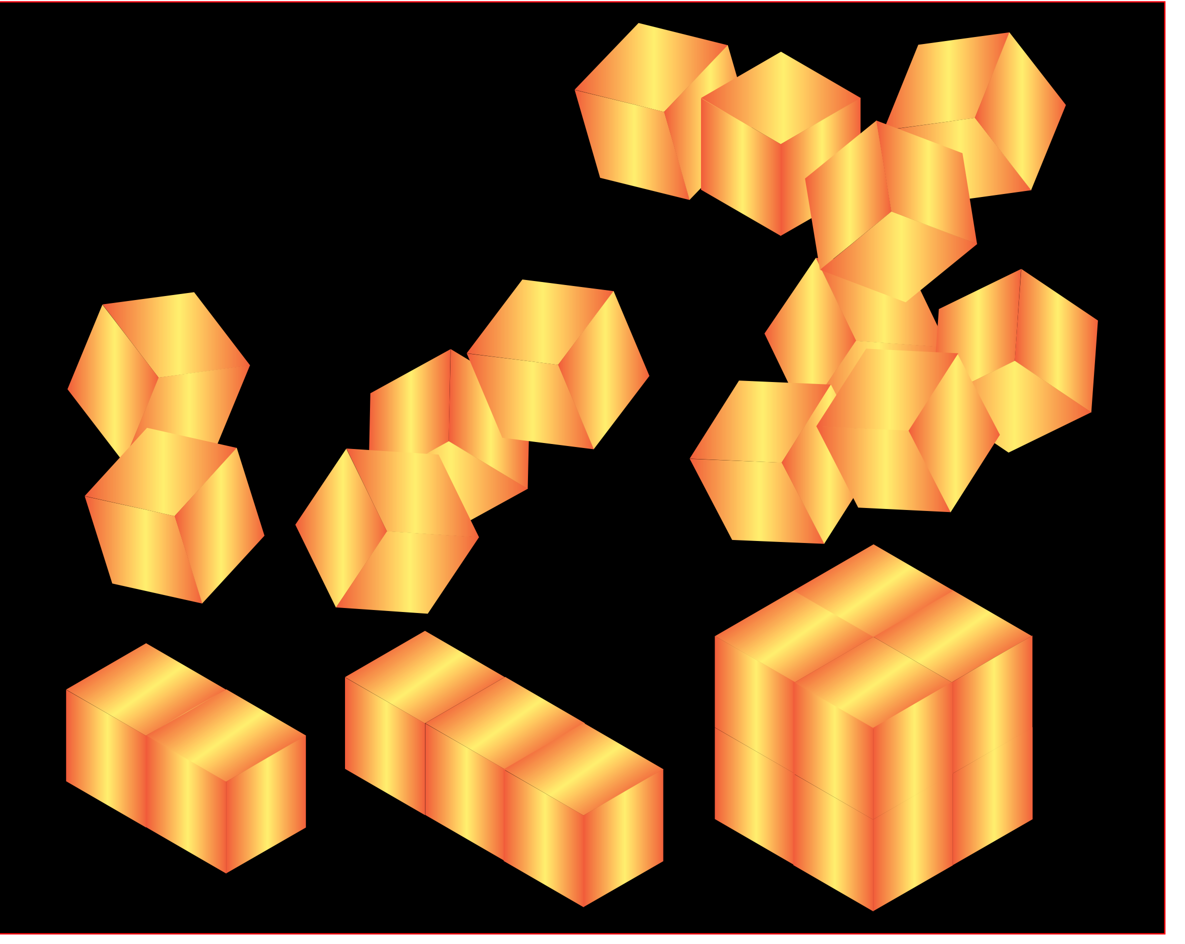
The limits to distortion can be found in mathematics as well. In topology, the way to change a shape is to distort it. Such distortions can be described mathematically: If a grid defines a shape, then a change to the grid will create a new shape. But the logic of the grid can change the shape only so far; push too much and you'll wind up with an impossible shape.

Although the easiest way to change a shape is to superimpose a square grid on the original and then reproduce the shape on a grid of different size, more interesting results can be obtained by redrawing the form on a distorted grid. Deliberate distortions have been found in pictorial representations from the earliest cave paintings to modern art. The sixteenth-century German artist Albrecht Dürer described various geometrical methods of changing the proportions of the human figure by means of altered coordinate systems. That particular method has the effect of producing grotesque but recognizable caricatures.

PLAYTHINK 763 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

CUBE TO CUBE

1. If a cube can be placed on a table in any of twenty-four different ways, how many different ways can two cubes be placed side by side on a table so that two faces touch each other?
2. When three cubes are placed side by side, what is the total number of different ways the cubes can be turned and still keep the same side by side arrangement?
3. Eight cubes can be stacked four on four to make a larger cube. If the cubes can turn in any way but still maintain their locations within the greater cube, what is the total number of ways the individual cubes can be turned?



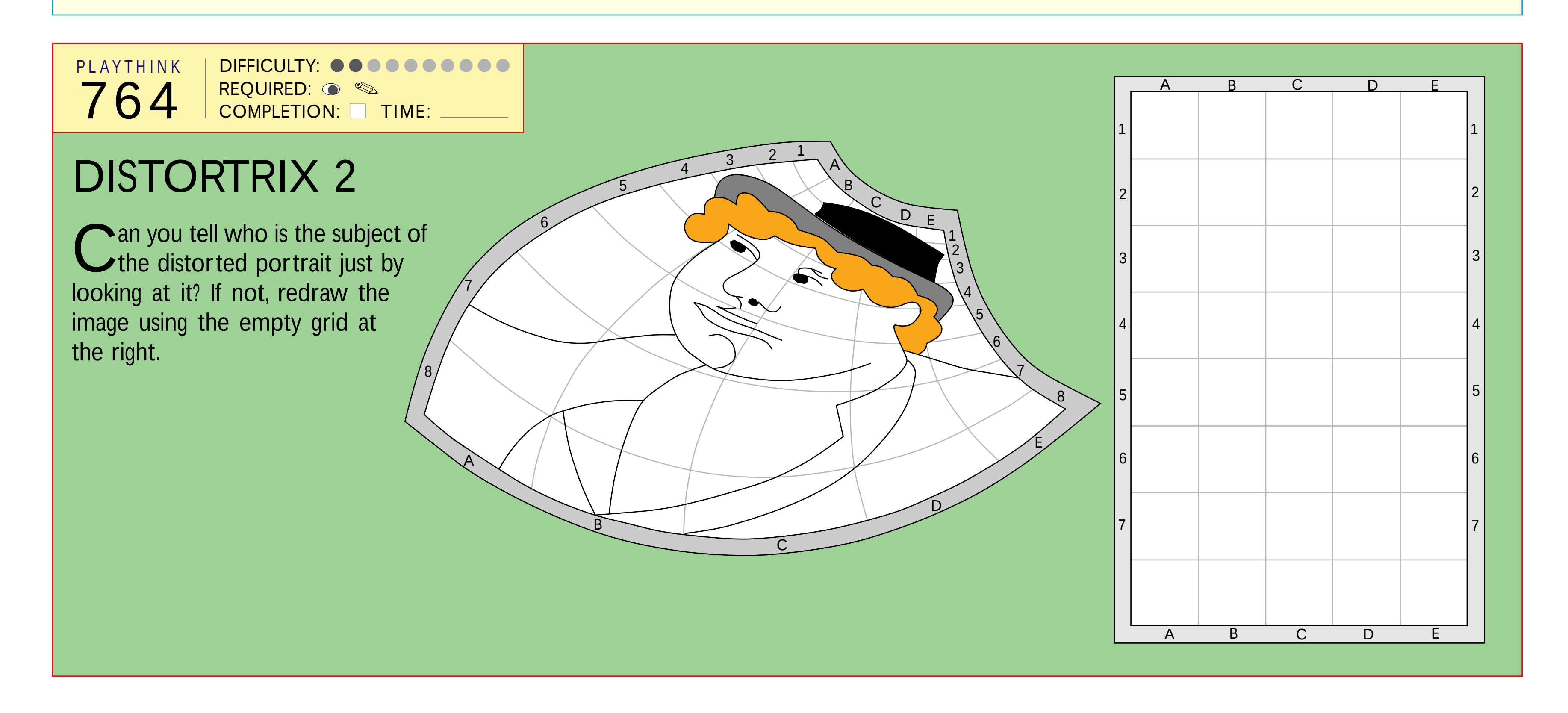
Anamorphic Distortions

When the human visual system is confronted with unusual projections, such as those found reflected in funhouse mirrors, it sometimes has difficulty piecing together the form of the original object. A simple but fascinating way to convey that sense of disorientation in a standard two-dimensional piece of art is through the so-called anamorphic projection.

Viewed from a standard perspective—with the observer's line of sight perpendicular to the picture—a piece of anamorphic art appears as a monstrous distortion. But the original image can be "formed again" (the translation of the Greek *anamorphe*) by viewing the picture on a slant or looking at its reflection in a cylindrical or conical mirror. Those who haven't previously encountered anamorphic art are usually amazed to see the undistorted reflection image seemingly pop out of nowhere.

The first slanted anamorphic image appears in the notebooks of Leonardo da Vinci, but anamorphic pictures were most popular about 300 years ago. Since then people have sometimes found it necessary to create such images for their protection. In England during the reigns of George I and George II, for instance, supporters of the outlawed and exiled pretender to the throne, Charles Edward Stuart, faced imprisonment for treason if they were found with a portrait of their preferred monarch, the "king over the water." Instead, they carried his anamorphic image.

Cognitive researchers have explored the underlying principle of anamorphic art. They asked experimental subjects to wear specially designed glasses that produced extreme topological deformations of the world around them—changing the perspective of things, for instance, or reversing their view of the world upside down or left to right. Their surprising finding was that not only did the subjects eventually adjust to their new "outlook" on the world but when they removed the glasses, their view of the unfiltered world was distorted, at least for a short period. Such experiments suggest that our visual system is more concerned with topologically invariant properties than with Euclidean ones.



PLAYTHINK 764

DIFFICULTY: ●●○○○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

DISTORTRIX 2

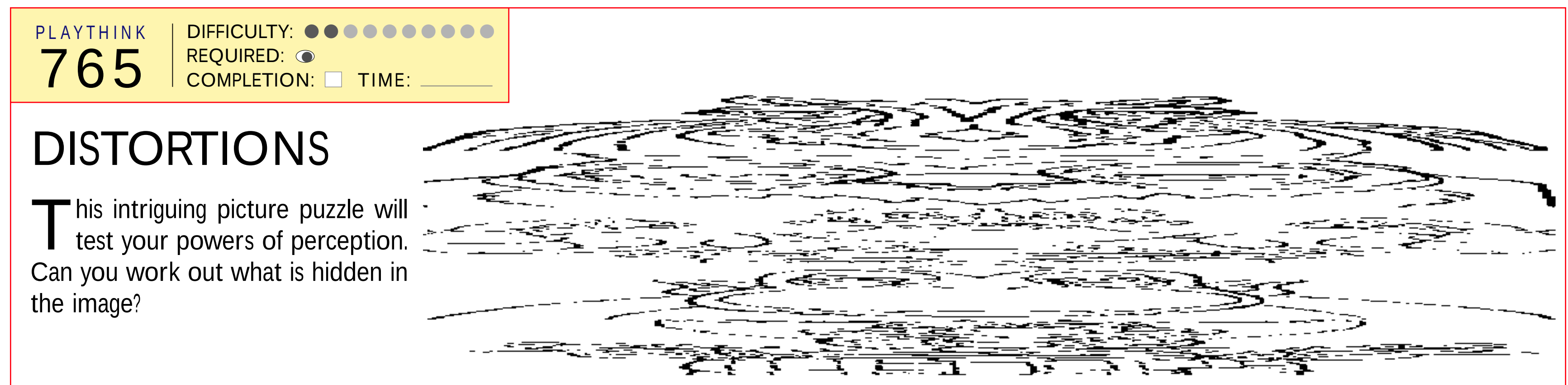
Can you tell who is the subject of the distorted portrait just by looking at it? If not, redraw the image using the empty grid at the right.

PLAYTHINK 765

DIFFICULTY: ●●○○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

DISTORTIONS

This intriguing picture puzzle will test your powers of perception. Can you work out what is hidden in the image?



PLAYTHINK 766

DIFFICULTY: ●●○○○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

DISTORTRIX 3

Can you tell who is the subject of the distorted portrait just by looking at it? If not, redraw the image using the empty grid at the right.

You can check your solution by placing a cylindrical mirror on the red circle. (Such a mirror can be made by placing shiny foil around a small tube.) The undistorted image will be seen in the mirror.

PLAYTHINK 767

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

TETRA-OCTA PYRAMID

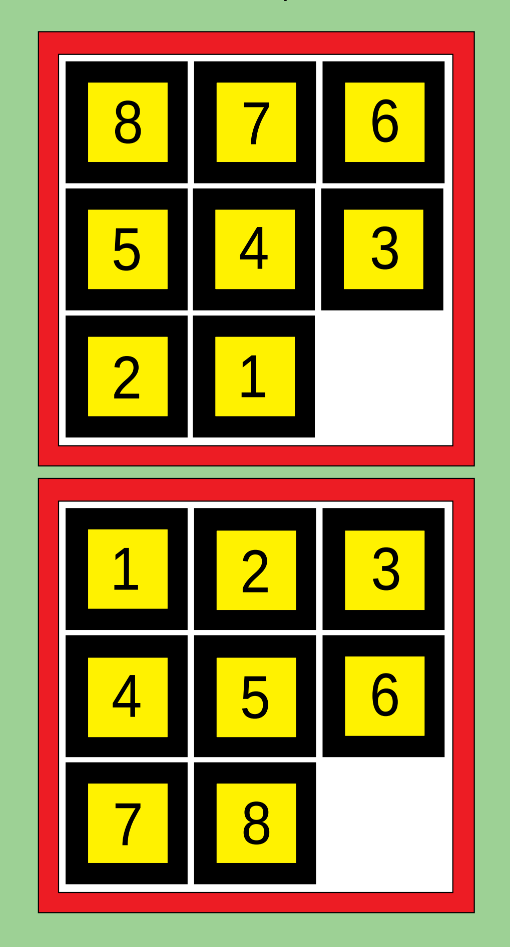
The pyramid shown below is made up of regular tetrahedrons and octahedrons packed together to fill the entire volume.

If the pyramid itself is a regular tetrahedron with edges three times that of a single tetrahedral building block, how many tetrahedrons and octahedrons are needed to form the pyramid?

PLAYTHINK 768 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: eye, pencil, paper, scissors | COMPLETION: ☐ TIME: ____

EIGHT-BLOCK SLIDING PUZZLE

The diagram at top shows a group of numbered blocks. Can you rearrange the blocks by sliding them into open spaces so that they form the ordered configuration below? If so, what is the smallest number of moves needed to accomplish this?

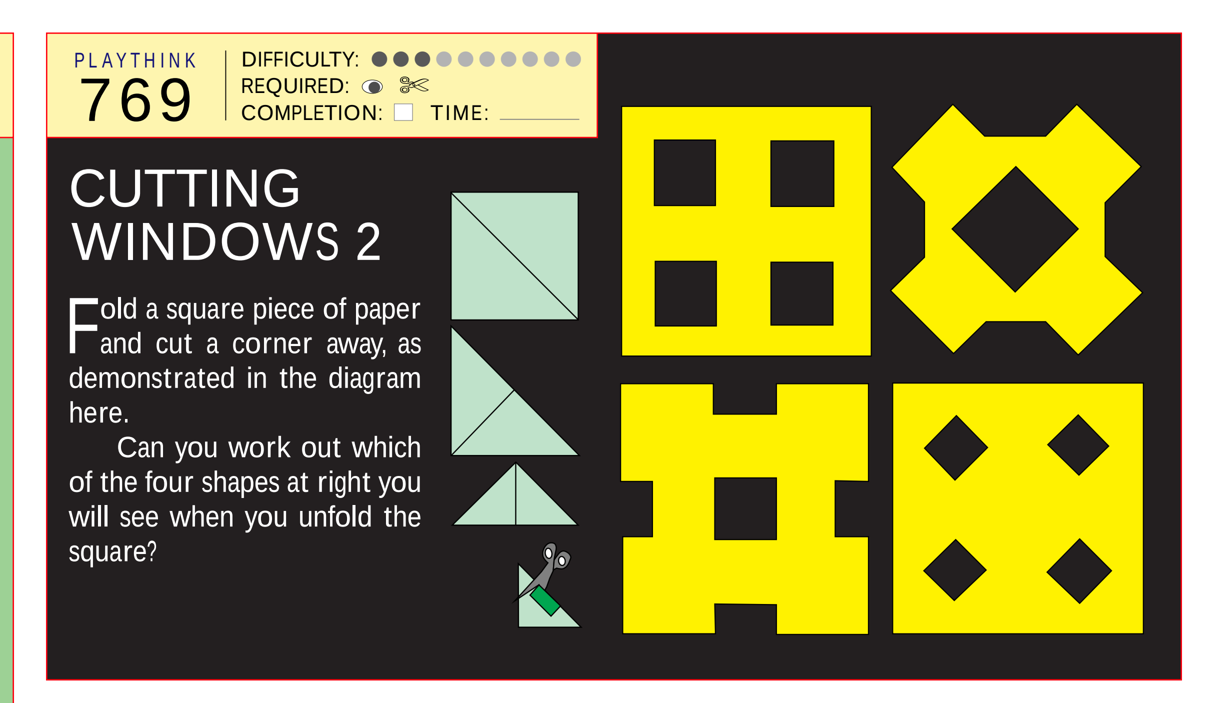


PLAYTHINK 769 | DIFFICULTY: ●●●○○○○○○○ | REQUIRED: eye, scissors | COMPLETION: ☐ TIME: ____

CUTTING WINDOWS 2

Fold a square piece of paper and cut a corner away, as demonstrated in the diagram here.

Can you work out which of the four shapes at right you will see when you unfold the square?

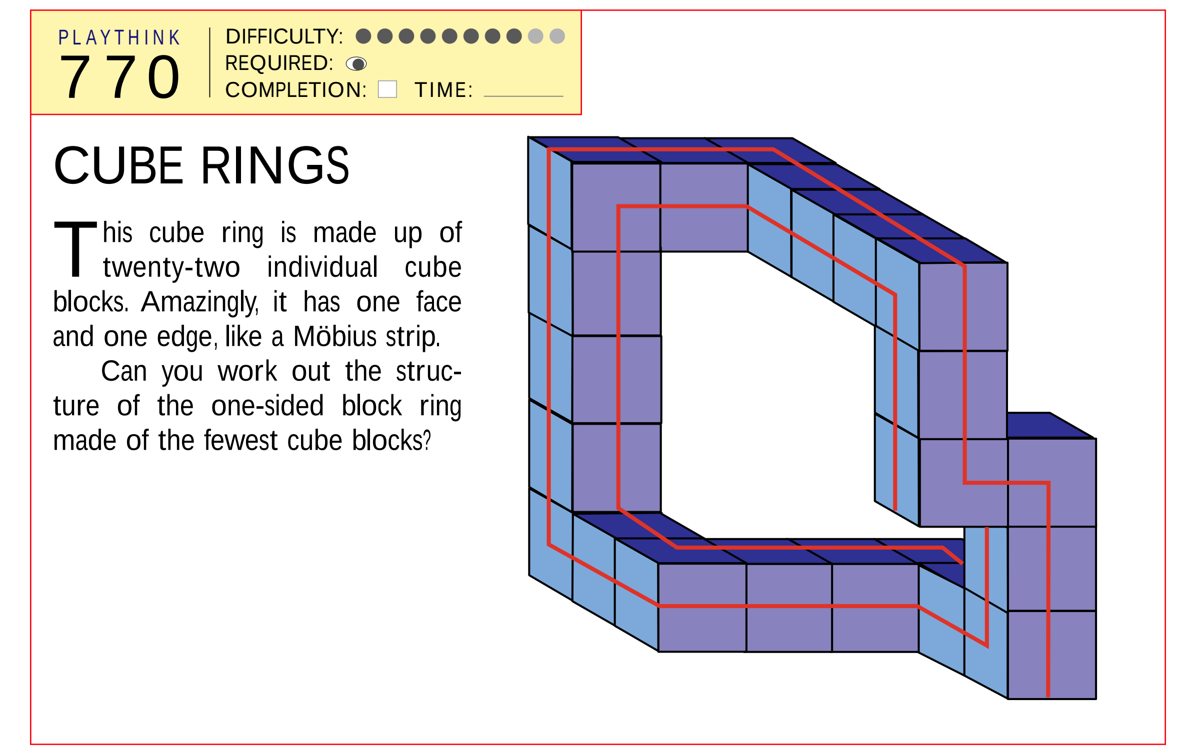


PLAYTHINK 770 | DIFFICULTY: ●●●●●●●●○○ | REQUIRED: eye | COMPLETION: ☐ TIME: ____

CUBE RINGS

This cube ring is made up of twenty-two individual cube blocks. Amazingly, it has one face and one edge, like a Möbius strip.

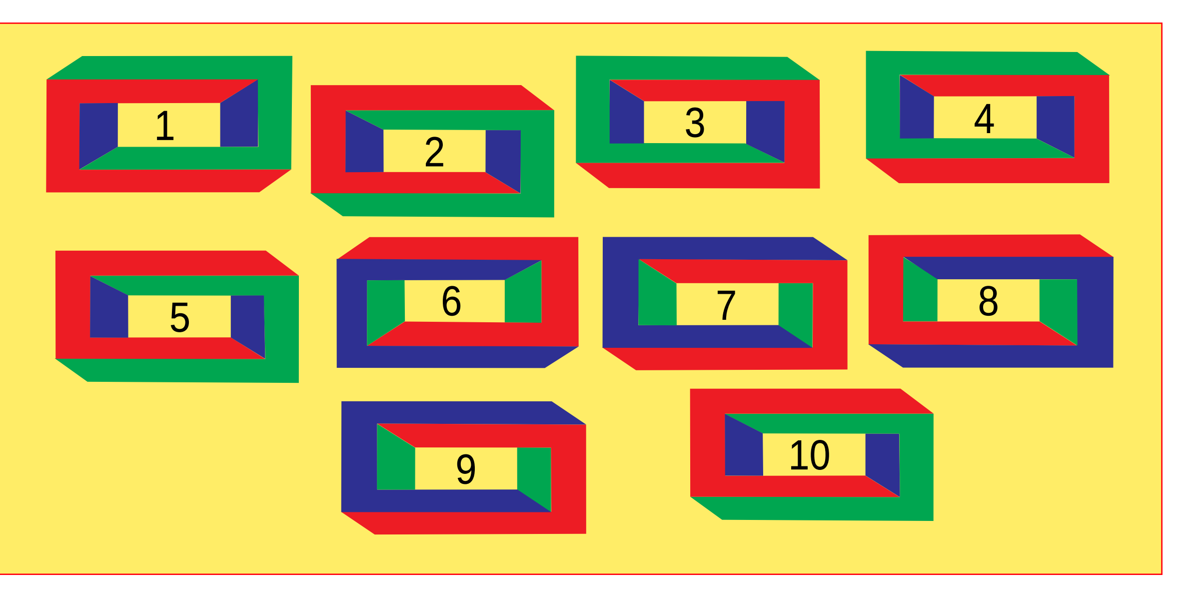
Can you work out the structure of the one-sided block ring made of the fewest cube blocks?



PLAYTHINK 771 | DIFFICULTY: ●●●○○○○○○○ | REQUIRED: eye | COMPLETION: ☐ TIME: ____

IMPOSSIBLE RECTANGLES

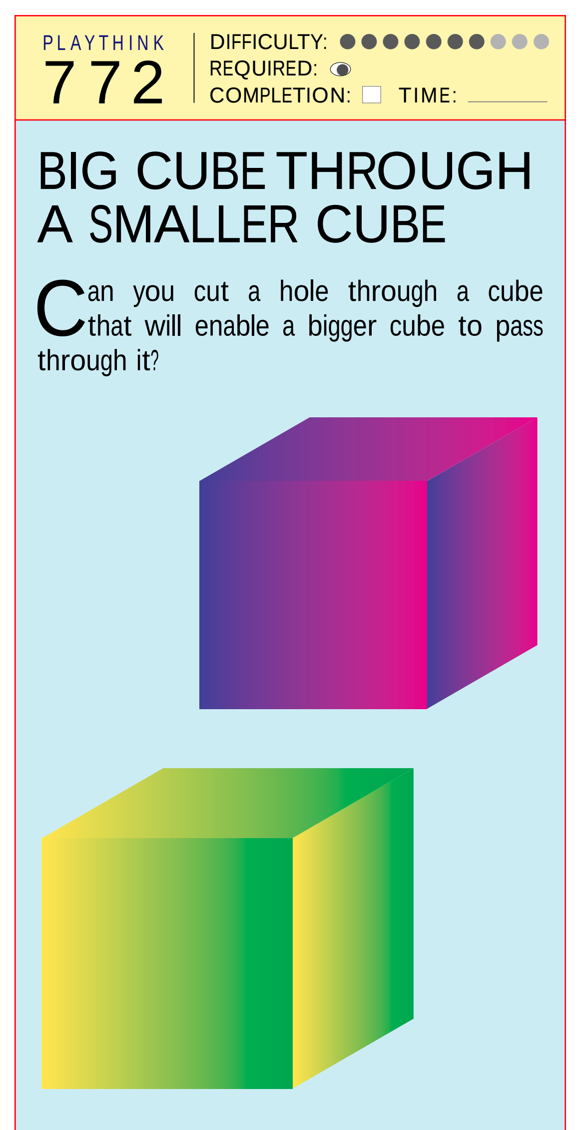
Of the ten figures shown here, five are identical, counting rotations but not reflections. And another set of three is identical, also counting rotations. Two of the figures are unique. Can you work out which two?



PLAYTHINK 772 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

BIG CUBE THROUGH A SMALLER CUBE

Can you cut a hole through a cube that will enable a bigger cube to pass through it?



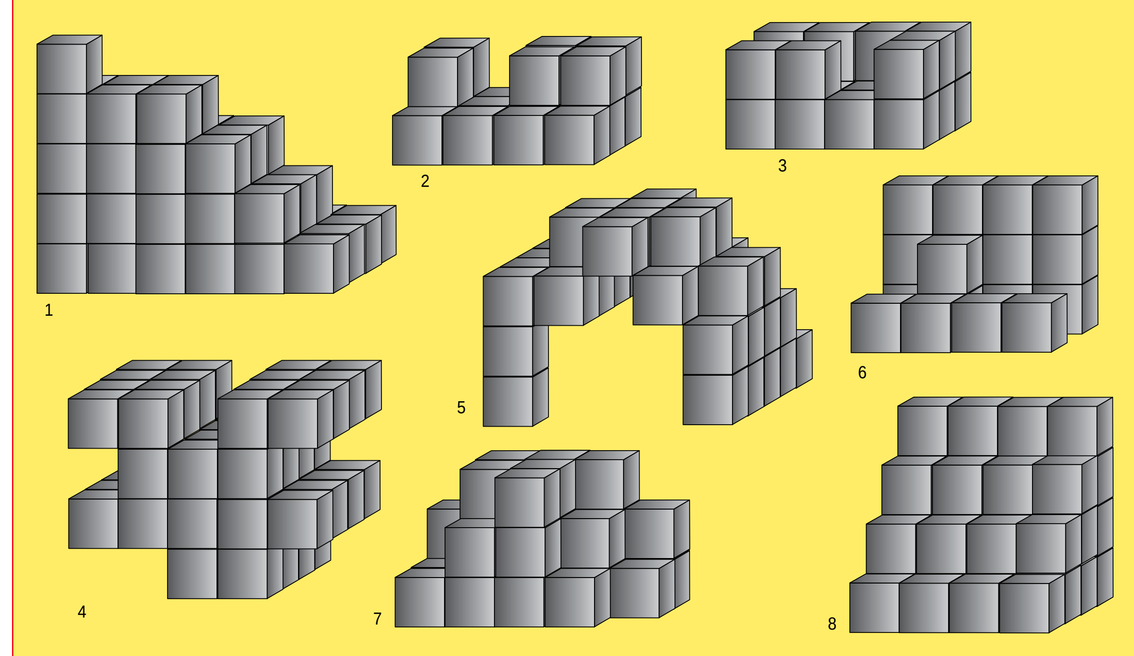
PLAYTHINK 773 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

COUNT THE CUBES

"Putting things into perspective" is such a common phrase that it's easy to forget that perspective does more than bring three-dimensional realism to a two-dimensional representation. It also helps us interpret things we can't see. That's because perspective lets us infer that objects follow certain geometric rules.

In the designs below, various combinations of cubes are stacked together. All the rows of cubes are complete unless you see them end. Most of the stacks are simple heaps, but some require you to understand that one or more rows continue, sight unseen, behind others. Such problems challenge your ability to judge spatial relationships.

Based on the visual evidence given, can you work out how many cubes make up each stack?



PLAYTHINK 774 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ 📄 ✂ | COMPLETION: ☐ TIME: ____

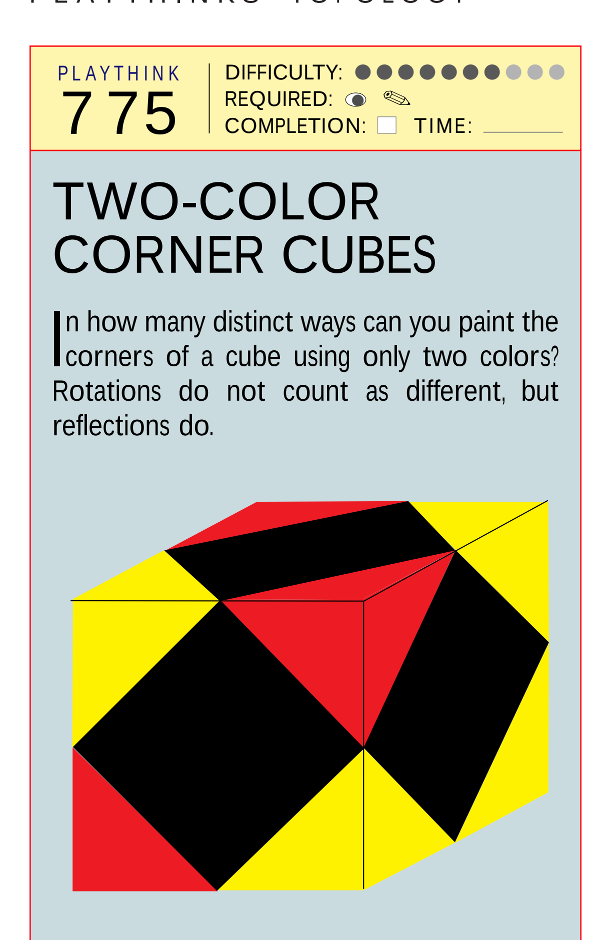
CROSSROADS 2

Place eleven counters in turn on the twelve circles. You must put each counter on an empty circle and then move the counter immediately to another empty circle connected to the first by a straight line.

PLAYTHINK 775 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ______

TWO-COLOR CORNER CUBES

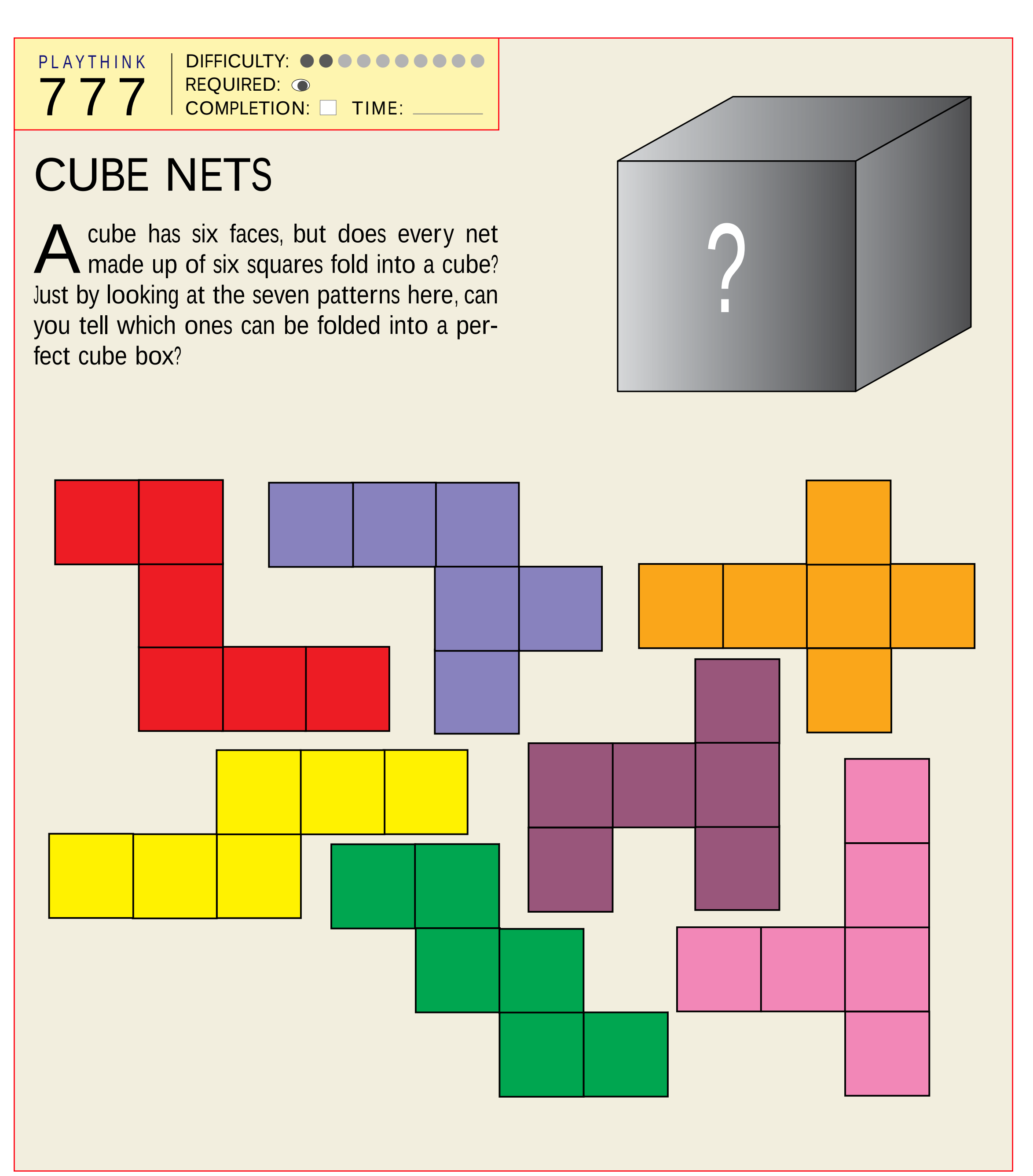
In how many distinct ways can you paint the corners of a cube using only two colors? Rotations do not count as different, but reflections do.



PLAYTHINK 777 | DIFFICULTY: ●●○○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ______

CUBE NETS

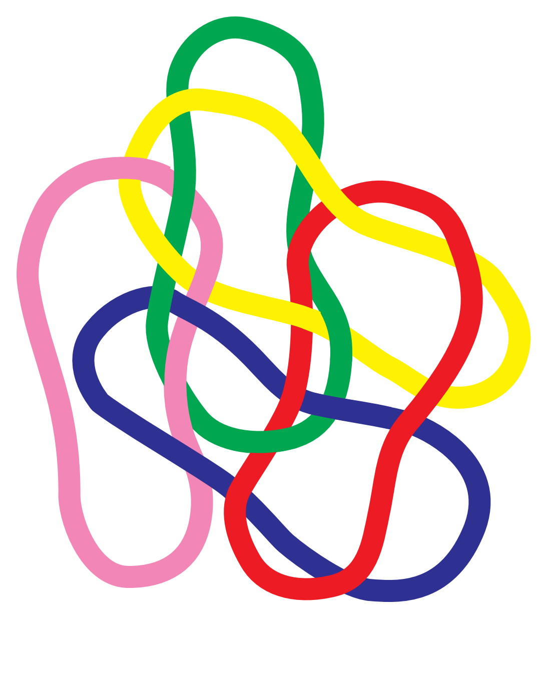
A cube has six faces, but does every net made up of six squares fold into a cube? Just by looking at the seven patterns here, can you tell which ones can be folded into a perfect cube box?



PLAYTHINK 776 | DIFFICULTY: ●●●○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ______

LINKED OR UNLINKED?

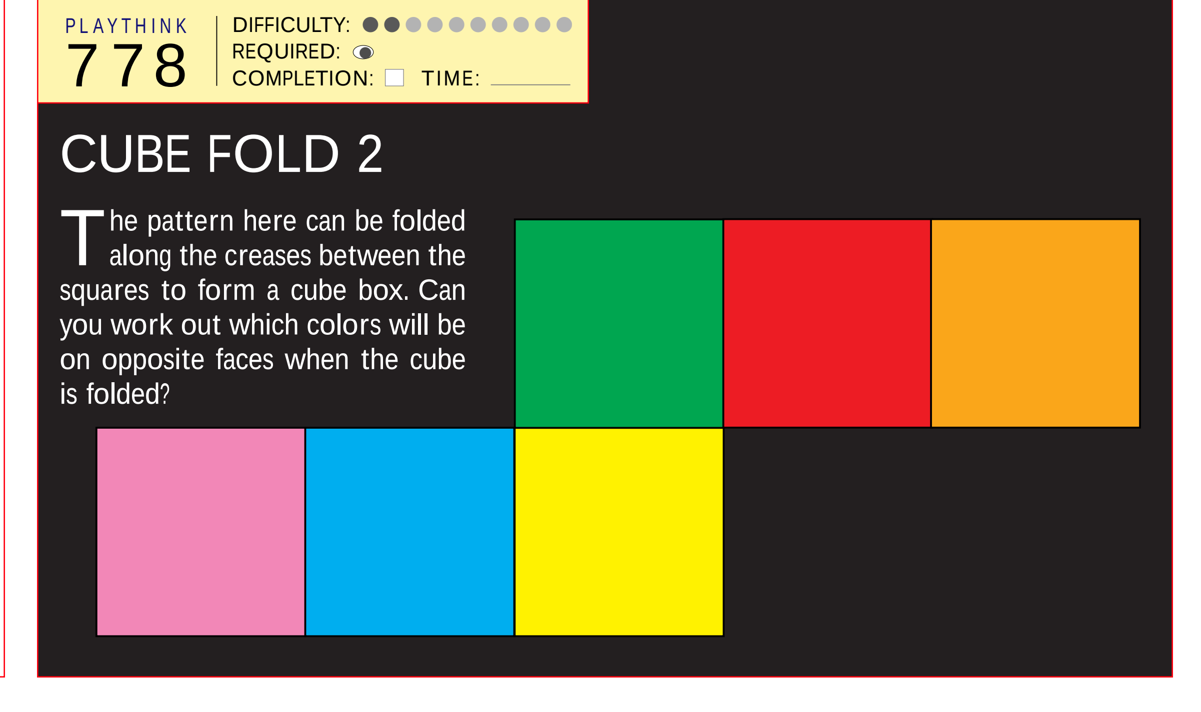
Which of the five loops must be cut so that the other loops will fall free?



PLAYTHINK 778 | DIFFICULTY: ●●○○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ______

CUBE FOLD 2

The pattern here can be folded along the creases between the squares to form a cube box. Can you work out which colors will be on opposite faces when the cube is folded?



PLAYTHINK 779

DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ______

FOUR COLOR SQUARES GAME

The object of this two-player game is to fill the entire game board with just four colors and with no two adjacent regions sharing a color.

Players select two of four colors—red, green, blue and yellow—and then take turns filling in one square at a time. Each newly colored section must touch at least one other colored square, but it cannot touch a square of the same color, even at the corner. (See sample game for guidance.) Play continues until no legal moves remain.

Scoring is straightforward: each two-by-two square filled in with one of a player's two colors counts as a point; each three-by-three square counts as two points and so on. One-by-one squares don't count.

PLAYER 1

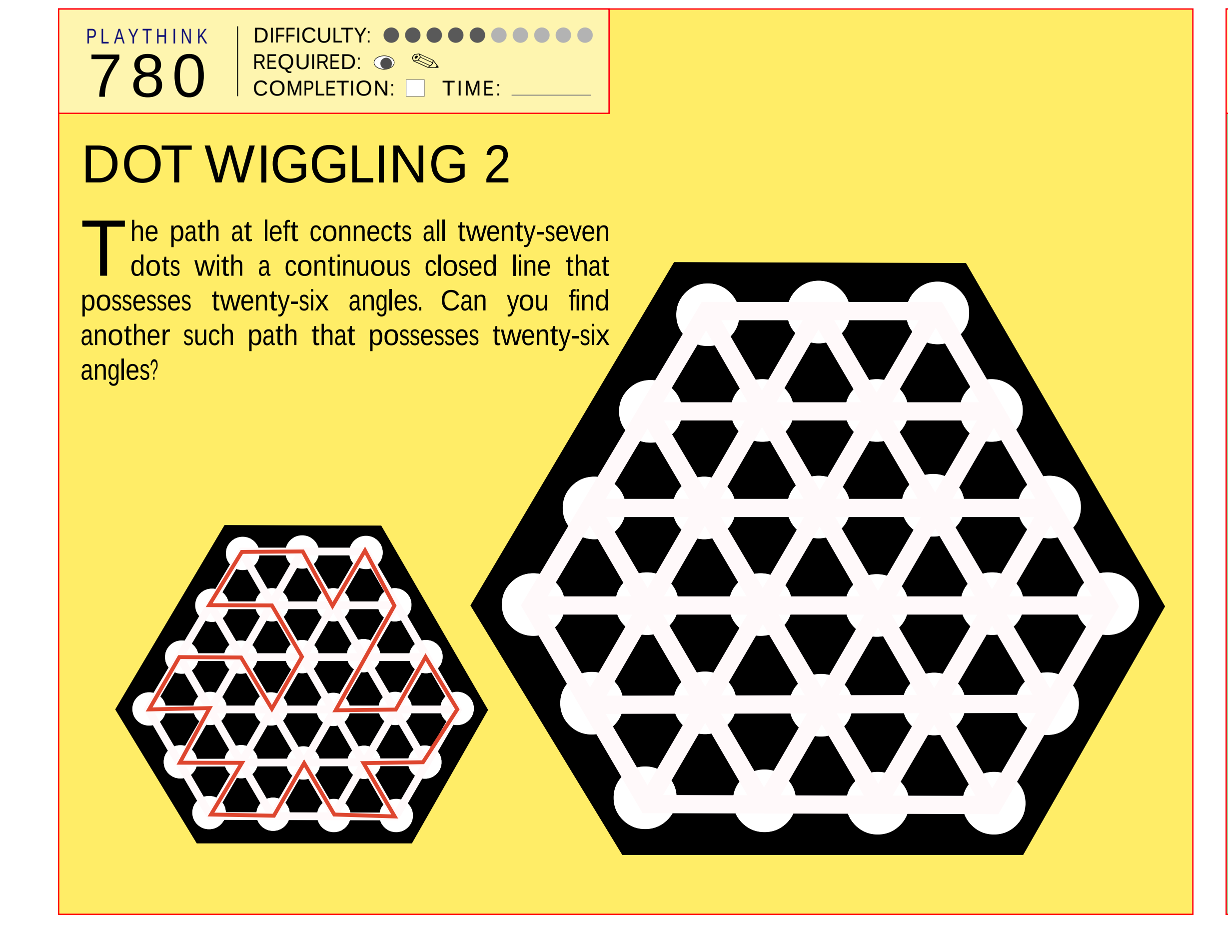
PLAYER 2

PLAYTHINK 780

DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ______

DOT WIGGLING 2

The path at left connects all twenty-seven dots with a continuous closed line that possesses twenty-six angles. Can you find another such path that possesses twenty-six angles?

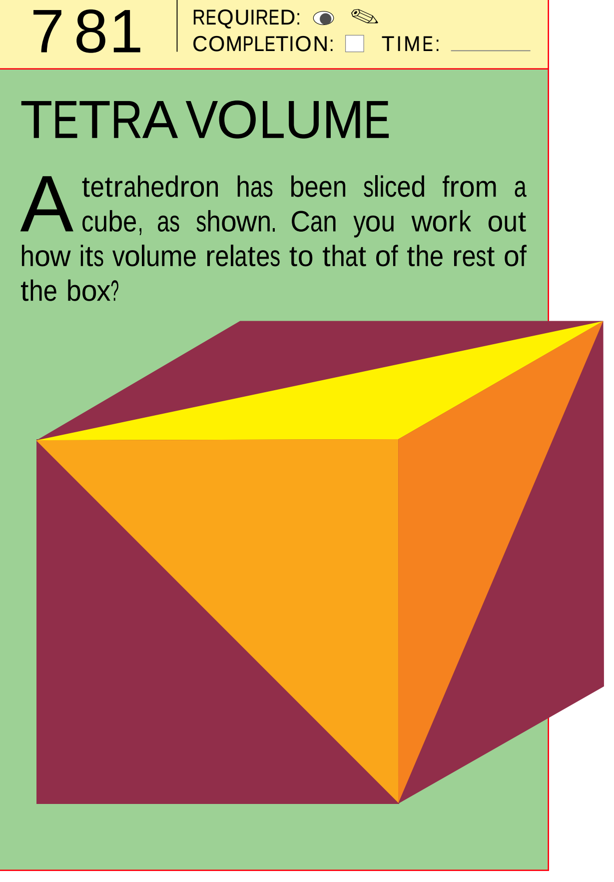


PLAYTHINK 781

DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ______

TETRA VOLUME

A tetrahedron has been sliced from a cube, as shown. Can you work out how its volume relates to that of the rest of the box?

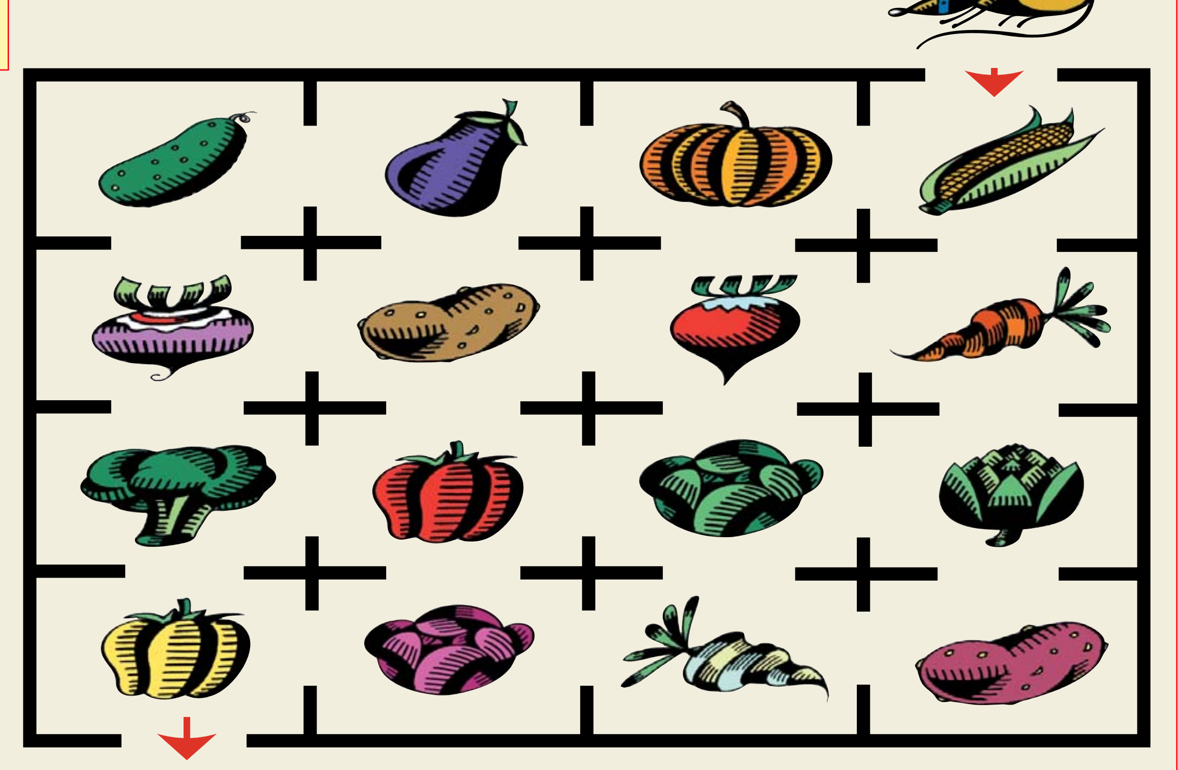


PLAYTHINK 782

DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ______

HUNGRY MOUSE

Can you find a path so that the mouse eats every vegetable and exits without entering any rooms twice?

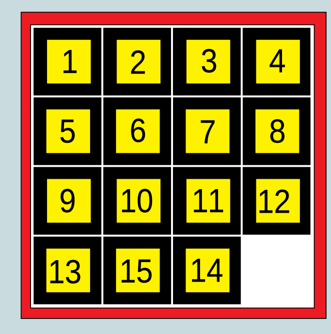


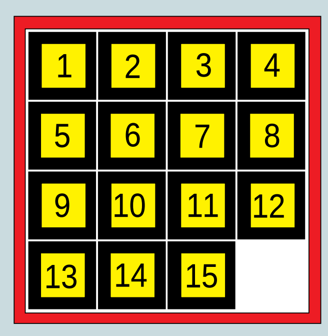
PLAYTHINK 783

DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ______

14-15 PUZZLE OF SAM LOYD

Using only sliding moves, can you rearrange the numbered tiles from the configuration on top to the perfectly ordered one on the bottom? How many moves does it take to exchange 14 and 15?





PLAYTHINK 784

DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ______

DAISY GAME

This is a two-person game; if no opponent is available, try figuring out yourself how you can *always* win.

Begin with all thirteen bees in the inward position, toward the center of the daisy. Each player may capture one bee or two adjacent bees each turn, by sliding the bee or bees outward on the petal. The player who captures the last bee wins. Can you work out a strategy so that if your opponent starts first, you will always win?


Science 12

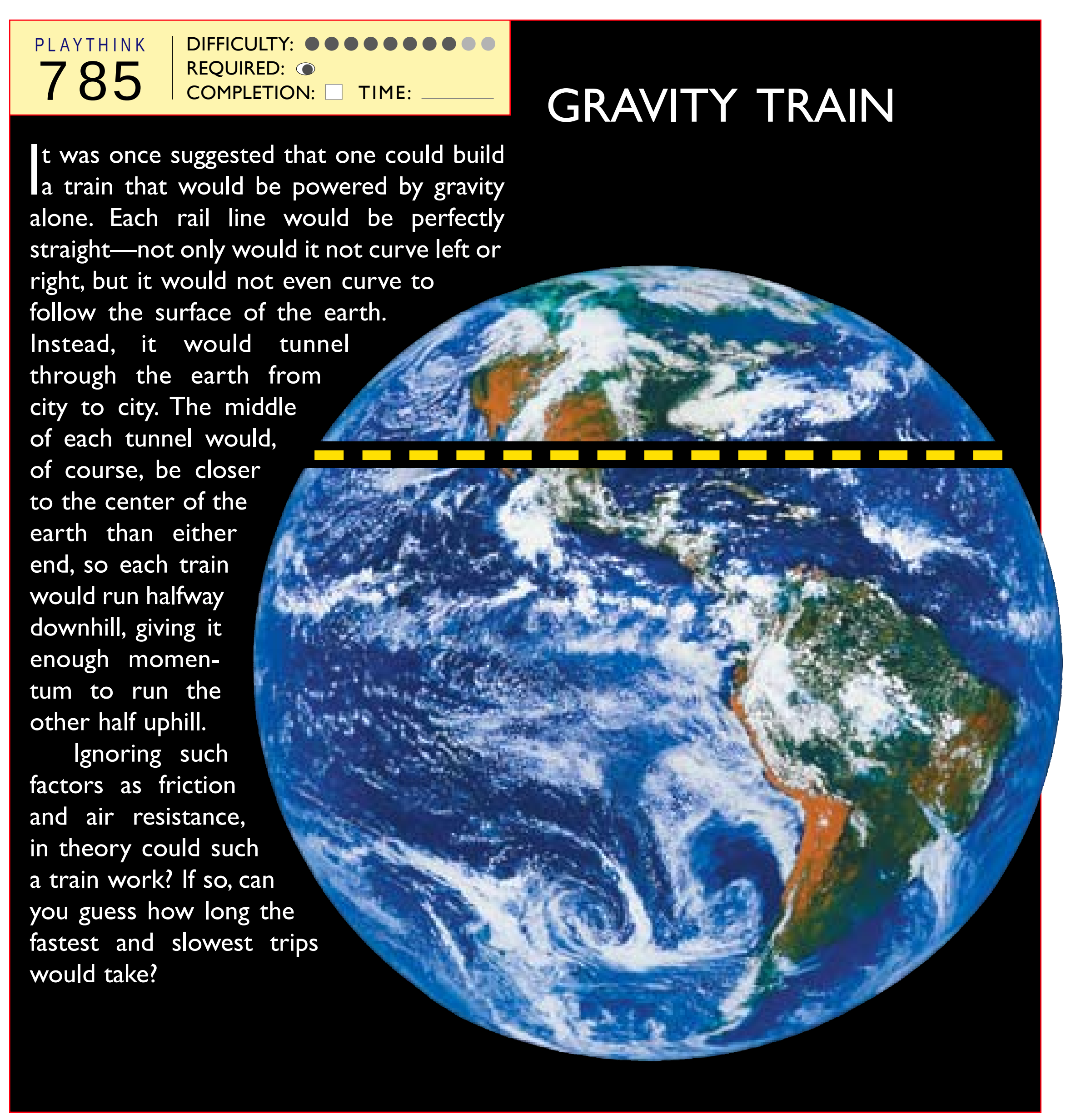
PLAYTHINK 785

DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

GRAVITY TRAIN

It was once suggested that one could build a train that would be powered by gravity alone. Each rail line would be perfectly straight—not only would it not curve left or right, but it would not even curve to follow the surface of the earth. Instead, it would tunnel through the earth from city to city. The middle of each tunnel would, of course, be closer to the center of the earth than either end, so each train would run halfway downhill, giving it enough momentum to run the other half uphill.

Ignoring such factors as friction and air resistance, in theory could such a train work? If so, can you guess how long the fastest and slowest trips would take?



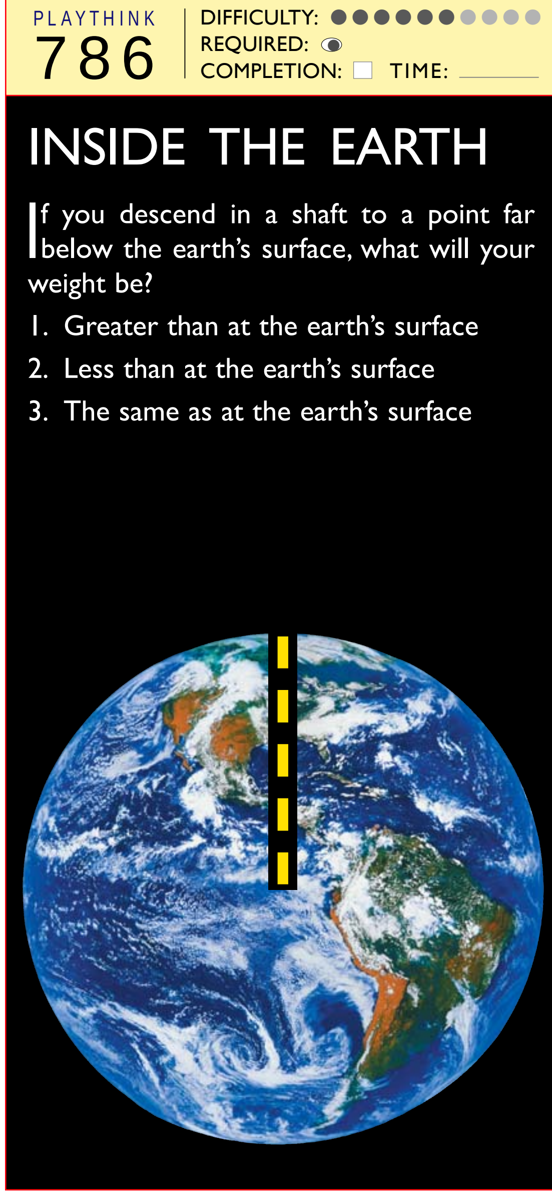
PLAYTHINK 786

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

INSIDE THE EARTH

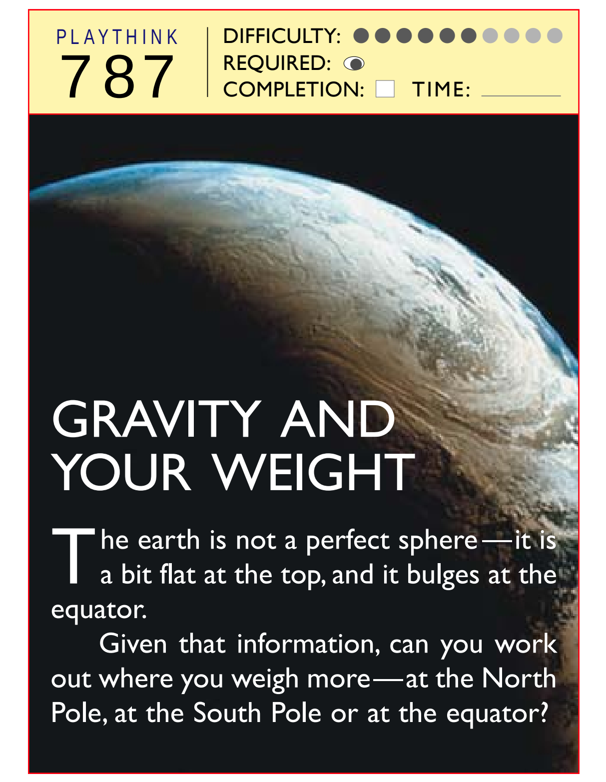
If you descend in a shaft to a point far below the earth's surface, what will your weight be?

1. Greater than at the earth's surface
2. Less than at the earth's surface
3. The same as at the earth's surface



PLAYTHINK 787

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____



GRAVITY AND YOUR WEIGHT

The earth is not a perfect sphere—it is a bit flat at the top, and it bulges at the equator.

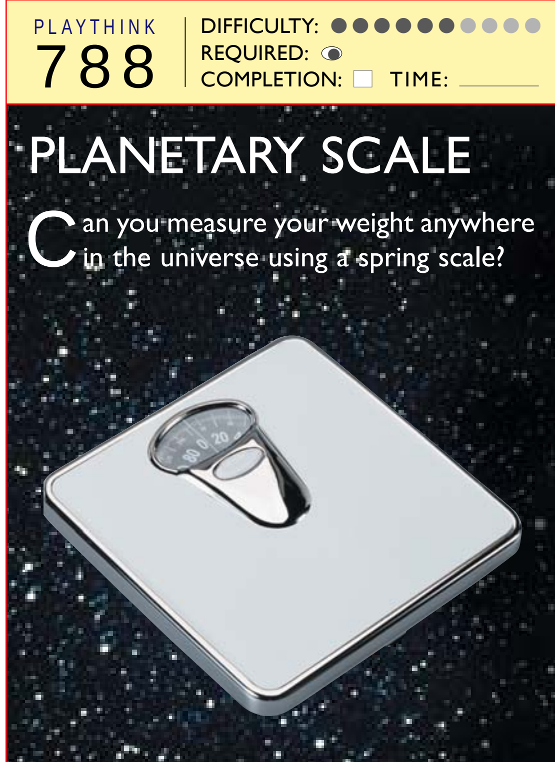
Given that information, can you work out where you weigh more—at the North Pole, at the South Pole or at the equator?

PLAYTHINK 788

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

PLANETARY SCALE

Can you measure your weight anywhere in the universe using a spring scale?

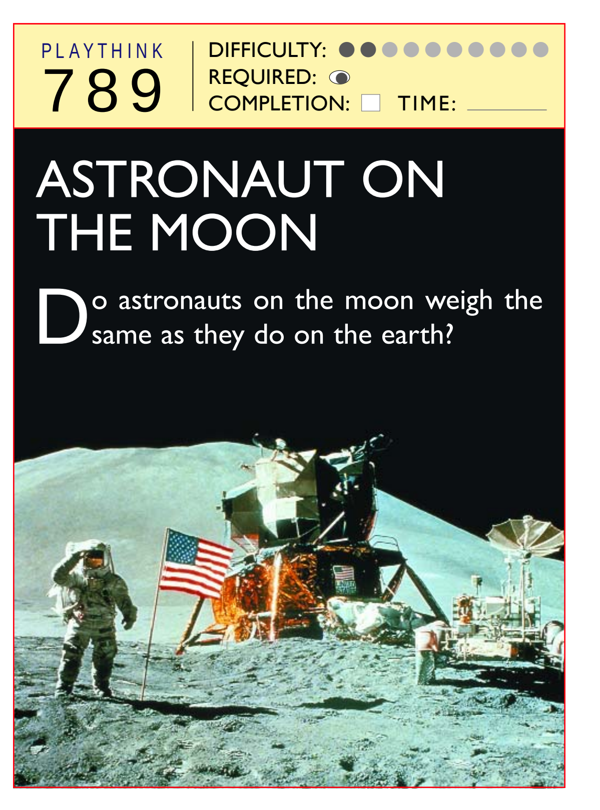


PLAYTHINK 789

DIFFICULTY: ●●○○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

ASTRONAUT ON THE MOON

Do astronauts on the moon weigh the same as they do on the earth?



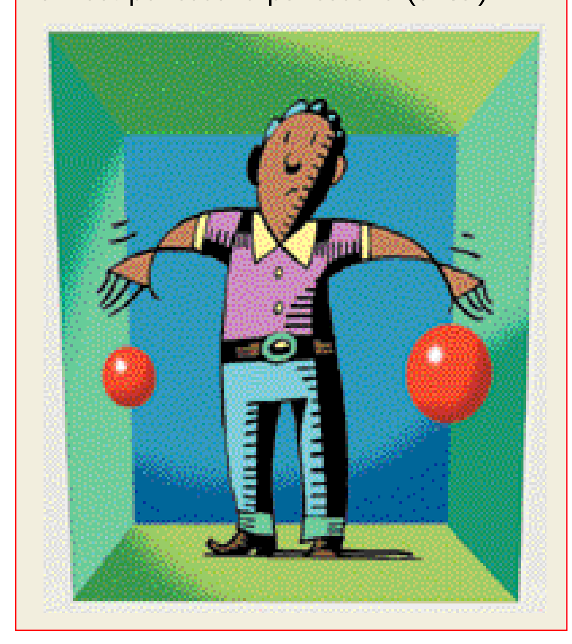
PLAYTHINK 790 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____



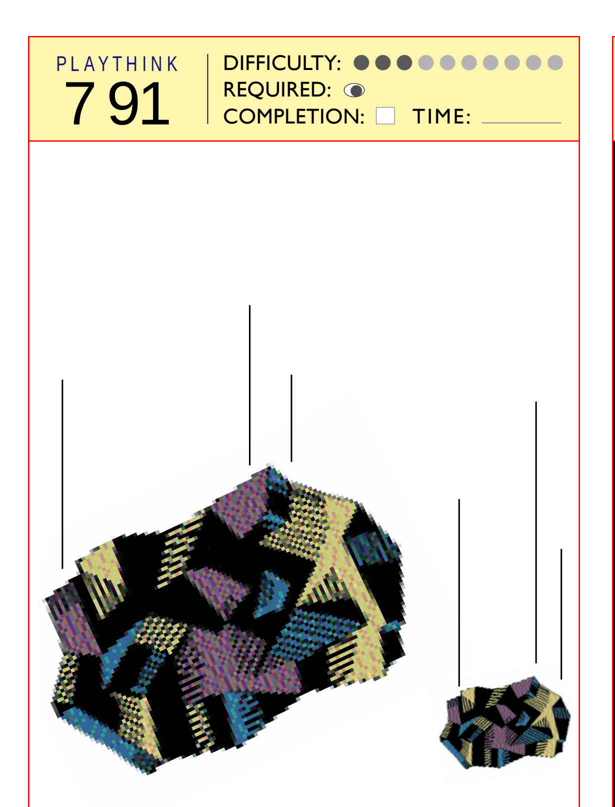
RELATIVITY OF GRAVITY

Imagine you are standing in a small, sealed, windowless room. You drop two objects of different mass, and they fall with the same acceleration and hit the floor at the same time.

Given this information, how can you tell for certain that you are in a room on Earth rather than in a room on a rocket that is undergoing a uniform acceleration equal to 32 feet per second per second ($32f/s^2$)?



PLAYTHINK 791 | DIFFICULTY: ●●●○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____



FALLING STONES

A large stone is 100 times heavier than a small rock, but when dropped at the same time, they fall with the same acceleration (ignoring air resistance). Why doesn't the large stone accelerate faster? Is it because of its weight, its energy, its surface area or its inertia?

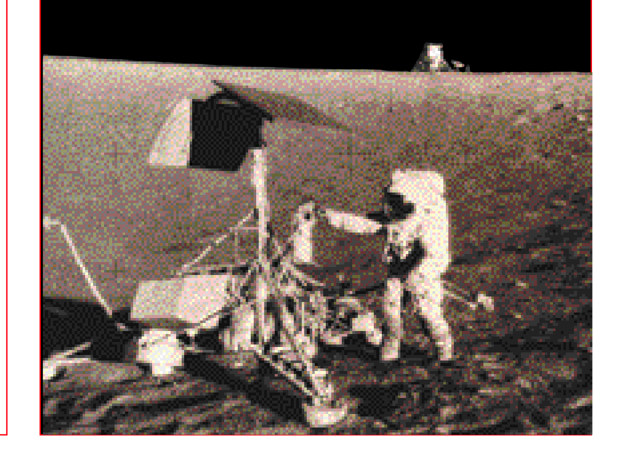
PLAYTHINK 792 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

FALLING OBJECTS

In 1971 the *Apollo 15* astronaut David Scott performed a famous experiment. He dropped a feather and a hammer at the same time—and they both dropped like the proverbial stone, undergoing the same acceleration. The reason is that he dropped them on the surface of the moon, which has no atmosphere and therefore no air resistance to slow the feather.

The belief that heavy objects fall faster than light ones dates back to Aristotle and dominated thinking until the Middle Ages. Galileo was the first to demonstrate that this belief was false by dropping objects off the tower of Pisa. Since then scientists have tried various ways to counteract the slowing effect of air resistance, though none has gone to as great a length as Scott.

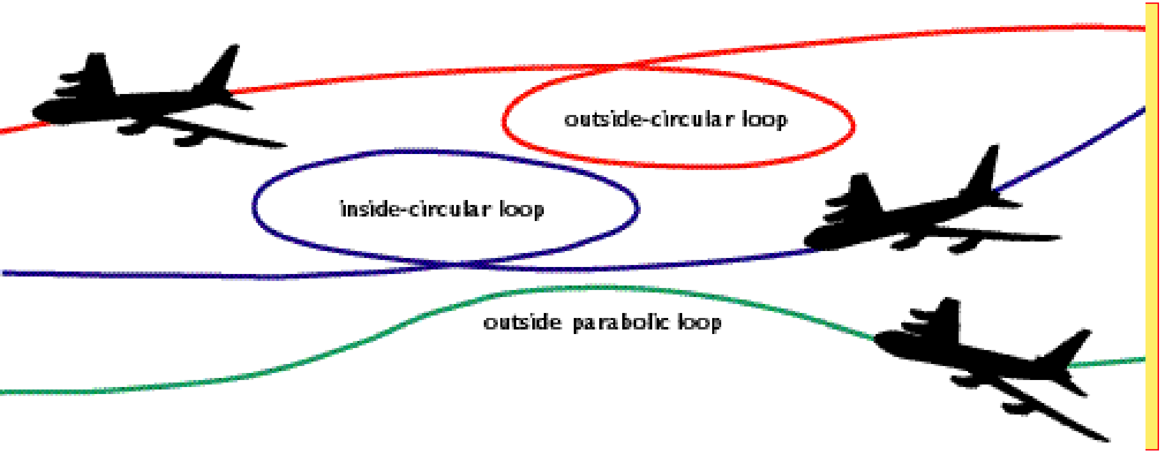
If you drop a coin and a small slip of paper at the same time, the coin will inevitably reach the ground first because of air resistance. Can you find a way to demonstrate that the coin and the paper ought to fall at the same rate in the absence of air resistance, even in a normal room?



PLAYTHINK 793 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

ANTIGRAVITY

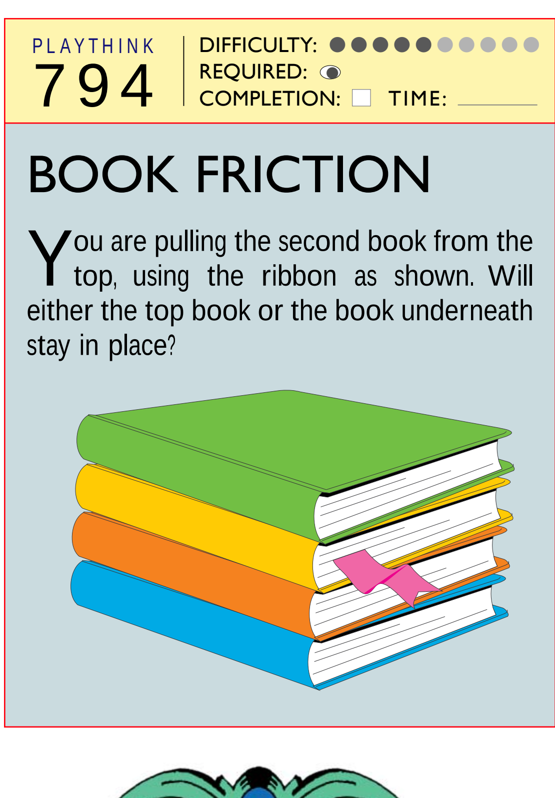
Astronauts in orbit feel weightless as they circle the earth. But the feeling of weightlessness can be achieved in an airplane that performs one of the maneuvers shown here. Can you work out which one?



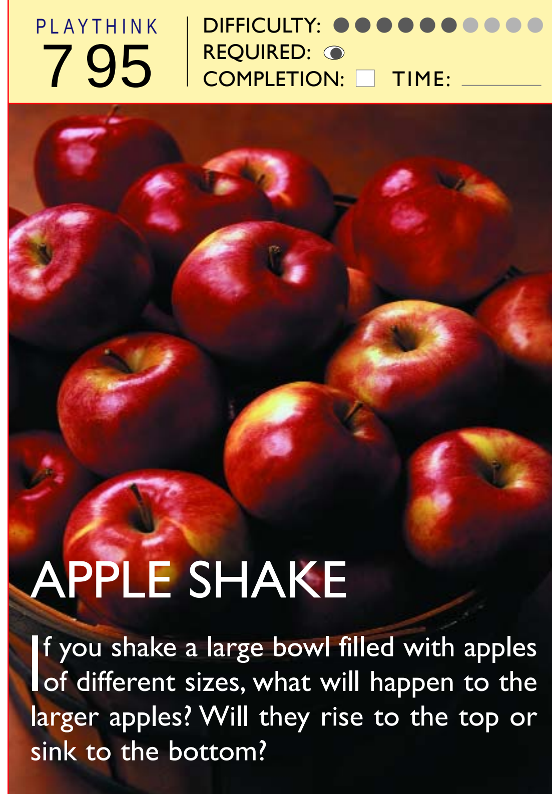
PLAYTHINK 794 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

BOOK FRICTION

You are pulling the second book from the top, using the ribbon as shown. Will either the top book or the book underneath stay in place?



PLAYTHINK 795 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____



APPLE SHAKE

If you shake a large bowl filled with apples of different sizes, what will happen to the larger apples? Will they rise to the top or sink to the bottom?

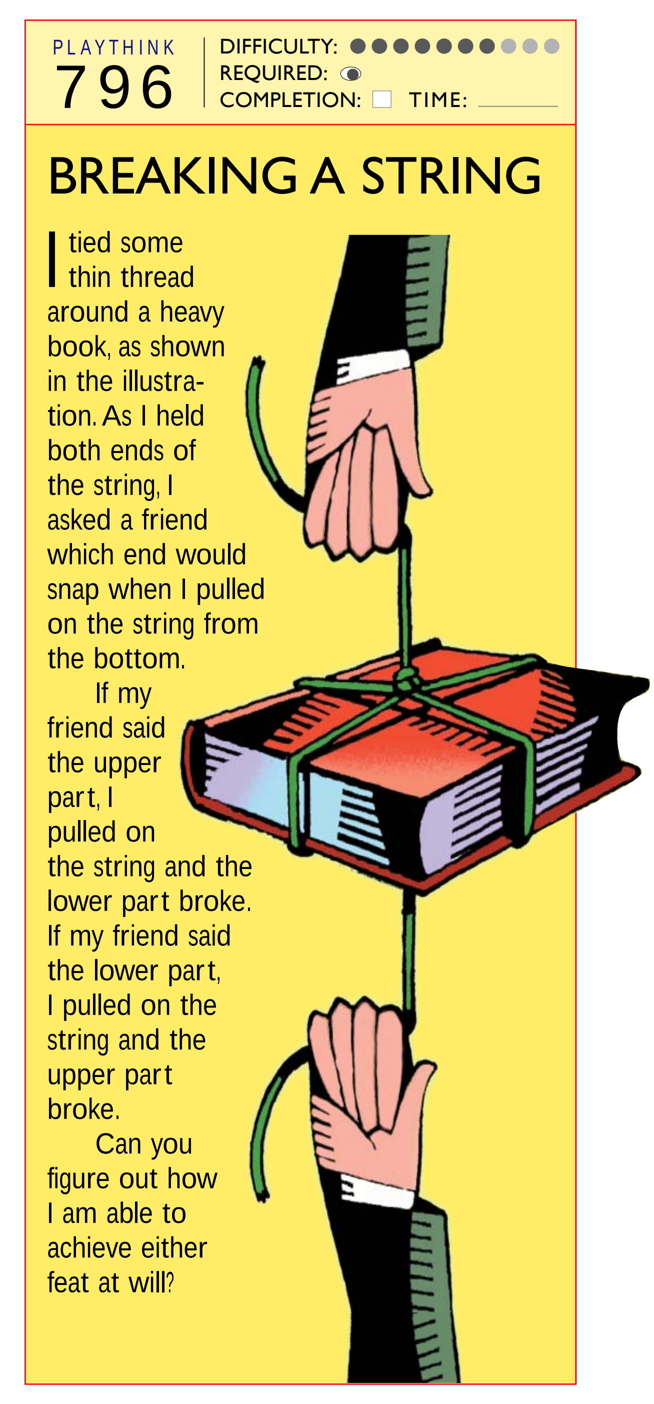
PLAYTHINK 796 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

BREAKING A STRING

I tied some thin thread around a heavy book, as shown in the illustration. As I held both ends of the string, I asked a friend which end would snap when I pulled on the string from the bottom.

If my friend said the upper part, I pulled on the string and the lower part broke. If my friend said the lower part, I pulled on the string and the upper part broke.

Can you figure out how I am able to achieve either feat at will?





PLAYTHINK 797 | DIFFICULTY: ●●●○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

BALLS BIG AND SMALL

If large steel balls are packed into a one-meter cube and small steel balls are packed into an identical cube, which weighs more?

Do you think it makes a difference that more small balls can be packed into the same space?

Center of Gravity

The center of gravity of an object isn't always at its center. Most standing lamps have a weighted base so that they won't tip over if you brush against them. Darts are often front-heavy so that they can be thrown with more accuracy.

To find the center of gravity of an irregularly shaped object, simply hang the object by a string from three different points. Because the center of gravity always seeks the lowest position it can reach, it will always be directly below the point from which it is hung—the place where those three vertical lines cross.

Many structures look unstable but are actually in equilibrium, demonstrating the fact that the center of gravity can sometimes exist outside the boundaries of the object itself. For a striking example, think of tightrope walkers, whose long, weighted poles allow them to retain a remarkable amount of stability even when walking upon a wire the diameter of a thumb.

PLAYTHINK 798

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

GOLD SMUGGLERS

Even though every passenger at the customs checkpoint had passed inspection, an observant officer stopped one passenger and reinspected one of his suitcases. The officer found a secret compartment filled with several heavy ingots.

Can you figure out what aroused the officer's suspicion?



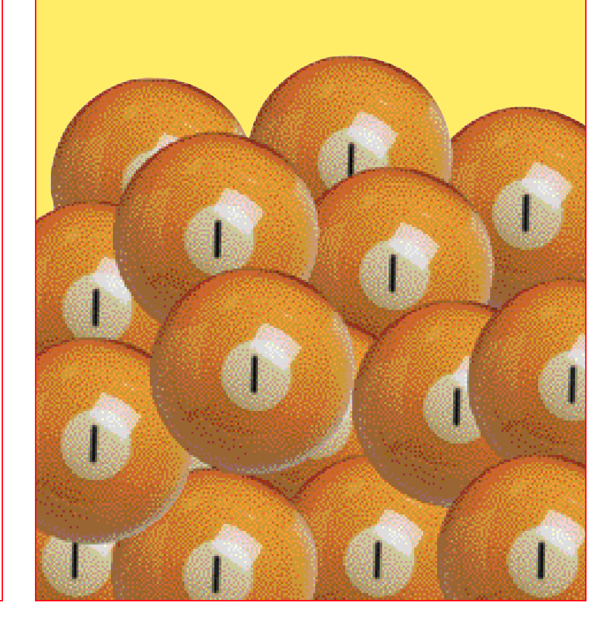
PLAYTHINK 799

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

ODDBALL

The owner of a pool hall is offered five bushels of colored balls, one each of red, blue, green, yellow and orange. All the balls weigh 100 grams, he learns, except for the balls of one color, which all weigh 110 grams.

The owner wants to use a spring scale that is accurate to within 10 grams to find out which color ball is too heavy. Can you work out the minimum number of weighings he must carry out to discover the color of the odd balls?

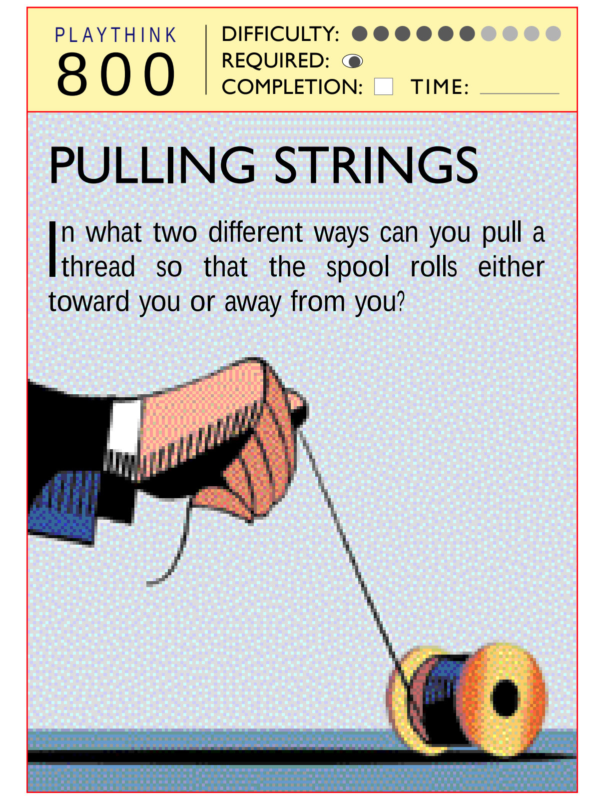


PLAYTHINK 800

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

PULLING STRINGS

In what two different ways can you pull a thread so that the spool rolls either toward you or away from you?



PLAYTHINK 801

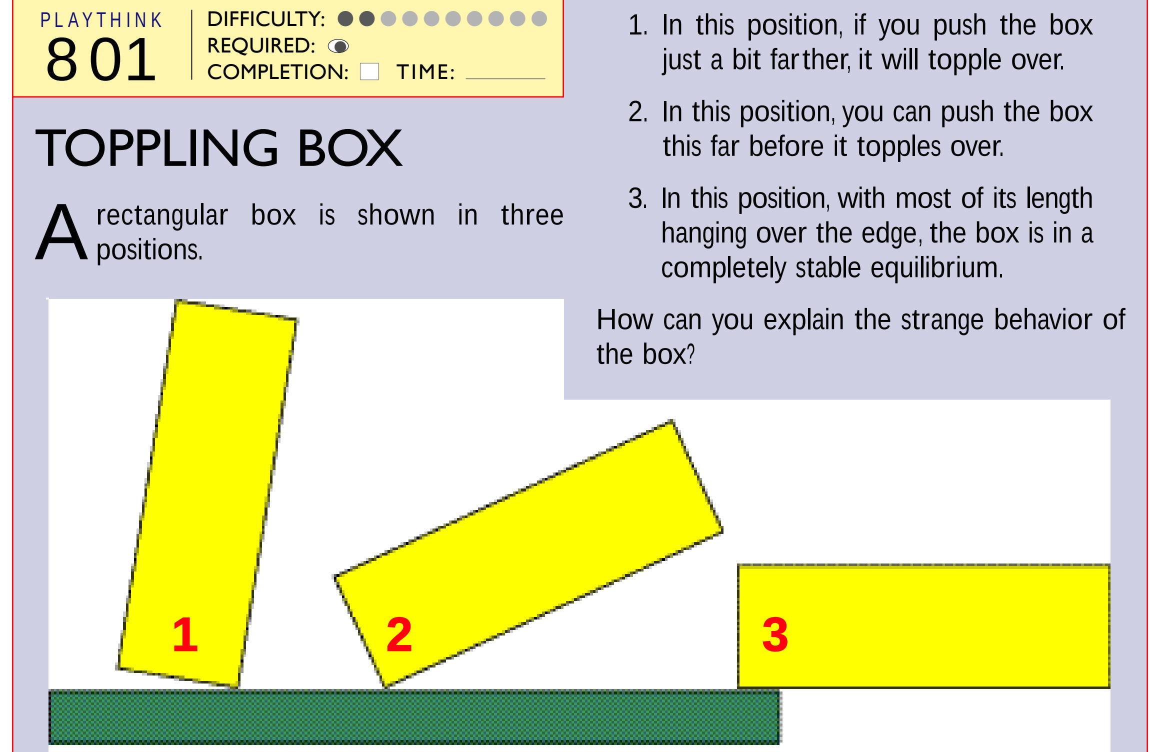
DIFFICULTY: ●●○○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

TOPPLING BOX

A rectangular box is shown in three positions.

1. In this position, if you push the box just a bit farther, it will topple over.
2. In this position, you can push the box this far before it topples over.
3. In this position, with most of its length hanging over the edge, the box is in a completely stable equilibrium.

How can you explain the strange behavior of the box?



PLAYTHINK 802
DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

PRIZE CATCH

You can measure weight—that is, the force of gravity—with a spring balance, the action of which depends directly on gravitational pull. The extension of a helical spring or even a simple rubber band is proportional to the force exerted on it. A twofold or threefold increase in weight leads to a twofold or threefold extension of the spring.

But because spring balances measure force, they don't always read what they mean. Take a look at the illustration here. The scale seems to show that the fisherman's prize marlin weighs 100 kilograms. Can you work out what the marlin really weighs?

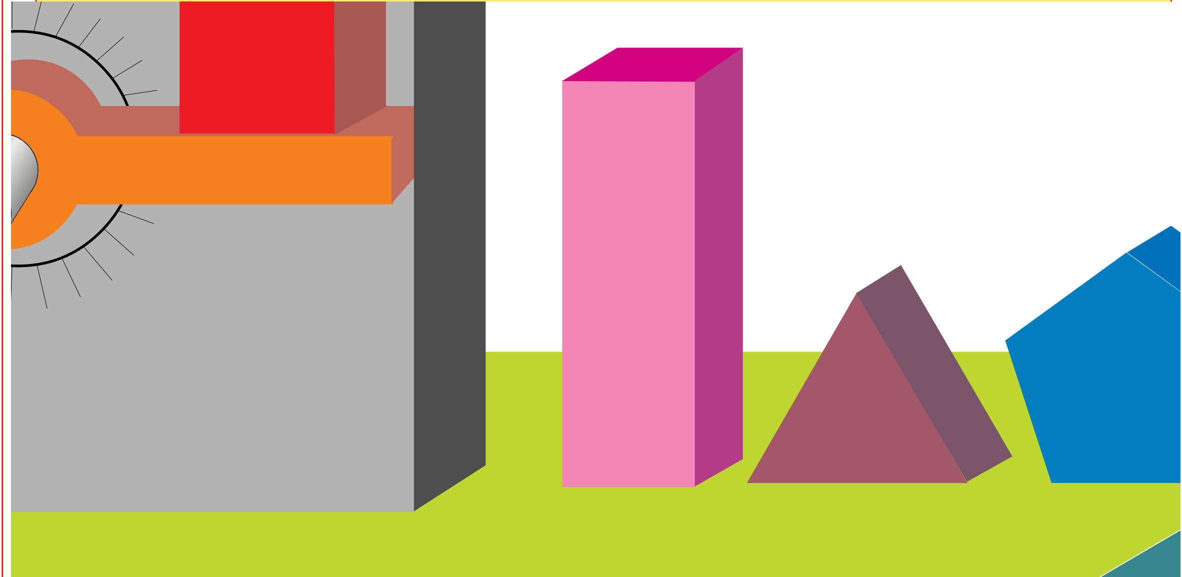


PLAYTHINK 803
DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

TOPPLING STABILITY

A very simple device can compare the toppling tendency of various shapes. Each shape is placed in succession on the testing platform; the platform slowly changes angle until the shape topples over. Simple.

Can you figure out which shape stayed on the platform the longest? That is, can you work out which of the shapes below has the greatest toppling stability?



PLAYTHINK 804
DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

STICK-BALANCING PARADOX

You and a friend can balance a yardstick on your index fingers, as shown in the illustration. Can you work out what will happen when you both try to slide your fingers toward the middle of the stick? What will happen if you start with both fingers in the middle and slide them toward the ends?

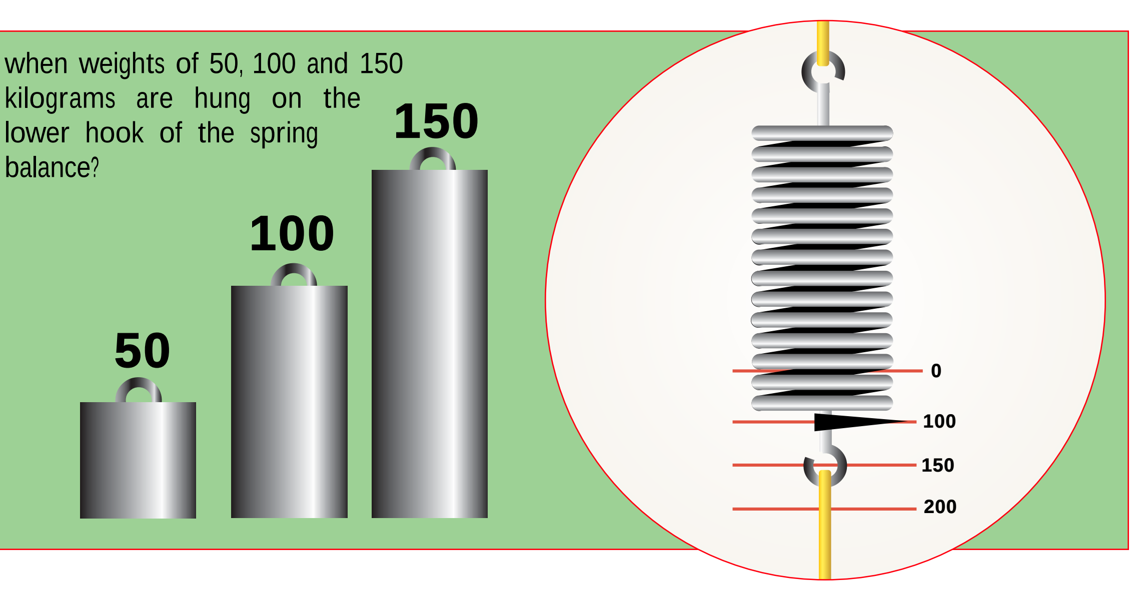


PLAYTHINK 805
DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

SPRING BALANCE

A spring balance is hung from the ceiling by a rope. A second rope attached to the floor is tied tightly to the balance, pulling on the spring so that the balance reads 100 kilograms.

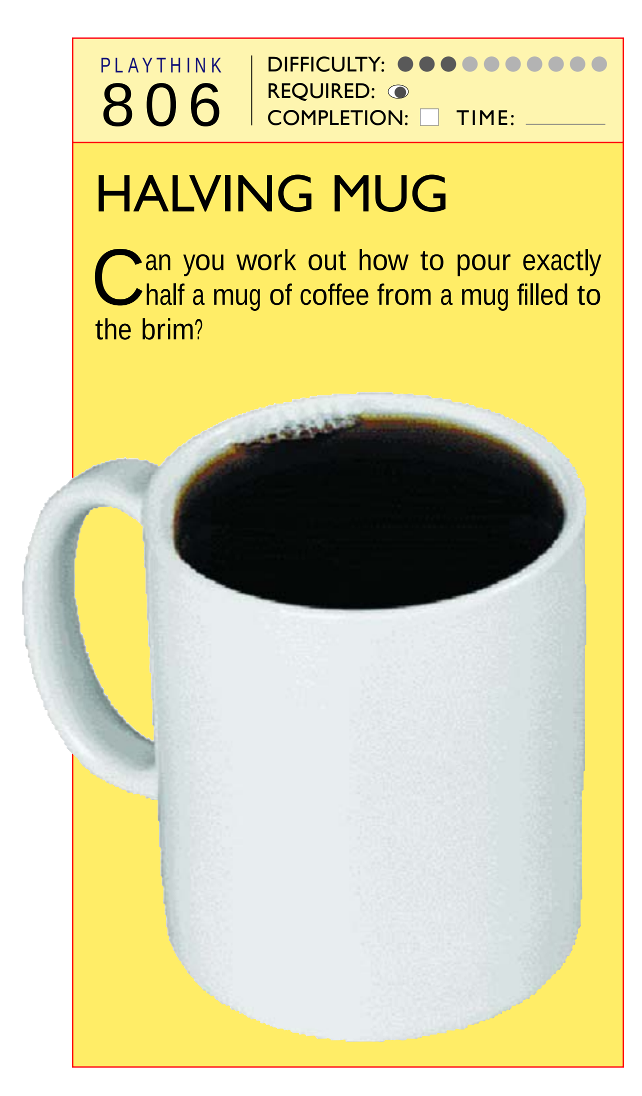
With the rope still attached, weights are hung from the hook and measured. Can you work out what the balance will read when weights of 50, 100 and 150 kilograms are hung on the lower hook of the spring balance?



PLAYTHINK 806 | DIFFICULTY: ●●●○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

HALVING MUG

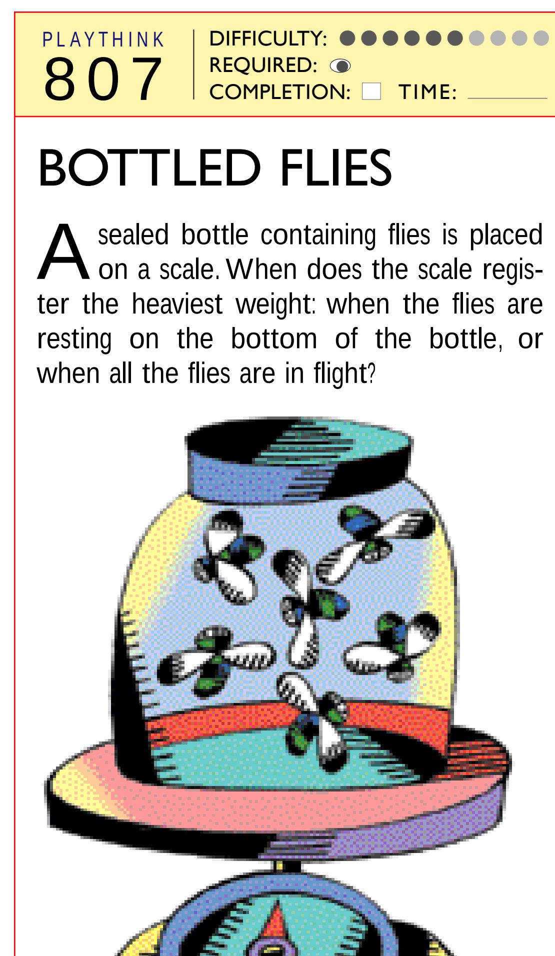
Can you work out how to pour exactly half a mug of coffee from a mug filled to the brim?



PLAYTHINK 807 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

BOTTLED FLIES

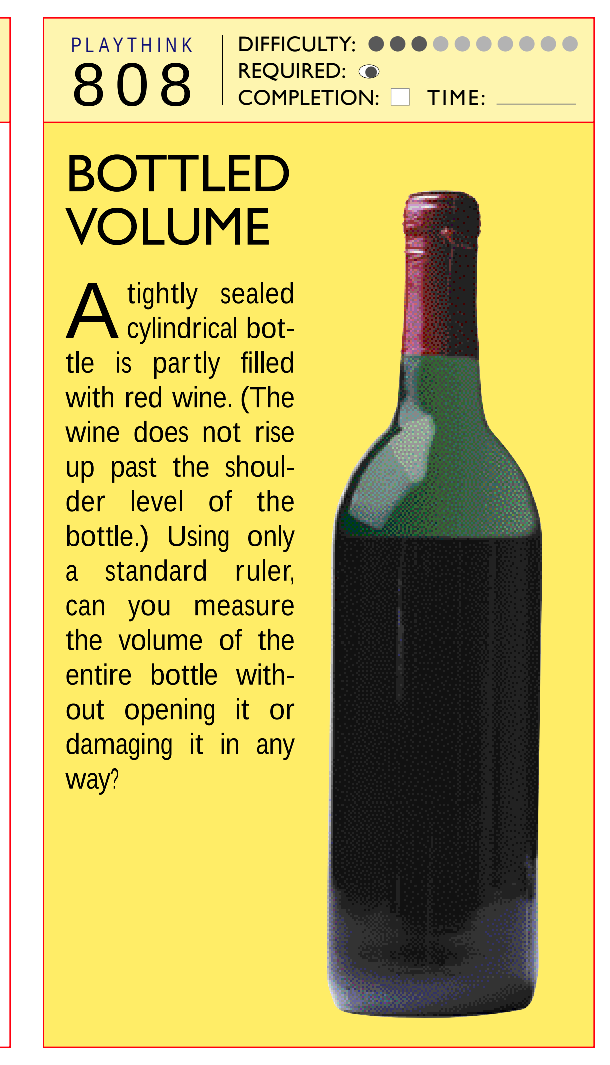
A sealed bottle containing flies is placed on a scale. When does the scale register the heaviest weight: when the flies are resting on the bottom of the bottle, or when all the flies are in flight?



PLAYTHINK 808 | DIFFICULTY: ●●●○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

BOTTLED VOLUME

A tightly sealed cylindrical bottle is partly filled with red wine. (The wine does not rise up past the shoulder level of the bottle.) Using only a standard ruler, can you measure the volume of the entire bottle without opening it or damaging it in any way?



PLAYTHINK 809 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____



LOST RING

You have sealed the ninth of nine identical parcels of precisely equal weights, only to discover that your diamond ring has accidentally fallen into one of the packages.

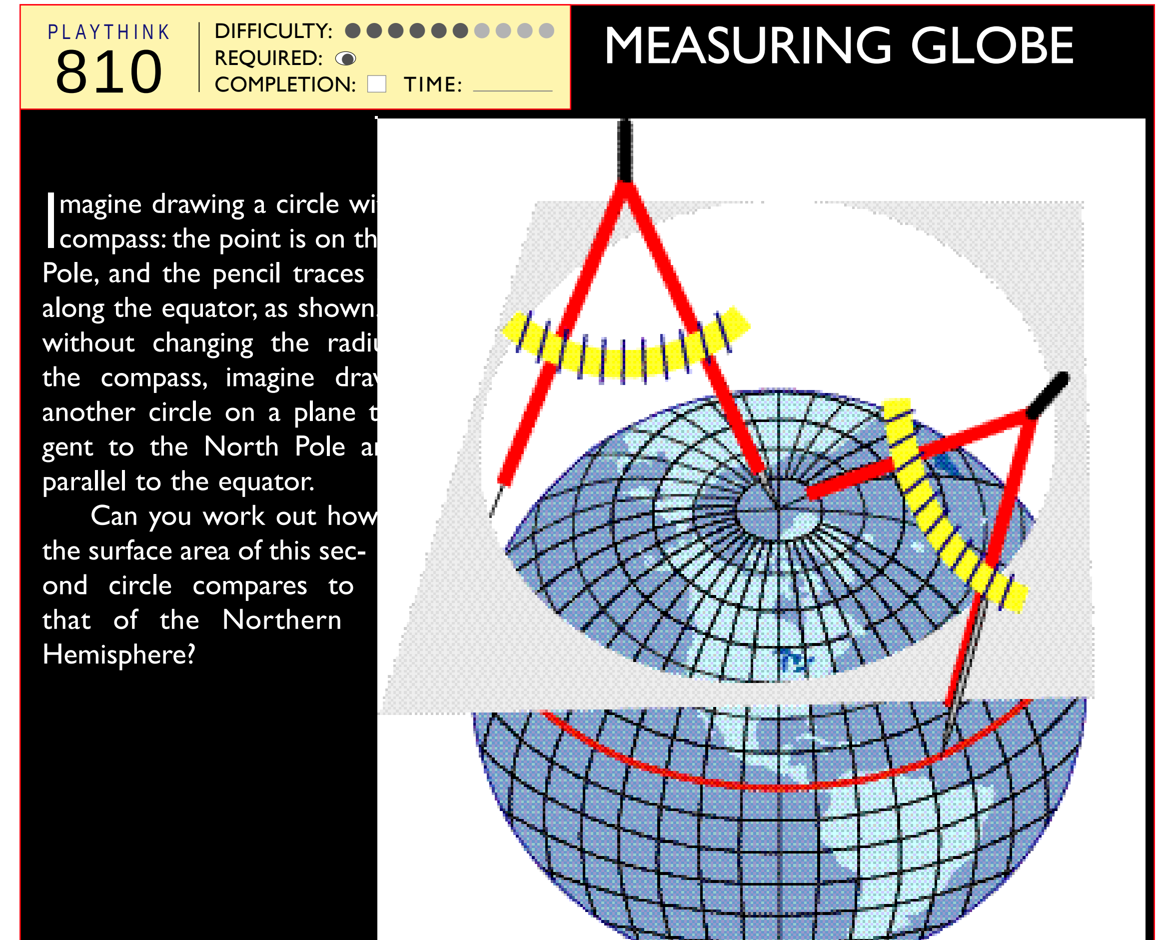
You don't want to unwrap every parcel. Can you work out how to find the parcel containing the ring with just two weighings on a balance scale?

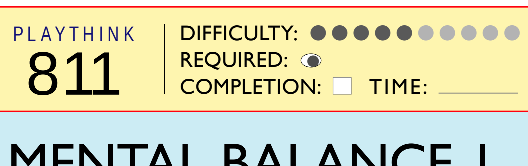
PLAYTHINK 810 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

MEASURING GLOBE

Imagine drawing a circle with a compass: the point is on the North Pole, and the pencil traces along the equator, as shown. Then without changing the radius of the compass, imagine drawing another circle on a plane tangent to the North Pole and parallel to the equator.

Can you work out how the surface area of this second circle compares to that of the Northern Hemisphere?



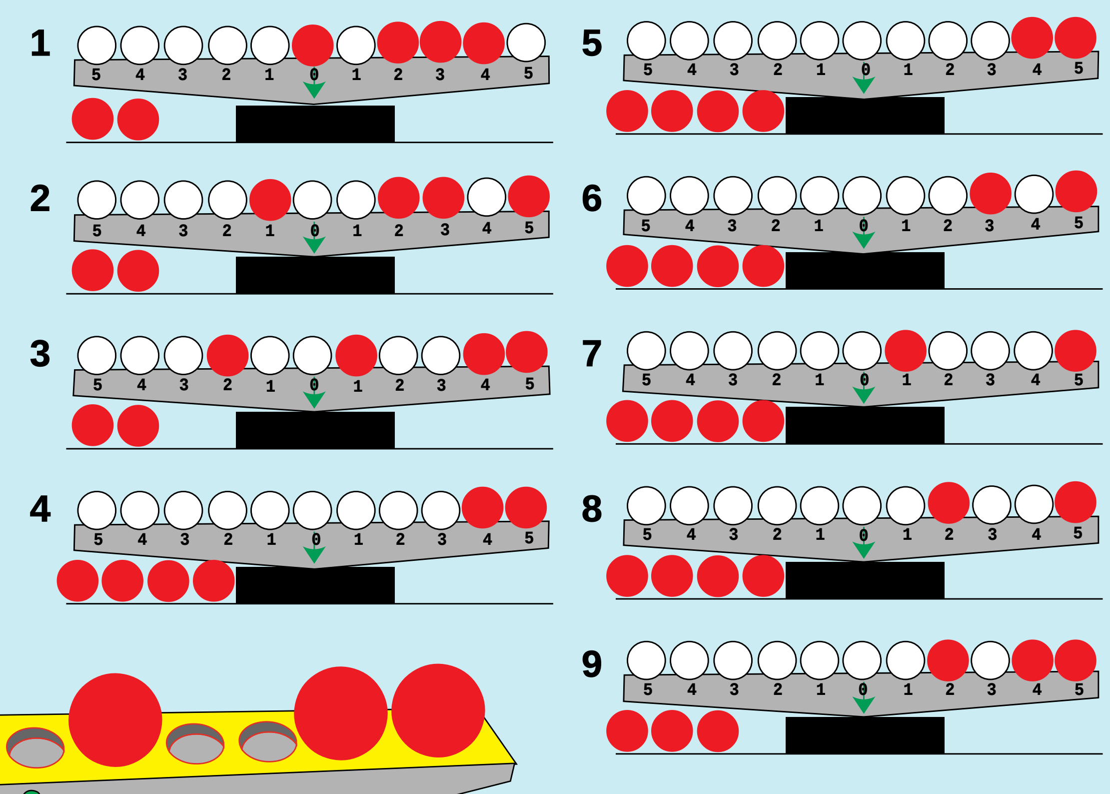


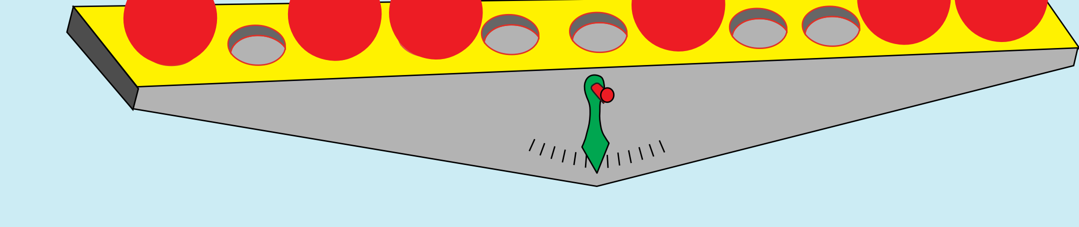
MENTAL BALANCE 1

The six identical steel balls are in perfect equilibrium in the balance shown below.

The balls may occupy any of the eleven positions on the balance; those positions are equally spaced and symmetrically arranged from the middle position. Simple equilibrium positions can be achieved through simple symmetrical configurations of the balls, but in these nine puzzles we will avoid such arrangements if at all possible.

In each puzzle some of the balls have already been placed on the balance. Can you place balls on the left side or in the middle position to achieve equilibrium?





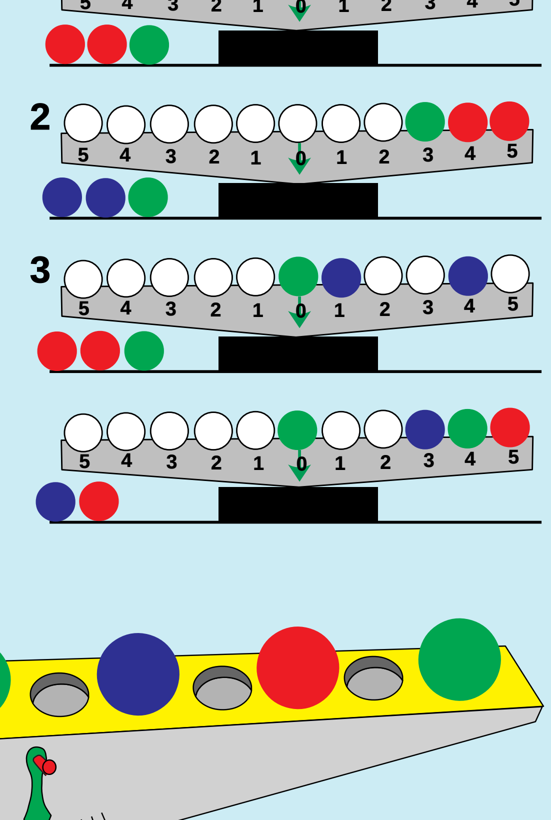
PLAYTHINK
812
DIFFICULTY:
REQUIRED:
COMPLETION: TIME:

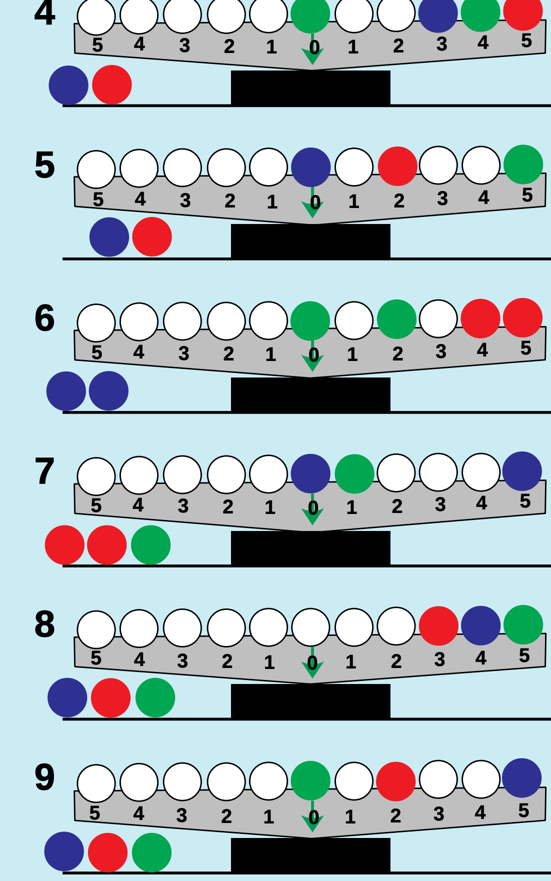
MENTAL BALANCE 2

The six steel balls in the balance are in perfect equilibrium. The set of balls comes with the following weights: green is 1 unit, red is 2 units and blue is 4 units.

The balls may occupy any of the eleven positions on the balance; those positions are equally spaced and symmetrically arranged from the middle position. Simple equilibrium positions can be achieved through simple symmetrical configurations of the balls, but in these nine puzzles we will avoid such arrangements if at all possible.

In each puzzle some of the balls have been placed on the right hand side of the balance. Can you place balls on the left side to achieve equilibrium?





Simple Machines

According to legend, the great Greek mathematician and engineer Archimedes once said, "Give me [a fulcrum and a] place on which to stand, and I will move the Earth," He meant it, too, so impressed was he by the enormous force produced by machines.

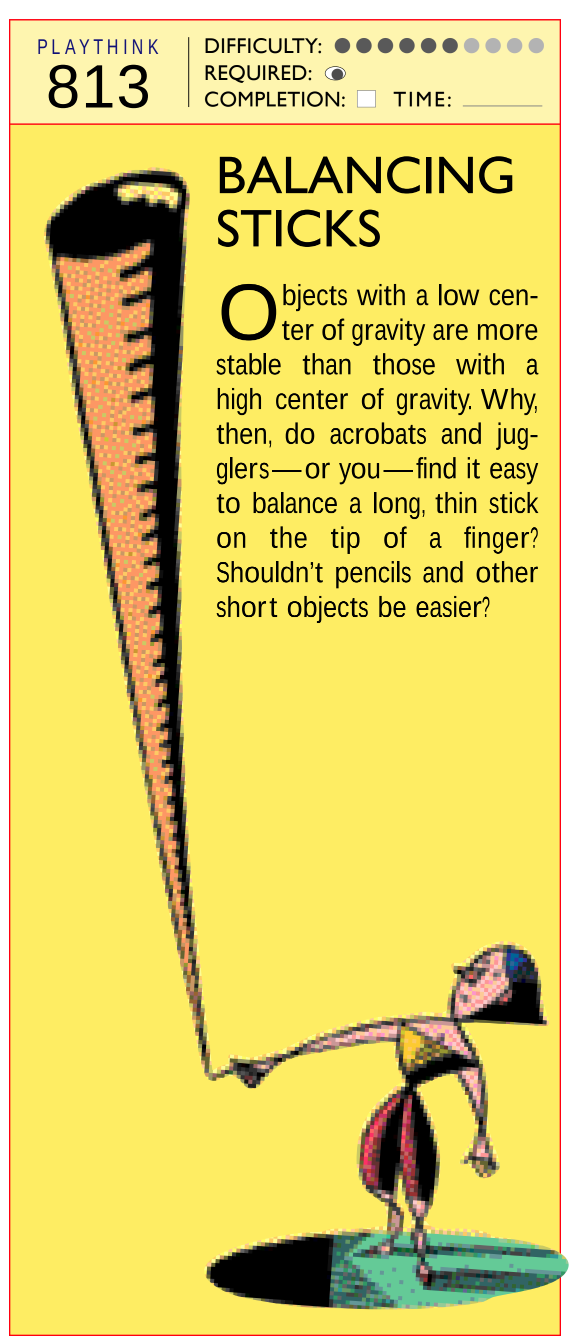
Prehistoric humans invented simple devices such as wedges and levers. The ancient Egyptians used roping and ramping to move enormous blocks of stone. The pulley was probably invented along with the first iron tools; Assyrian art from the eighth century B.C. shows the pulley in common use. But it was the ancient Greeks who actually studied simple machines deeply enough to group them into five categories: the lever, the wheel and axle, the pulley, the wedge and the screw.

Simple machines are extensions of the human body, originally invented to augment the muscular efforts of men and animals. Today they are everywhere—and no longer simple!

PLAYTHINK 813 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

BALANCING STICKS

Objects with a low center of gravity are more stable than those with a high center of gravity. Why, then, do acrobats and jugglers—or you—find it easy to balance a long, thin stick on the tip of a finger? Shouldn't pencils and other short objects be easier?

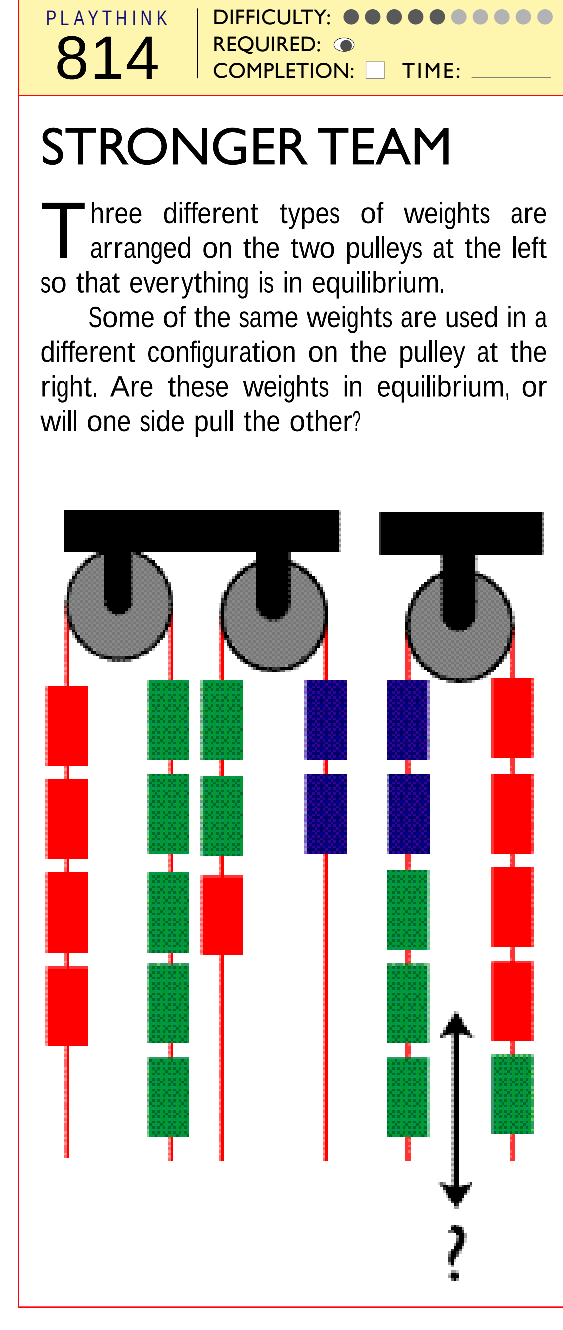


PLAYTHINK 814 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

STRONGER TEAM

Three different types of weights are arranged on the two pulleys at the left so that everything is in equilibrium.

Some of the same weights are used in a different configuration on the pulley at the right. Are these weights in equilibrium, or will one side pull the other?



PLAYTHINK 815 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

WATERING CANS

Which watering can holds more water?



PLAYTHINK 816 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

EGG OF COLUMBUS

Christopher Columbus is said to have stood an egg on its pointy end when he first crossed the equator. I thought of that story several years ago when I saw an ingenious equilibrium toy; The challenge was to re-create Columbus's feat. But as much as I tried, the egg would not balance. Shaking the egg did not reveal any moving parts. In fact, the only way the egg would balance was if one followed the instructions on the box:

1. Hold the egg with the pointed end up for at least thirty seconds.
2. Turn the egg over and wait for another ten seconds, then place it on the pointed end.

The egg would then balance beautifully. It would stay balanced for about fifteen seconds. After that period anyone else who tried to balance the egg would have no luck unless he or she knew the secret of the egg.

From the above description, can you work out the mysterious inner structure of this deeply puzzling egg?

PLAYTHINK 817 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

FIVE-MINUTE EGG

You must boil an egg for exactly five minutes, but all you have is a four-minute timer and a three-minute timer. Can you work out how to use these two timers to measure five minutes?

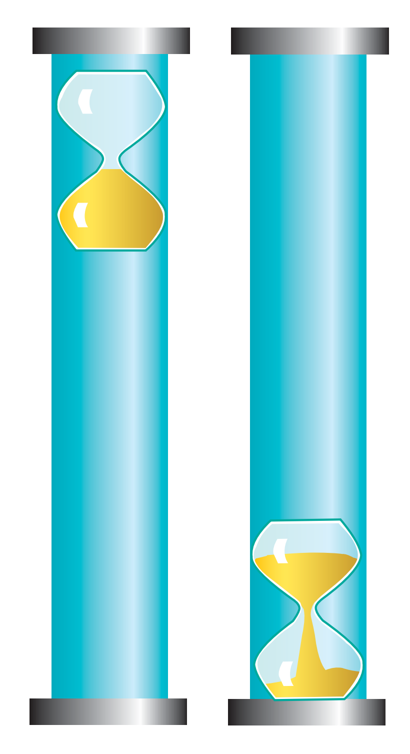


PLAYTHINK 818 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

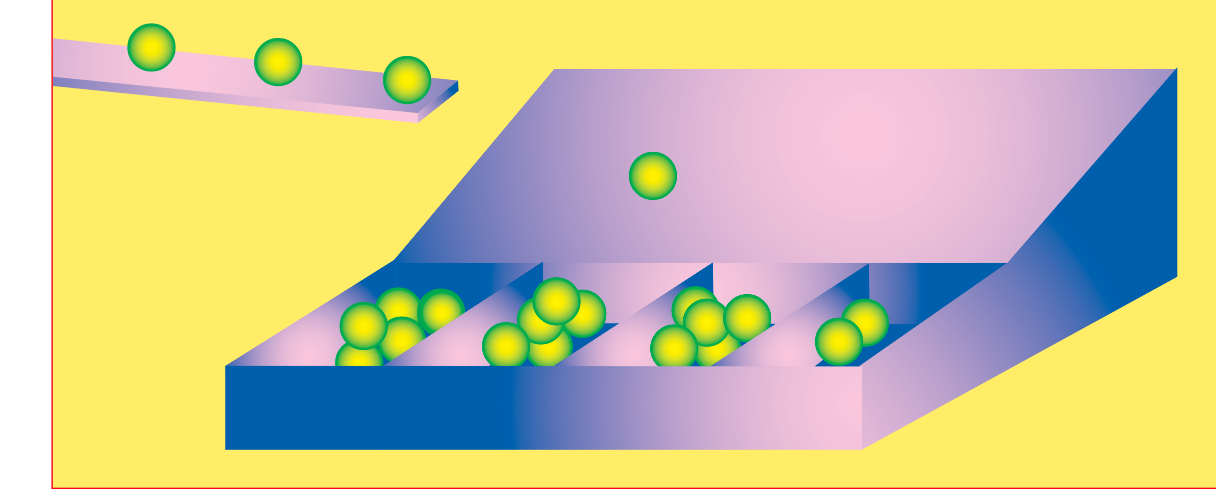
HOURGLASS PARADOX

A small, enclosed hourglass floats in a sealed, water-filled cylinder, as shown in the diagram. Turn the cylinder over and, surprisingly, the hourglass will not float back to the top. It will sit at the bottom until most of the sand has passed to the lower compartment. Only then will the hourglass float to the top.

Can you work out what delays the floating of the hourglass?



PLAYTHINK 819 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____



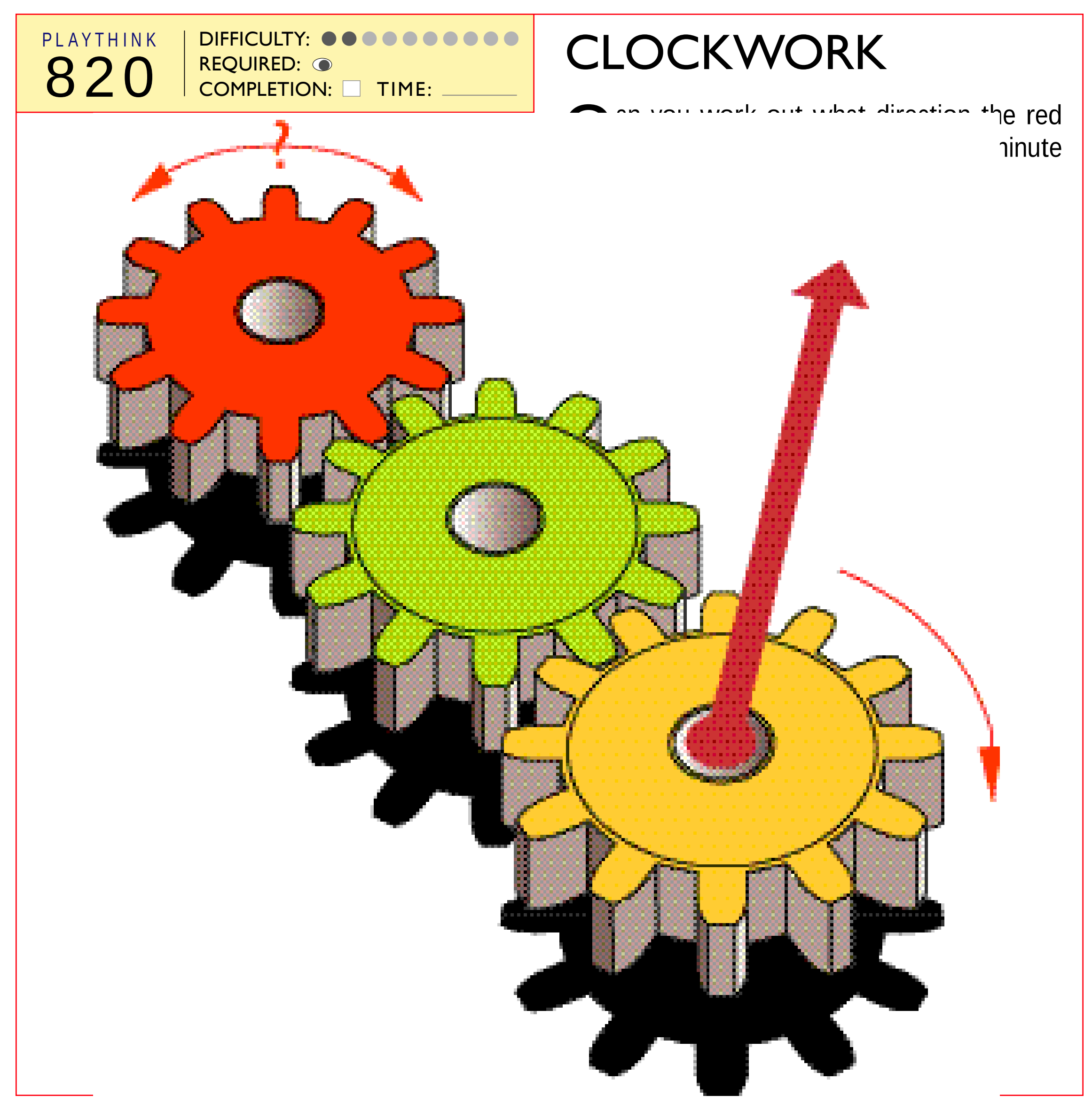
BALL-SORTING DEVICE

A steady supply of balls of the same size but four different weights rolls down a chute. From the chute the balls drop onto the slanted coarse surface of the sorting box. The device easily sorts the balls into four weight groups, eliminating the tedious job of weighing each ball.

From the illustration, can you tell which compartment collects the heaviest balls?

PLAYTHINK 820 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

CLOCKWORK

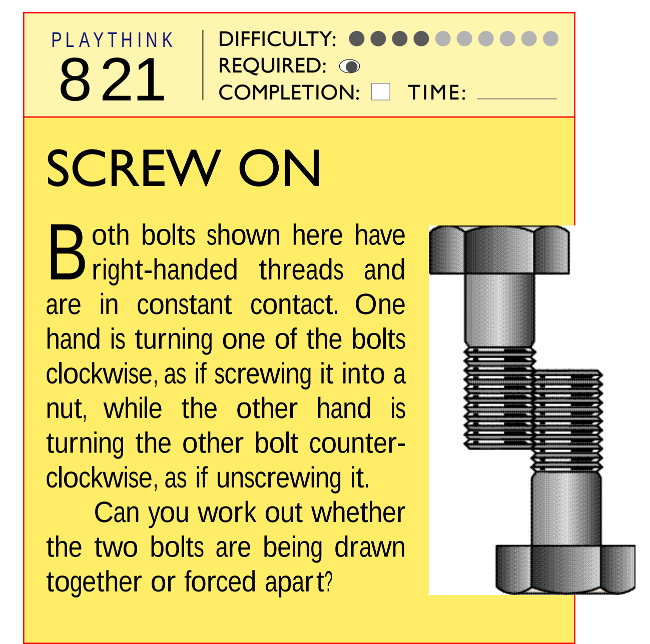


PLAYTHINK 821 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

SCREW ON

Both bolts shown here have right-handed threads and are in constant contact. One hand is turning one of the bolts clockwise, as if screwing it into a nut, while the other hand is turning the other bolt counter-clockwise, as if unscrewing it.

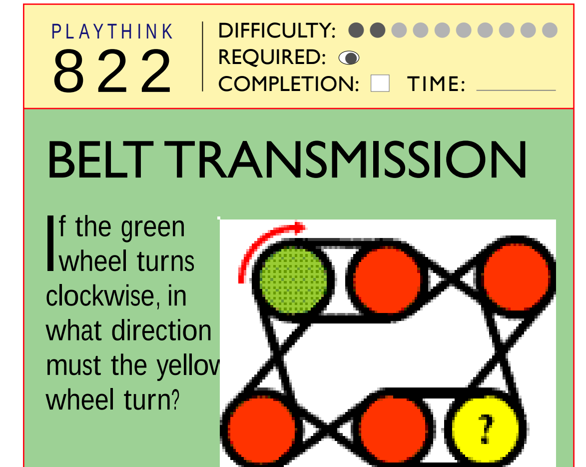
Can you work out whether the two bolts are being drawn together or forced apart?



PLAYTHINK 822 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

BELT TRANSMISSION

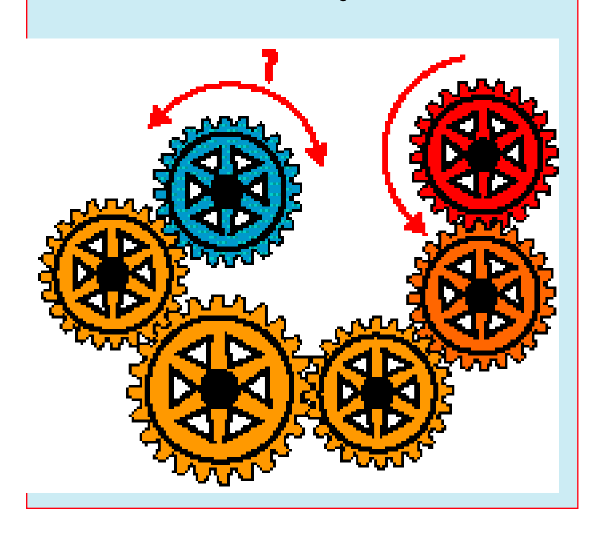
If the green wheel turns clockwise, in what direction must the yellow wheel turn?



PLAYTHINK 823 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

GEAR TRAIN 1

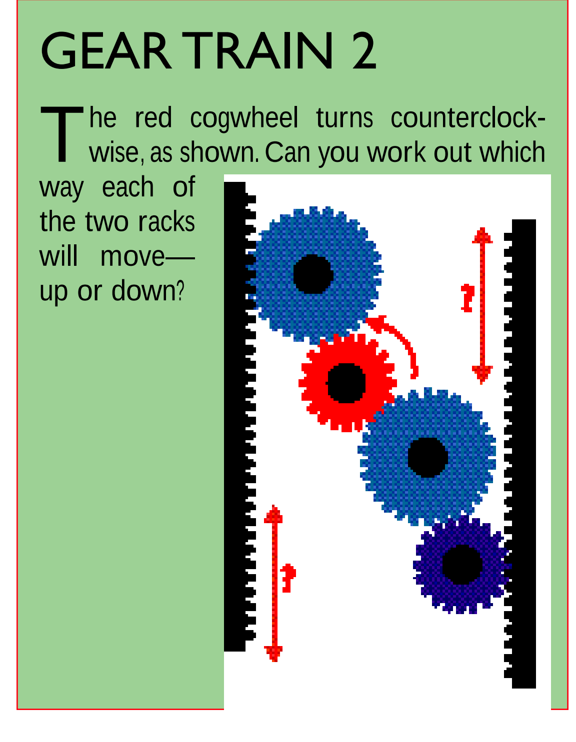
The red cogwheel turns in a counter-clockwise direction, as shown. In what direction will the blue cogwheel turn?



PLAYTHINK 824 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

GEAR TRAIN 2

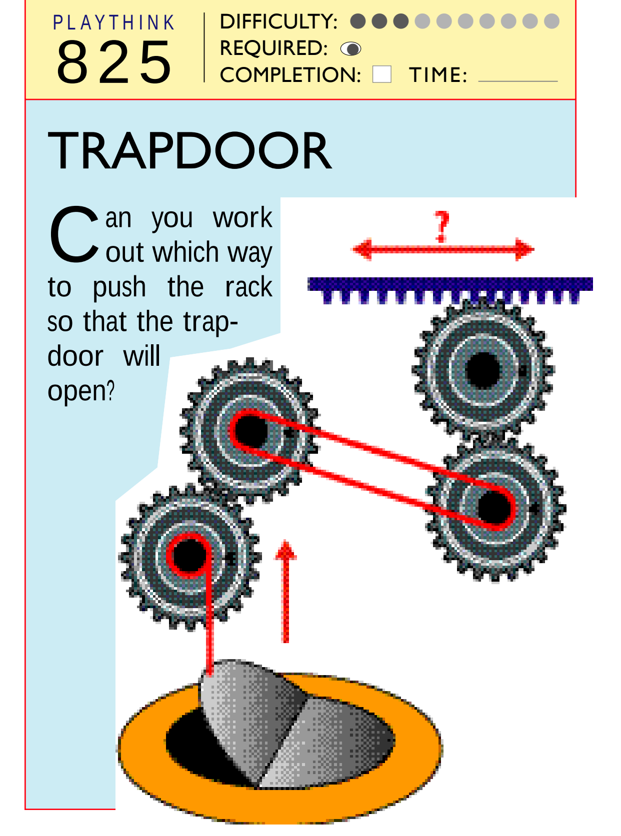
The red cogwheel turns counterclockwise, as shown. Can you work out which way each of the two racks will move—up or down?

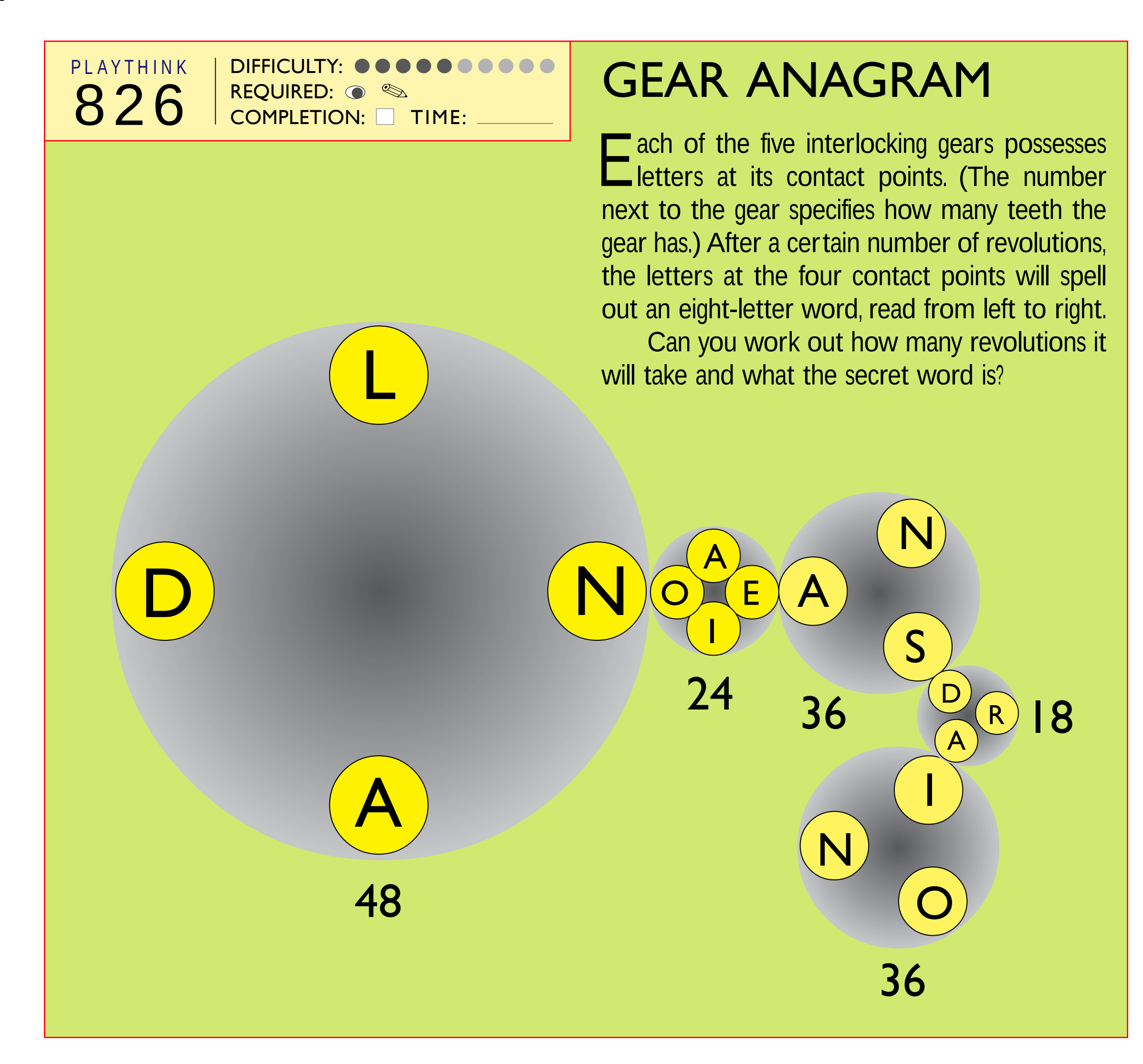


PLAYTHINK 825 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

TRAPDOOR

Can you work out which way to push the rack so that the trapdoor will open?



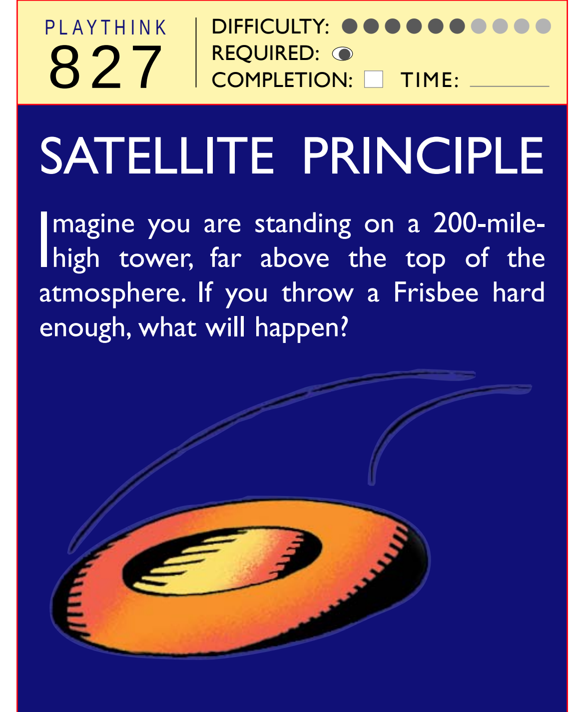


PLAYTHINK 826 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ______

GEAR ANAGRAM

Each of the five interlocking gears possesses letters at its contact points. (The number next to the gear specifies how many teeth the gear has.) After a certain number of revolutions, the letters at the four contact points will spell out an eight-letter word, read from left to right.

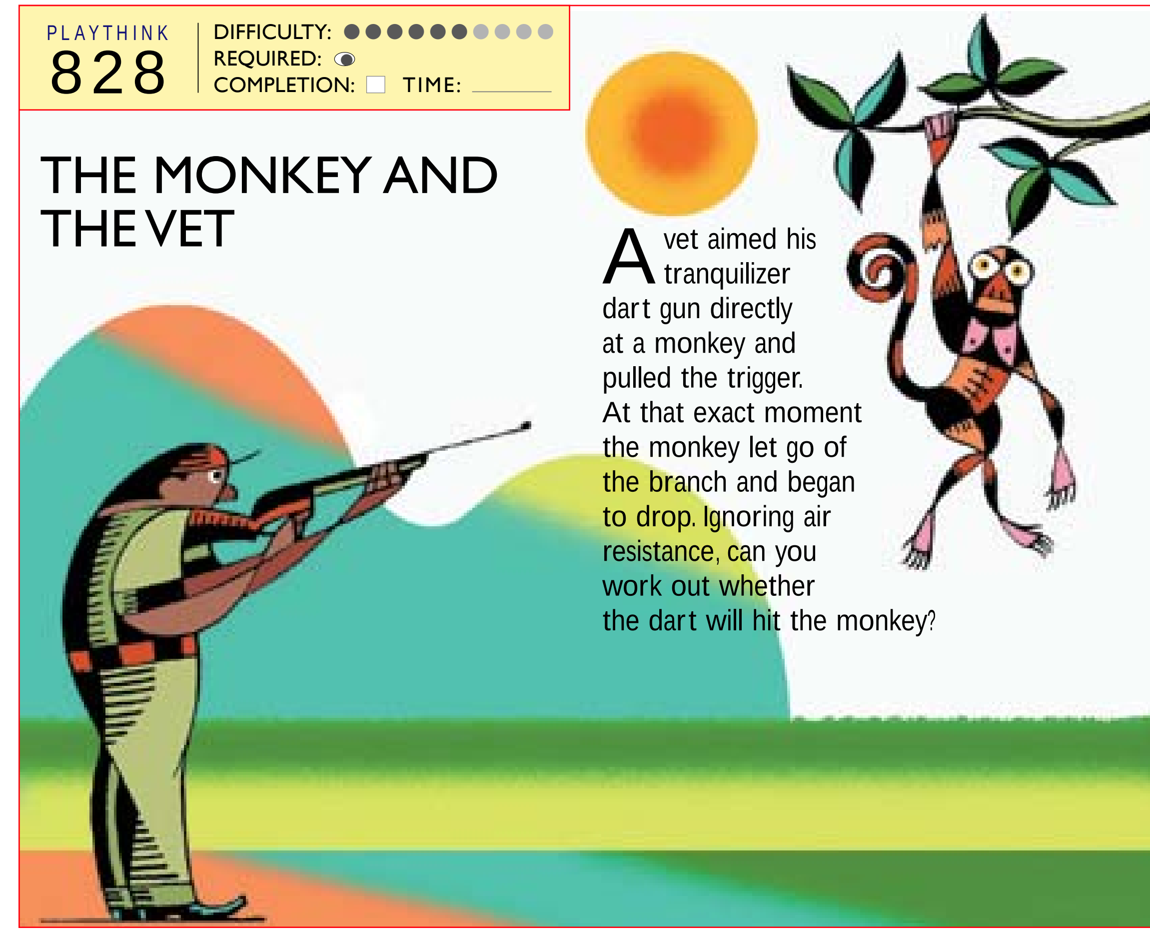
Can you work out how many revolutions it will take and what the secret word is?



PLAYTHINK 827 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ______

SATELLITE PRINCIPLE

Imagine you are standing on a 200-mile-high tower, far above the top of the atmosphere. If you throw a Frisbee hard enough, what will happen?



PLAYTHINK 828 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ______

THE MONKEY AND THE VET

A vet aimed his tranquilizer dart gun directly at a monkey and pulled the trigger. At that exact moment the monkey let go of the branch and began to drop. Ignoring air resistance, can you work out whether the dart will hit the monkey?

PLAYTHINK 829 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ______

JOGGING FLY

Every morning two joggers start out 10 kilometers apart, on either end of a trail. The moment the joggers start running toward the middle of the trail, a fly that sat on the head of one of the joggers flies straight toward the other; once the fly reaches the second jogger, it turns around and heads back toward the first. This back-and-forth flying continues until the two joggers meet.

If each jogger runs at a constant 5 kilometers an hour and the fly travels at 10 kilometers an hour, can you work out how many kilometers the fly covered before the joggers met?

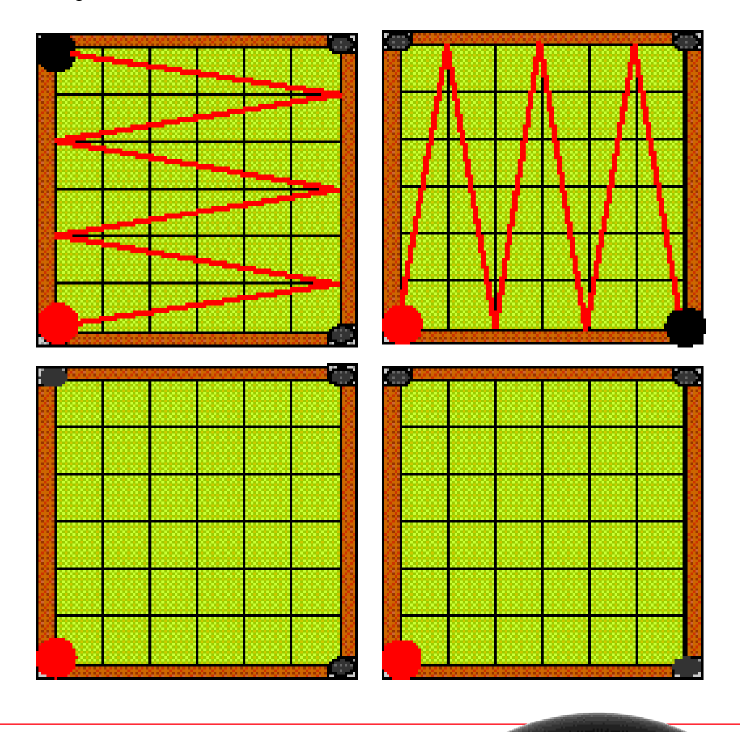
PLAYTHINK 830 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

ON THE REBOUND 1

You have cleared the pool table of all but your last ball and are on the verge of victory. To celebrate, you plan to sink the last ball in as complex a way as possible, with at least two bounces off the side cushions.

Figuring out where to aim the ball for such a complex trajectory is tricky work. It often helps to have a grid superimposed over the table; the lines can be used as aiming markers at the edge of the table, and the squares can help you measure the angles at which the ball strikes the cushion. (You know, after all, that the angle at which the ball strikes the cushion is identical to the angle at which it rebounds.)

Either of the paths shown below would be too easy—they use only two side cushions. Can you work out a path for the balls that would take it from the bottom left-hand corner, off three cushions, and into either the top left-hand pocket or the bottom right-hand pocket?

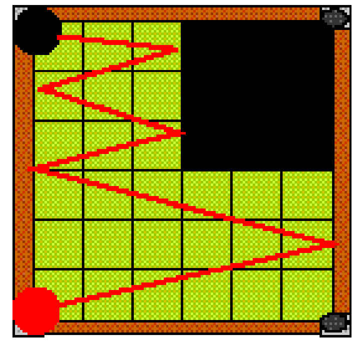


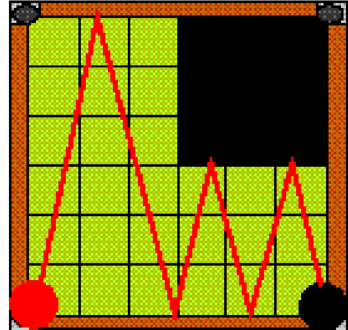
PLAYTHINK 831 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

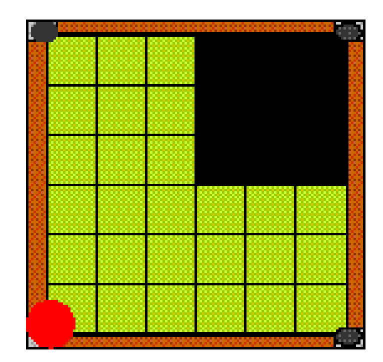
ON THE REBOUND 2

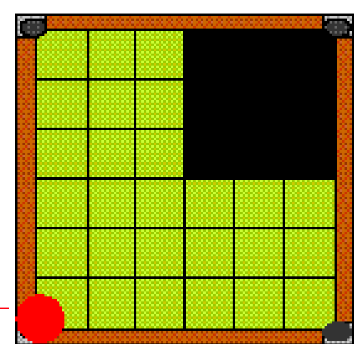
Playing pool on an L-shaped table is a challenge, but even then, sinking the ball from the lower left-hand corner to either the upper left or the lower right pocket is easy. The top two diagrams show how it can be done.

But to make things interesting, can you find a way to sink the ball into those pockets by bouncing it off at least four of the six sides? The ball should make five bounces before going into the upper left-hand pocket and seven bounces before going into the lower right-hand pocket.







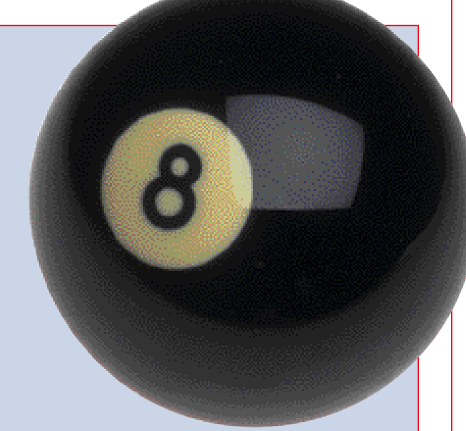


PLAYTHINK 832 | DIFFICULTY: ●●●●●●●●○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

1.

2.

3.



ON THE REBOUND 3

Perhaps you would like to try your luck with even more irregular tables. Starting with the ball in the lower left-hand corner, can you work out how to sink the ball in each instance? You must observe certain restrictions on each shot:

1. Three bounces, each on a different side
2. Seven bounces
3. Thirteen bounces and six different sides

The ball may travel as long as necessary to sink in the pocket.

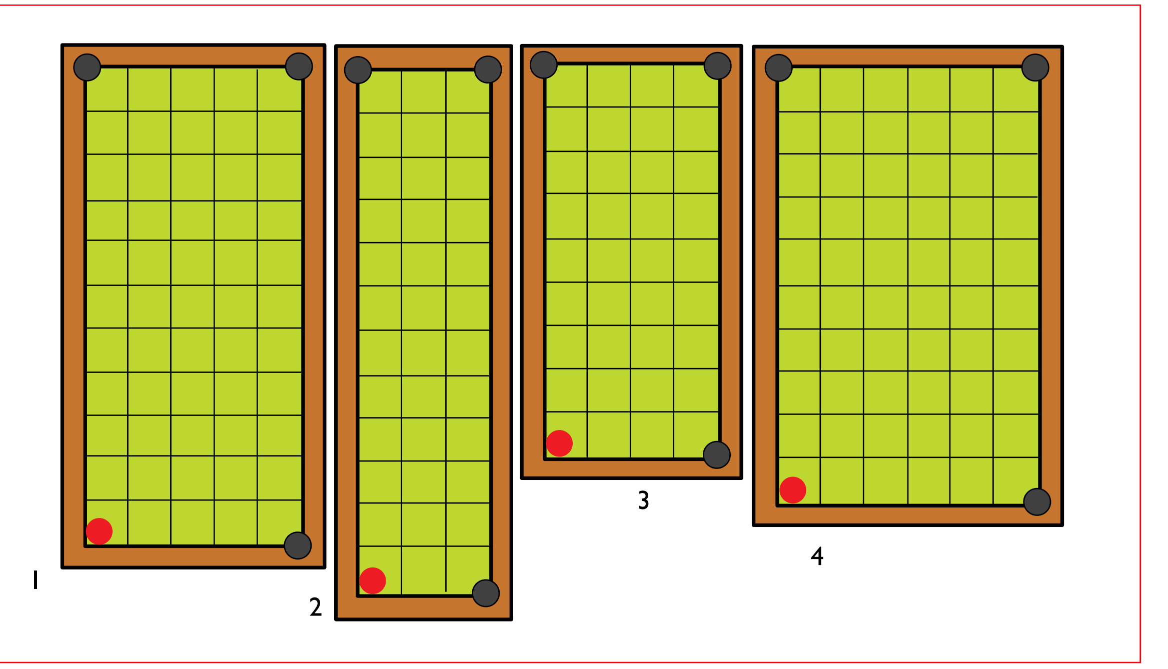
PLAYTHINK 833

DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

REFLECTED BALLS

When a ball hits a side cushion, it rebounds at the same angle at which it struck. With that knowledge, skilled billiard players know the exact path of a cue ball before they hit it.

A number of pool tables of different shapes and sizes are shown here. Can you trace the path of a ball in the lower left-hand corner that has been struck at a 45-degree angle? Can you predict which pocket the ball will land in, based on the dimensions of the individual table?

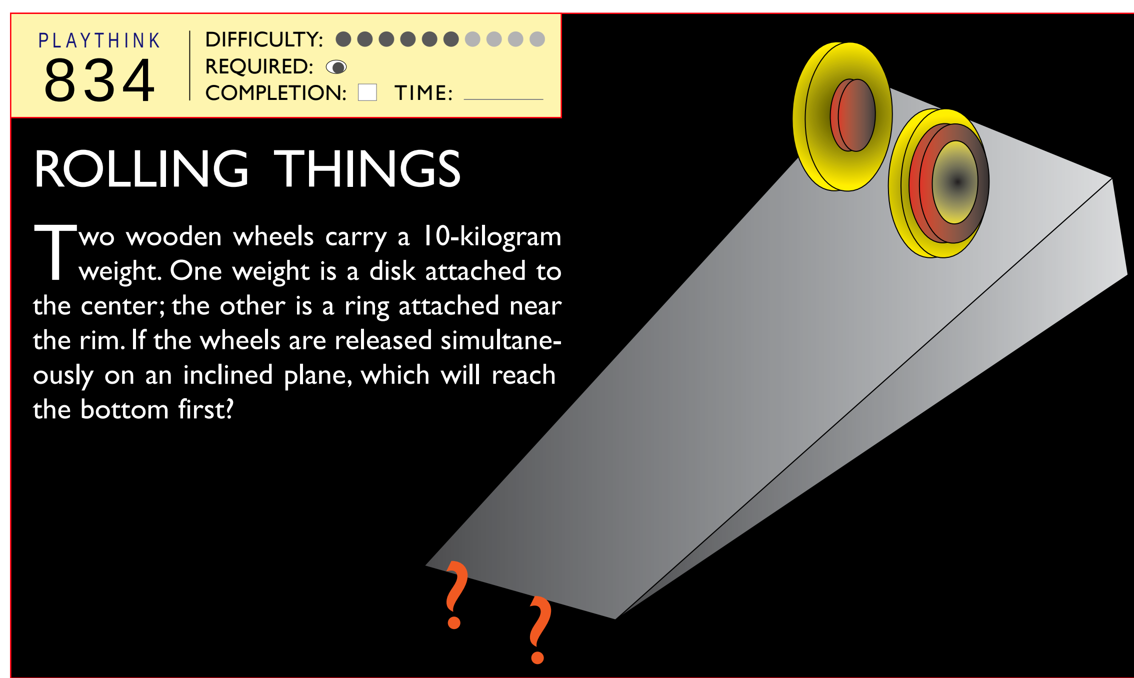


PLAYTHINK 834

DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

ROLLING THINGS

Two wooden wheels carry a 10-kilogram weight. One weight is a disk attached to the center; the other is a ring attached near the rim. If the wheels are released simultaneously on an inclined plane, which will reach the bottom first?

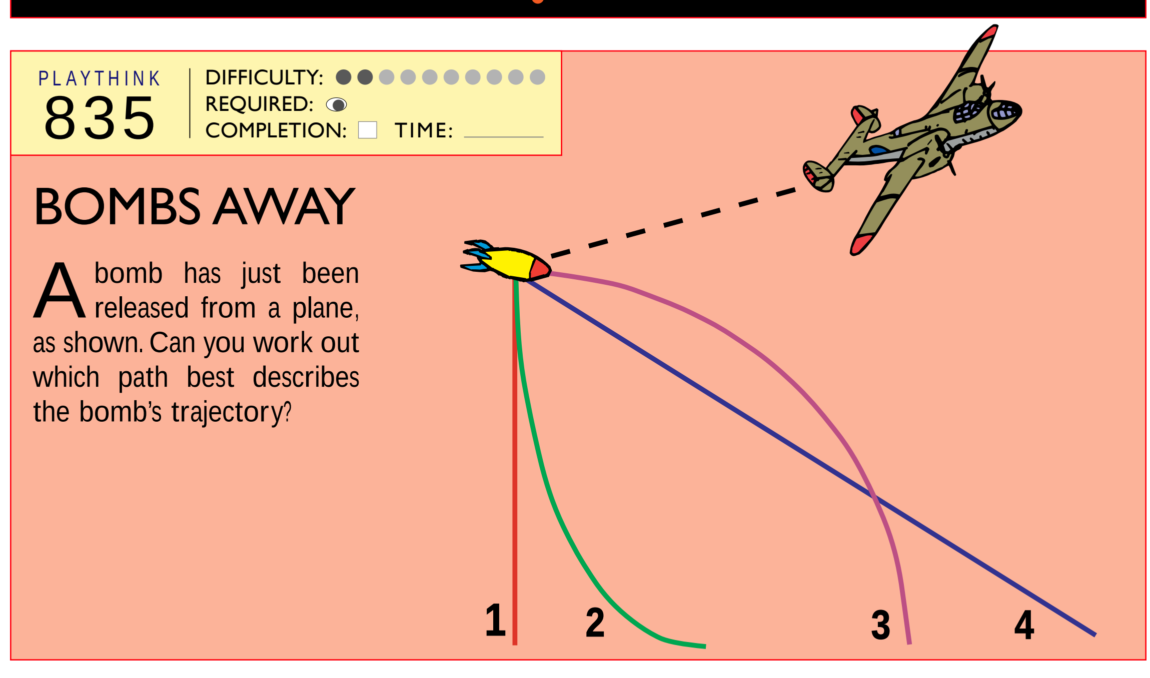


PLAYTHINK 835

DIFFICULTY: ●●○○○○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ____

BOMBS AWAY

A bomb has just been released from a plane, as shown. Can you work out which path best describes the bomb's trajectory?



PLAYTHINK 836

DIFFICULTY: ●●●●○○○○○○
REQUIRED:
COMPLETION: ☐ TIME: ____



WALKING THE DOG

During his daily walk, Mr. Smith exercises his dog, Punk, by throwing a Frisbee for him to retrieve. If Mr. Smith wants Punk to run as far as possible during the walk, in what direction should he throw the Frisbee?

PLAYTHINK 837 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

FOLDING LADDER

A folding ladder is placed on the floor with one leg supported by a stick, as shown. A bowling ball rests in the rungs near the end of the leg. A short distance away a bucket is firmly tied to the leg, and near the pivot a heavy weight rests on the leg. The idea behind the setup is a simple one:

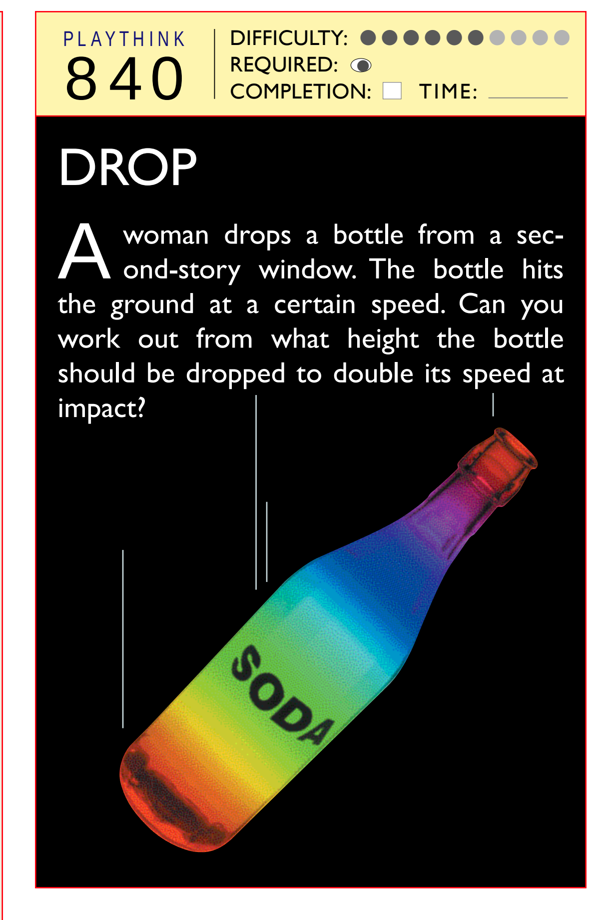
You pull the stick away, the ladder collapses, and the ball lands in the bucket.

Can such a trick work? Won't all the objects fall at the same rate?

PLAYTHINK 840 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

DROP

A woman drops a bottle from a second-story window. The bottle hits the ground at a certain speed. Can you work out from what height the bottle should be dropped to double its speed at impact?



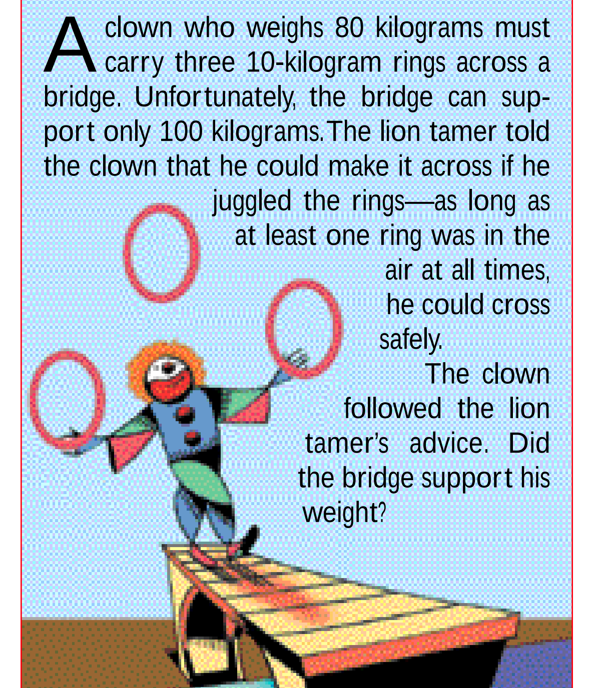
PLAYTHINK 841 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____



JUGGLER

A clown who weighs 80 kilograms must carry three 10-kilogram rings across a bridge. Unfortunately, the bridge can support only 100 kilograms. The lion tamer told the clown that he could make it across if he juggled the rings—as long as at least one ring was in the air at all times, he could cross safely.

The clown followed the lion tamer's advice. Did the bridge support his weight?



PLAYTHINK 838 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____



FROG IN THE WELL

A frog falls into the bottom of a 20-meter well. In its struggle to get out, the frog advances 3 meters up the slimy walls of the well; during the night when it rests, the frog slips back 2 meters.

Can you work out how many days it takes for the frog to escape?

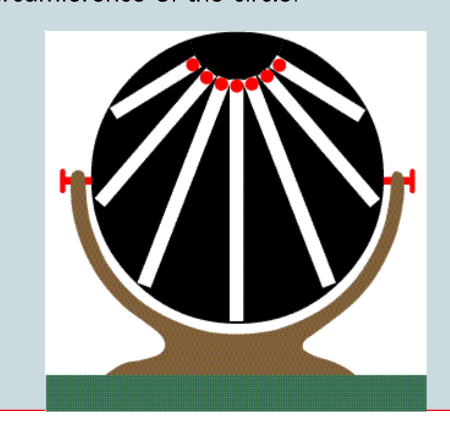
PLAYTHINK 839 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____



RADIAL DESCENT

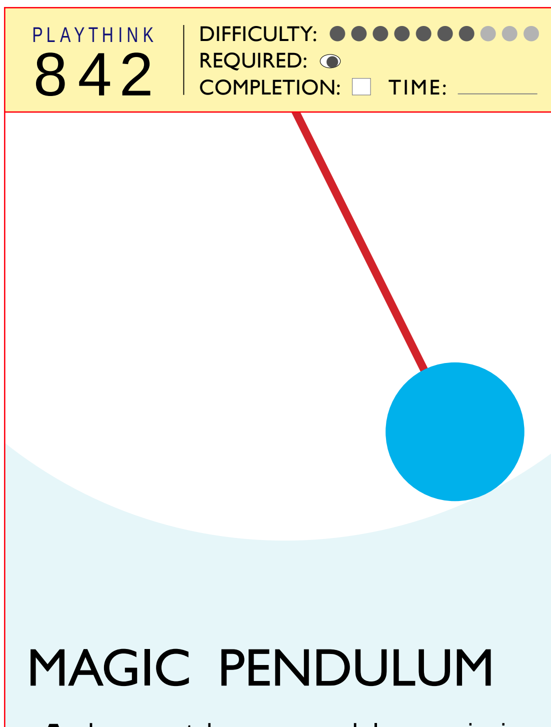
The diagram below shows an experimental device invented by Galileo, in which identical balls are released simultaneously at slanted angles along the chord of a circle. The device can be adjusted to any angle, from horizontal to vertical.

As each ball follows its track, can you work out which will be the first to reach the circumference of the circle?



PLAYTHINK 842

DIFFICULTY: REQUIRED: COMPLETION: TIME:



MAGIC PENDULUM

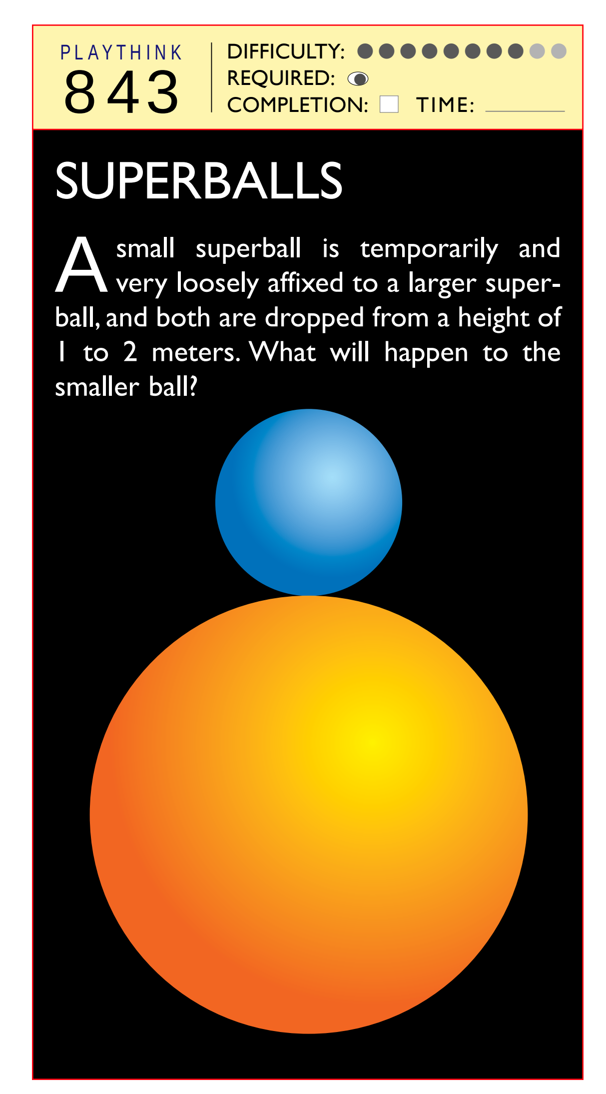
A boy watches a pendulum swinging through a plane. He is wearing a broken pair of sunglasses—the right lens is missing. Can you work out how he will perceive the motion of the pendulum?

PLAYTHINK 843

DIFFICULTY: REQUIRED: COMPLETION: TIME:

SUPERBALLS

A small superball is temporarily and very loosely affixed to a larger superball, and both are dropped from a height of 1 to 2 meters. What will happen to the smaller ball?



PLAYTHINK 844

DIFFICULTY: REQUIRED: COMPLETION: TIME:

FOUCAULT'S PENDULUM

Is it possible to watch the earth rotate?

One of the important properties of a pendulum is that once it is set in motion, it will continue to swing through the same plane unless a force acts on it. This is a property of inertia.

That fact became the basis of one of the most beautiful science demonstrations ever performed. The French physicist Jean-Bernard Foucault was invited to arrange a scientific exhibit as part of the Paris Exhibition in 1851. From the dome of the Pantheon, Foucault hung a pendulum consisting of 61 meters of piano wire and a 27-kilogram cannonball. On the floor below the ball, he sprinkled a layer of fine sand. A stylus fixed to the bottom of the ball traced the path in the sand, thus recording the movement of the pendulum.

At the end of an hour, the line in the sand had moved 11 degrees and 18 minutes.

If the pendulum stayed in the same plane, how could it trace different paths in the sand?

Jean-Bernard Foucault

PLAYTHINK 845

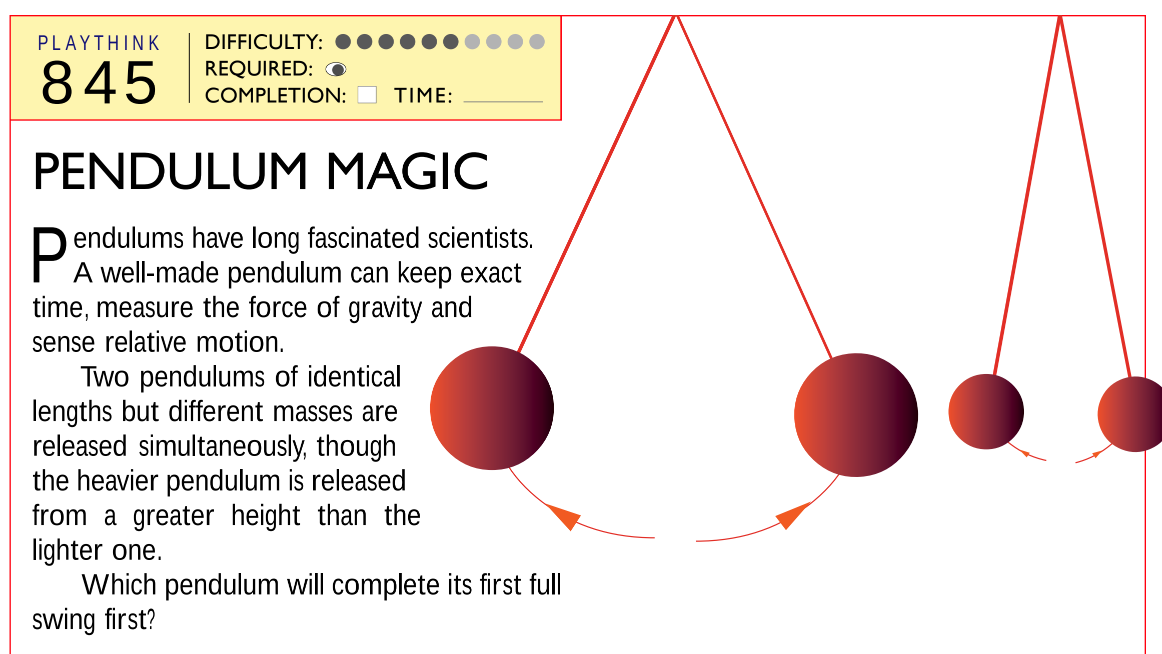
DIFFICULTY: REQUIRED: COMPLETION: TIME:

PENDULUM MAGIC

Pendulums have long fascinated scientists. A well-made pendulum can keep exact time, measure the force of gravity and sense relative motion.

Two pendulums of identical lengths but different masses are released simultaneously, though the heavier pendulum is released from a greater height than the lighter one.

Which pendulum will complete its first full swing first?

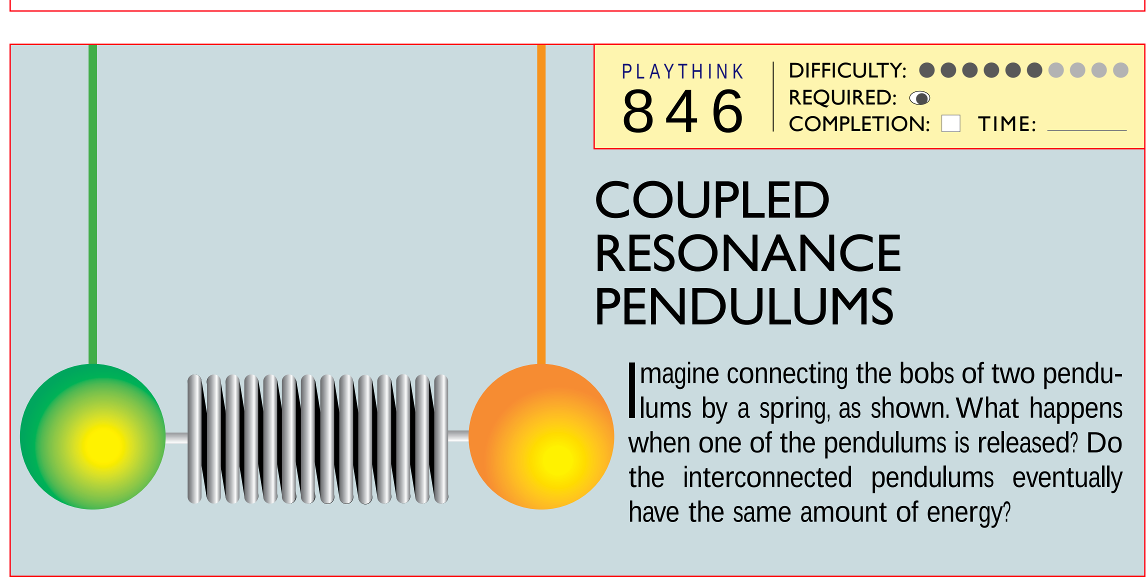


PLAYTHINK 846

DIFFICULTY: REQUIRED: COMPLETION: TIME:

COUPLED RESONANCE PENDULUMS

Imagine connecting the bobs of two pendulums by a spring, as shown. What happens when one of the pendulums is released? Do the interconnected pendulums eventually have the same amount of energy?



PLAYTHINK 847 | DIFFICULTY: ●●●●●●●●○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

PECKING WOODPECKER

You may have se variation of toy. Begin w the woodpe at the top o the rod. If y lift back the woodpecke and release the woodpe will peck th rod and slo descend to bottom. Ca you explain behavior?

PLAYTHINK 848 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

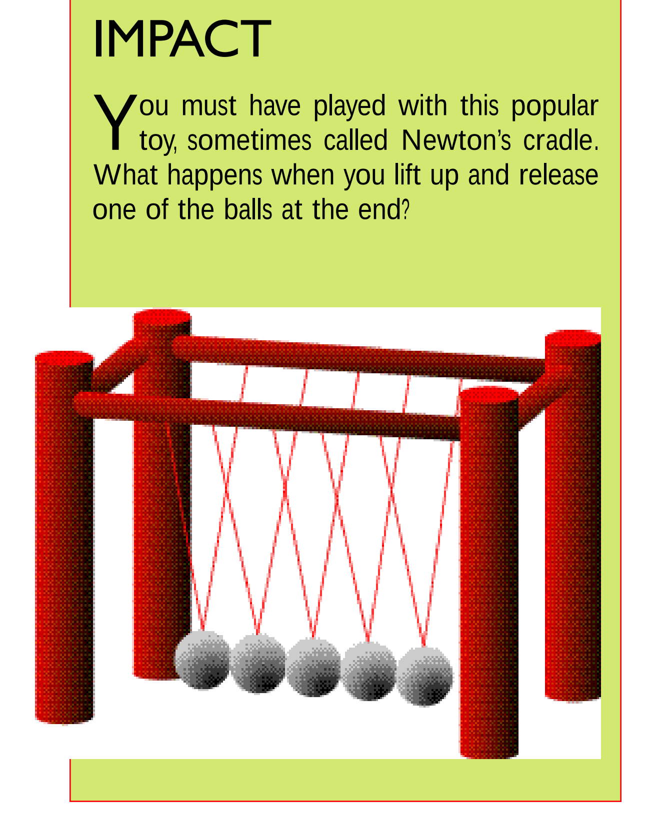
CIRCLING WEIGHT

A ball on a string is swung in a circle at constant speed. Do the ball's velocity and acceleration stay the same? Can you work out what would happen to the ball if the string suddenly snapped?

PLAYTHINK 849 | DIFFICULTY: ●●●●●●●●○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

IMPACT

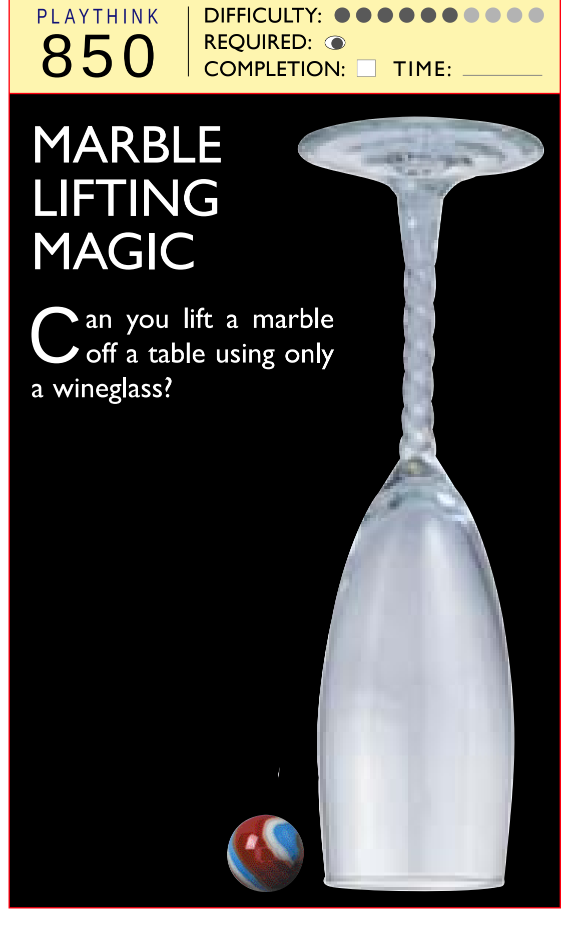
You must have played with this popular toy, sometimes called Newton's cradle. What happens when you lift up and release one of the balls at the end?



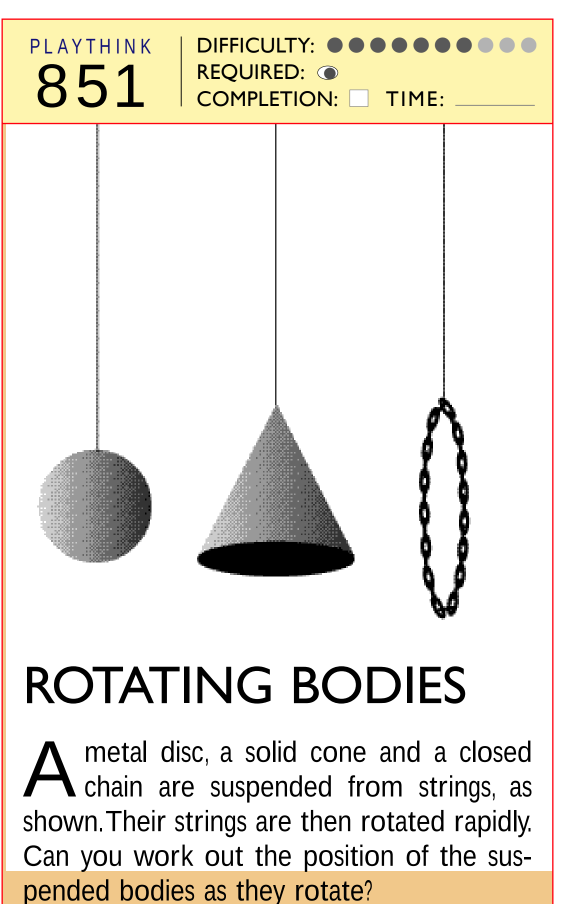
PLAYTHINK 850 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

MARBLE LIFTING MAGIC

Can you lift a marble off a table using only a wineglass?



PLAYTHINK 851 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____



ROTATING BODIES

A metal disc, a solid cone and a closed chain are suspended from strings, as shown. Their strings are then rotated rapidly. Can you work out the position of the suspended bodies as they rotate?

Gyroscopes

Bicycle wheels, Frisbees, yo-yos, tops—all these spinning objects illustrate the curious properties of the gyroscope, as does any solid object rotating about a fixed point.

A gyroscope has a certain rotational momentum that depends on its mass, on the square of the distance of the individual particles of mass to the axis of rotation, and on the speed of rotation (properties we understand courtesy of Newton's laws of motion). To increase the rotational momentum, a gyroscope can be designed as a disc with a thickened rim; that will concentrate most of its mass as far from the axis of rotation as possible.

The most significant feature of a gyroscope is the way it conserves its momentum and the direction of its rotational axis. As long as no external forces act upon the gyroscope, it will keep the direction of its axis constant in space. It can be used, therefore, to stabilize movement, as well as to measure the change of orientation in three-dimensional space.

PLAYTHINK 852 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

HUMAN GYRO 1

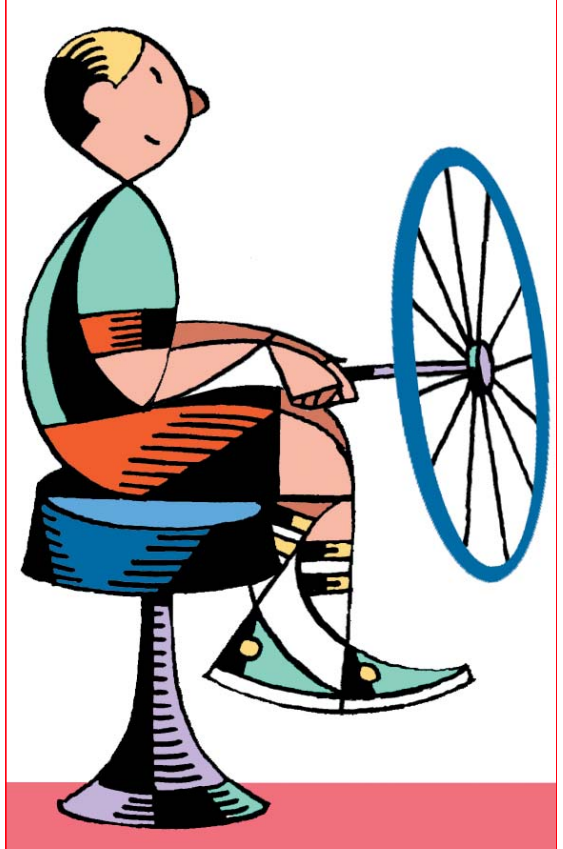
Can you work out what will happen when a boy who sits on a freely rotating stool holds a spinning bicycle tire as shown?



PLAYTHINK 853 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

HUMAN GYRO 2

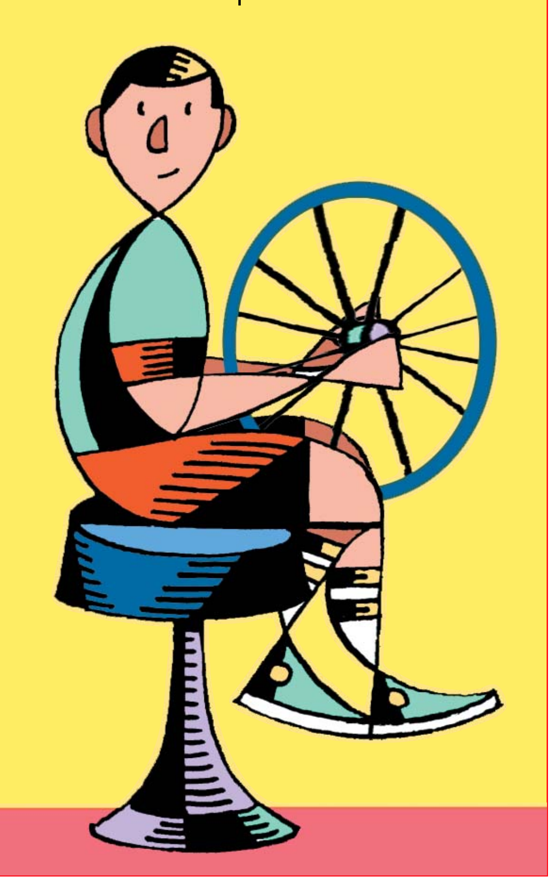
Can you work out what will happen when a boy who sits on a freely rotating stool holds a spinning bicycle tire as shown?



PLAYTHINK 854 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

HUMAN GYRO 3

A boy sitting on a freely rotating stool holds a spinning bicycle tire vertically with both hands as shown. Can you work out what he should do so that his stool will begin to turn left? Will pushing the handle forward with his right hand and backward with his left accomplish this?



PLAYTHINK 855 DIFFICULTY: ●●●●●●●○○○ REQUIRED: 👁 COMPLETION: ☐ TIME: ____

CENTRIPETAL FORCE



Rotating rides, such as the rotating vertical cylinder shown here, are popular at carnivals. Riders stand with their backs to the wall as the cylinder begins to spin. When the maximum spin rate is reached, the floor drops away. Amazingly, the riders remain stuck to the wall.

Can you work out why this occurs?

PLAYTHINK 857 DIFFICULTY: ●●●●●●●○○○ REQUIRED: 👁 COMPLETION: ☐ TIME: ____

ICE SKATING

A figure skater spins on the ice with her arms held wide open. What happens when she brings her hands to her chest?

PLAYTHINK 856 DIFFICULTY: ●●●●●●●○○○ REQUIRED: 👁 COMPLETION: ☐ TIME: ____

GOLF BALLS

Why does a golf ball have a dimpled surface?

PLAYTHINK 858 DIFFICULTY: ●●●●○○○○○○ REQUIRED: 👁 COMPLETION: ☐ TIME: ____

BALL GAME CAROUSEL

Two jugglers stand on a rapidly rotating carousel, and one throws a ball straight to the other. Can you work out the ball's trajectory and show where it will land?



Branched Structures

Whenever one area has an advantage over adjacent areas when it comes to getting more matter, heat, light, or some other requisite for growth, the resultant structure shows signs of the growth of isolated individual sections feeding into a branched form. This is best illustrated by the common tree or a riverbed, but is also found in electric discharges, corrosion and crystal growth.

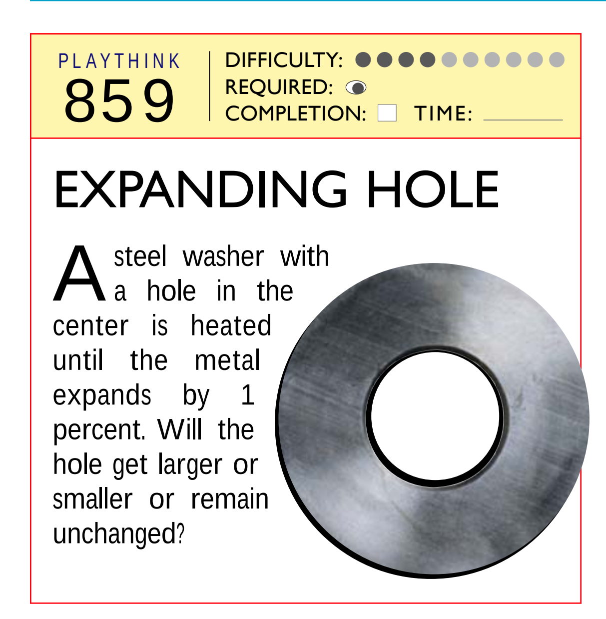
All such structures start from a point and grow linearly, but they eventually stop as the branches interfere with others already present.

Trees and lungs and river deltas all have the same principle: distribution. And they have all produced the same solution: branching.

PLAYTHINK 859 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ | TIME: ____

EXPANDING HOLE

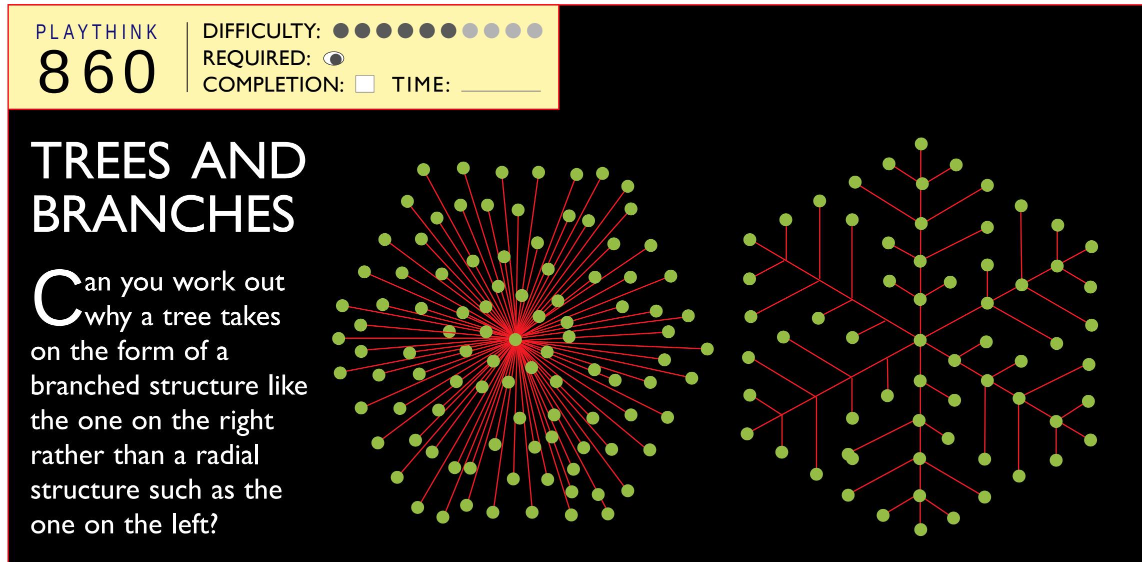
A steel washer with a hole in the center is heated until the metal expands by 1 percent. Will the hole get larger or smaller or remain unchanged?



PLAYTHINK 860 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ | TIME: ____

TREES AND BRANCHES

Can you work out why a tree takes on the form of a branched structure like the one on the right rather than a radial structure such as the one on the left?

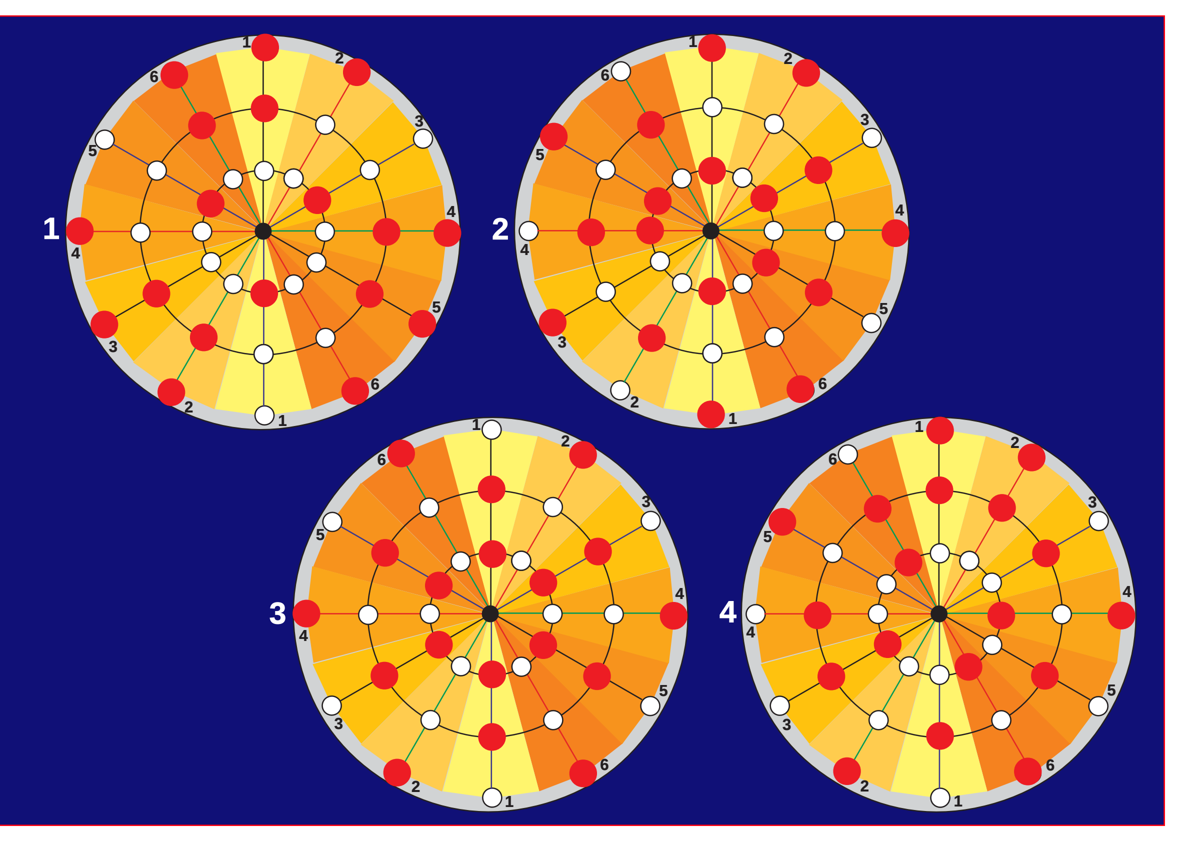


PLAYTHINK 861 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ | TIME: ____

BALANCING PLATFORM

At many hands-on-science exhibits, you can find balancing platforms that pivot at their centers. The idea is for groups of people to position themselves so that the platforms stay in a balanced position of equilibrium.

Imagine people of uniform weight as red circles distributed in four different configurations on the balancing platform, as shown. Can you work out which arrangements are in equilibrium?



Cracks and Dried Mud

Cracking is inevitably sequential rather than simultaneous. As a result, when a crack is formed, it will typically join an existing crack by forming the ubiquitous three-rayed intersection. The formation of a four-rayed intersection is highly unlikely, though not impossible, since it is improbable that two new cracks would intersect an existing crack from opposite sides at exactly the same point.

It is often possible to determine which of the two lines appeared earlier: the older of the two cracks passes right through the point of junction. Thus, we can follow the splits and eventually find the beginning of the whole system of cracks.

Bubbles and rocks, different as they may seem, break up according to the same principles. Since both are elastic, they divide into segments that meet at 120-degree angles.

When a material is inelastic, like the glaze on a bowl, it cracks first along lines that intersect at right angles. Then, when tension is reduced and elasticity restored, secondary cracks occur, as they do in mud or rock, along lines that run at 120-degree angles.

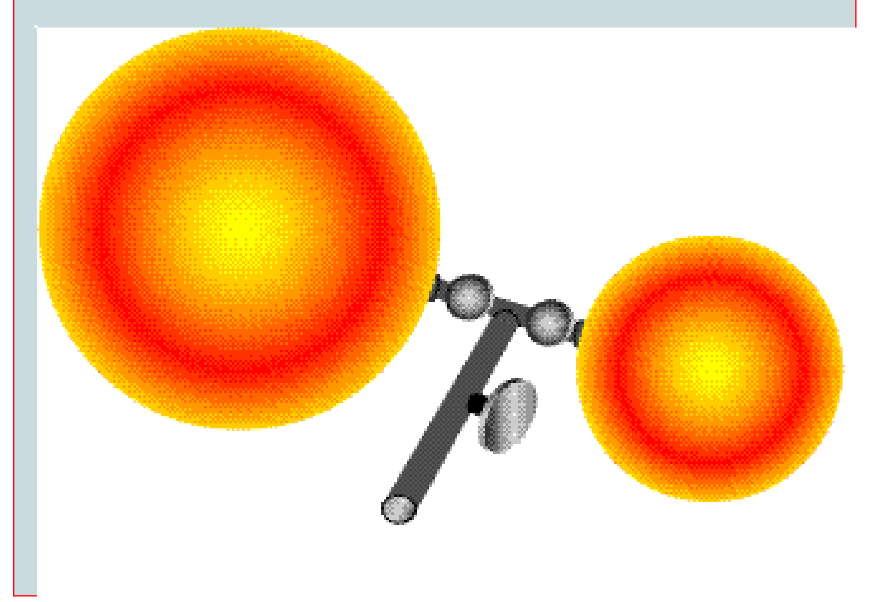
The patterns of sun-dried mud seem to be quite irregular; nevertheless, they show right angles. This can be explained by assuming that the breaking up of a layer of mud is an effect of contraction: the crack has to follow the line of least work. Because work is proportional to the areas of the sections, the lines must minimize the surfaces laid open by the fissure. The lines will be at right angles if the mud is homogenous. Variations in the thickness of the layer account for the curvature of the lines.

PLAYTHINK 862 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

SOAP BUBBLES

Two soap bubbles of different sizes are blown up in succession. The inlet between the two is closed while they are being blown up. Then the outside inlet is sealed and the passage between the two bubbles is opened.

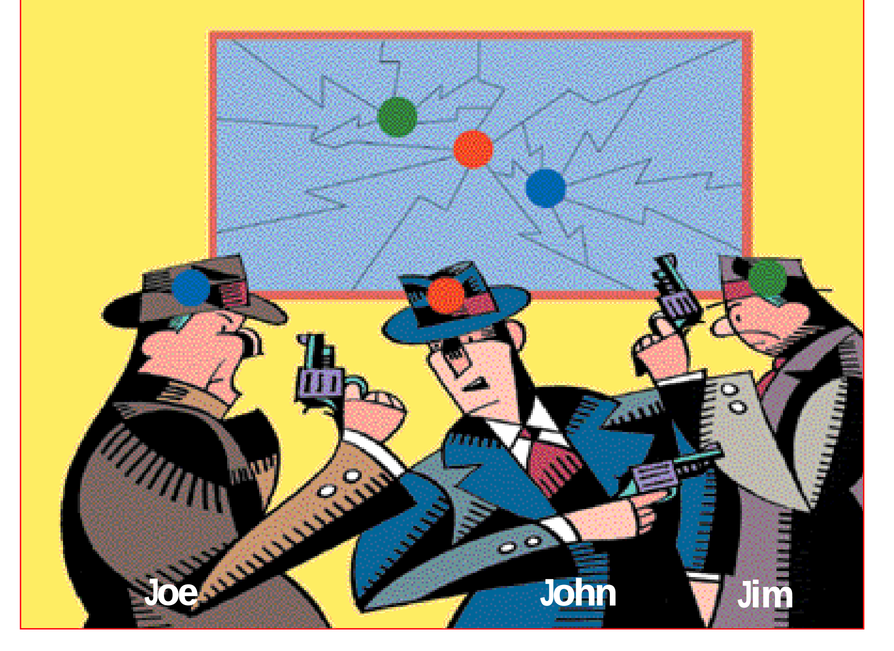
Can you work out what will happen? Will the smaller bubble grow until the two are of equal size?



PLAYTHINK 863 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: | COMPLETION: ☐ TIME: ____

WHO FIRED THE FIRST SHOT?

Look at the scene as a detective would: The three men each fired a shot. The holes from their shots match the colored dots on their hats. From this information, can you work out who fired the first shot—Joe, John or Jim?

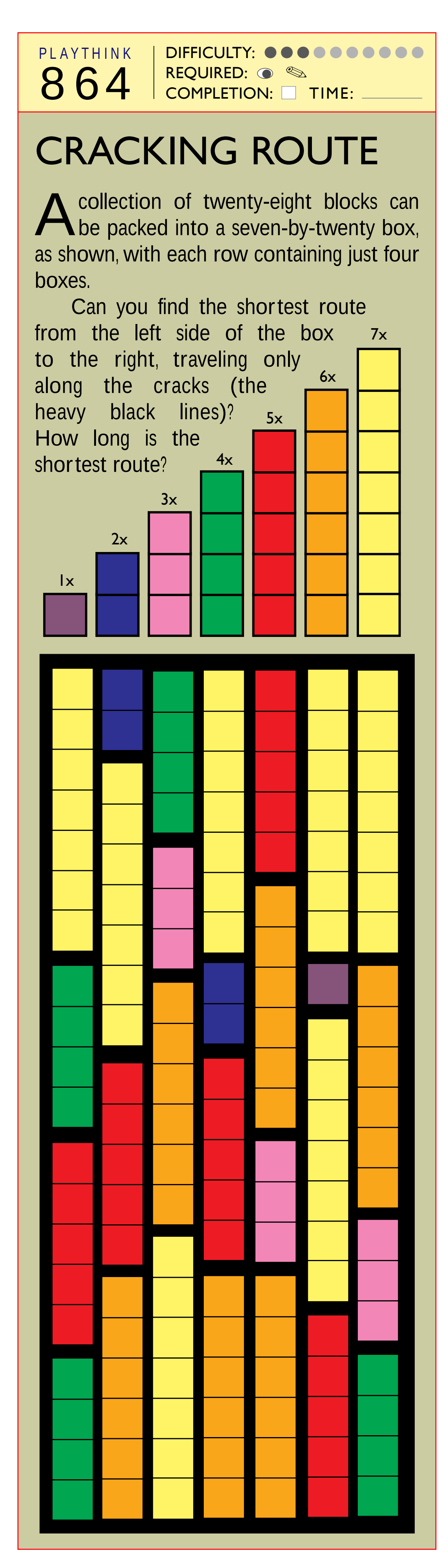


PLAYTHINK 864 | DIFFICULTY: ●●●○○○○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ______

CRACKING ROUTE

A collection of twenty-eight blocks can be packed into a seven-by-twenty box, as shown, with each row containing just four boxes.

Can you find the shortest route from the left side of the box to the right, traveling only along the cracks (the heavy black lines)? How long is the shortest route?



PLAYTHINK 865 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ______

AIR RESISTANCE

Place a long, thin strip of wood on a table so that about 10 centimeters extend beyond the edge. Lay a few sheets of newspaper over the strip and smooth the paper down to press out all the air from under the newsprint.

Then strike the extended end of the strip. Can you guess what will happen?



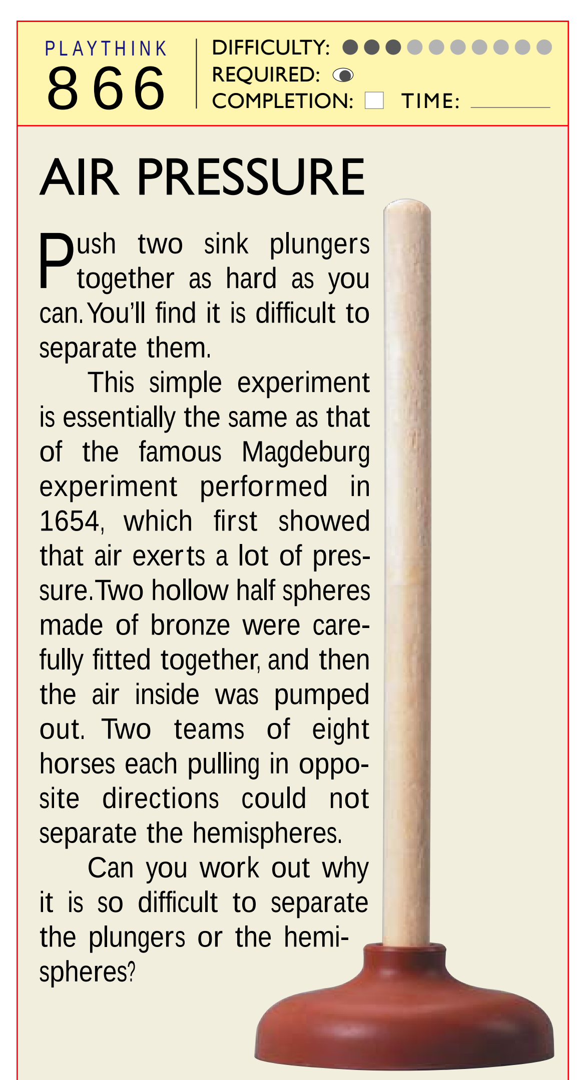
PLAYTHINK 866 | DIFFICULTY: ●●●○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ______

AIR PRESSURE

Push two sink plungers together as hard as you can. You'll find it is difficult to separate them.

This simple experiment is essentially the same as that of the famous Magdeburg experiment performed in 1654, which first showed that air exerts a lot of pressure. Two hollow half spheres made of bronze were carefully fitted together, and then the air inside was pumped out. Two teams of eight horses each pulling in opposite directions could not separate the hemispheres.

Can you work out why it is so difficult to separate the plungers or the hemispheres?

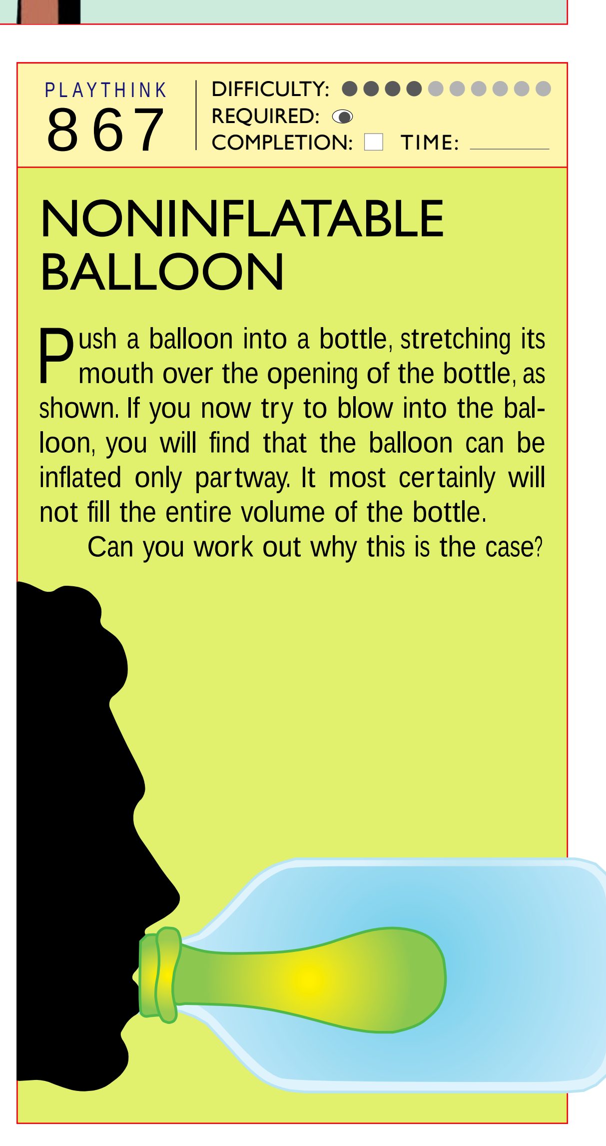


PLAYTHINK 867 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ______

NONINFLATABLE BALLOON

Push a balloon into a bottle, stretching its mouth over the opening of the bottle, as shown. If you now try to blow into the balloon, you will find that the balloon can be inflated only partway. It most certainly will not fill the entire volume of the bottle.

Can you work out why this is the case?



PLAYTHINK 868 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____



TRAIN DANGER

Why is it especially dangerous to stand too near the edge of a platform when an express train passes through a station at high speed?

PLAYTHINK 869 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

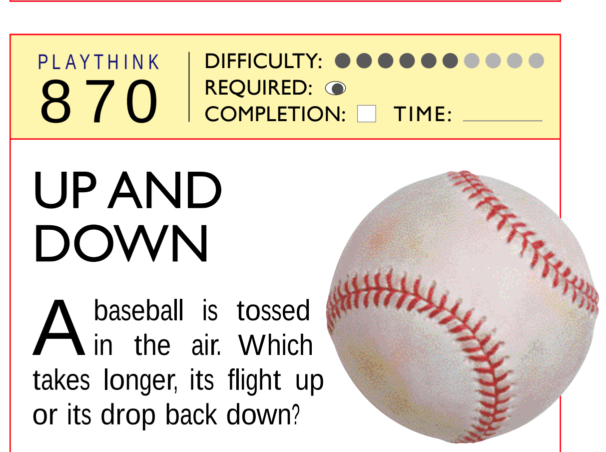


BERNOULLI'S SURPRISE

Two lightweight beach balls are suspended a short distance from each other, as shown. Can you guess what will happen if you blow air between the two balls?

PLAYTHINK 870 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

UP AND DOWN



A baseball is tossed in the air. Which takes longer, its flight up or its drop back down?

Fluid Mechanics

Why do most high-speed airplanes have the same general look? It's because they are all subject to the same sorts of intense forces, and the common design is the one that best accommodates them. The designs of aircraft, rockets and ship hulls are based on the principles of fluid mechanics—principles that also help to explain the circulation of blood, meteorology and oceanography.

The general term *fluid* includes any substance that has no rigidity. Fluids have no definite length or shape—they assume the shape of the vessel that contains them. Thus, both liquids and gases are considered fluids. A distinction can be made between the two: a liquid has a surface and thus a definite volume, while gases have no such volume and expand to fill a container of any size.

The motion of fluids is very complex, which is why engineers need wind tunnels and computer simulations to help them design the most efficient shapes for aircraft and automobiles. The evolution of the scientific understanding of fluid mechanics can be seen in the design of automobiles over the decades: boxy shapes have given way to streamlined silhouettes as ignorance has given way to knowledge.

PLAYTHINK 871 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____



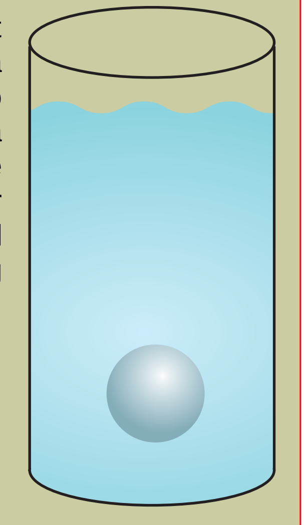
AIRPLANE FLIGHT

Why is the top of an airplane wing curved?

PLAYTHINK 872 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

ASCENDING BALL

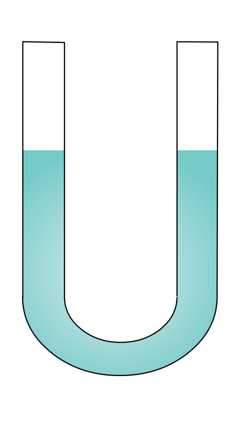
Will the time it takes for a Ping-Pong ball to rise to the top of a cylinder of water be different if the water in the cylinder is still or if it is swirling around?



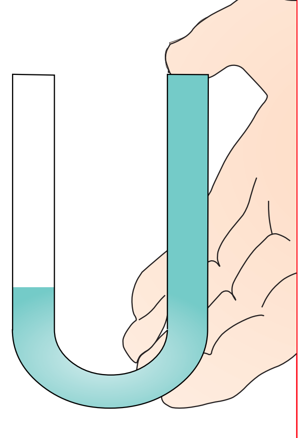
PLAYTHINK 873 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

U-TUBE

Pour water into a transparent U-shaped tube, as shown. Put your thumb on one end of the tube, then carefully tilt the tube back until the water touches your thumb. Press your thumb over the end to make a tight seal.



When you return the tube to an upright position, the water will remain touching your thumb. The water level will be unbalanced, as shown in the illustration. Can you explain what causes the water level to be unbalanced?

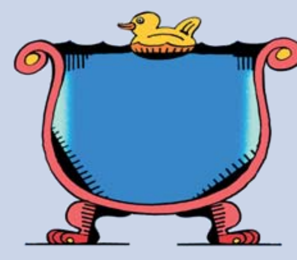


PLAYTHINK 874 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

BATH

Imagine you are in a bathtub checking to see how much weight your toy duck can carry before it sinks. You place a heavy metal ring on the duck, and it doesn't sink. Then the ring slips off and falls to the bottom of the tub.

When the ring falls, does the water level in the tub go up, go down or stay the same?



PLAYTHINK 875 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

AIR JET

Put a Ping-Pong ball inside a small funnel. Then tip your head back and blow as hard as you can. Rather than being blown to the ceiling, the ball remains suspended in the air. The harder you blow, the higher it will float above the funnel. Can you work out what causes such strange behavior?



PLAYTHINK 876 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

BLOWING CANDLES

What happens when you blow between two burning candles?



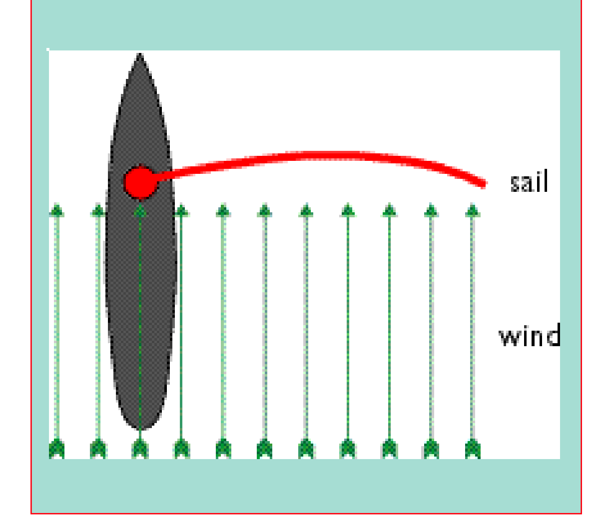


PLAYTHINK 877

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

SAILING 1

Suppose you are sailing directly downwind in a 40-kilometer-per-hour wind. If your sail makes a 90-degree angle with the keel of the boat, what is the fastest speed you can achieve?

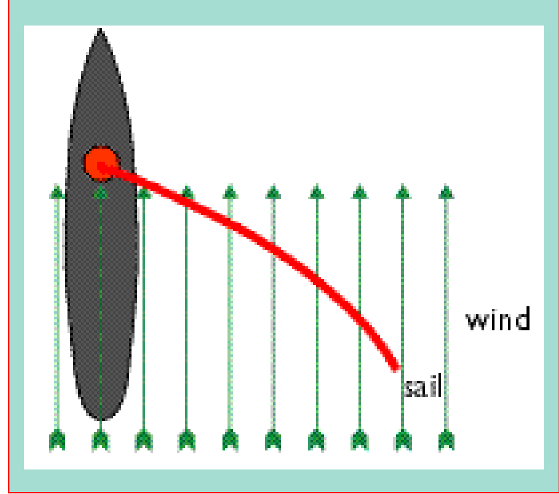


PLAYTHINK 878

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

SAILING 2

Suppose you are sailing directly downwind in a 40-kilometer-per-hour wind. If your sail makes less than a 90-degree angle with the keel of the boat, what is the fastest speed you can achieve?

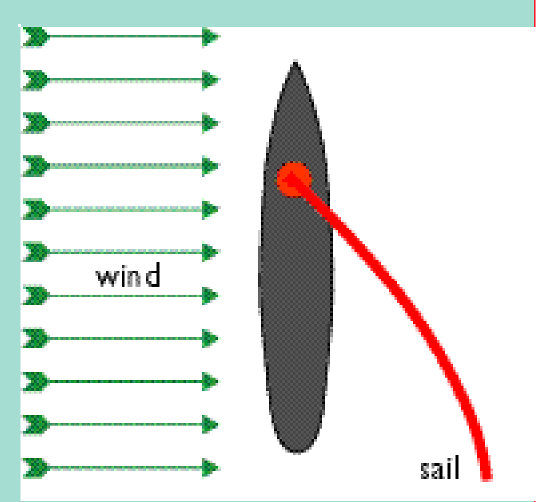


PLAYTHINK 879

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

SAILING 3

Suppose you are sailing exactly across a 40-kilometer-per-hour wind. If your sail makes less than a 90-degree angle with the keel of the boat, will you sail faster or slower than you would in a tailwind?



PLAYTHINK 880

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

SAILING 4

Which of the four boats moves forward with the greatest speed?

1 2 3 4

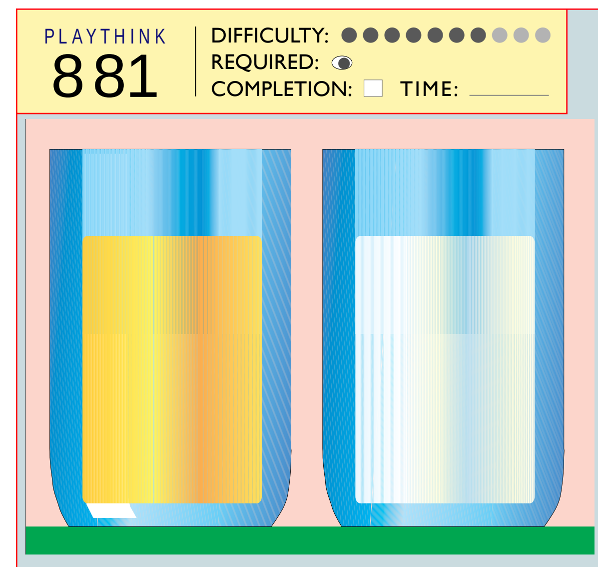
PLAYTHINK 881

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

TEA WITH MILK

You have two glasses, one exactly half full of tea, the other exactly half full of milk. Take a teaspoon of milk from the glass and stir it into the tea. Then take a teaspoon of the tea-milk mixture and stir that into the glass with the milk.

Can you tell whether there is more milk in the tea than there is tea in the milk? Or is there more tea in the milk than milk in the tea?



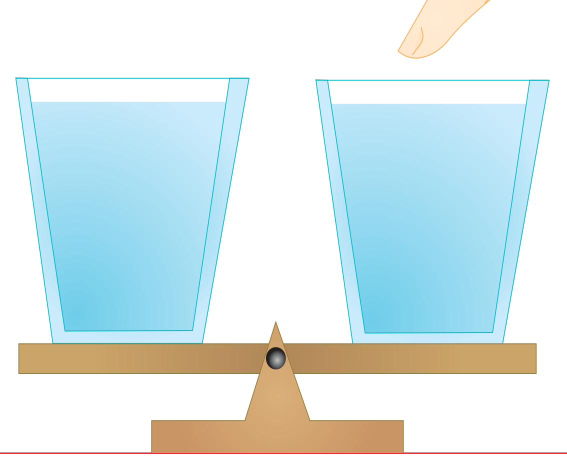
PLAYTHINK 882

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

FINGER IN THE GLASS

Two glasses filled with water are balanced on a scale, as shown. What happens to the scale when you stick your finger in one of the glasses? Will that side of the balance tip, as if it were heavier?

How would the result change if your finger were made of heavy metal?

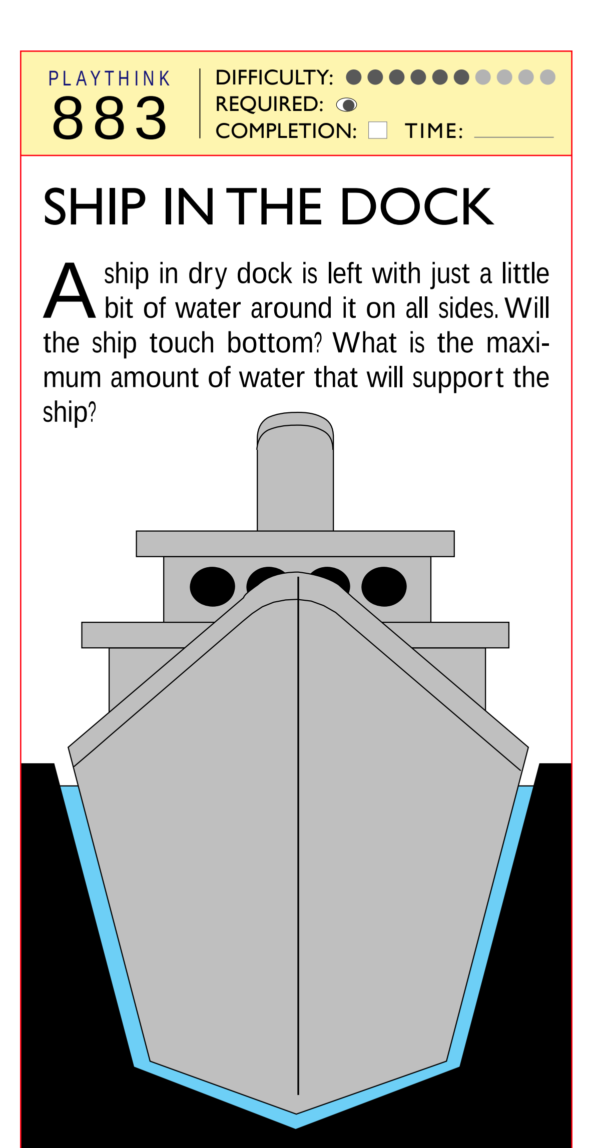


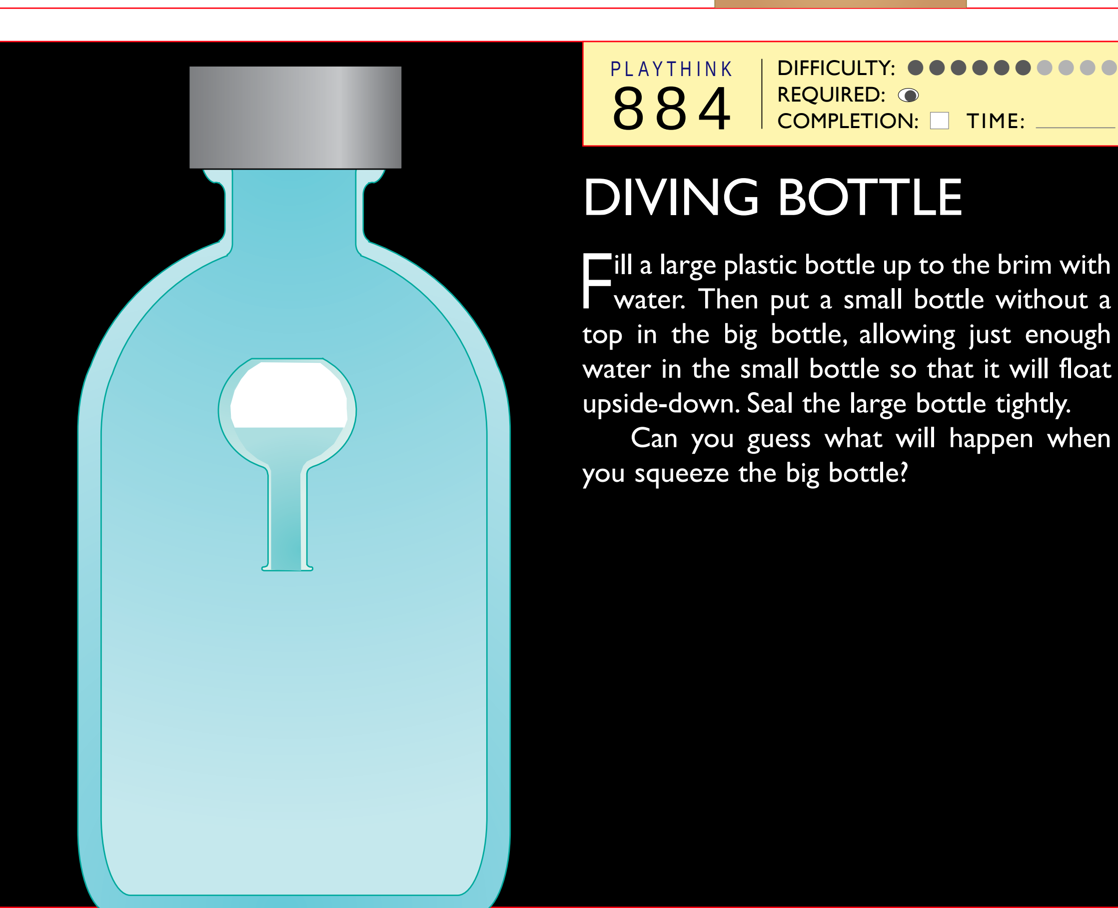
PLAYTHINK 883

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

SHIP IN THE DOCK

A ship in dry dock is left with just a little bit of water around it on all sides. Will the ship touch bottom? What is the maximum amount of water that will support the ship?





PLAYTHINK 884

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

DIVING BOTTLE

Fill a large plastic bottle up to the brim with water. Then put a small bottle without a top in the big bottle, allowing just enough water in the small bottle so that it will float upside-down. Seal the large bottle tightly.

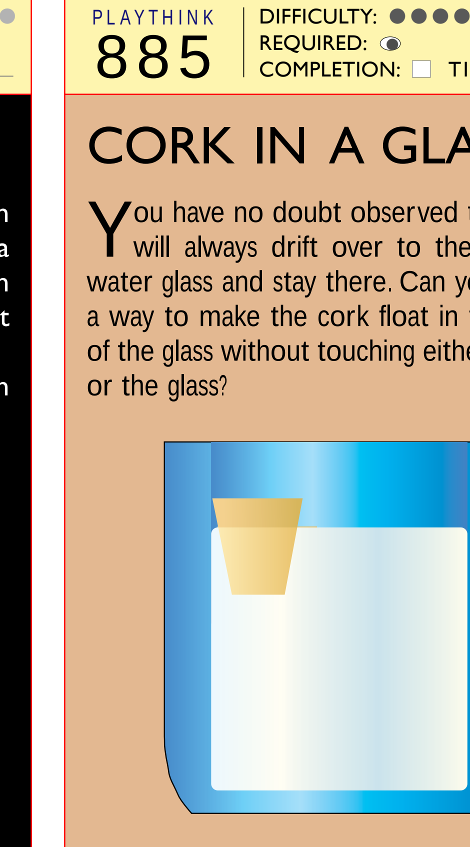
Can you guess what will happen when you squeeze the big bottle?

PLAYTHINK 885

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

CORK IN A GLASS

You have no doubt observed that a cork will always drift over to the side of a water glass and stay there. Can you think of a way to make the cork float in the middle of the glass without touching either the cork or the glass?

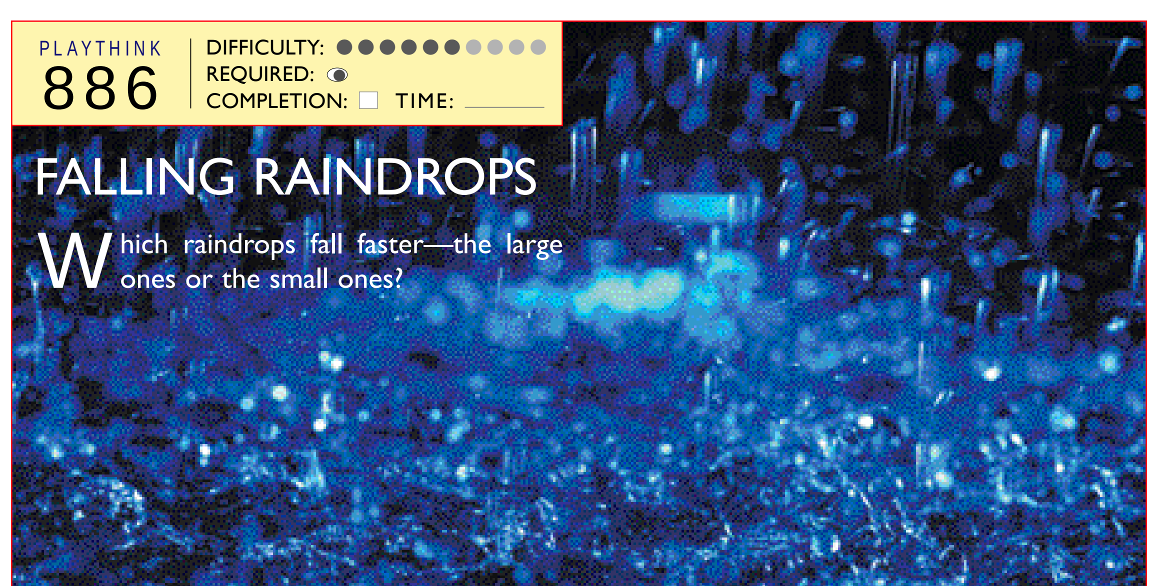


Surface Tension

Why are soap bubbles round? For the same reason that most drops of water are round. Molecules far from the surface of a liquid can be attracted evenly in all directions, but the ones near the surface will be pulled back into the liquid by other molecules. This attraction creates a tendency to minimize the surface area, which becomes as small as possible and behaves like an elastic film. This is surface tension.

Soap has the tendency to reduce the surface tension of water, which is the reason it can pull molecules from a body of water to create soap bubbles and films. When they form, both soap bubbles and drops of liquid contract into the shape that has the least surface area—a sphere, since it is the geometric solid that has the least surface for the same volume.

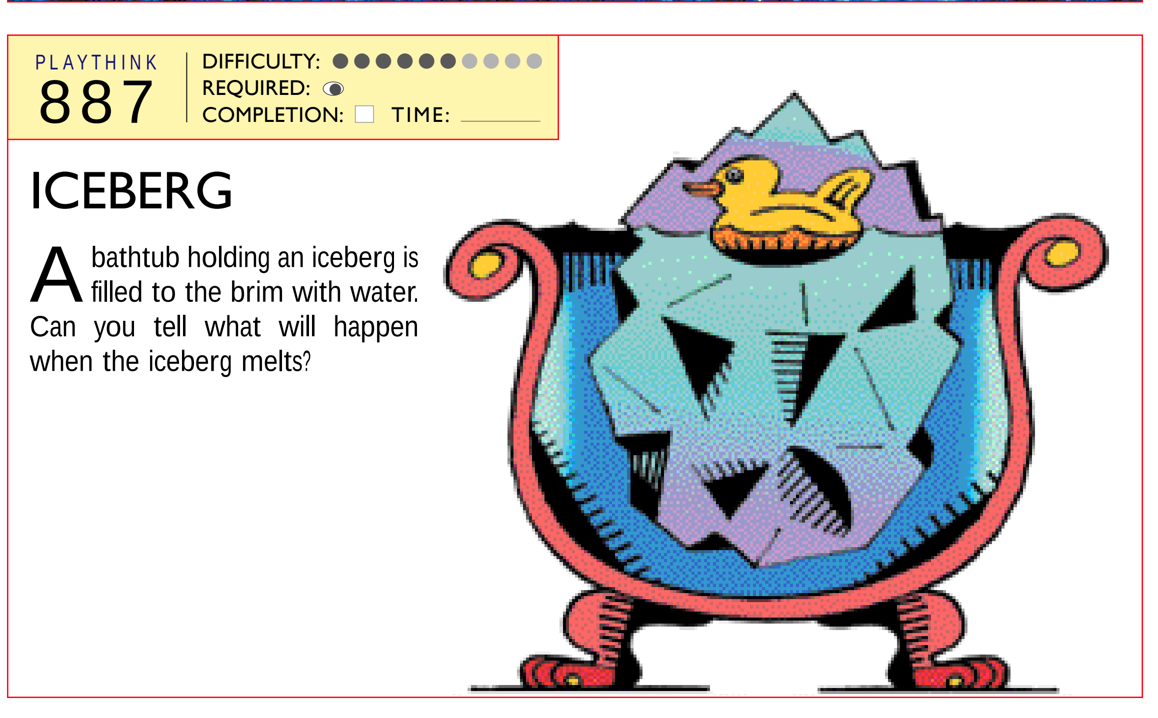
Surface tension is not the same for all liquids: the force is much stronger in water than in oil. The surface tension of mercury, on the other hand, is about seven times stronger than that of water. That's why spilled mercury forms spherical beads on a tabletop.



PLAYTHINK 886 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

FALLING RAINDROPS

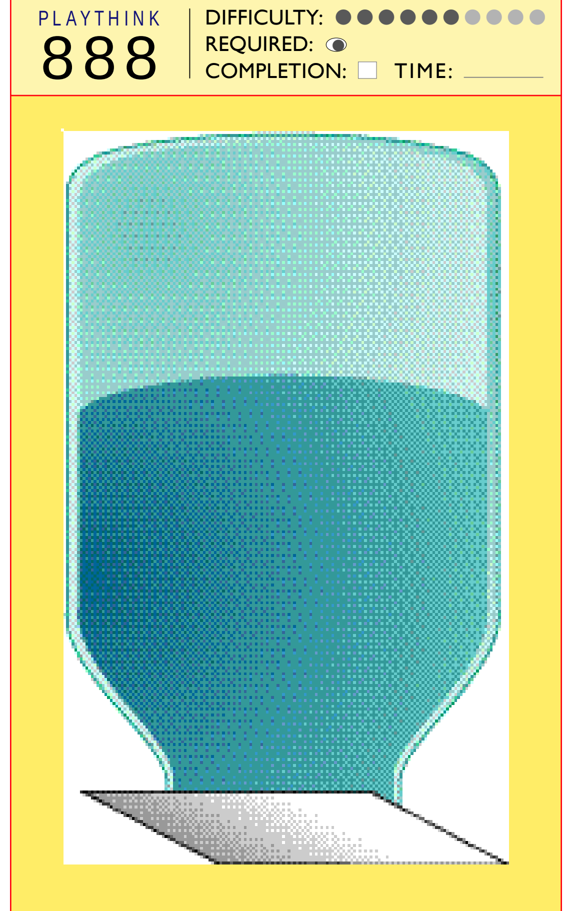
Which raindrops fall faster—the large ones or the small ones?



PLAYTHINK 887 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

ICEBERG

A bathtub holding an iceberg is filled to the brim with water. Can you tell what will happen when the iceberg melts?



PLAYTHINK 888 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

INVERTED BOTTLE

You may have seen this demonstration: The mouth of a jar or bottle full of water is covered with a piece of paper. When the bottle is inverted, the paper remains on the opening and the water does not pour out.

Can you explain why this trick works?

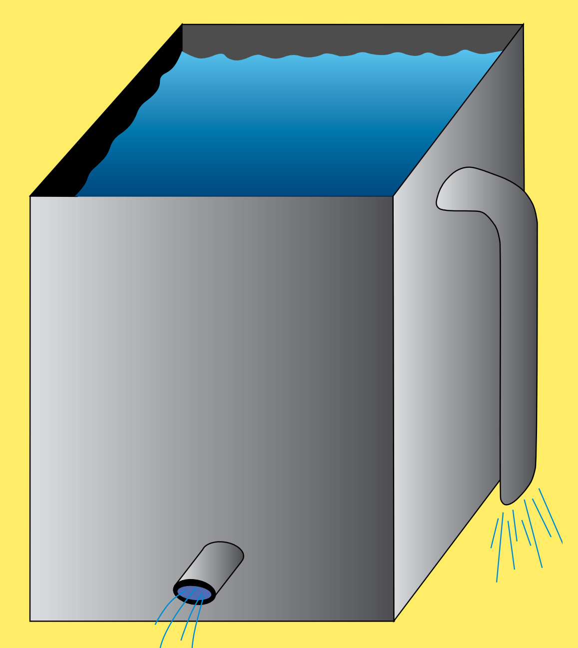
PLAYTHINK 889

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

STORAGE TANK

A tank has two identical holes used to drain water from it. One hole is at the bottom of the tank; the other is a downspout that has a drain hole near the top of the tank but discharges the water at the same level as the other drain.

Ignoring complicating factors such as friction, can you work out which hole discharges water at the faster rate?



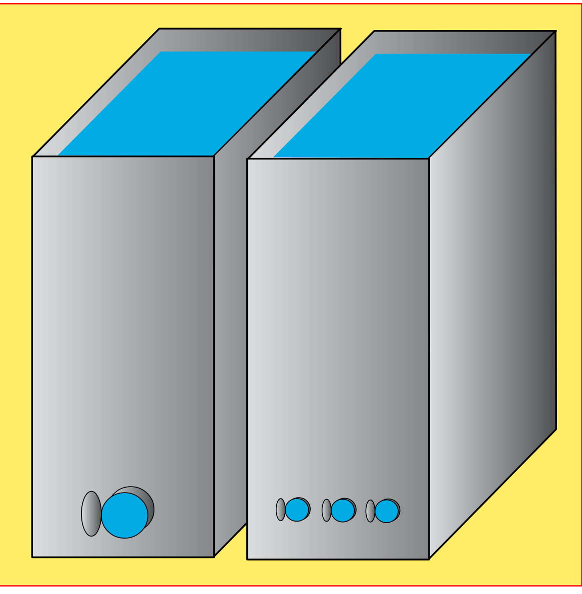
PLAYTHINK 890

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

STORAGE TANKS

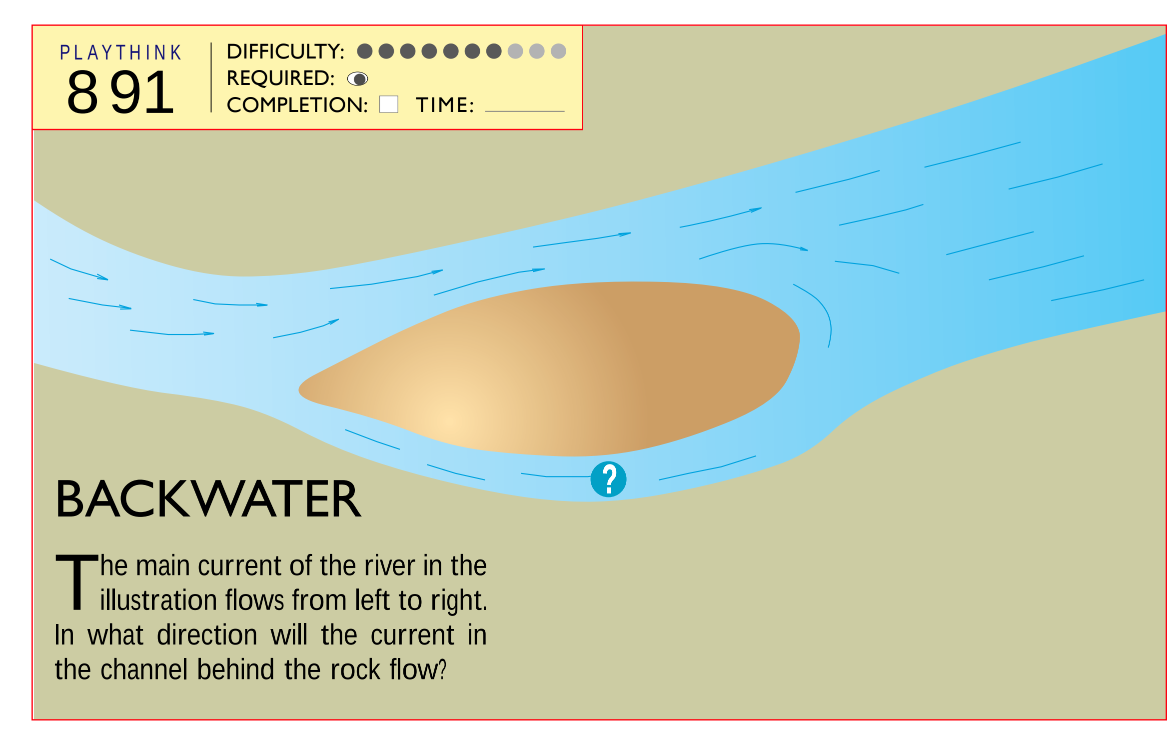
Two water tanks are identical in every way except for the size and number of their drains. One tank has one drain that is 6 centimeters across. The other drain has three drains, each 2 centimeters across.

If you open all the drains simultaneously, can you work out which tank will empty first?



PLAYTHINK 891

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____



BACKWATER

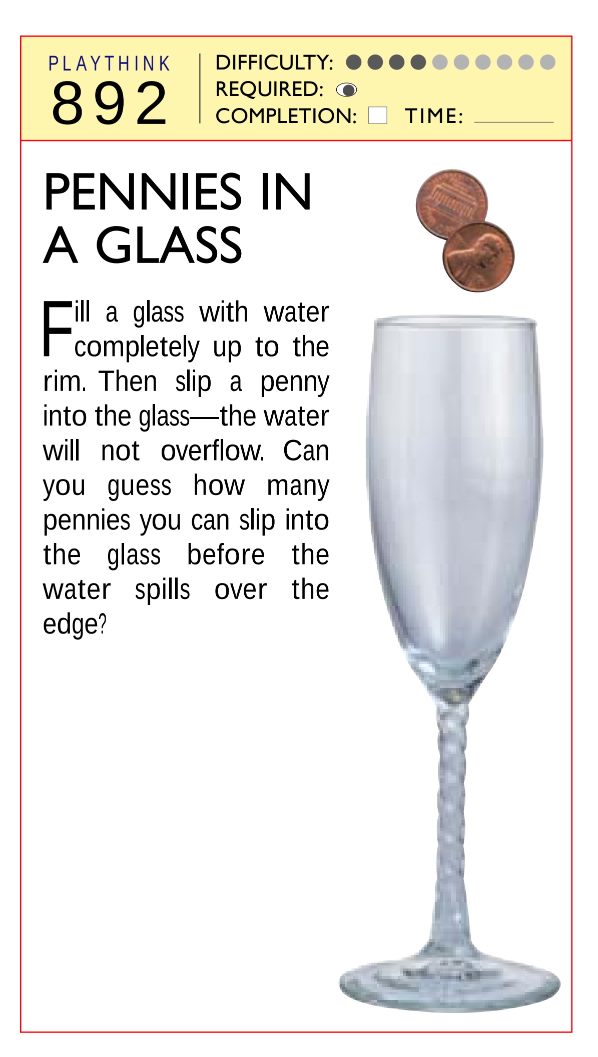
The main current of the river in the illustration flows from left to right. In what direction will the current in the channel behind the rock flow?

PLAYTHINK 892

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

PENNIES IN A GLASS

Fill a glass with water completely up to the rim. Then slip a penny into the glass—the water will not overflow. Can you guess how many pennies you can slip into the glass before the water spills over the edge?



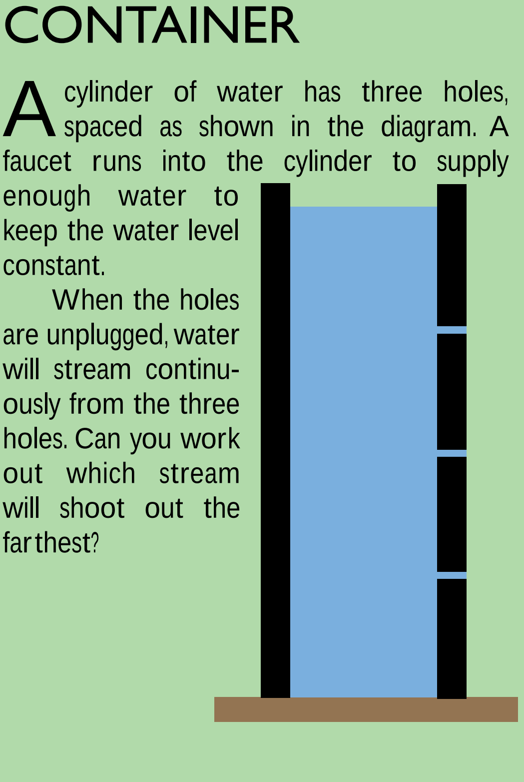
PLAYTHINK 893

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

MARIONETTE'S CONTAINER

A cylinder of water has three holes, spaced as shown in the diagram. A faucet runs into the cylinder to supply enough water to keep the water level constant.

When the holes are unplugged, water will stream continuously from the three holes. Can you work out which stream will shoot out the farthest?

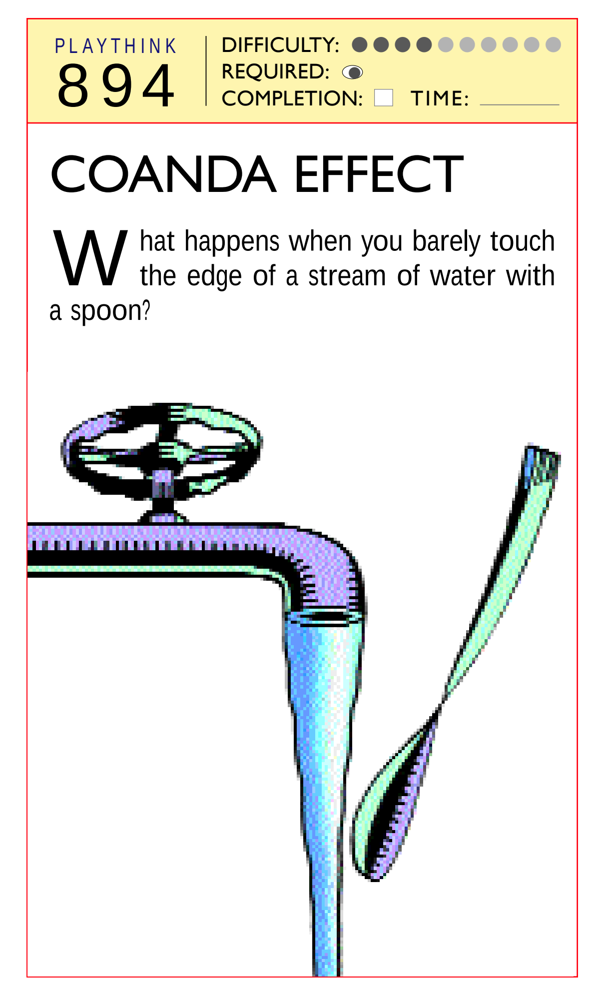


PLAYTHINK 894

DIFFICULTY: ●●●●○○○○○○
REQUIRED: ◉
COMPLETION: ☐ TIME: ____

COANDA EFFECT

What happens when you barely touch the edge of a stream of water with a spoon?

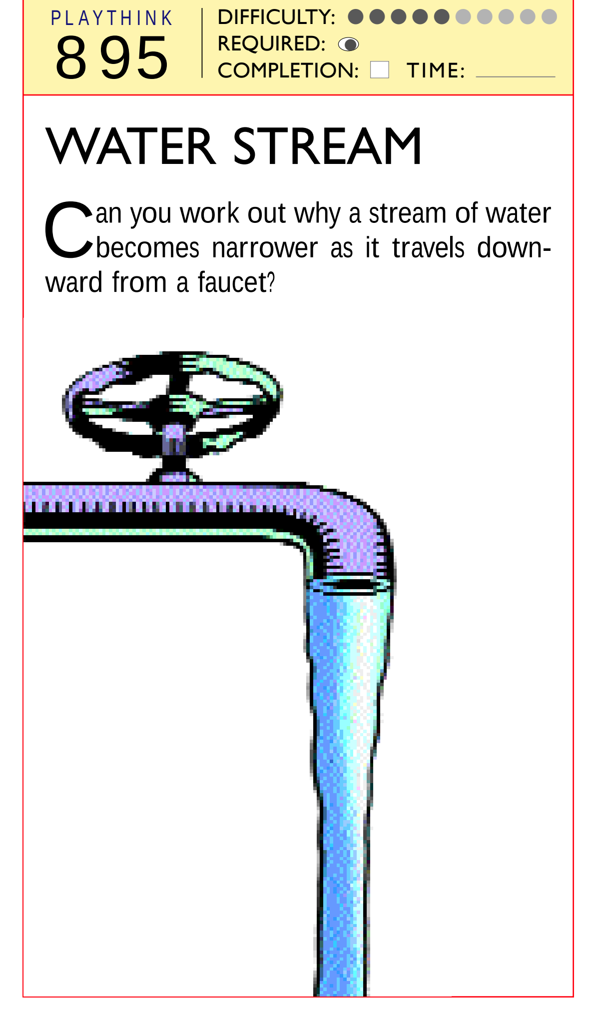


PLAYTHINK 895

DIFFICULTY: ●●●●●○○○○○
REQUIRED: ◉
COMPLETION: ☐ TIME: ____

WATER STREAM

Can you work out why a stream of water becomes narrower as it travels downward from a faucet?



PLAYTHINK 896

DIFFICULTY: ●●●●●●○○○○
REQUIRED: ◉
COMPLETION: ☐ TIME: ____

MUSICAL TUBE

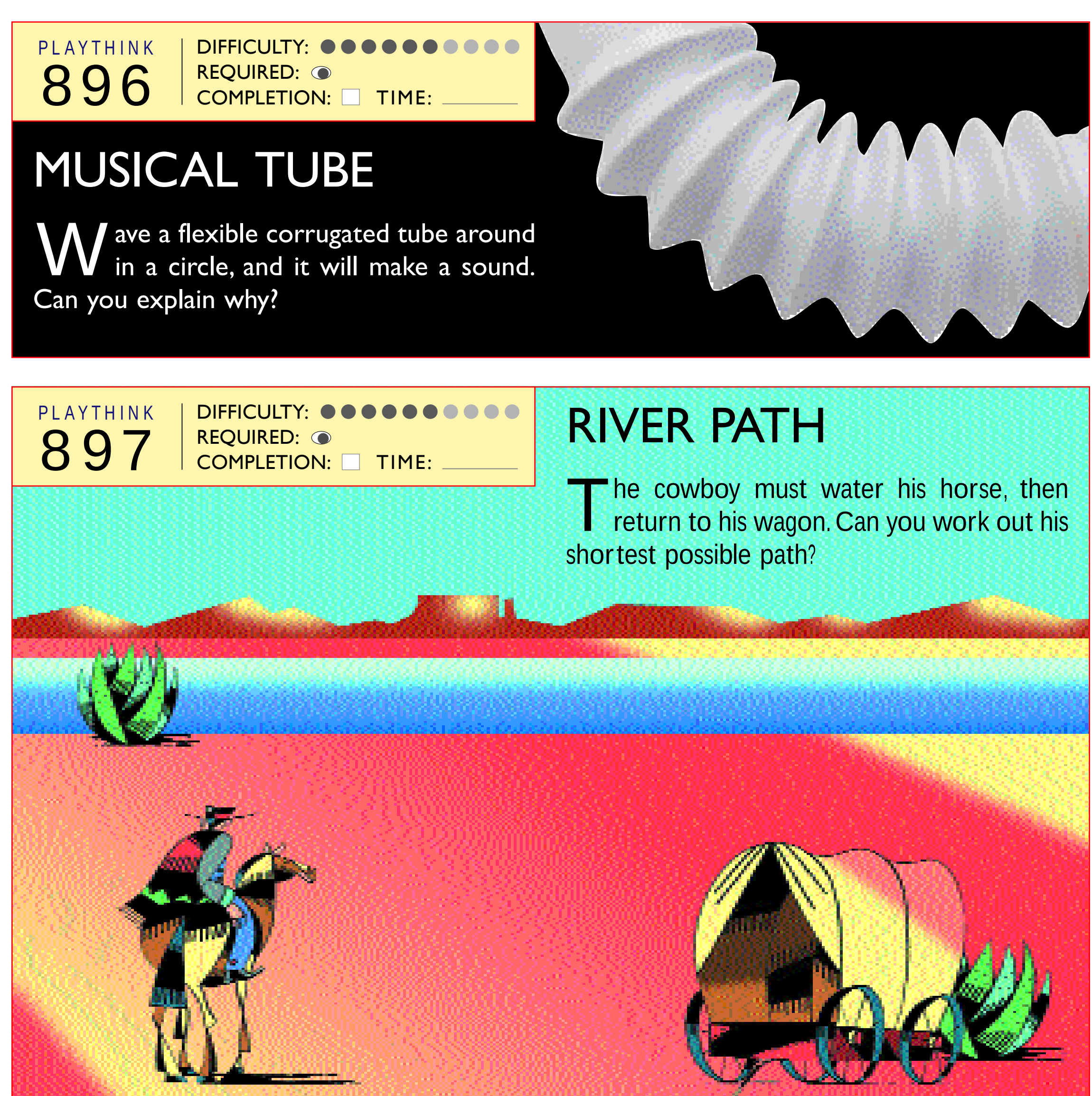
Wave a flexible corrugated tube around in a circle, and it will make a sound. Can you explain why?

PLAYTHINK 897

DIFFICULTY: ●●●●●●○○○○
REQUIRED: ◉
COMPLETION: ☐ TIME: ____

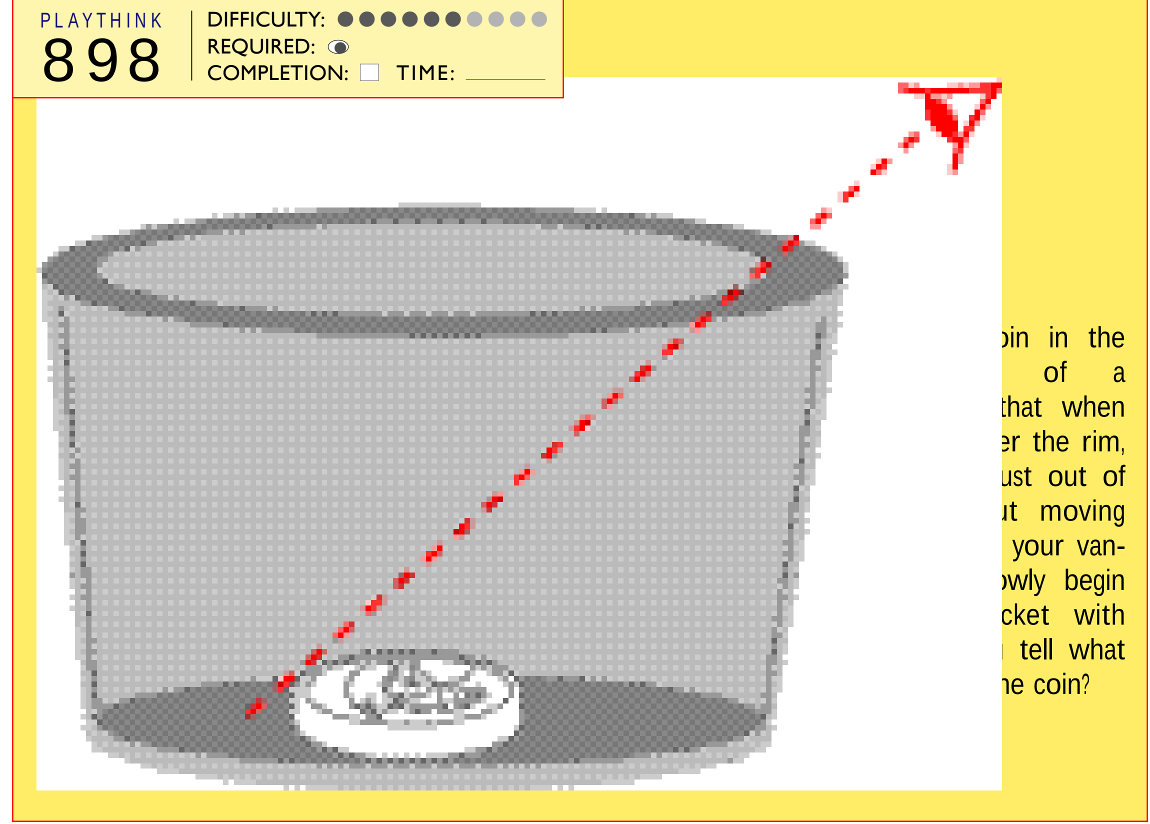
RIVER PATH

The cowboy must water his horse, then return to his wagon. Can you work out his shortest possible path?



PLAYTHINK 898

DIFFICULTY: ●●●●●●○○○○
REQUIRED: ◉
COMPLETION: ☐ TIME: ____



oin in the
of a
that when
er the rim,
ust out of
ut moving
your van-
owly begin
cket with
tell what
he coin?

PLAYTHINK 899 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

MAGNIFIER IN WATER

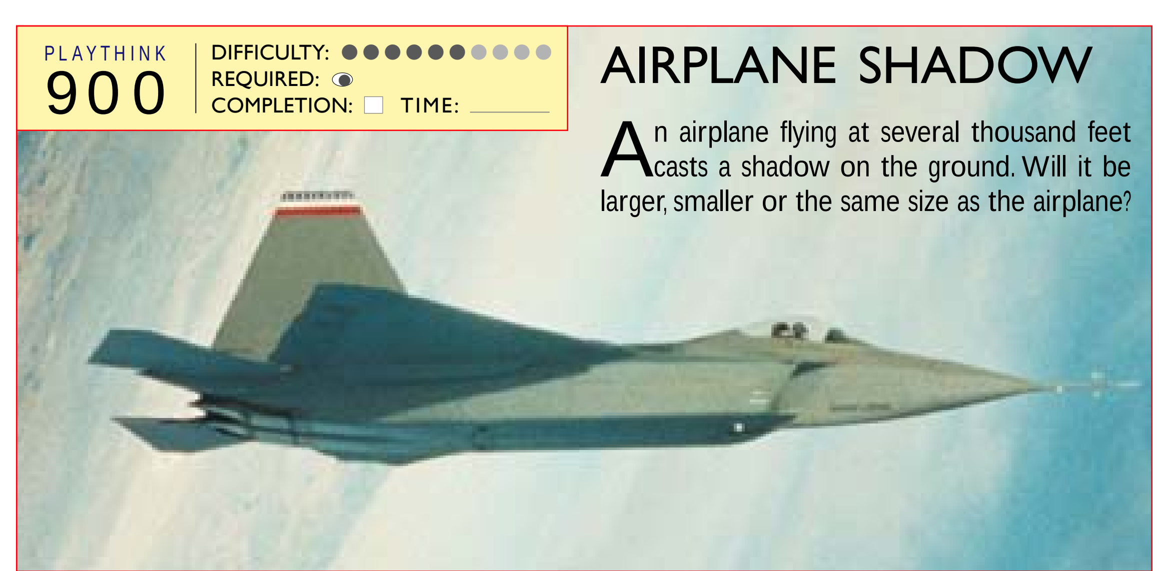
Will a magnifying glass enlarge the image of the knife more if the lens is placed underwater?



PLAYTHINK 900 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

AIRPLANE SHADOW

An airplane flying at several thousand feet casts a shadow on the ground. Will it be larger, smaller or the same size as the airplane?



PLAYTHINK 901 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

MAGNIFYING ANGLE

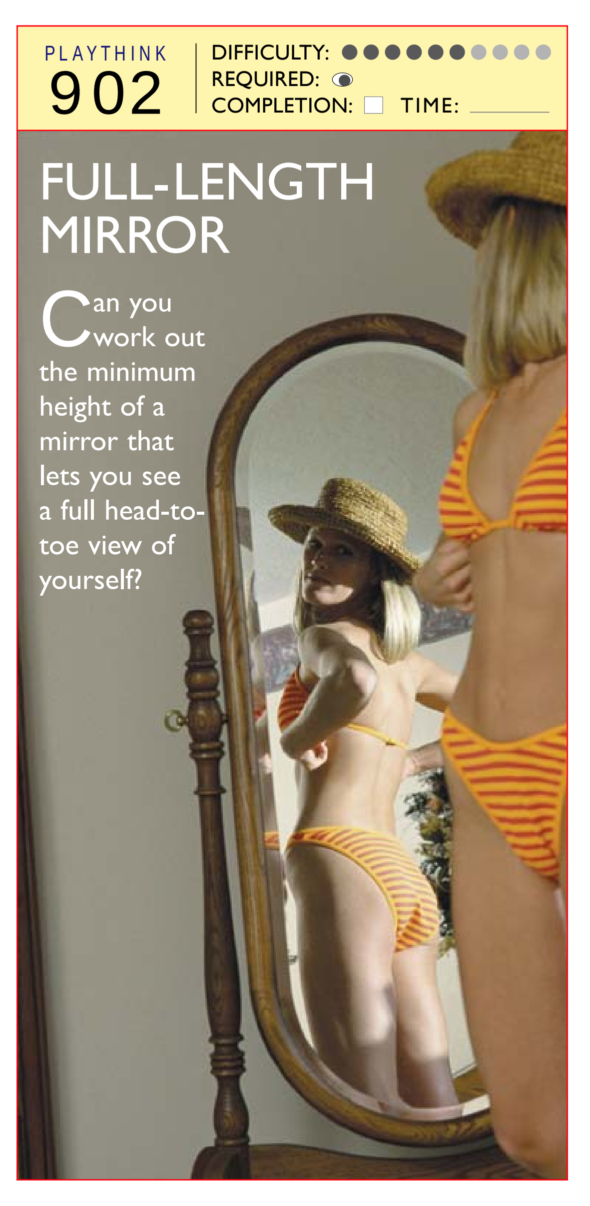
If you view an angle of 15 degrees through a lens that magnifies every dimension by three, can you work out how large the angle will appear?



PLAYTHINK 902 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

FULL-LENGTH MIRROR

Can you work out the minimum height of a mirror that lets you see a full head-to-toe view of yourself?



PLAYTHINK 903

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

REFLECTION OFF A MIRROR

A ray of light from point A bounces off the surface of a plane mirror and reaches point B.

Can you find the point of reflection on the mirror?



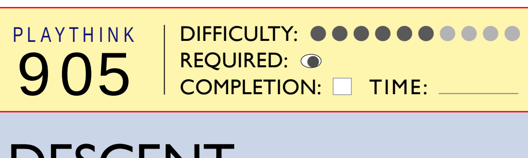
PLAYTHINK 904

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

MIRROR LABYRINTH

...six entrances to the labyrinth, ...ked with an arrow. All the walls ...n are covered with mirrors, and ...the reflections, you can make ...n each entrance to one of the

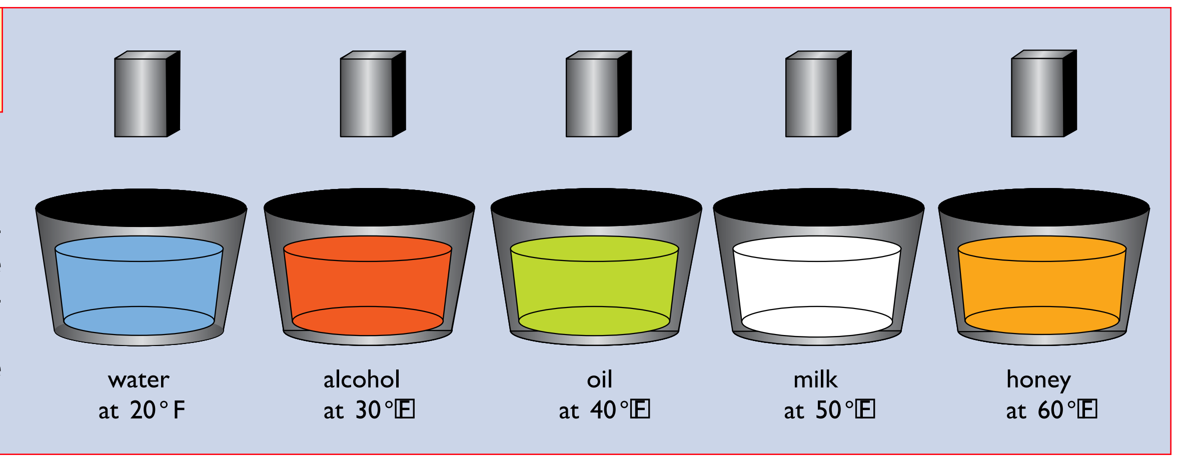
...work out which entrance leads ...which animal? (You must enter ...n the direction of the arrow.)



DESCENT

Identical lead weights are dropped simultaneously into each of five containers that are filled with different substances at the temperatures listed for each.

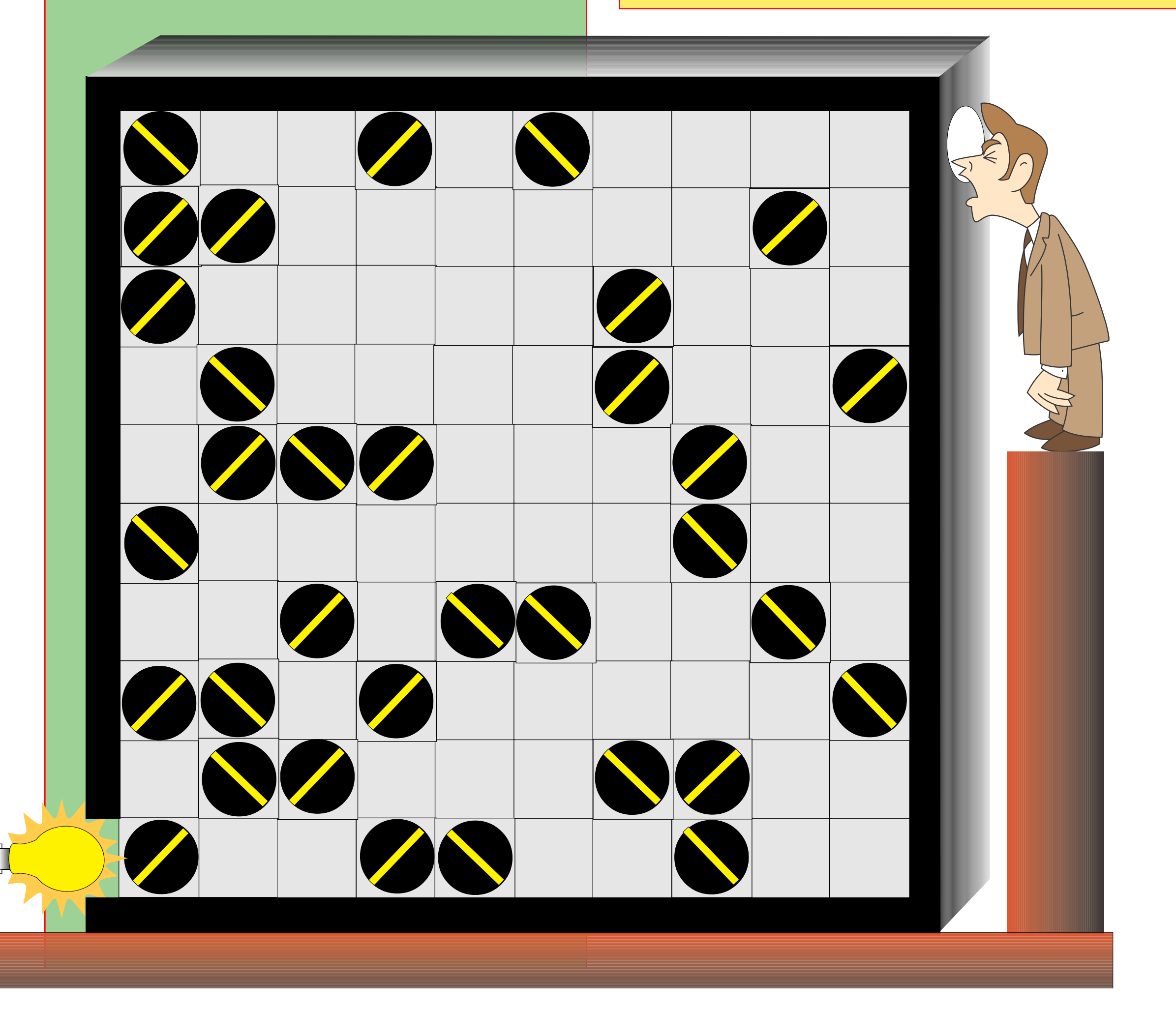
In which container will the weight take the longest to reach the bottom?



PLAYTHINK 906 — DIFFICULTY: REQUIRED: COMPLETION: TIME:

SUPER PERISCOPE

If you rotate ten of the double-sided mirrors by 90 degrees each, you will be able to see the reflection of the lightbulb from the porthole in the top right-hand corner. Can you work out which ten mirrors must be moved?



PLAYTHINK 907 — DIFFICULTY: REQUIRED: COMPLETION: TIME:

ARCHIMEDES'S MIRRORS

Mirrors are found in common yet seemingly miraculous objects in science, magic and everyday life. Telescopes, optical scanners and the box in which the magician saws a lady in half all employ mirrors.

One of the most imaginative uses of mirrors is credited to the ancient Greek scientist Archimedes. According to writings from the period, Archimedes used mirrors to repel a fleet of Roman ships that besieged the city of Syracuse in 214 B.C. He is supposed to have used the mirrors to focus the rays of the sun on the ships, setting them on fire.

Is such a feat really possible?

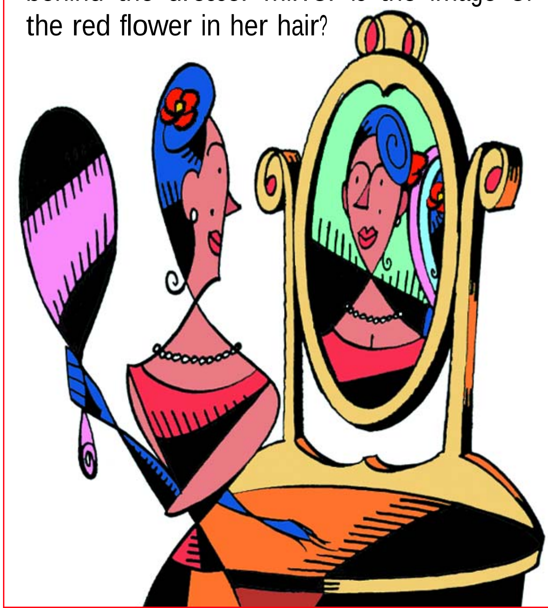
Archimedes

PLAYTHINK 908 — DIFFICULTY: REQUIRED: COMPLETION: TIME:



FASHION MIRROR

The model stands 2 meters from the dresser mirror and holds a hand mirror half a meter behind her head. How far behind the dresser mirror is the image of the red flower in her hair?




Perception
13

Missing Cubes

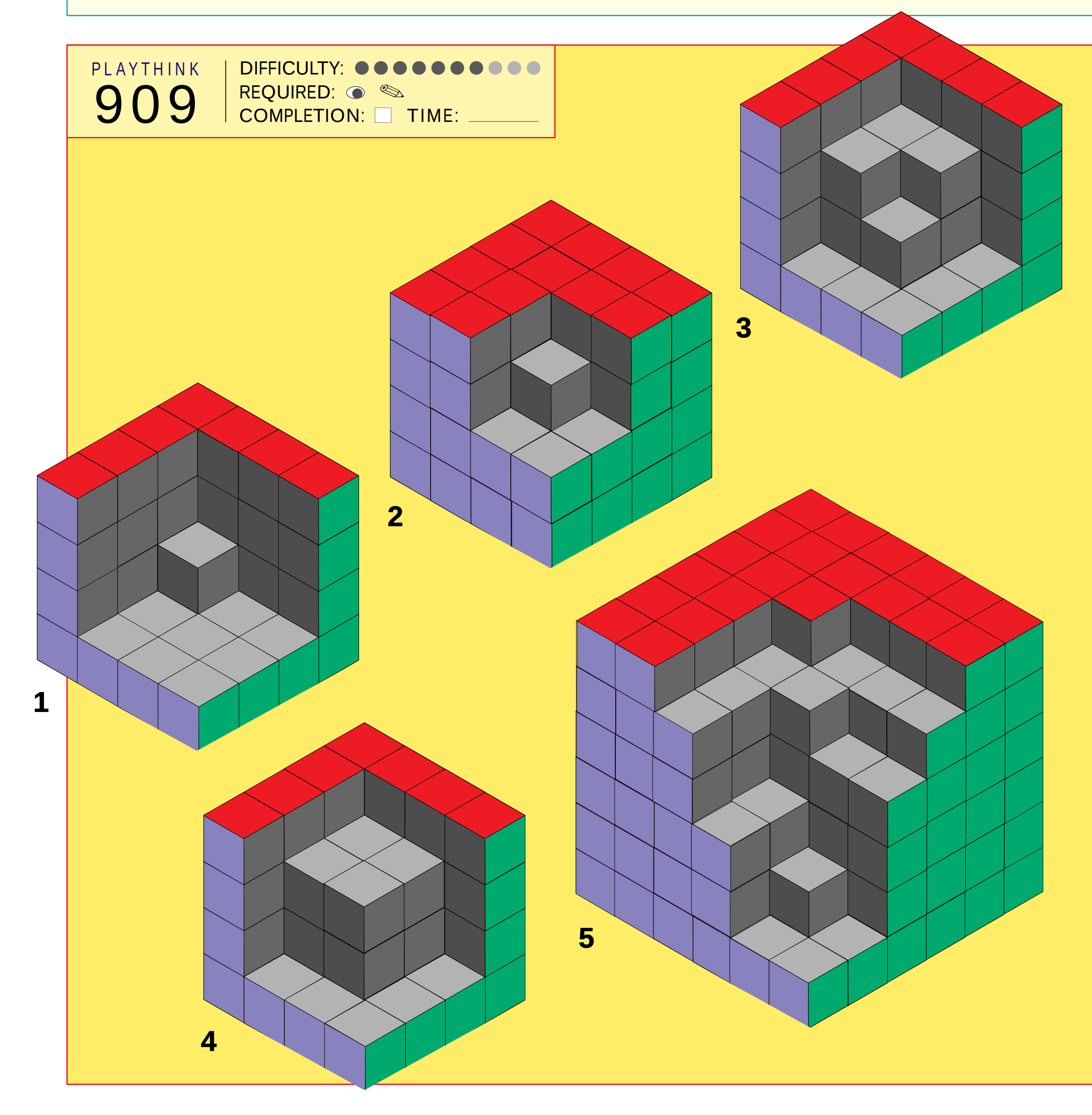
You have no doubt seen a room in which a man seems to shrink as he walks from one end to the other. As you quickly realize, the man isn't shrinking—he's walking away from you in a specially constructed room designed to mask the perception of depth.

There are no such tricks included in the following puzzles. These problems depend upon our ability to perceive depth—the three-dimensional effect afforded by perspective—projected into two-dimensional illustrations. Although this ability was either unknown or ignored before the Middle Ages, the effect is so well understood at present that computers can be programmed to recognize three-dimensional objects (such as a particular programmer's facial features) at any angle. And holograms, which don't employ perspective but capture three-dimensional information about an object from the light bouncing off it, are found in demonstrations of science, breathtaking works of art and commercial security systems. Indeed, this last use is now the most common: many a credit card carries a little hologram on its face.

Certainly there are cases when perspective can be misleading. All the same, there is a shortcut to finding the solution to these problems, one that emerges from the vagaries of perspective.

PLAYTHINK 909

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______



MISSING CUBES

The five cubes illustrated here are missing parts. Can you work out how many unit cubes are missing in each case?

Once you have calculated the total number of missing cubes, you should note that some of the missing unit cubes are colored red, blue or green on some of their faces, while others are completely gray. Can you fill in the scorecard below with the number of cubes that fall into each category? Can you work out a visual shortcut for finding that information?

Score Box

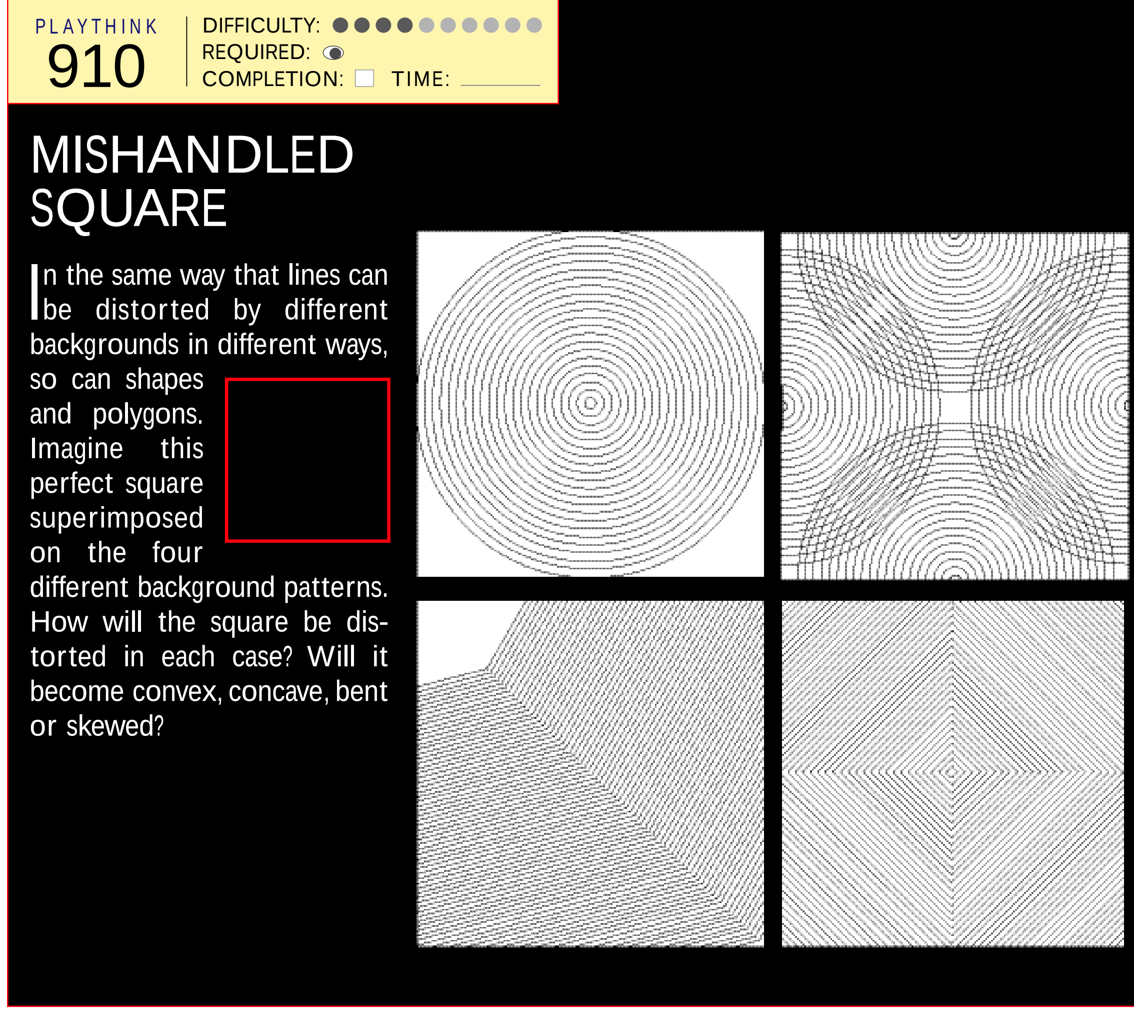
Missing Cubes	1	2	3	4	5
Cubes colored on three sides					
Cubes colored on two sides					
Cubes colored on one side					
Cubes not colored					
Totals					

PLAYTHINK 910

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

MISHANDLED SQUARE

In the same way that lines can be distorted by different backgrounds in different ways, so can shapes and polygons. Imagine this perfect square superimposed on the four different background patterns. How will the square be distorted in each case? Will it become convex, concave, bent or skewed?

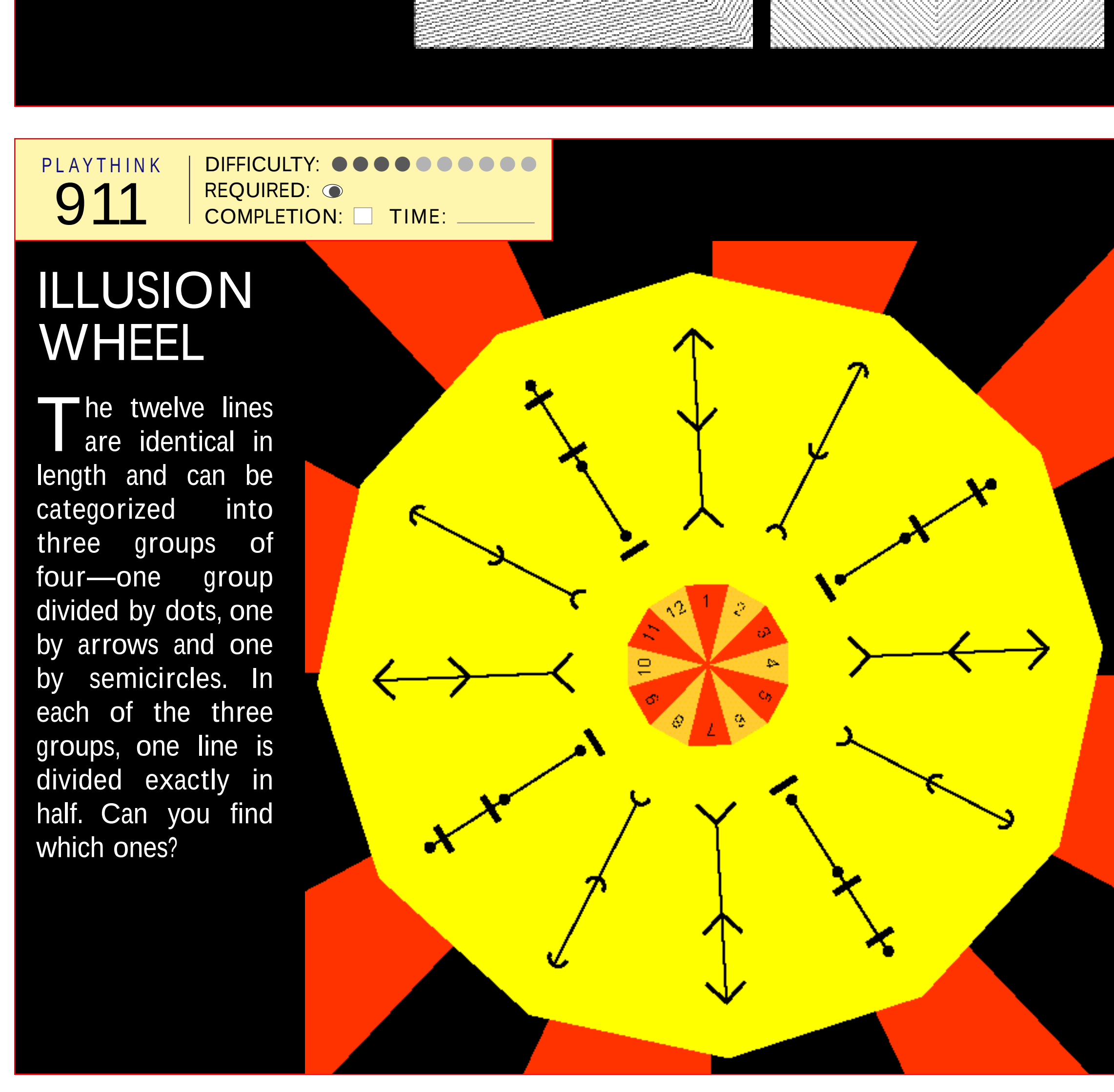


PLAYTHINK 911

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

ILLUSION WHEEL

The twelve lines are identical in length and can be categorized into three groups of four—one group divided by dots, one by arrows and one by semicircles. In each of the three groups, one line is divided exactly in half. Can you find which ones?

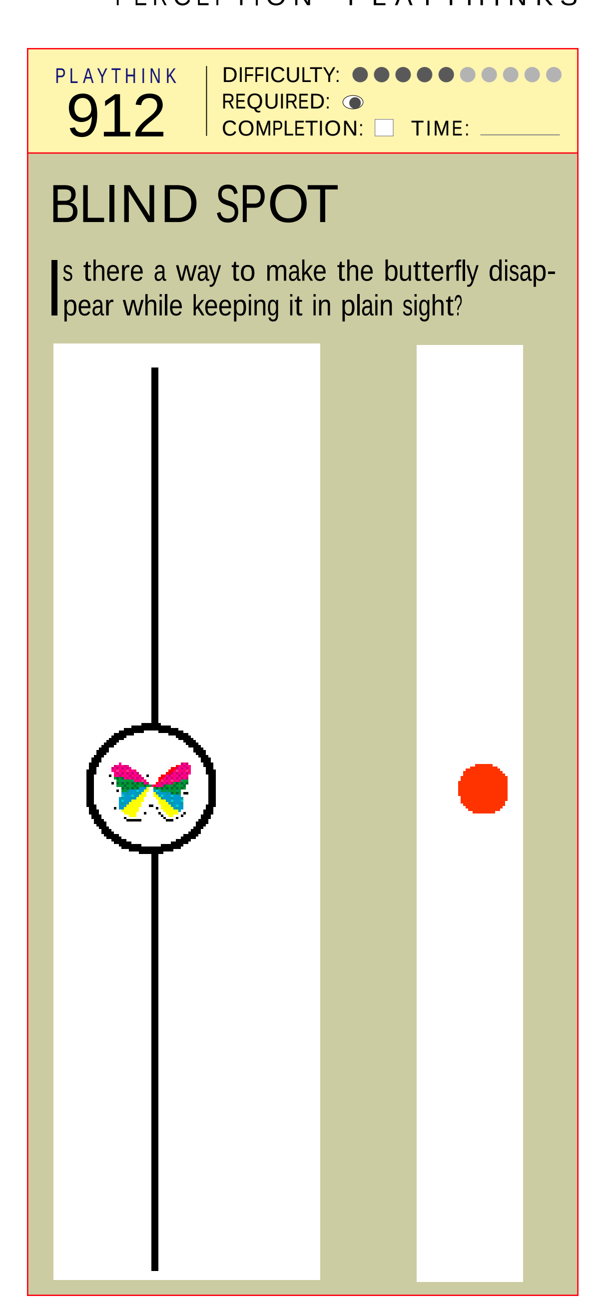


PLAYTHINK 912

DIFFICULTY: ●●●●●○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

BLIND SPOT

Is there a way to make the butterfly disappear while keeping it in plain sight?

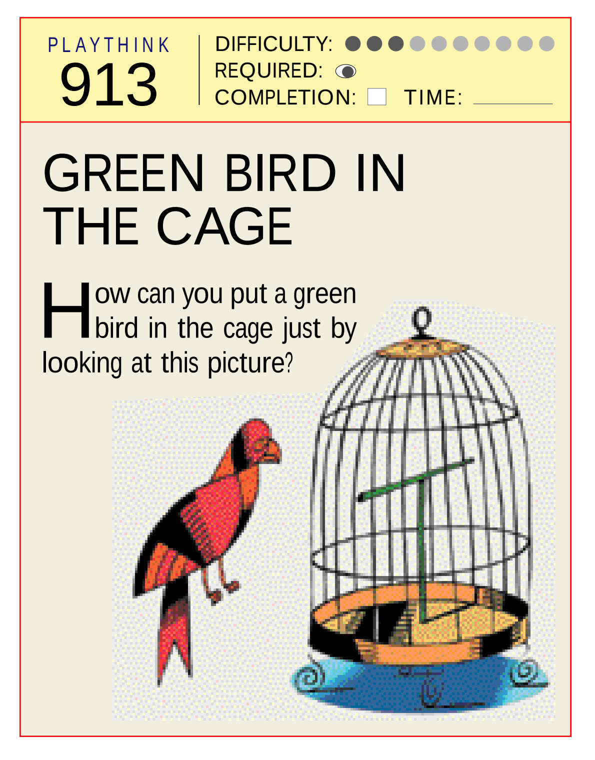


PLAYTHINK 913

DIFFICULTY: ●●●○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

GREEN BIRD IN THE CAGE

How can you put a green bird in the cage just by looking at this picture?



PLAYTHINK 914

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____



WHITE KNIGHT

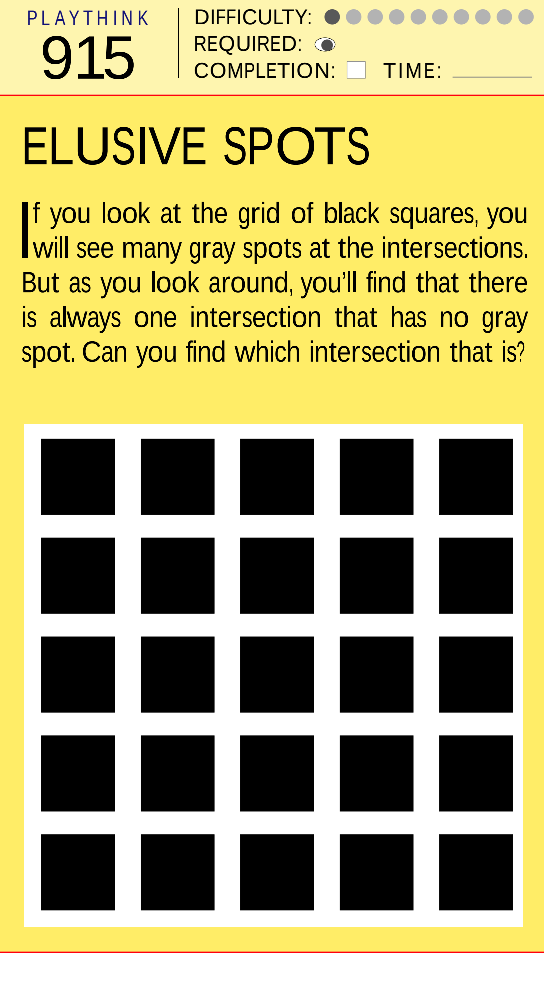
How can you turn the black knight on a white horse into a white knight on a black horse?

PLAYTHINK 915

DIFFICULTY: ●○○○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

ELUSIVE SPOTS

If you look at the grid of black squares, you will see many gray spots at the intersections. But as you look around, you'll find that there is always one intersection that has no gray spot. Can you find which intersection that is?



PLAYTHINK 916

DIFFICULTY: ●●●○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

DRACULA'S COFFIN

Can you work out which lid goes with which coffin?



PLAYTHINK 917

DIFFICULTY: ●●●○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

CROSSING LINES

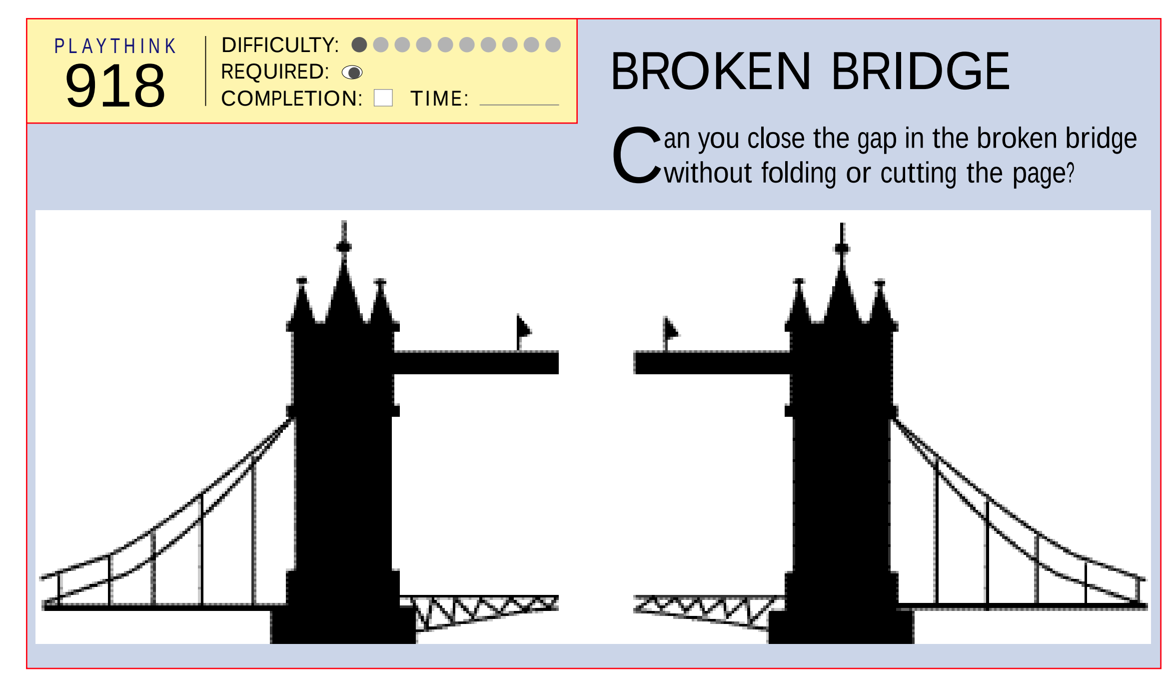
How can you look at the two intersecting lines to see more than two lines?

PLAYTHINK 918

DIFFICULTY: ●○○○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

BROKEN BRIDGE

Can you close the gap in the broken bridge without folding or cutting the page?



PLAYTHINK 919

DIFFICULTY: ●●●○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

SHADOW PROFILES

There are three identical profiles. How quickly can you find them?

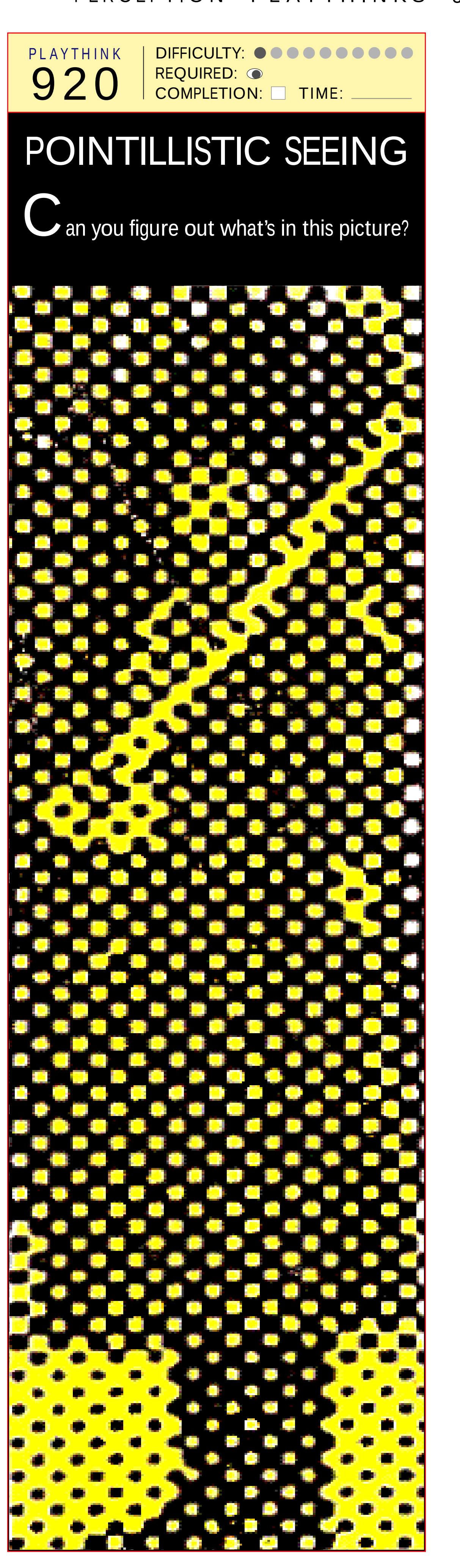


PLAYTHINK 920

DIFFICULTY: ●○○○○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

POINTILLISTIC SEEING

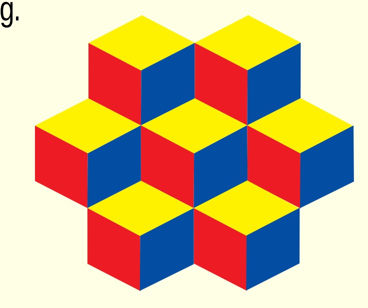
Can you figure out what's in this picture?



How Many Cubes?

This famous optical illusion is a striking demonstration of the power of the mind to change the orientation of objects. In the same illustration you can see either seven whole cubes or three cubes and several parts of incomplete cubes. If you have a hard time seeing the three-cube orientation, turn the page upside down.

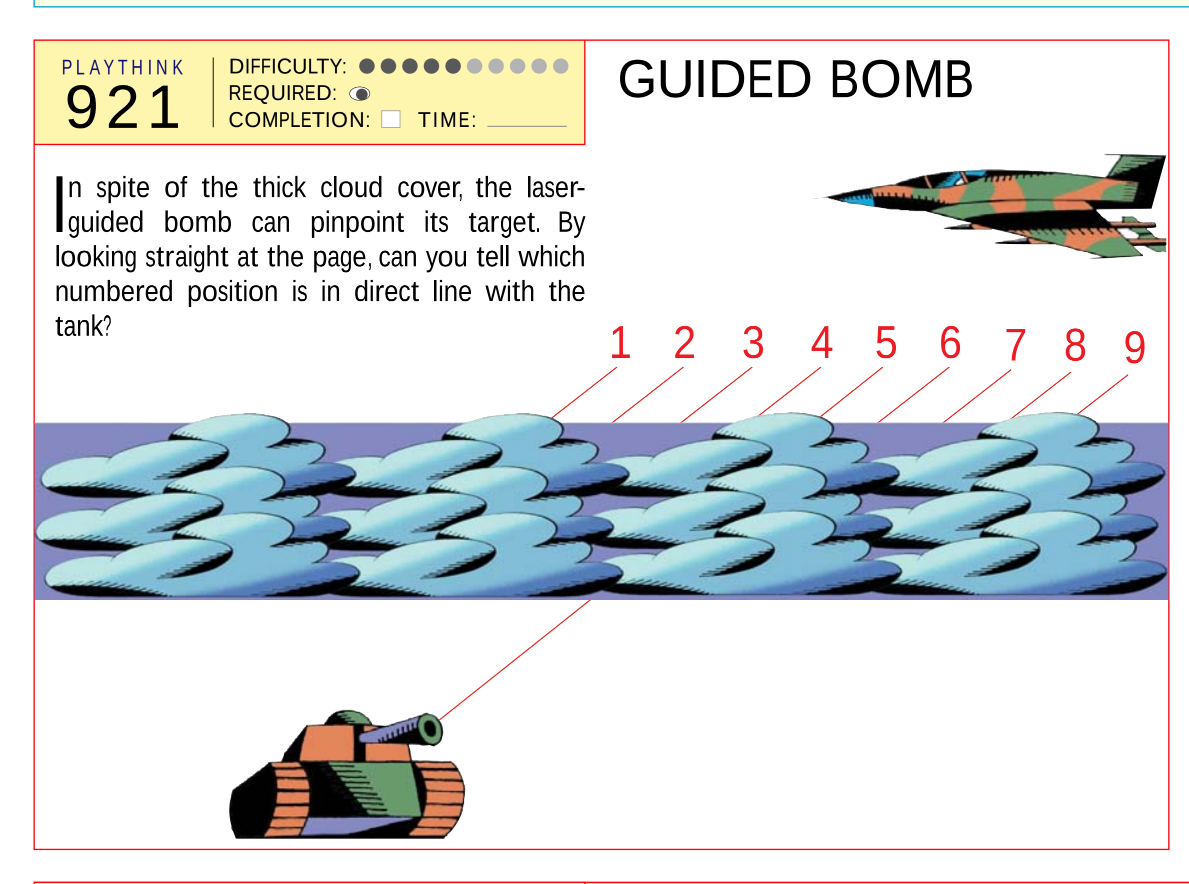
Changing the point of view in this way—or being able to see things the way more literal-minded people can't—is the essence of creative thinking.



PLAYTHINK 921 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

GUIDED BOMB

In spite of the thick cloud cover, the laser-guided bomb can pinpoint its target. By looking straight at the page, can you tell which numbered position is in direct line with the tank?



PLAYTHINK 922 | DIFFICULTY: ●○○○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

MISSING SLICE

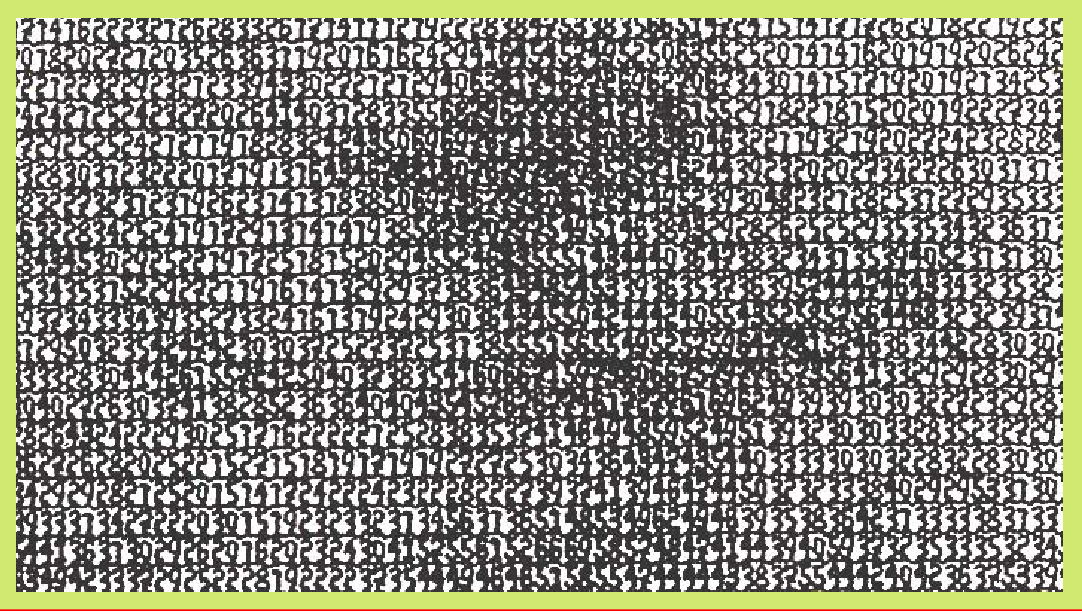
Can you find the missing slice of cake?



PLAYTHINK 923 | DIFFICULTY: ●●●○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

DIGITS

Can you tell what the pattern of digits represents?



Point of View

Painters often find that the most difficult shape to capture is the familiar one. To see an object as a pure form rather than as a clock or an apple, artists go to great lengths to alter their perception. Many artists will study a still life arrangement through a mirror—or backward, through their legs—to obtain a fresh view of the subject. Such a new perspective can enable them to discover an innovative way to capture a subject's form on canvas.

That artists must struggle to overcome their built-in perception is evidence that our conscious mind stores three-dimensional images, memorizing and categorizing everything we see. Such images are generally available for recall in a way that makes comparison and recognition possible even from unfamiliar angles. This ability is so constant and automatic that it is totally unremarkable—except when that ability is lost. The victims of certain forms of brain damage that impair the normal ability to compare and recognize shapes find, in fact, that everyday life becomes nearly impossible without that aspect of human consciousness.

PLAYTHINK 924

DIFFICULTY: ●●●●●●●●●●

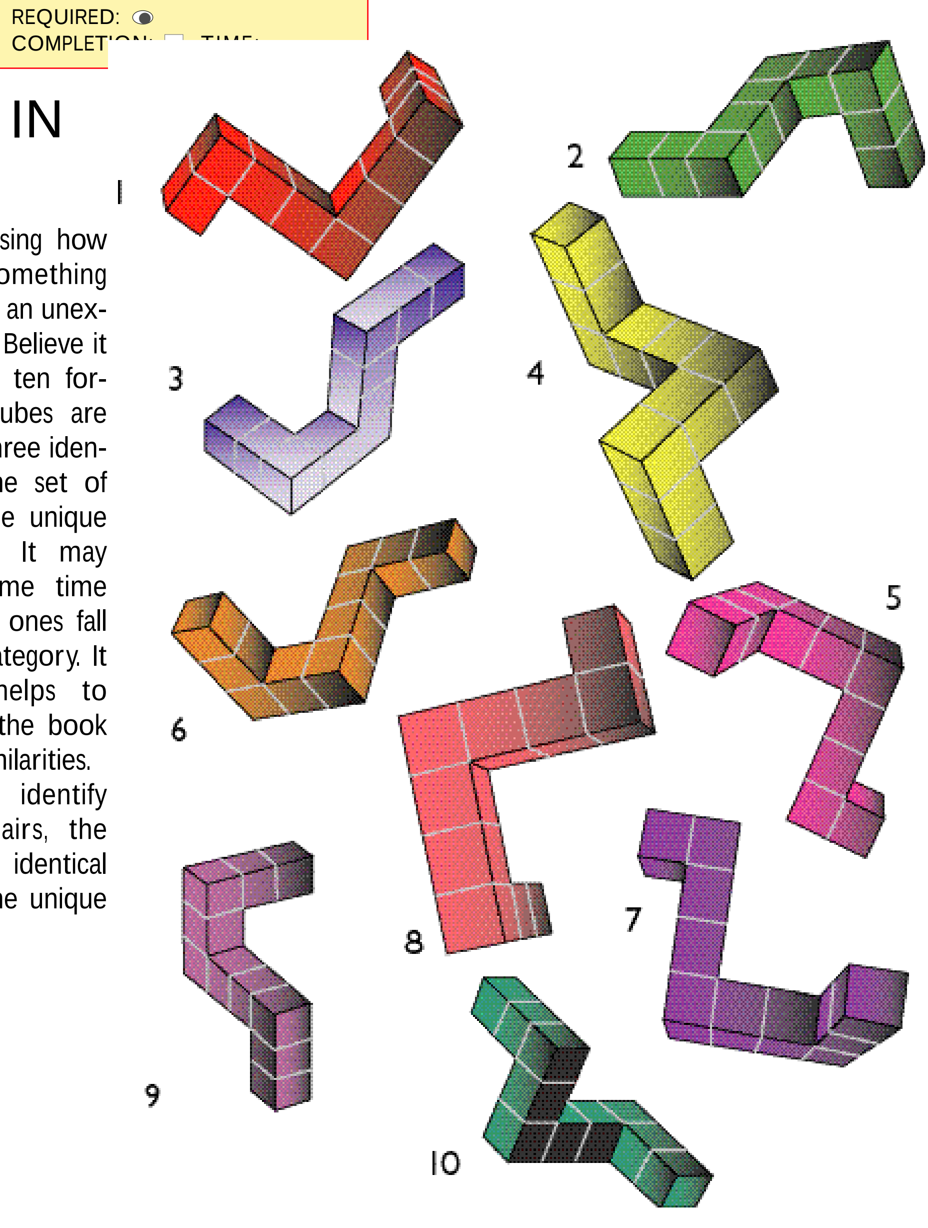
REQUIRED: 👁

COMPLETION: ☐ TIME:

CUBES IN SPACE

Isn't it surprising how different something can look from an unexpected angle? Believe it or not, these ten formations of cubes are made up of three identical pairs, one set of three and one unique configuration. It may take you some time to see which ones fall into which category. It sometimes helps to actually turn the book to find the similarities.

Can you identify the three pairs, the set of three identical shapes and the unique configuration?



PLAYTHINK 925

DIFFICULTY: ●●●●●●●●●●

REQUIRED: 👁

COMPLETION: ☐ TIME: ____

UPSIDE-DOWN WORDS

Place a mirror along the red line. The words in the top frame will appear reversed right to left, as is normal. But the words in the bottom frame will appear upside-down. Can you explain this?

PLACE A MIRROR VERTICALLY ON THE LEFT RED LINE. WORDS IN THE TOP FRAME WILL BE REVERSED RIGHT-LEFT (BUT NOT UPSIDE-DOWN). WORDS AT THE BOTTOM FRAME ARE NOT ONLY REVERSED, BUT ALSO TURNED UPSIDE-DOWN. CAN YOU EXPLAIN WHY ?

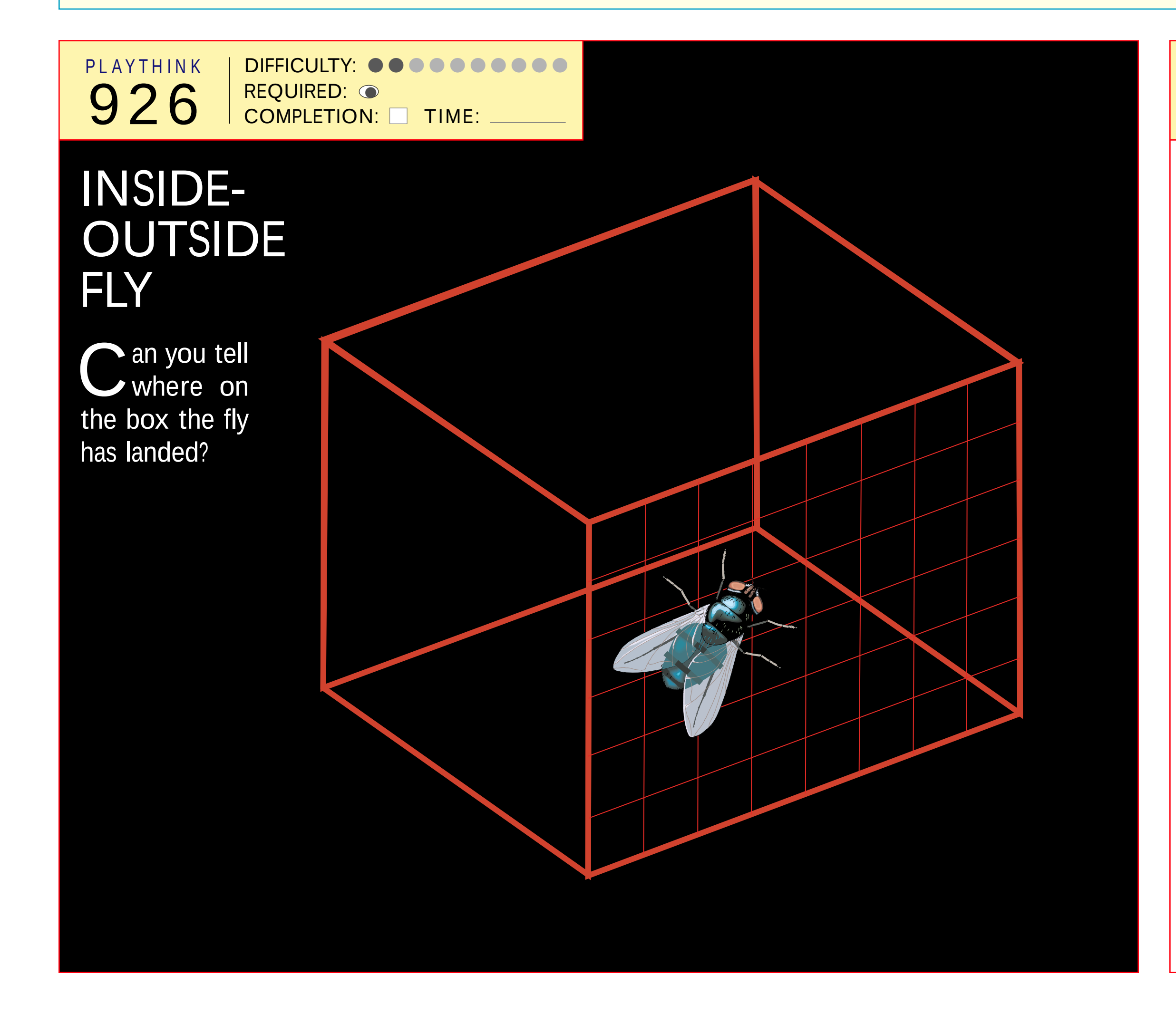
BOOKIE EXCEEDED HIKED ICEBOX CHOKED COED BOBBED DECK BEECED COD HID BOXED DODO BOB CHOKED COCO EXCEEDED BOOKIE HIKED ICEBOX DID CHOICE BOOKED OBOE HEEDED OX HID COKE EXHOED BOOHOO DOCKED

The Limits of Seeing

Most people experience seeing as a passive "taking-in" process. But, in fact, perception is an active pattern-seeking process that is closely allied to the act of thinking. The brain is as much a "seeing" organ as the eye. Optical illusions take advantage of the tendency of the human brain to see things as it thinks they should be—based on previous experience—rather than as they are. Although this normalizing property of our perceptional system is widely engaged in science, math, art, design and architecture, the ease with which we can be fooled by a simple optical illusion should be a warning to the general unreliablility of our observations. (Remember that if you should ever have to listen to "eyewitness testimony.") We can be made to perceive things to be larger than they actually are, register depth in a two-dimensional flat surface, see colors in a monochromatic pattern or experience motion in a static image.

There is a limit to the reliability of our senses, and no amount of practice can ever make them good enough for some special tasks. One solution to that problem is to find ways to extend our senses, to invent devices capable of perceiving and recording information without error. Although no one has created a perfect system for doing this, cameras and recorders have proven to be much more reliable and free from bias than even the best human observer.

This human tendency to be tripped up by our perceptions has long been a source of play for the makers of optical illusions (and of inspiration for Op Art painters). For as long as humans have been playing with lines and shapes, colors and patterns, it has been known that we can see disappearing cubes or lines where there are, in fact, none.



PLAYTHINK 926 | DIFFICULTY: ●●○○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

INSIDE-OUTSIDE FLY

Can you tell where on the box the fly has landed?



PLAYTHINK 927 | DIFFICULTY: ●○○○○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

BEFORE-AFTER

Below is a portrait of a newlywed couple. Can you find an image that shows how happy they will be in a few years?


Bonus Round
14

PLAYTHINK 928

DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

MIXED-UP BLUEPRINTS

The architects want to begin construction on the eight buildings shown on the wall panel, but the blueprints, which show either the front view or the top view of a building, have been mixed up with blueprints for other projects. Can you match each building with its blueprint?

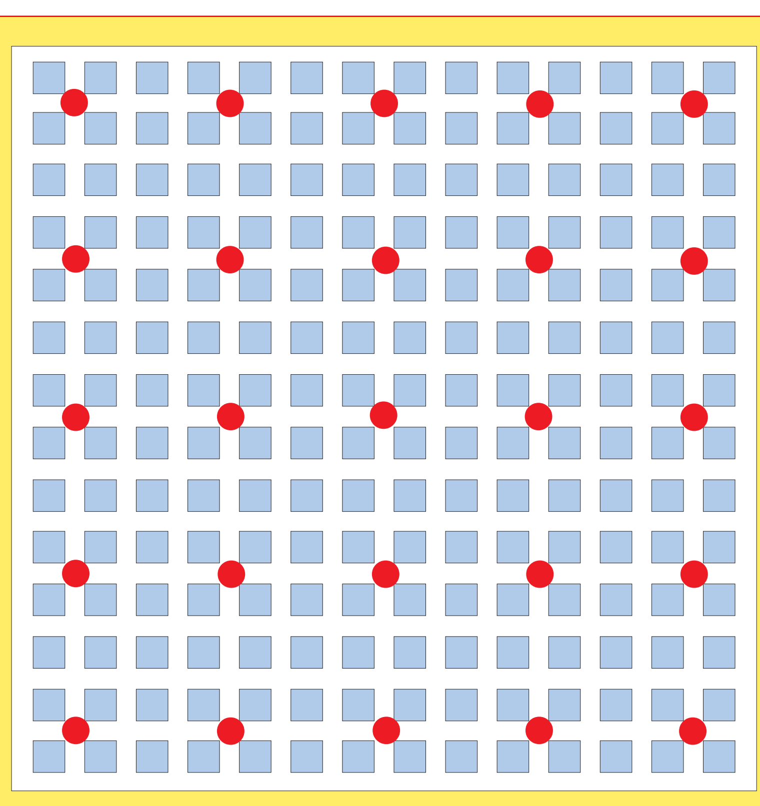
PLAYTHINK 929

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

EXHIBITION WIRING

An architect is examining his design for the placement of the electrical outlets in an exhibition hall. The hall is divided into identical unit blocks, and the client needs each intersection to be no more than three blocks from an electrical outlet.

His initial design, shown here, used twenty-five electrical outlets, but the architect is certain that there is a more economical solution. Is he right? Can you find the design that provides the fewest number of outlets yet puts no intersection more than three blocks from an outlet?

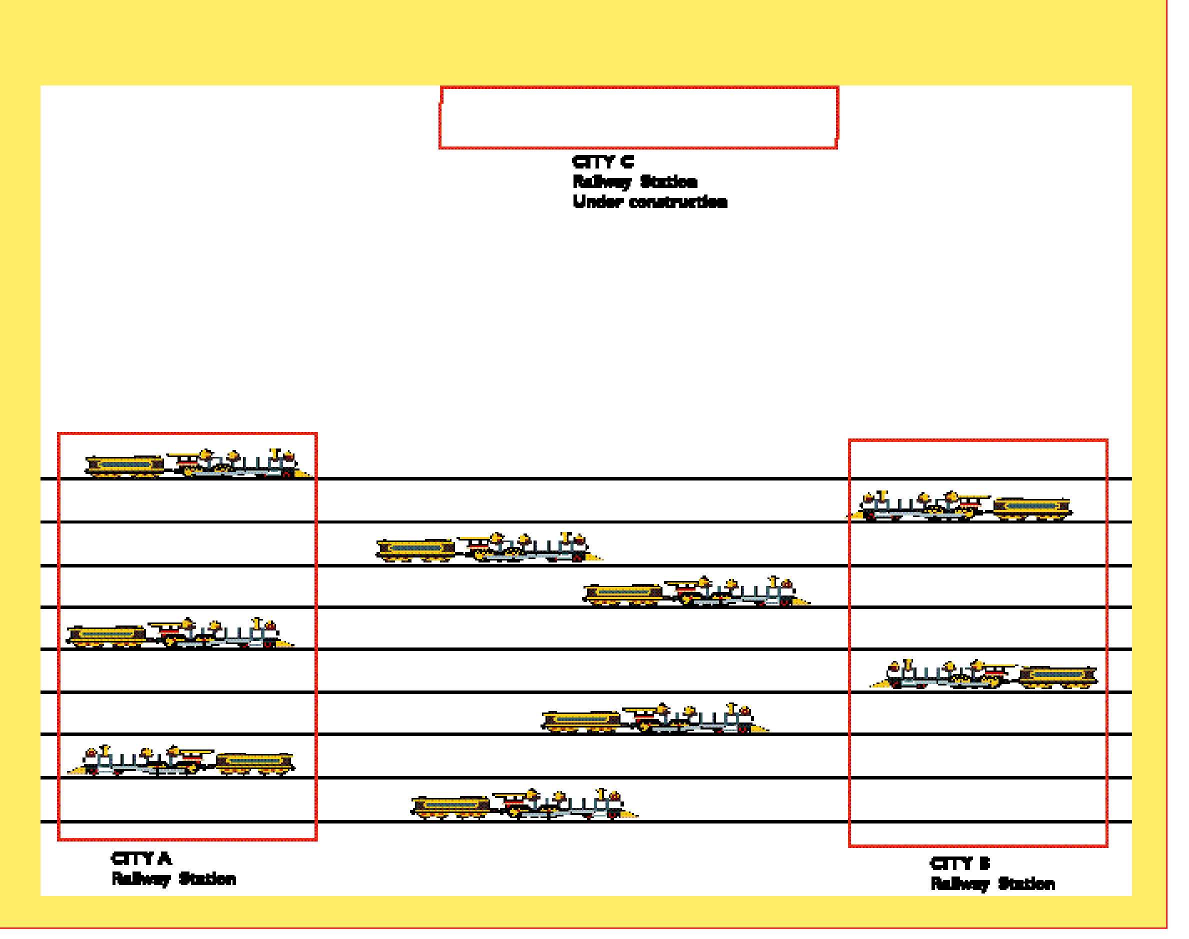


PLAYTHINK 930 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

FLATLAND RAILWAY

Nine straight parallel railway tracks run between two cities in Flatland. The tracks can connect the two cities without any intersections, which is advantageous for scheduling purposes. The leaders of a third city that is not in line with the existing rail lines have asked for some of the tracks to be re-laid so that their city can be connected to the other two by at least two tracks.

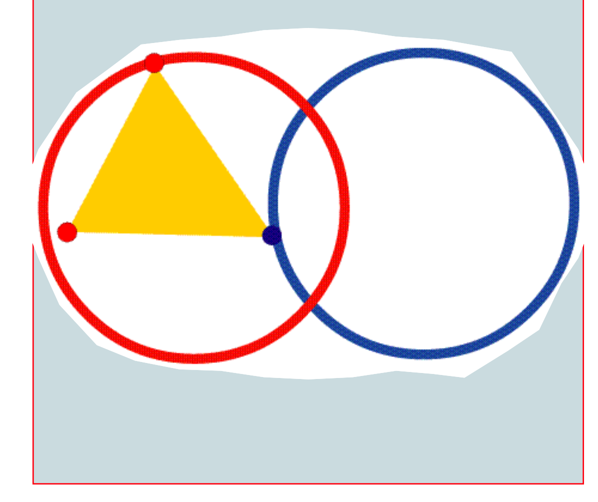
The tracks will be laid out so that one set is parallel in one direction, another is parallel in another direction, and a third set is parallel in a third direction. Can you work out how to design the rail system so that you create the smallest number of intersections?



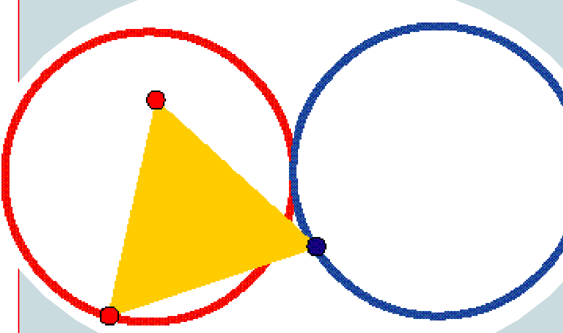
PLAYTHINK 931 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ ✂ | COMPLETION: ☐ TIME: ____

MOVING TRIANGLE 2

Two vertices of the triangle are constrained to move along the circumferences of the intersecting circles. As the triangle tips follow their circular paths, the third vertex traces out a complex shape. Can you determine what shape the vertex traces?



PLAYTHINK 932 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 ✎ ✂ | COMPLETION: ☐ TIME: ____



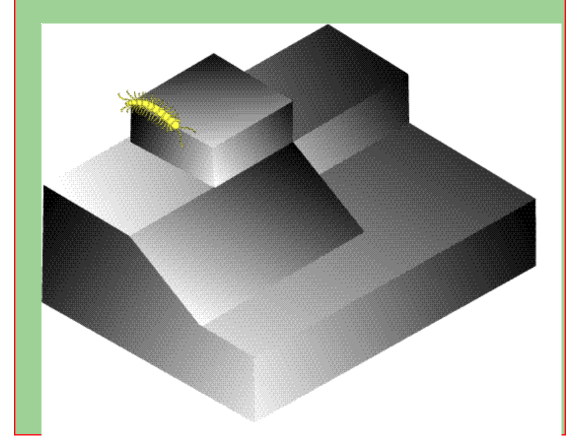
MOVING TRIANGLE 3

Two vertices of the triangle are constrained to move along the circumferences of the two touching circles. As the triangle tips follow their circular paths, the third vertex traces out a complex shape. Can you work out what shape the vertex traces?

PLAYTHINK 933 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

CRAWLING CENTIPEDE

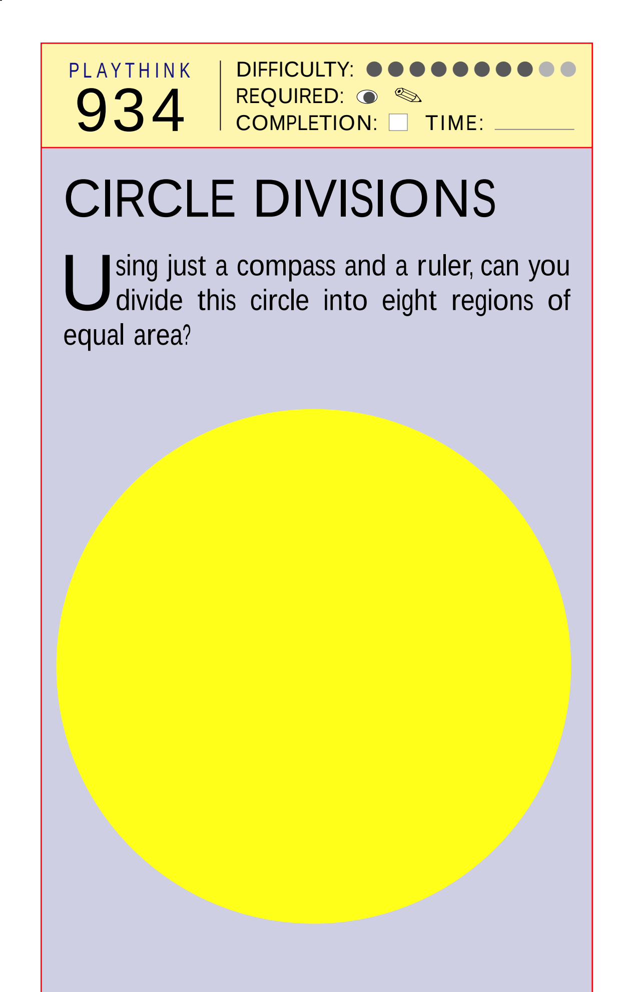
A centipede sits at the top corner of a three-dimensional solid structure, as shown. Can you find a route along the edges for the bug so that it visits each corner once and only once while not traveling along any edge more than once? (Note that its path will not include every edge.)



PLAYTHINK 934
DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ______

CIRCLE DIVISIONS

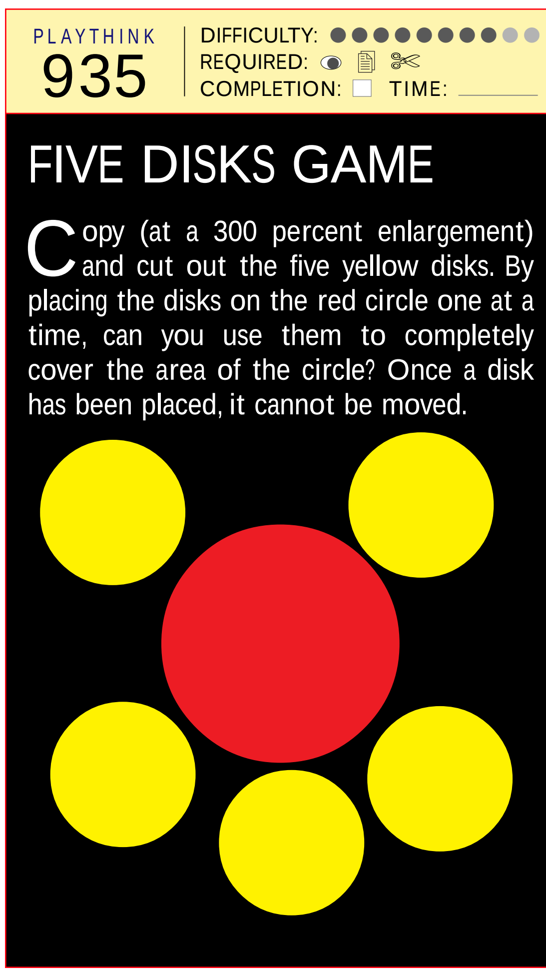
Using just a compass and a ruler, can you divide this circle into eight regions of equal area?



PLAYTHINK 935
DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ______

FIVE DISKS GAME

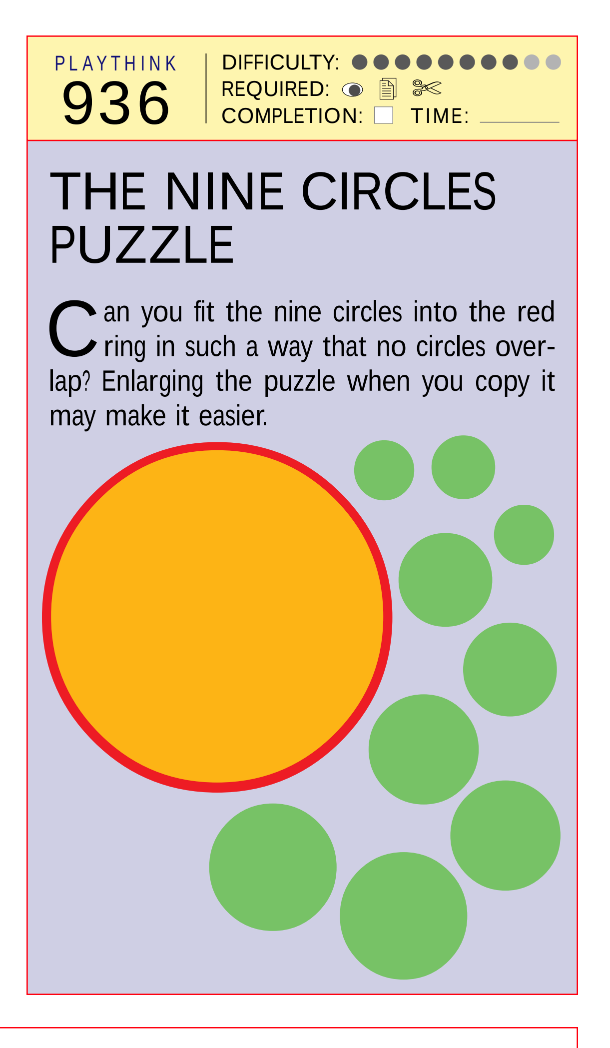
Copy (at a 300 percent enlargement) and cut out the five yellow disks. By placing the disks on the red circle one at a time, can you use them to completely cover the area of the circle? Once a disk has been placed, it cannot be moved.



PLAYTHINK 936
DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ______

THE NINE CIRCLES PUZZLE

Can you fit the nine circles into the red ring in such a way that no circles overlap? Enlarging the puzzle when you copy it may make it easier.

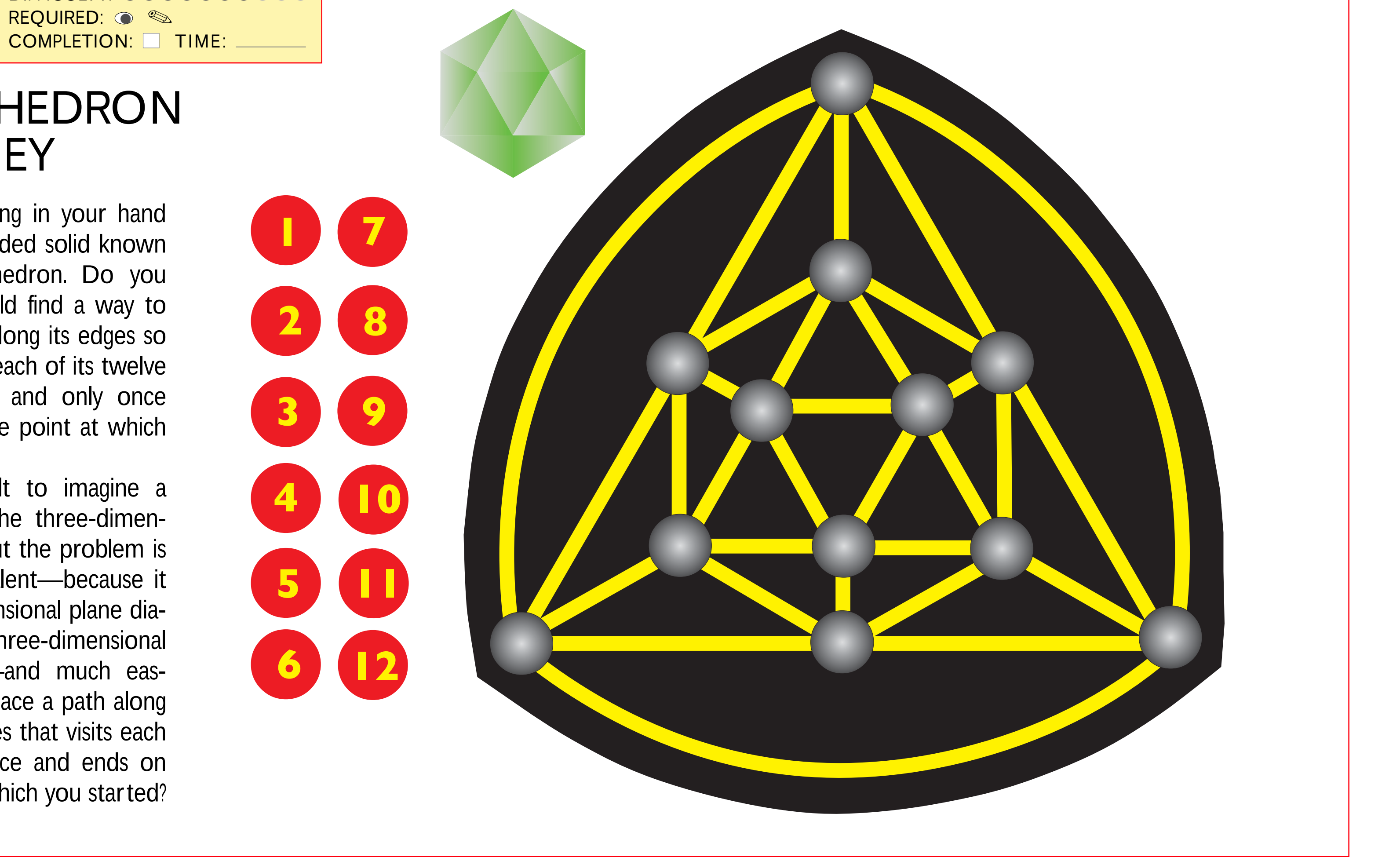


PLAYTHINK 937
DIFFICULTY: ●●●●●●●●●●
REQUIRED:
COMPLETION: ☐ TIME: ______

ICOSAHEDRON JOURNEY

Imagine holding in your hand the twenty-sided solid known as the icosahedron. Do you think you could find a way to trace a path along its edges so that you visit each of its twelve corners once and only once and end at the point at which you began?

It's difficult to imagine a solution for the three-dimensional solid. But the problem is exactly equivalent—because it is a two-dimensional plane diagram of the three-dimensional icosahedron—and much easier. Can you trace a path along the yellow lines that visits each circle only once and ends on the circle at which you started?

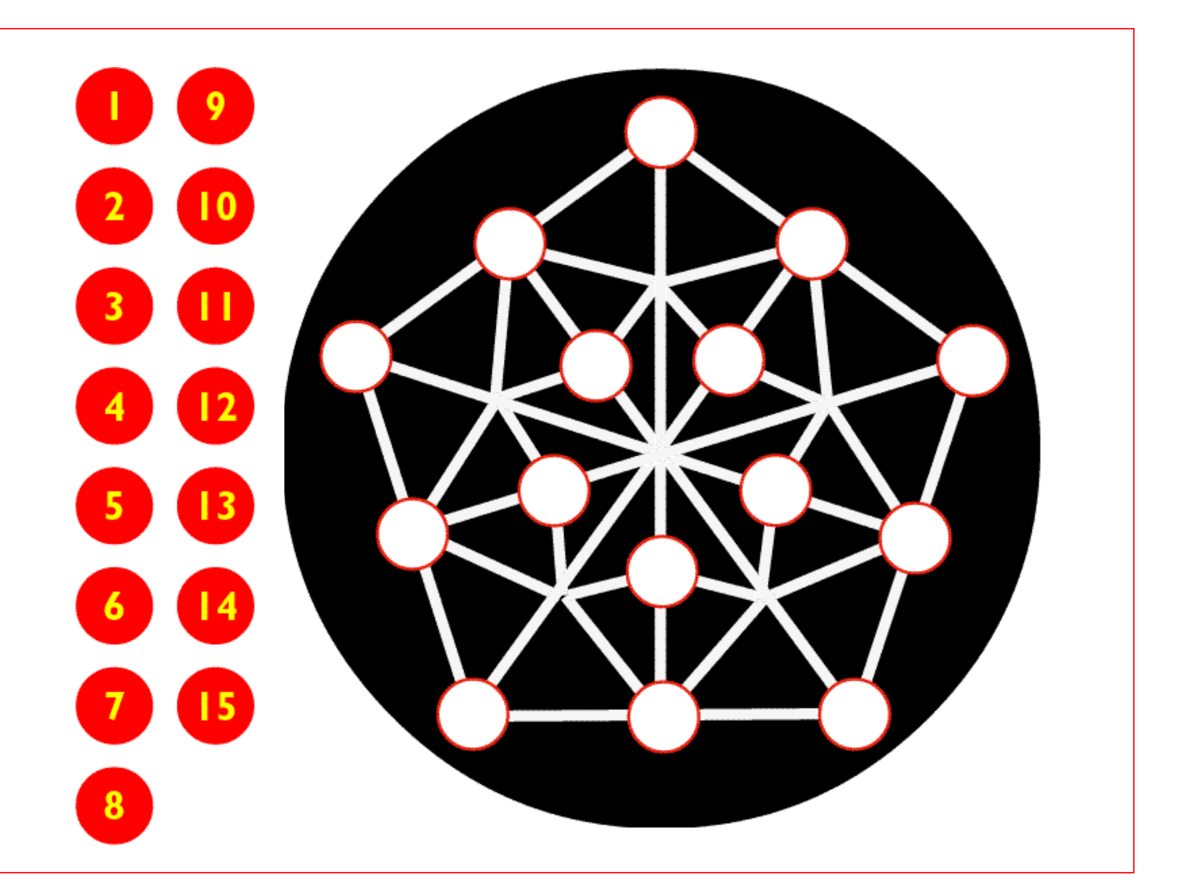


PLAYTHINK 938 | DIFFICULTY: ●●●●●●●●○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

TRAVELING IN CIRCLES

Only those who can follow the rules posted in this pentagonal garden are allowed to walk here. First, you may walk only along the paths. Second, you must visit each of the fifteen circles only once and leave a numbered marker to show the order of your visits. Third, as you leave each circle after the first one you decide to visit (which can be any of them), you must change directions so that you are not moving in a straight line. Fourth, you may walk along a path only once.

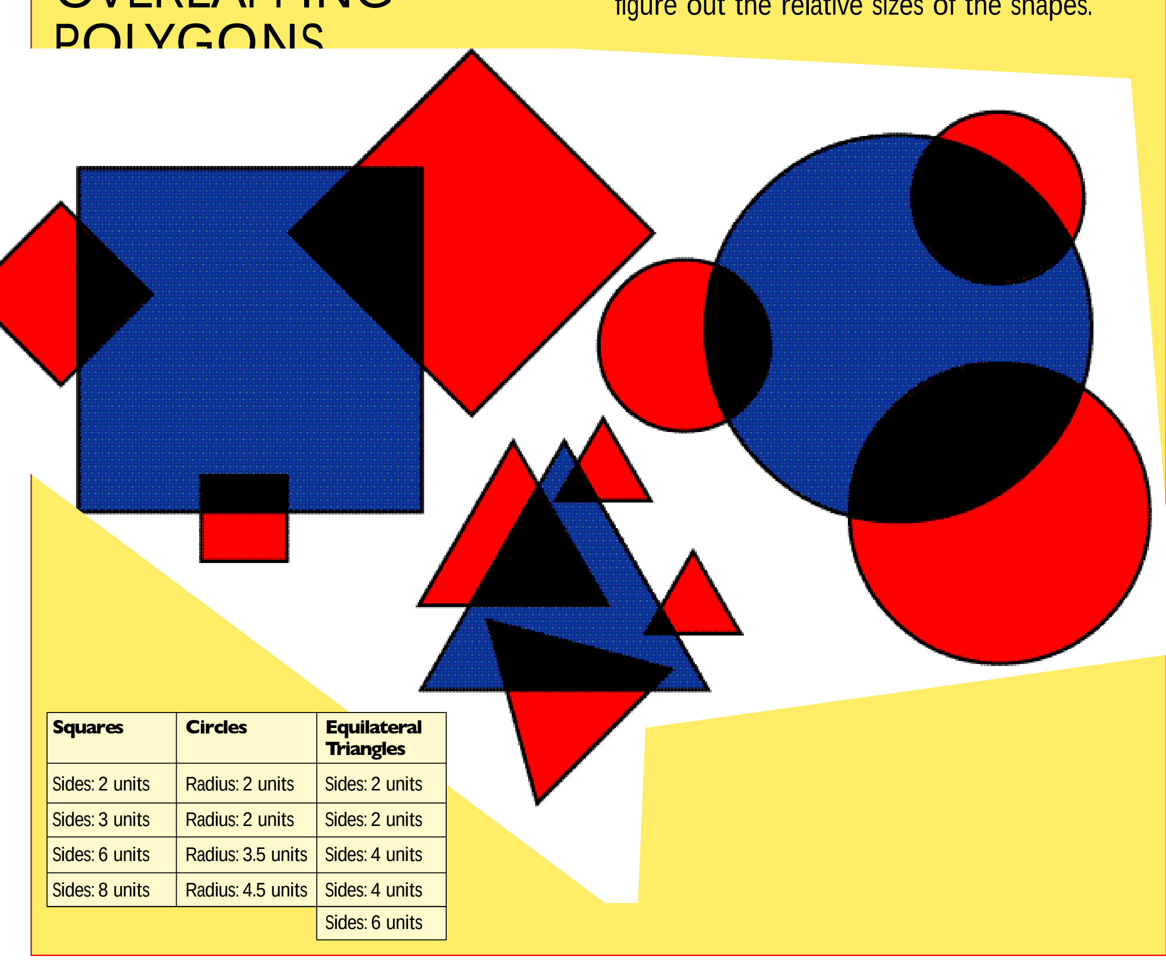
Can you find a route that will enable you to walk in the garden?



PLAYTHINK 939 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

OVERLAPPING POLYGONS

For each of the sets of overlapping shapes, can you work out which is larger: the sum of the uncovered red areas or the uncovered blue area in the middle? Refer to the box to figure out the relative sizes of the shapes.

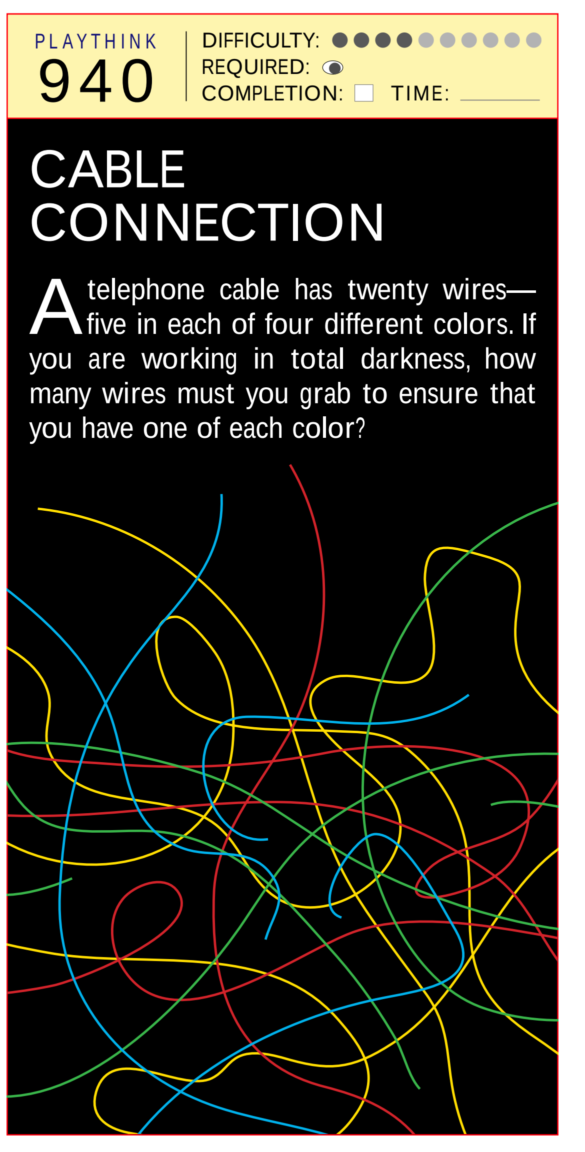


Squares	Circles	Equilateral Triangles
Sides: 2 units	Radius: 2 units	Sides: 2 units
Sides: 3 units	Radius: 2 units	Sides: 2 units
Sides: 6 units	Radius: 3.5 units	Sides: 4 units
Sides: 8 units	Radius: 4.5 units	Sides: 4 units
		Sides: 6 units

PLAYTHINK 940 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

CABLE CONNECTION

A telephone cable has twenty wires—five in each of four different colors. If you are working in total darkness, how many wires must you grab to ensure that you have one of each color?

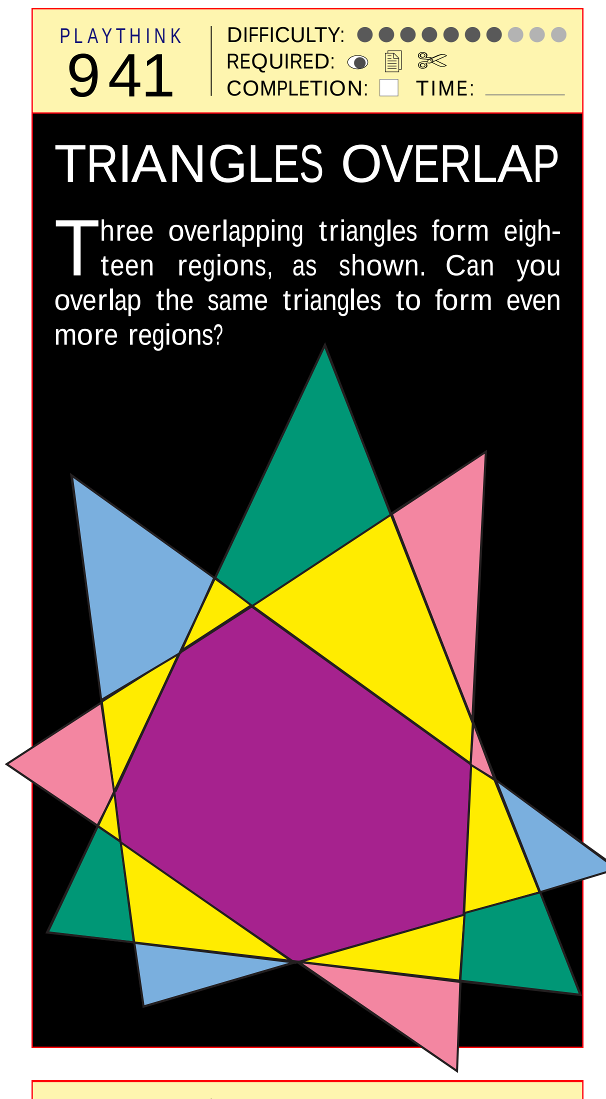


PLAYTHINK 941

DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁 📄 ✂
COMPLETION: ☐ TIME: ____

TRIANGLES OVERLAP

Three overlapping triangles form eighteen regions, as shown. Can you overlap the same triangles to form even more regions?



PLAYTHINK 942

DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁 ✏
COMPLETION: ☐ TIME: ____

WINNING HORSES

If seven horses have entered a race, how many different ways can the first three places be filled?

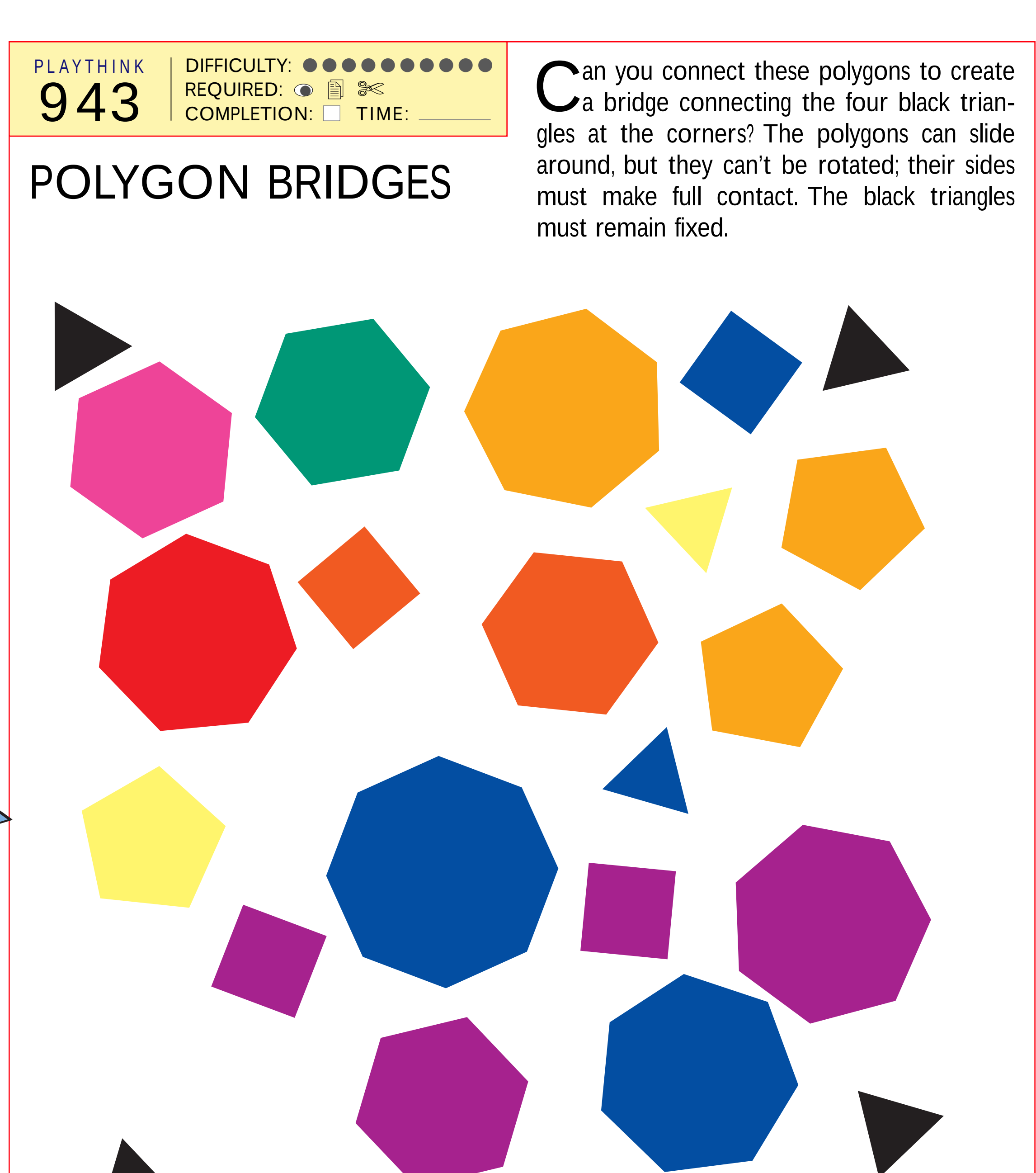


PLAYTHINK 943

DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁 📄 ✂
COMPLETION: ☐ TIME: ____

POLYGON BRIDGES

Can you connect these polygons to create a bridge connecting the four black triangles at the corners? The polygons can slide around, but they can't be rotated; their sides must make full contact. The black triangles must remain fixed.

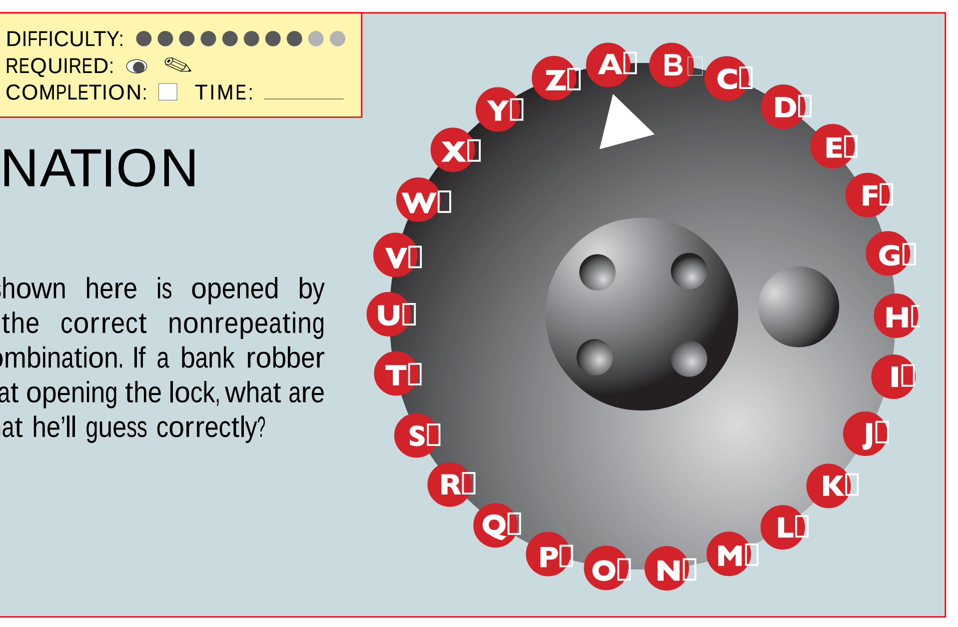


PLAYTHINK 944

DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁 ✏
COMPLETION: ☐ TIME: ____

COMBINATION LOCK

The lock shown here is opened by selecting the correct nonrepeating three-letter combination. If a bank robber has one guess at opening the lock, what are the chances that he'll guess correctly?



PLAYTHINK 945 | DIFFICULTY: ●●●●○○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ | TIME: ____

COVERED TRIANGLE

A right triangle is cut out of paper and folded as shown. Can you work out the relationship of the area visible in red to that of the original triangle?

PLAYTHINK 946 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ | TIME: ____

MY CLASS

In a class of fifteen boys, fourteen have blue eyes, twelve have black hair, eleven are overweight and ten are tall. Can you work out how many tall, overweight, black-haired, blue-eyed boys there must be?

PLAYTHINK 947 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ | TIME: ____

LICENSE PLATES

In many countries automobile license plates take the form shown here: one letter, followed by three numbers, followed by three letters.

In such a country how many different license plates are possible?

A 234 HIL

PLAYTHINK 948 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ | TIME: ____

WALKING DOGS

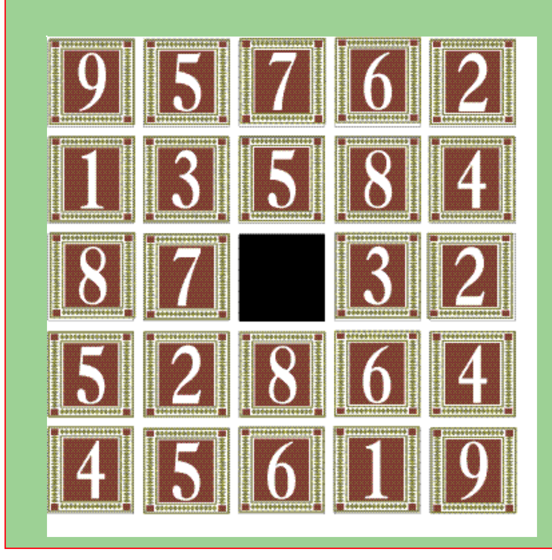
Beatrice has six dogs to walk. If she walks them two at a time, how many different pairs of dogs can she take out?



PLAYTHINK 949 | DIFFICULTY: ●●●●●●●●○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ | TIME: ____

MAGIC GRID MATRIX 1

Examine this matrix of numbers. Can you divide it into eight parts in such a way that the digits in each part will add up to the same total?



9	5	7	6	2
1	3	5	8	4
8	7		3	2
5	2	8	6	4
4	5	6	1	9

PLAYTHINK 950 | DIFFICULTY: ●●●●●●●●●○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ | TIME: ____

MAGIC GRID MATRIX 2

Can you divide this matrix along the grid lines into sixteen identical parts? No two parts may have the same numbers, and the sum of the numbers in each part must total 34.



2	3	13	16	3	2	14	15
5	8	10	11	4	9	10	11
11	10	9	4	13	16	1	2
16	13	4	1	14	7	9	6
4	5	10	13	5	8	10	11
3	6	11	16	2	14	8	10
12	10	9	3	11	5	3	15
15	13	4	2	16	12	5	1

PLAYTHINK 951

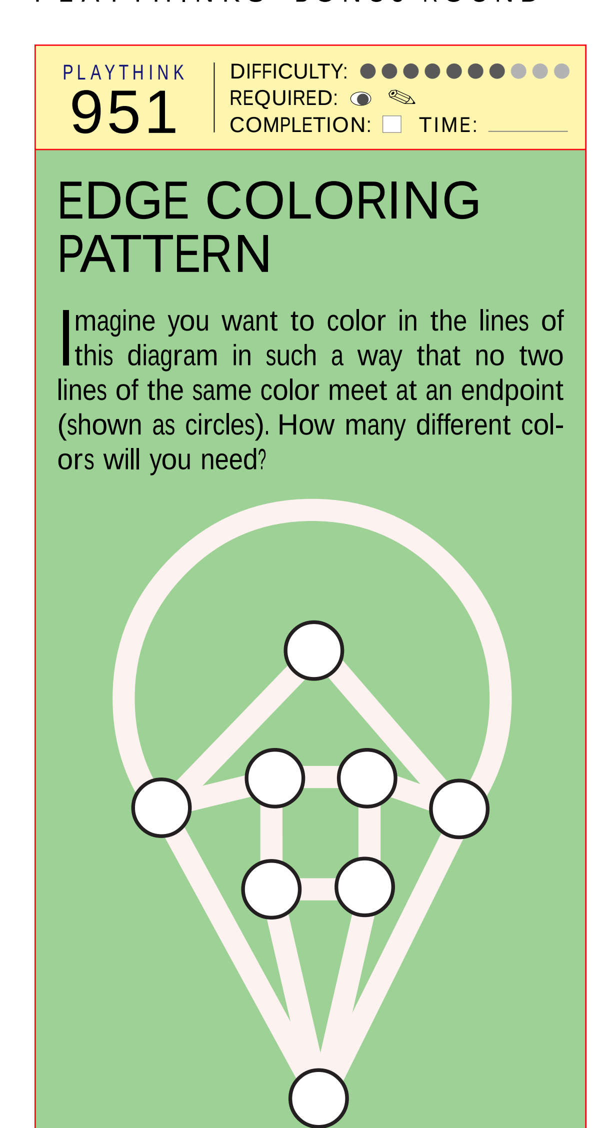
DIFFICULTY: ●●●●●●●●○○

REQUIRED:

COMPLETION: ☐ TIME: ______

EDGE COLORING PATTERN

Imagine you want to color in the lines of this diagram in such a way that no two lines of the same color meet at an endpoint (shown as circles). How many different colors will you need?



PLAYTHINK 952

DIFFICULTY: ●●●●●●●●●○○

REQUIRED:

COMPLETION: ☐ TIME: ______

CUBES IN PERSPECTIVE 2

If you rearrange these twenty-five tiles without rotating them, you can create an image containing a number of cubes. How many cubes does that image contain?



PLAYTHINK 953

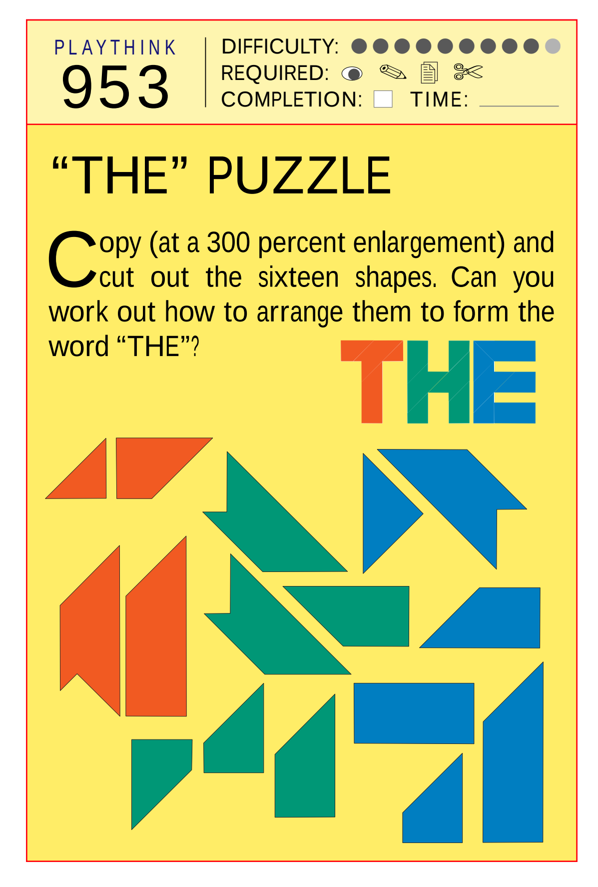
DIFFICULTY: ●●●●●●●●●○

REQUIRED:

COMPLETION: ☐ TIME: ______

"THE" PUZZLE

Copy (at a 300 percent enlargement) and cut out the sixteen shapes. Can you work out how to arrange them to form the word "THE"?



PLAYTHINK 954

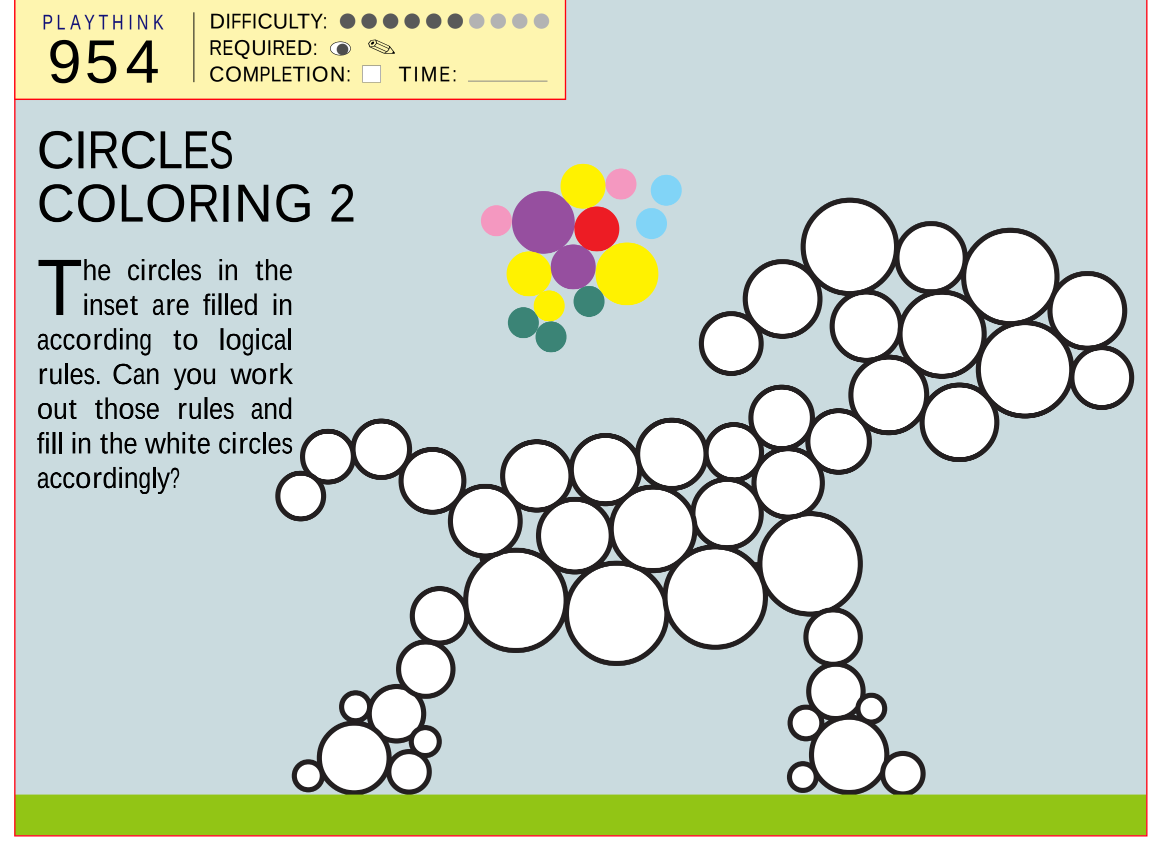
DIFFICULTY: ●●●●●●○○○○

REQUIRED:

COMPLETION: ☐ TIME: ______

CIRCLES COLORING 2

The circles in the inset are filled in according to logical rules. Can you work out those rules and fill in the white circles accordingly?

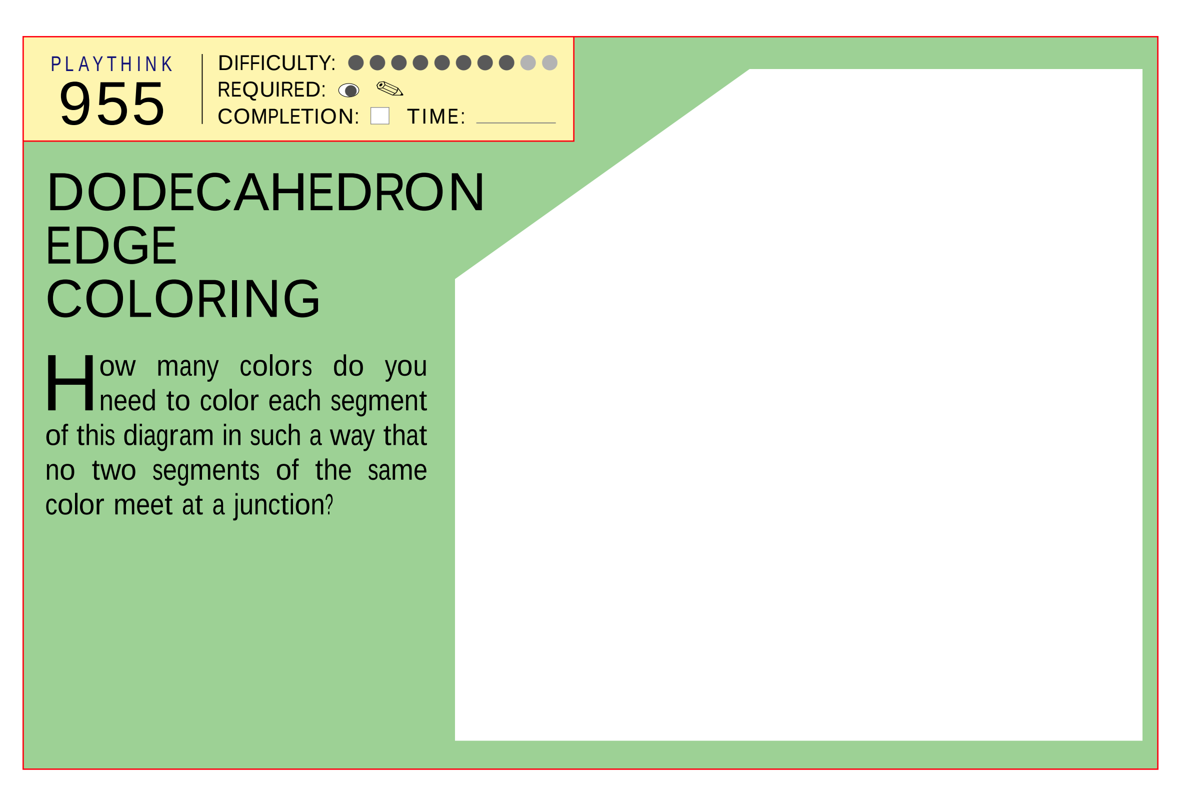


PLAYTHINK 955

DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

DODECAHEDRON EDGE COLORING

How many colors do you need to color each segment of this diagram in such a way that no two segments of the same color meet at a junction?



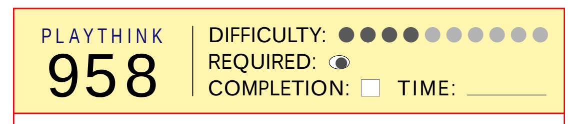
PLAYTHINK 957

DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

THREE-DIGIT NUMBER

A toy robot has a three-digit electronic display on its front. The only digits it can display are 1, 2 and 3. Can you work out how many different numbers the robot can display?



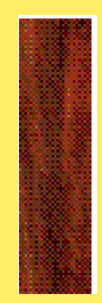


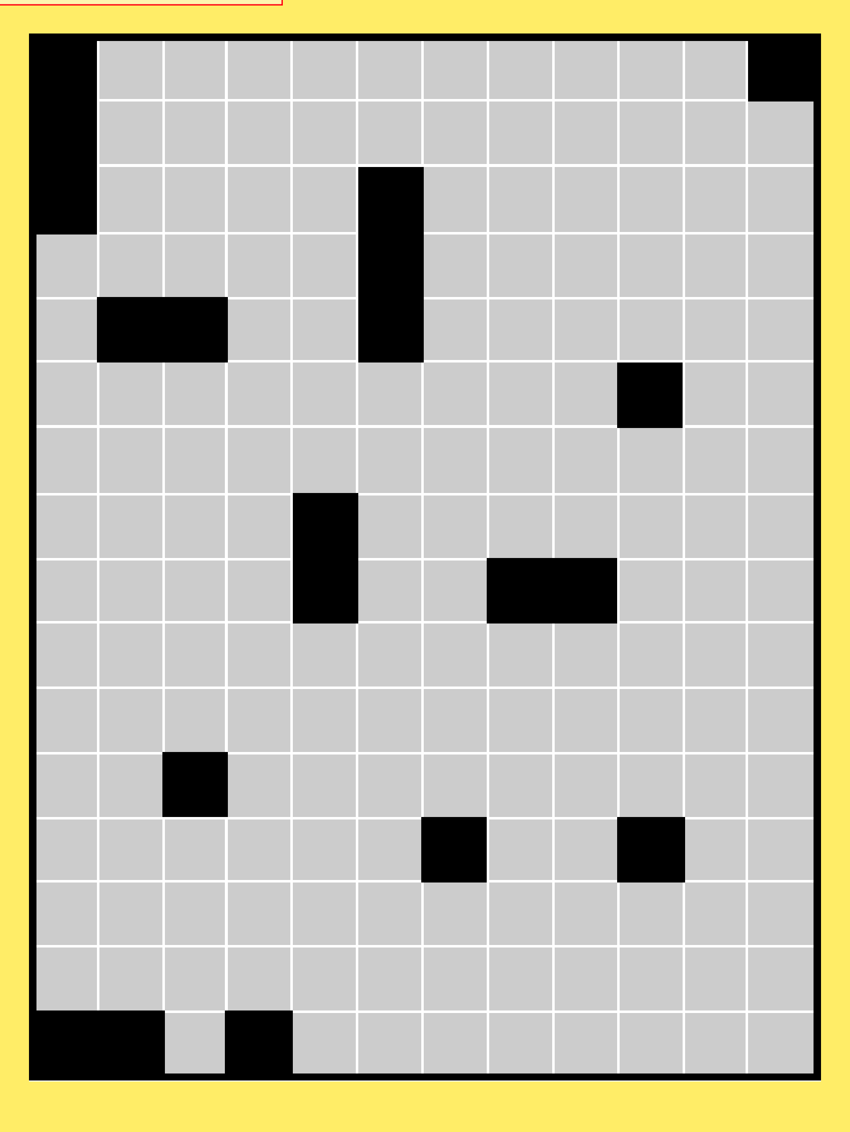
PLAYTHINK 956

DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

PARQUET

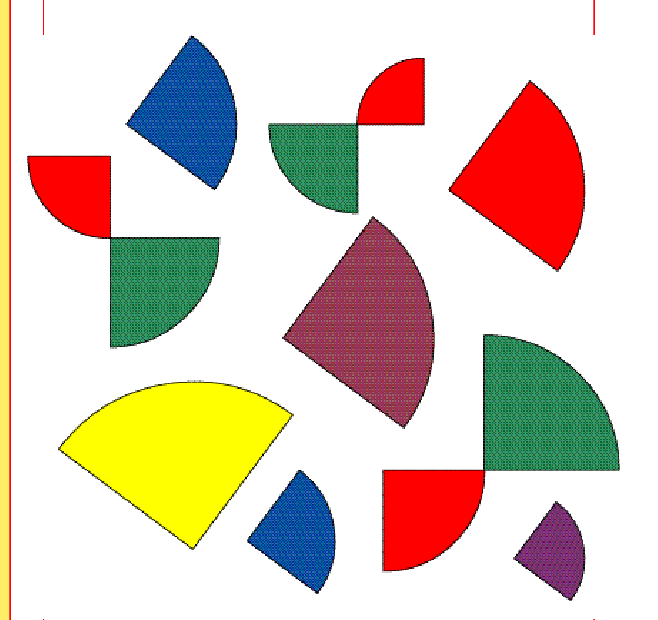
The floor plan of an odd room is shown at right; the black squares and rectangles indicate where columns and fixtures fit into the floor. Can you find a way to completely cover the floor with uncut wooden planks that are 1 unit by 4 units?





PLAYTHINK 958

DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁
COMPLETION: ☐ TIME: ______



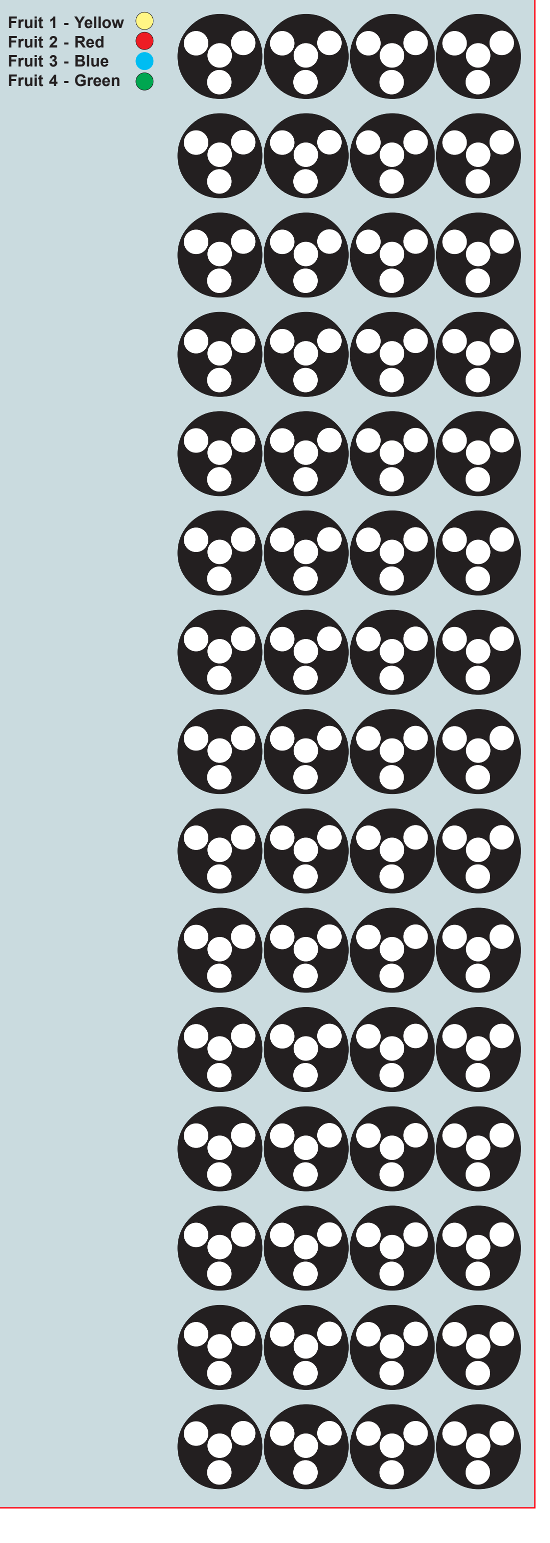
EQUAL AREAS 2

This diagram shows three pairs of quarter circles in contact with each other, as well as a number of isolated quarter circles of various sizes. It turns out that the sum of the area of each pair of quarter circles is equal to the area of one of the single quarter circles shown. Can you work out which quarter circle goes with which pair? Can you guess which geometric property ensures that the areas are exactly equal?

PLAYTHINK 959 DIFFICULTY: ●●●●●●●●●○ REQUIRED: COMPLETION: ☐ TIME: ______

FRUITS ON FOUR PLATES

A hostess has four pieces of fruit and four identical unlabeled plates. Can you find all the different ways she can serve those four pieces of fruit? You can use the blank diagram below and four colors of pencil to help you find them all.

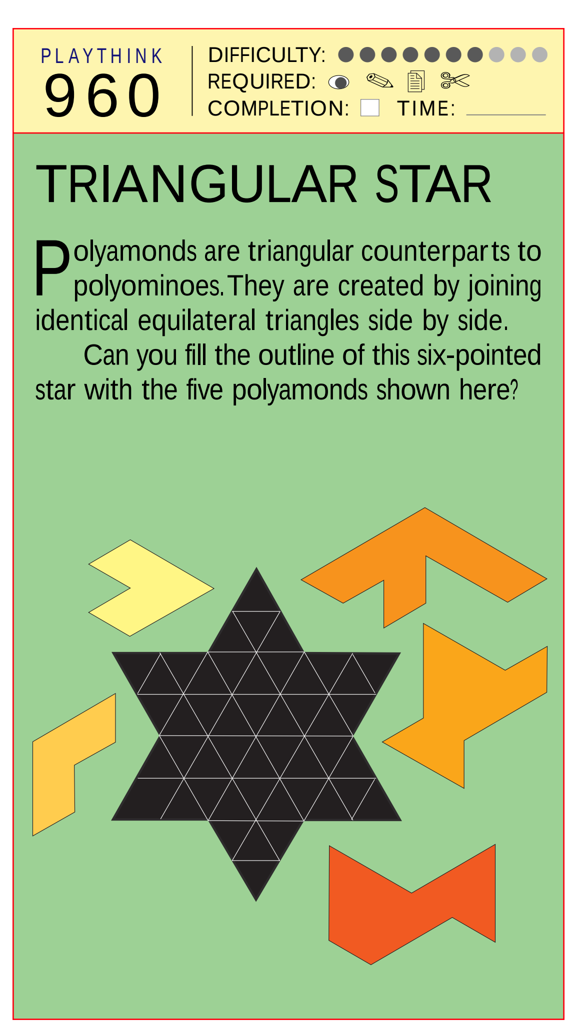


PLAYTHINK 960 DIFFICULTY: ●●●●●●●○○○ REQUIRED: COMPLETION: ☐ TIME: ______

TRIANGULAR STAR

Polyamonds are triangular counterparts to polyominoes. They are created by joining identical equilateral triangles side by side.

Can you fill the outline of this six-pointed star with the five polyamonds shown here?

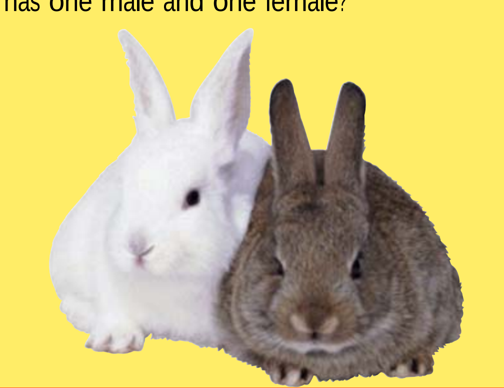


PLAYTHINK 961 DIFFICULTY: ●●●●●●●●○○ REQUIRED: COMPLETION: ☐ TIME: ______

FIBONACCI RABBITS

In 1202 Leonardo Fibonacci, a twenty-seven-year-old Italian mathematician, published a book called *Liber Abaci*. In that groundbreaking work, he wrote the following puzzle:

Every month a breeding pair of rabbits (one male, one female) produces one new pair of rabbits—also one male, one female. That new pair begins breeding two months later. How many pairs of rabbits can be produced from a single pair of rabbits in one year, assuming no rabbits die and every pair has one male and one female?

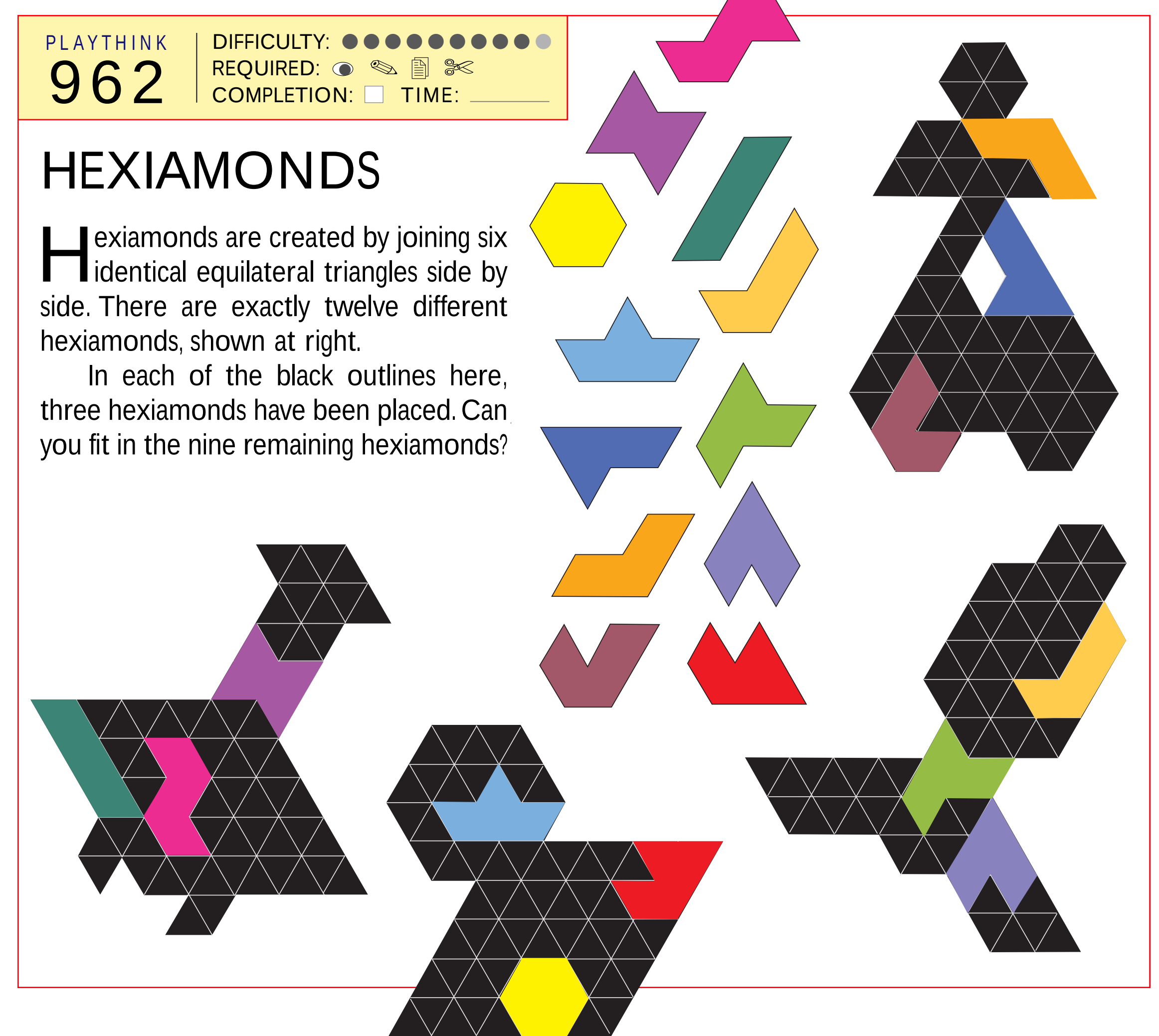


PLAYTHINK 962 DIFFICULTY: ●●●●●●●●●○ REQUIRED: COMPLETION: ☐ TIME: ______

HEXIAMONDS

Hexiamonds are created by joining six identical equilateral triangles side by side. There are exactly twelve different hexiamonds, shown at right.

In each of the black outlines here, three hexiamonds have been placed. Can you fit in the nine remaining hexiamonds?



PLAYTHINK 963

DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁 ✎ 📄 ✂
COMPLETION: ☐ TIME: ______

PENTAHEX HONEYCOMB

All 22 different ways to join five regular hexagons edge to edge are shown here. Such combinations are called pentahexes.

If you're by yourself, try to cover the entire 110-hex board below with the 22 possible pentahexes.

If you're with a friend, take turns placing pentahexes along the grid lines of the board. The last player able to place a shape successfully wins the game.

PLAYTHINK 964

DIFFICULTY: ●●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

SUM-FREE GAME

In this two-person game, which I heard described in a lecture by American graph theorist Frank Harary, players alternate placing consecutive numbers (starting with 1) in either of the two columns. A player can place a number in a column only if it does not already possess two numbers that add up to that number. For example, in this sample game, the player whose turn it is must play an 8 but is blocked from doing so because the first column contains 1 and 7 and the second column contains 3 and 5. The last player to place a number wins.

Can you work out the moves for player 2 so she or he will win every time no matter what player 1 does?

Can you work out the longest possible game?

Sample

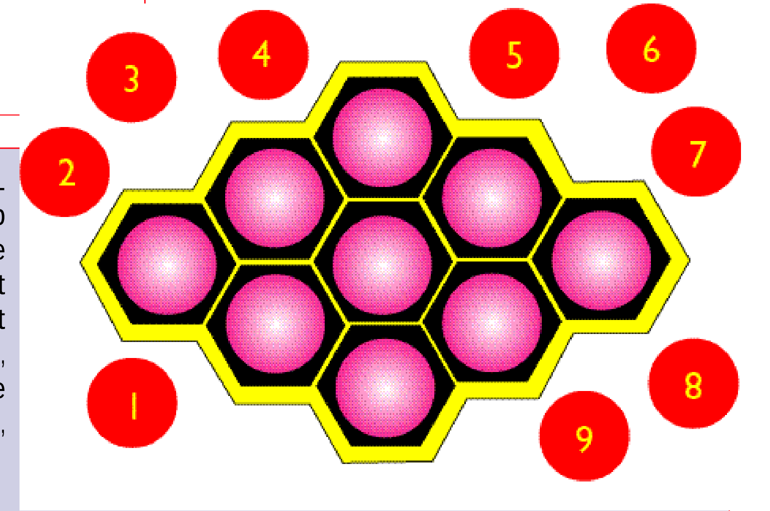
COLUMN 1	COLUMN 2
1	3
2	5
4	6
7	

PLAYTHINK 965

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

MATHEMAGIC HONEYCOMB

Can you place the numbers 1 through 9 in this honeycomb so that, for any given hexagon, the sum of the numbers in the adjacent hexagons will be a multiple of that hexagon's number? For example, if a hexagon contains a 5, the adjacent hexes must total 5, 10, 15, 25 and so on.

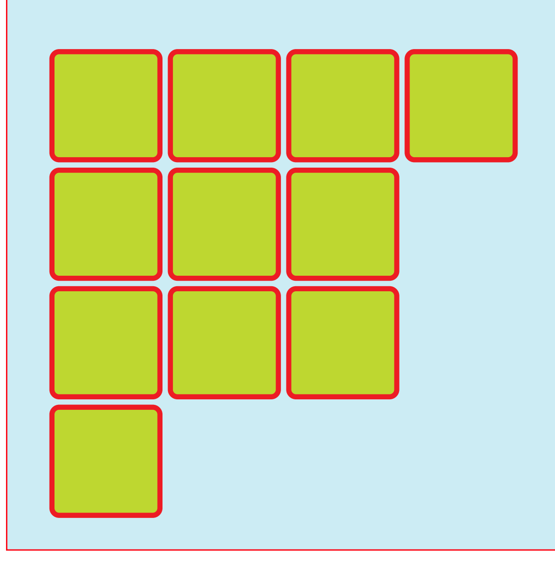


PLAYTHINK 966 | DIFFICULTY: ●●●●●●●●○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

AN ARRAY OF SOLDIERS

Each of eleven army units (represented here by the green squares) has an identical number of soldiers. If you add the general to the total number, the soldiers can be rearranged to form a single perfect array of fighting personnel.

What's the minimum number of soldiers that must be in each army unit? How many soldiers—including the general—are in the array?



PLAYTHINK 967 | DIFFICULTY: ●●●●●●●●○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

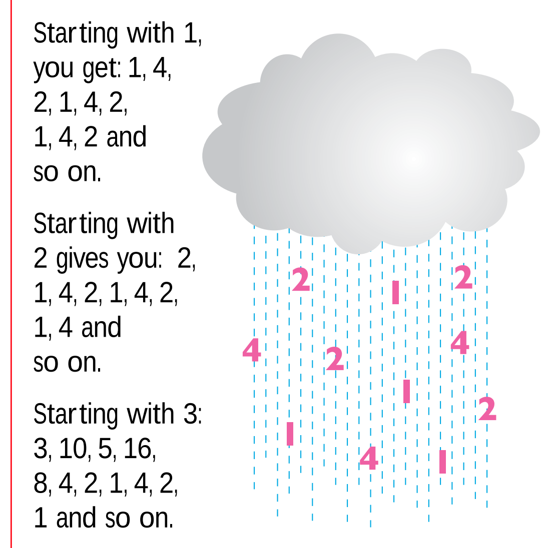
HAILSTONE NUMBERS

Think of a number. If it is odd, triple it and add 1; if it is even, divide it by 2. Apply this rule to each new number you obtain. Can you see what eventually happens?

Starting with 1, you get: 1, 4, 2, 1, 4, 2, 1, 4, 2 and so on.

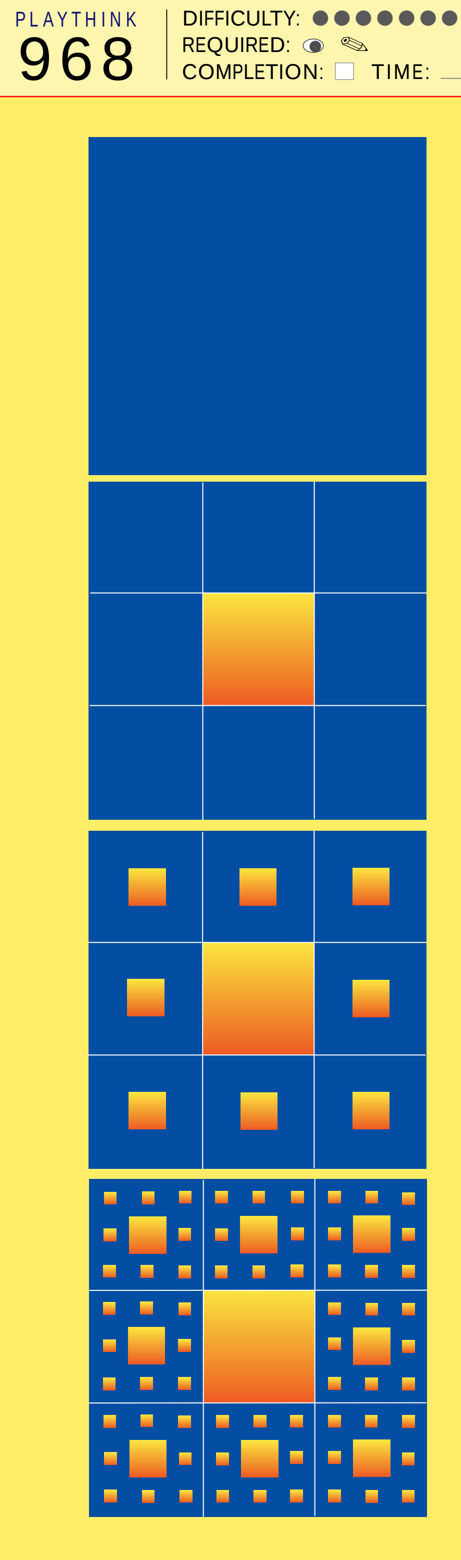
Starting with 2 gives you: 2, 1, 4, 2, 1, 4, 2, 1, 4 and so on.

Starting with 3: 3, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1 and so on.



It quickly becomes apparent that the above sequences get stuck in a loop of 1-4-2-1-4-2. But will every sequence run into that inescapable routine? Test your idea by starting with 7.

PLAYTHINK 968 | DIFFICULTY: ●●●●●●●●○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____



SQUARES IN SQUARES

A blue square is divided into nine smaller squares, and the middle one is painted gold. The eight remaining blue squares are divided into nine, and the middle square of those is painted gold. If this process continues indefinitely, can you work out the eventual area of the gold squares in relation to the original blue square?

PLAYTHINK 969 | DIFFICULTY: ●●●●●●●●●○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

MAXIMUM OVERHANG

Nine identical planks, each 1 meter in length, are stacked with the bottom plank nailed to the floor, as shown. Can you move the eight other planks to achieve the maximum overhang for the top plank? Will that overhang be sufficient for the mouse to cross over the planks to reach the cheese on a platform 1.4 meters away?

1 meter

1.4 meters

PLAYTHINK 970

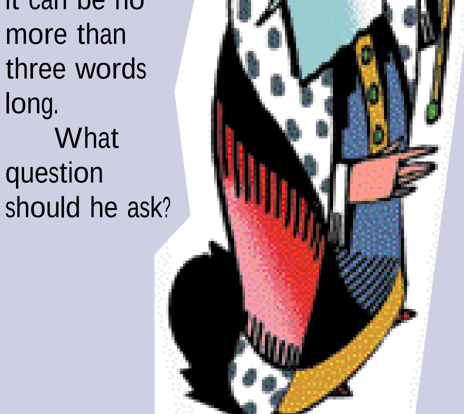
DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

TRUTH AND MARRIAGE

The king has two daughters—the virtuous Amelia, who always tells the truth, and the wicked Leila, who always lies. One of them is married, and one of them is not—but the king has kept the details of the marriage a secret, even down to which of the daughters is wedded.

To find a suitable mate for the other daughter, the king has organized a joust. The winner gets to name which of the daughters he wants to marry; if she is available, they will wed the next day. The man who wins asks the king if he may talk to the daughters. The king says he may ask one of the daughters one question, but it can be no more than three words long.

What question should he ask?



PLAYTHINK 971

DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____



HOTEL INFINITY

This problem is a favorite introduction to the weirdness of infinite numbers:

You are the manager of the Hotel Infinity, an inn that has an infinite number of rooms. No matter how crowded the hotel is, you know that you can always make room for one more guest: you simply move the person in room 1 to room 2, the person in room 2 to room 3, the person in room 3 to room 4 and so on. After all the guests have been moved, you check the new guest into room 1.

Unfortunately, just as you are about to go off-duty, a group of people arrive for a convention. The topic must be very popular, because there are an infinite number of new guests. If you already *have* an infinite number of guests, how can you accommodate the newcomers?

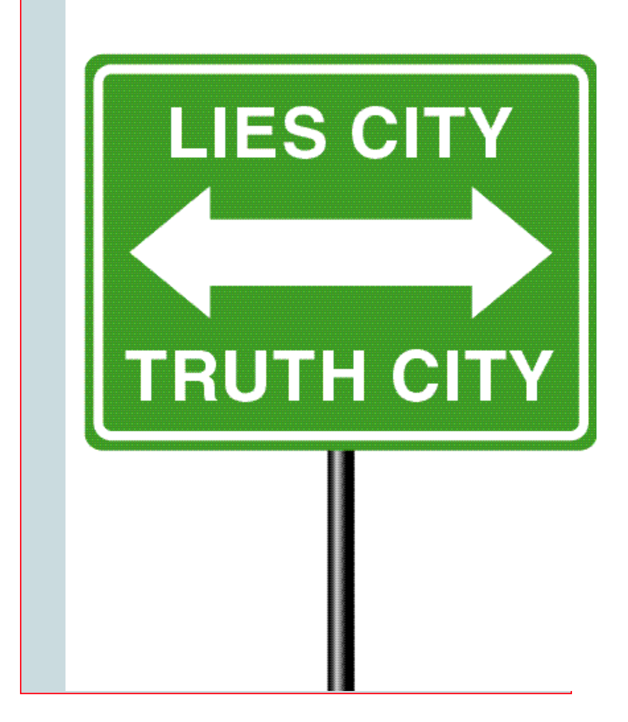
PLAYTHINK 972

DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

TRUTH CITY

You are on your way to Truth City, where the inhabitants always tell the truth. At one point you reach a fork in the road, with one branch leading to Truth City and the other leading to Lies City, where the citizens are all liars. The road signs at the junction are, as you can imagine, confusing, but there is a man standing there from whom you can ask directions. The only problem is, you don't know where he is from—the city where everyone always gives the right answer or the city where everyone lies.

If you have time to ask him only one question, what question will ensure that you will be headed in the right direction?

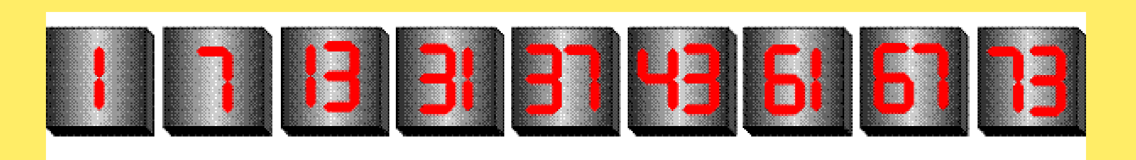


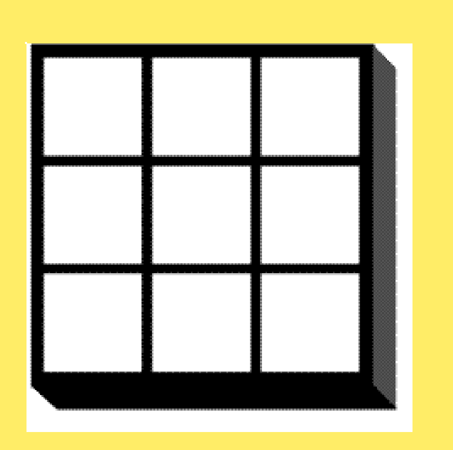
PLAYTHINK 973

DIFFICULTY: ●●●●●●●●●●
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

MAGIC PRIMES SQUARE

Can a magic square be made up of only prime numbers and 1? Henry Ernest Dudeney, the greatest English puzzle inventor, was the first to construct such a square, using the numbers 1, 7, 13, 31, 37, 43, 61, 67 and 73. Can you fit those numbers into a three-by-three grid to form a magic square?





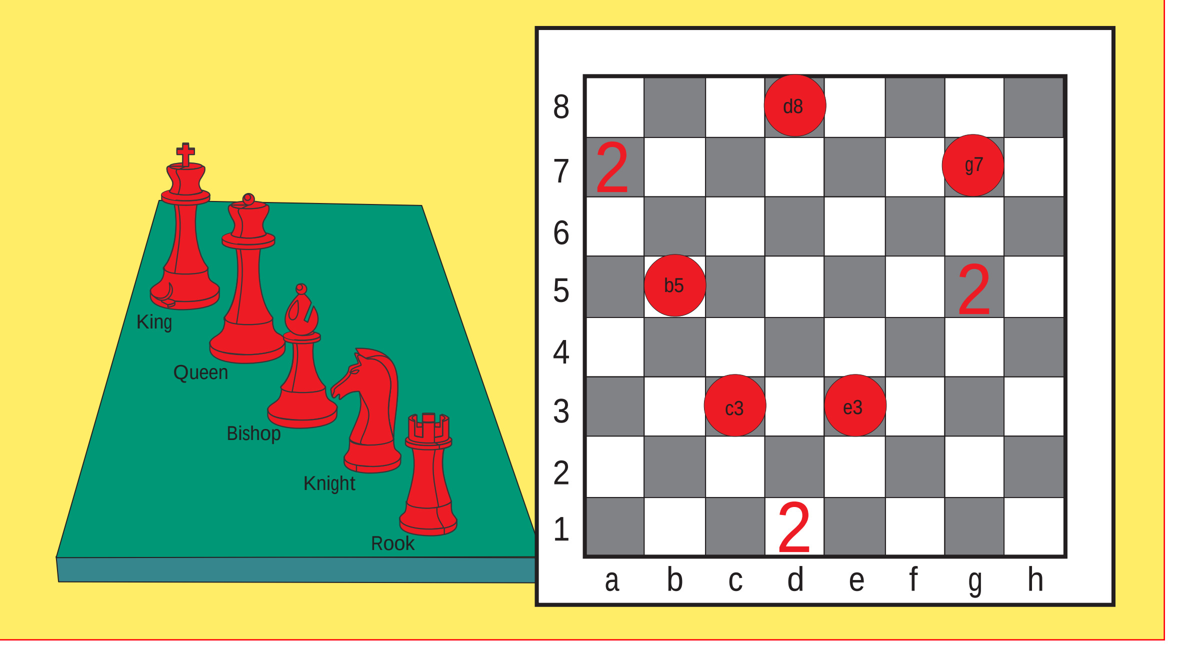
PLAYTHINK 974

DIFFICULTY: ●●●●●●●●●○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

GUESS CHESS

Five chess pieces—a king, a queen, a bishop, a knight and a rook—are to be placed on a chessboard. The pieces must each occupy a square marked with a red circle, and they must be placed in such a way that two of the pieces can attack the squares marked with a red 2.

Can you work out where each of the pieces must be placed?

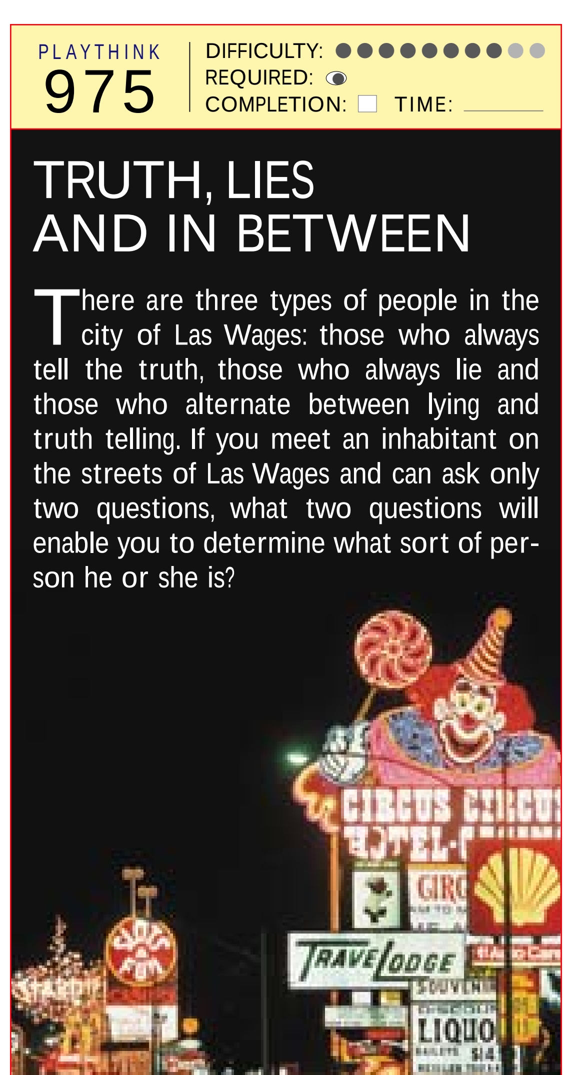


PLAYTHINK 975

DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

TRUTH, LIES AND IN BETWEEN

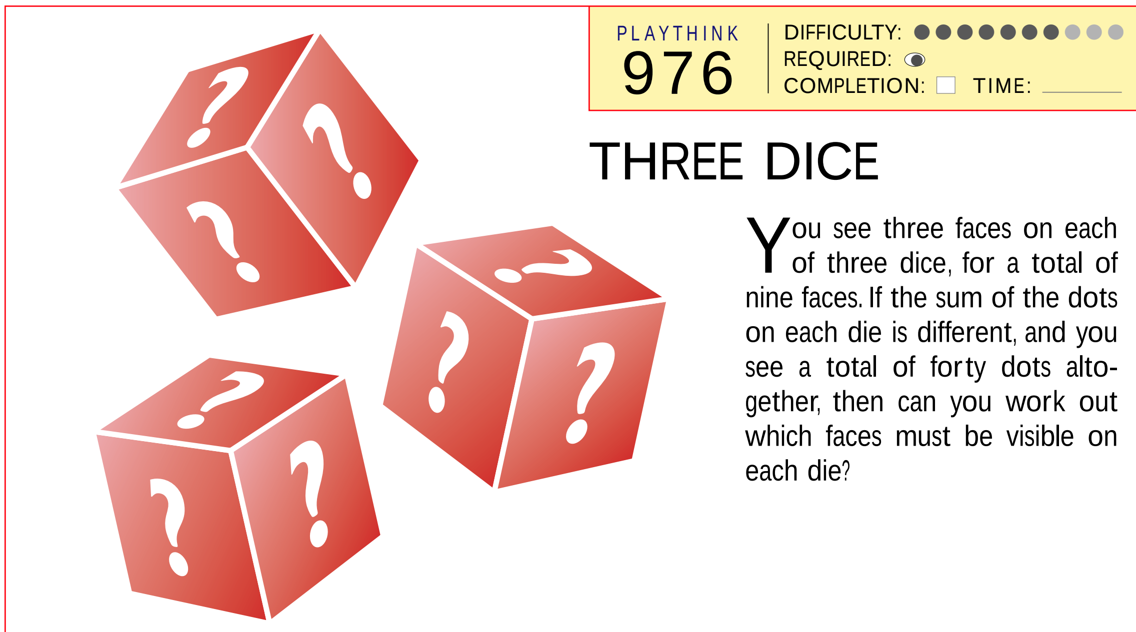
There are three types of people in the city of Las Wages: those who always tell the truth, those who always lie and those who alternate between lying and truth telling. If you meet an inhabitant on the streets of Las Wages and can ask only two questions, what two questions will enable you to determine what sort of person he or she is?



PLAYTHINK 976

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

THREE DICE



You see three faces on each of three dice, for a total of nine faces. If the sum of the dots on each die is different, and you see a total of forty dots altogether, then can you work out which faces must be visible on each die?

PLAYTHINK 977

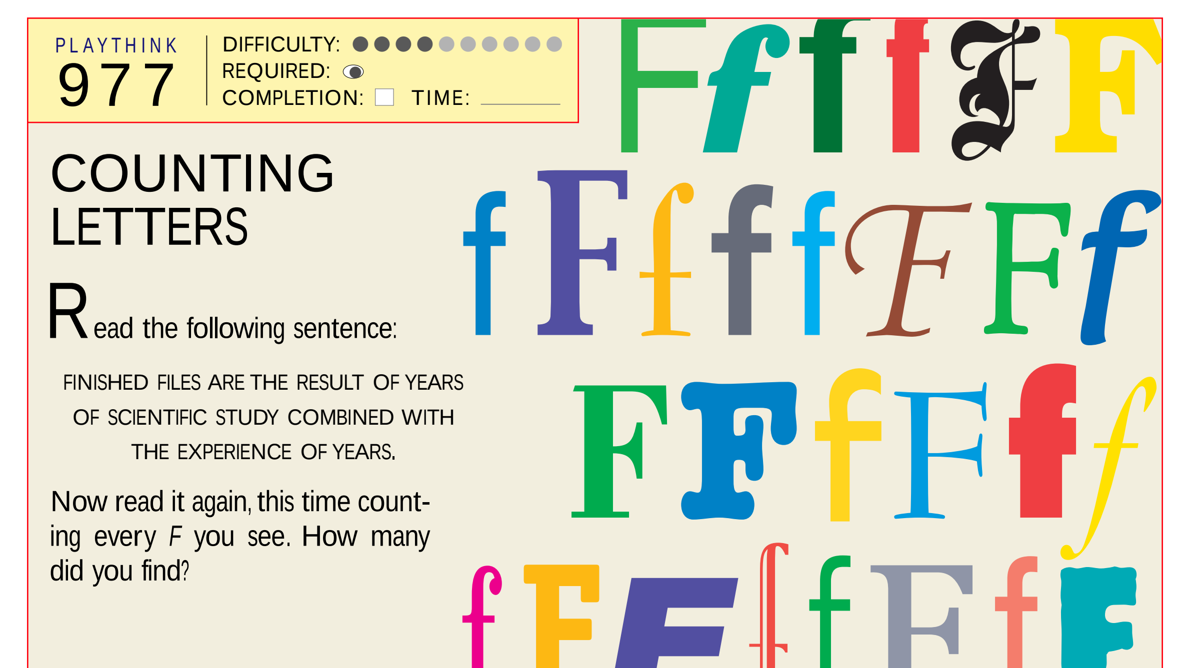
DIFFICULTY: ●●●●○○○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ______

COUNTING LETTERS

Read the following sentence:

FINISHED FILES ARE THE RESULT OF YEARS OF SCIENTIFIC STUDY COMBINED WITH THE EXPERIENCE OF YEARS.

Now read it again, this time counting every *F* you see. How many did you find?

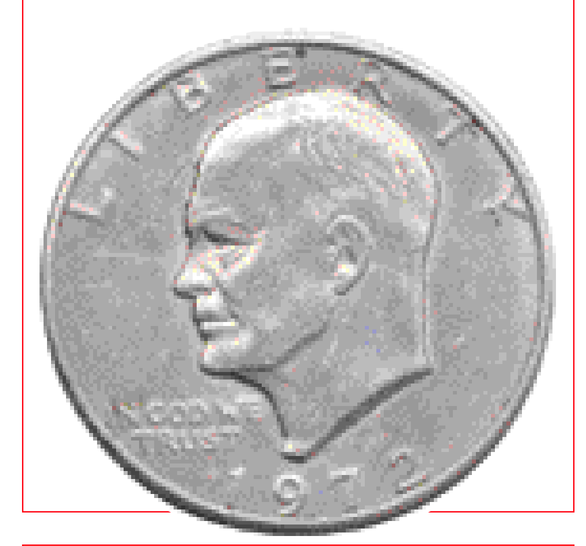


PLAYTHINK 978

DIFFICULTY: ●●●●●●●●●○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

FLIPPING COIN GAME

Two boys play a simple game: They take turns flipping a coin, and the first to throw "heads" wins. Can you work out whether one player can gain an advantage even if the coin is fair?

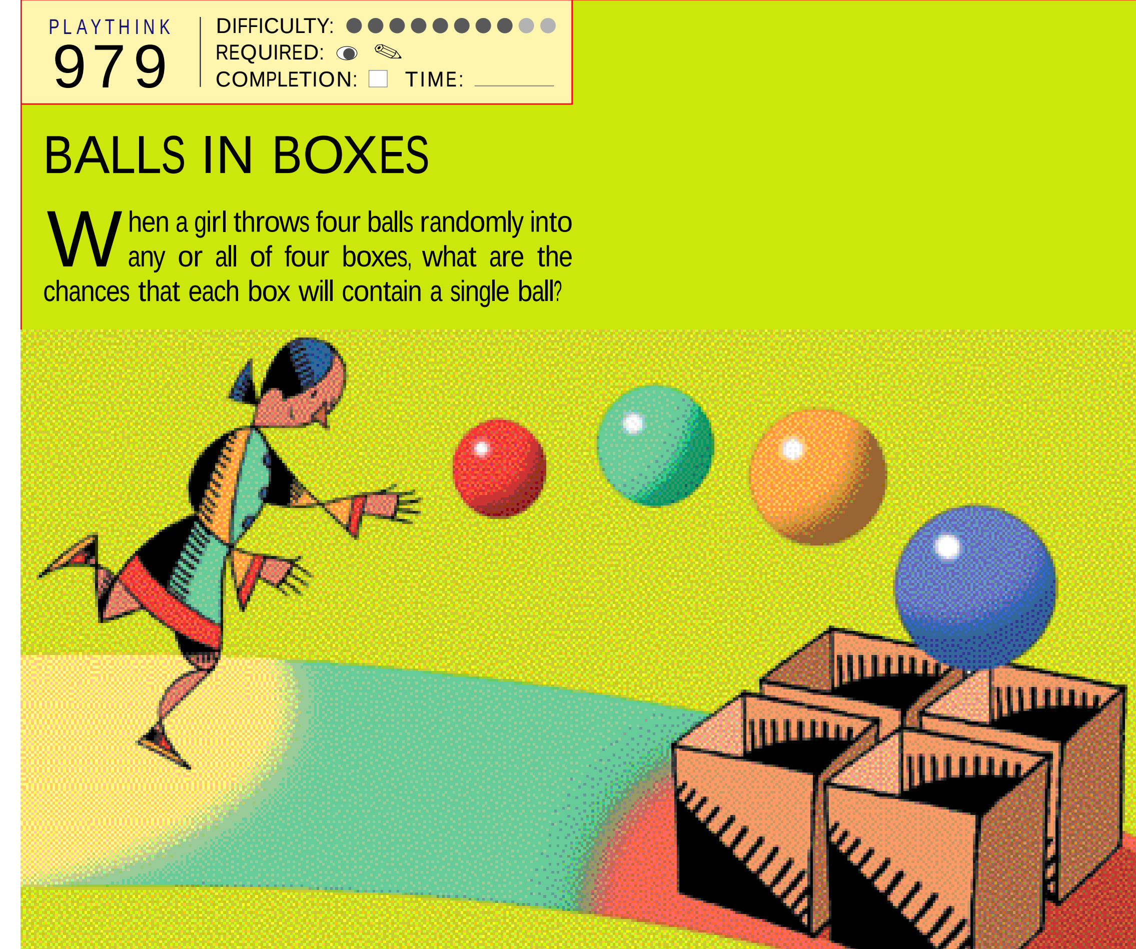


PLAYTHINK 979

DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

BALLS IN BOXES

When a girl throws four balls randomly into any or all of four boxes, what are the chances that each box will contain a single ball?



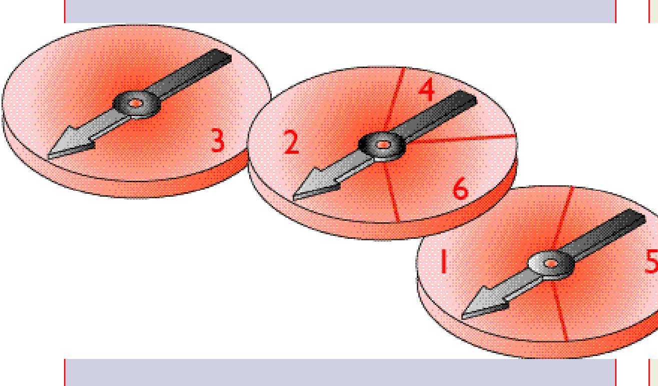
PLAYTHINK 980

DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

SPINNERS GAME

The object of this game is simple: spin the higher number. You and your opponent may each choose one of three spinners. The first spinner has only one number, 3. The second spinner is divided 56 percent for number 2, 22 percent for number 4 and 22 percent for number 6. The third spinner is divided 51 percent for number 1 and 49 percent for number 5.

Can you work out which spinner is the best to choose?



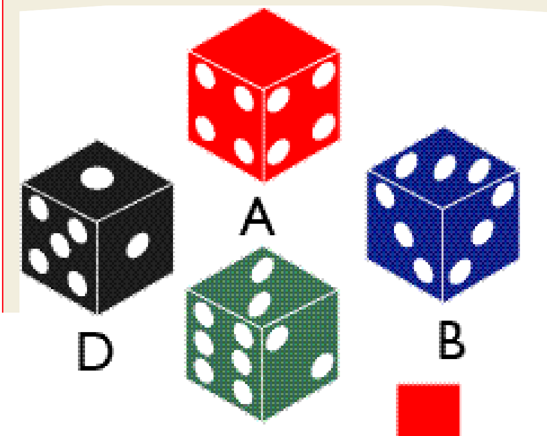
PLAYTHINK 981

DIFFICULTY: ●●●●●●●●●○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ______

NONTRANSITIVE DICE

The mathematical property of transitivity states that if A is greater than B, and B is greater than C, then A is greater than C. But certain games appear to flout this logic. One common nontransitive game is "Rock, Paper, Scissors," the children's game that displays circular logic: Scissors cut paper, paper wraps rock, rock breaks scissors.

A special set of dice, shown here, also displays this nontransitive logic. If you play a two-person betting game with these dice, always allow your opponent to select his or her die first. No matter which die your opponent chooses, you can select a die that will give you an advantage. Can you work out how?

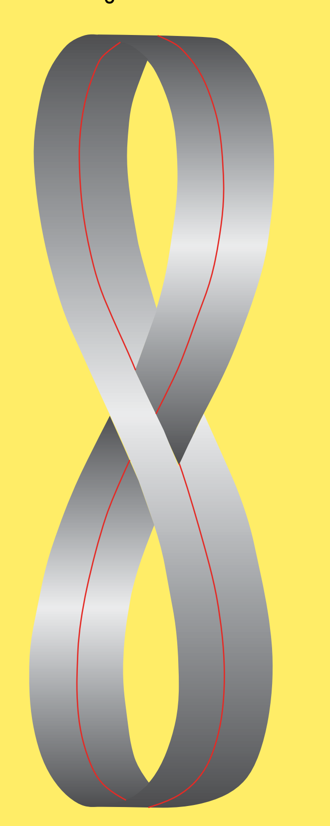


A B C D

PLAYTHINK 982 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✂ | COMPLETION: ☐ TIME: ____

SLOTTED BAND

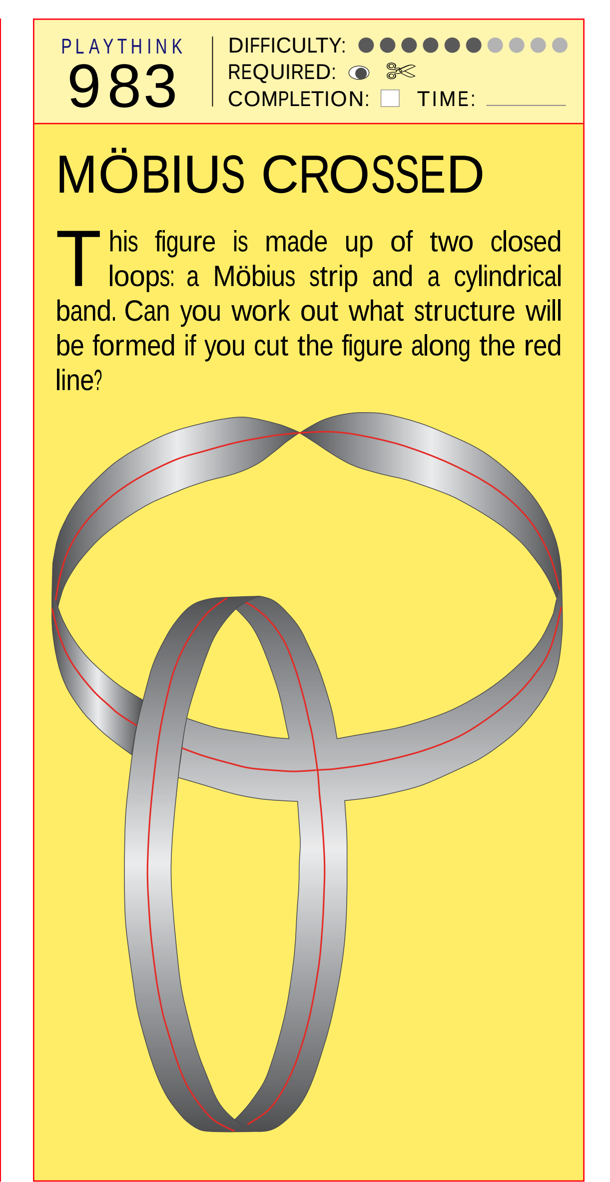
This strip passes through itself, as shown. Can you work out what will happen if you divide it along the red line?



PLAYTHINK 983 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✂ | COMPLETION: ☐ TIME: ____

MÖBIUS CROSSED

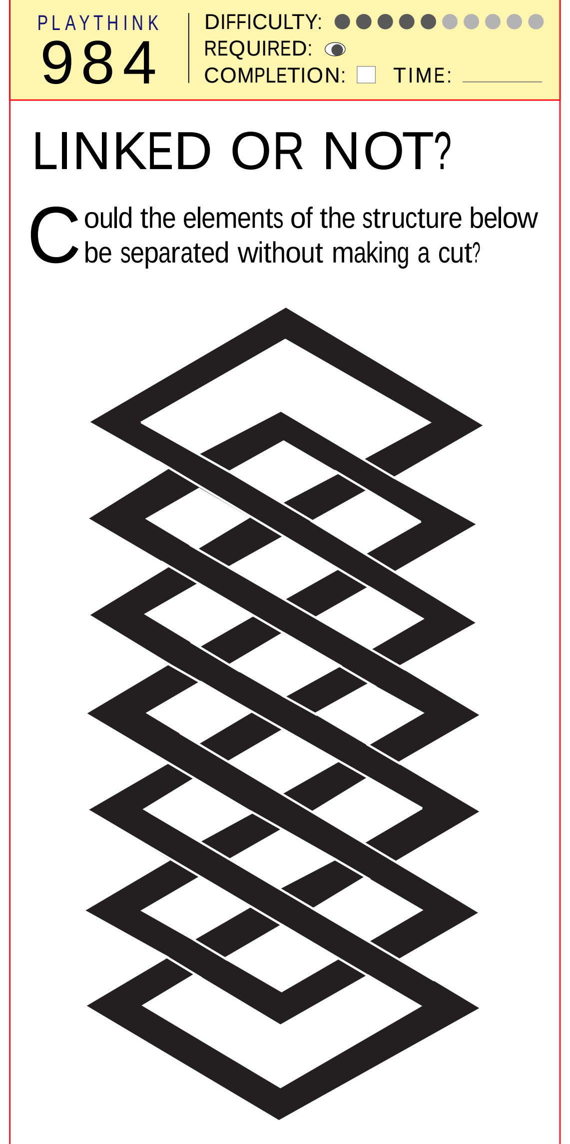
This figure is made up of two closed loops: a Möbius strip and a cylindrical band. Can you work out what structure will be formed if you cut the figure along the red line?



PLAYTHINK 984 | DIFFICULTY: ●●●●●○○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

LINKED OR NOT?

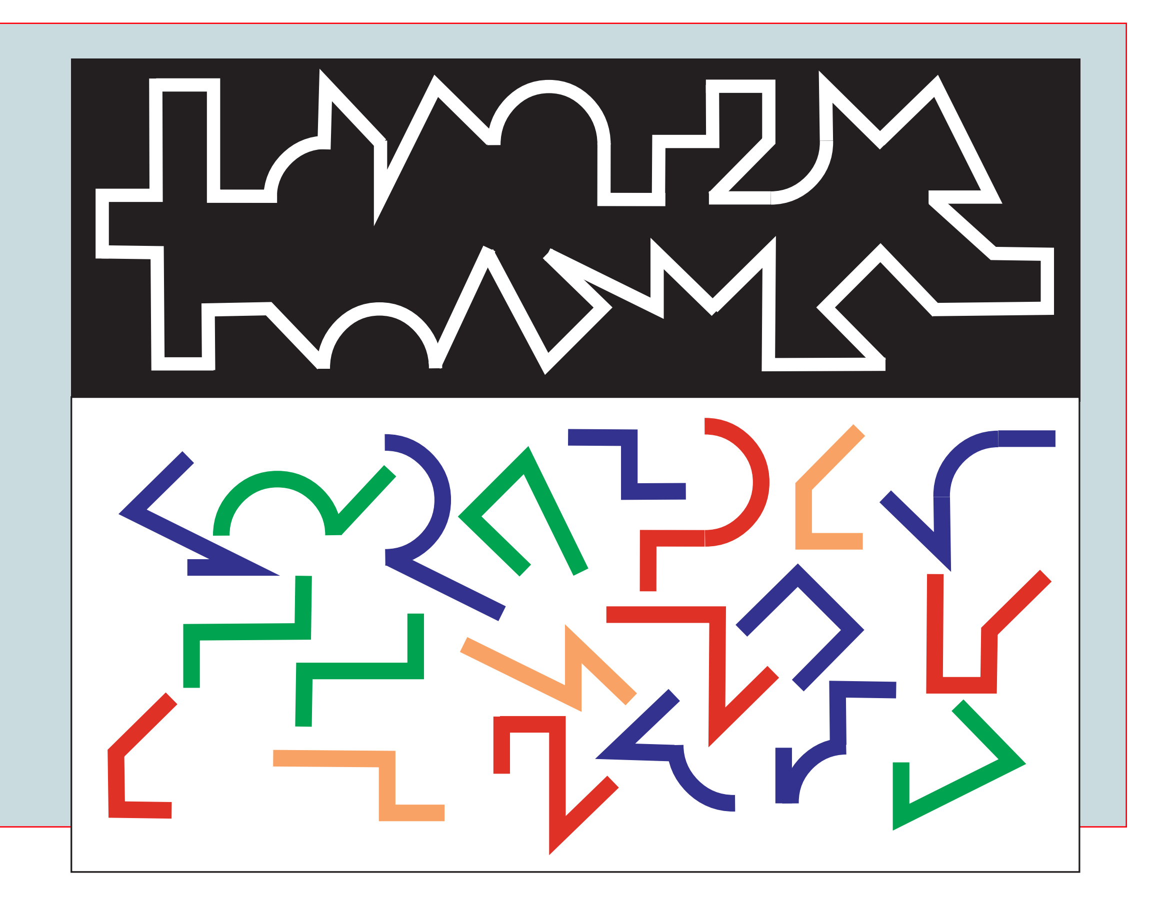
Could the elements of the structure below be separated without making a cut?



PLAYTHINK 985 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

LINKS

The closed white line is made up of sixteen links, each of which is shown separately and in color. The separate links may be in a different orientation than they appear in the line, but none of the links overlap. Can you color in the line according to the colors of the sixteen links?



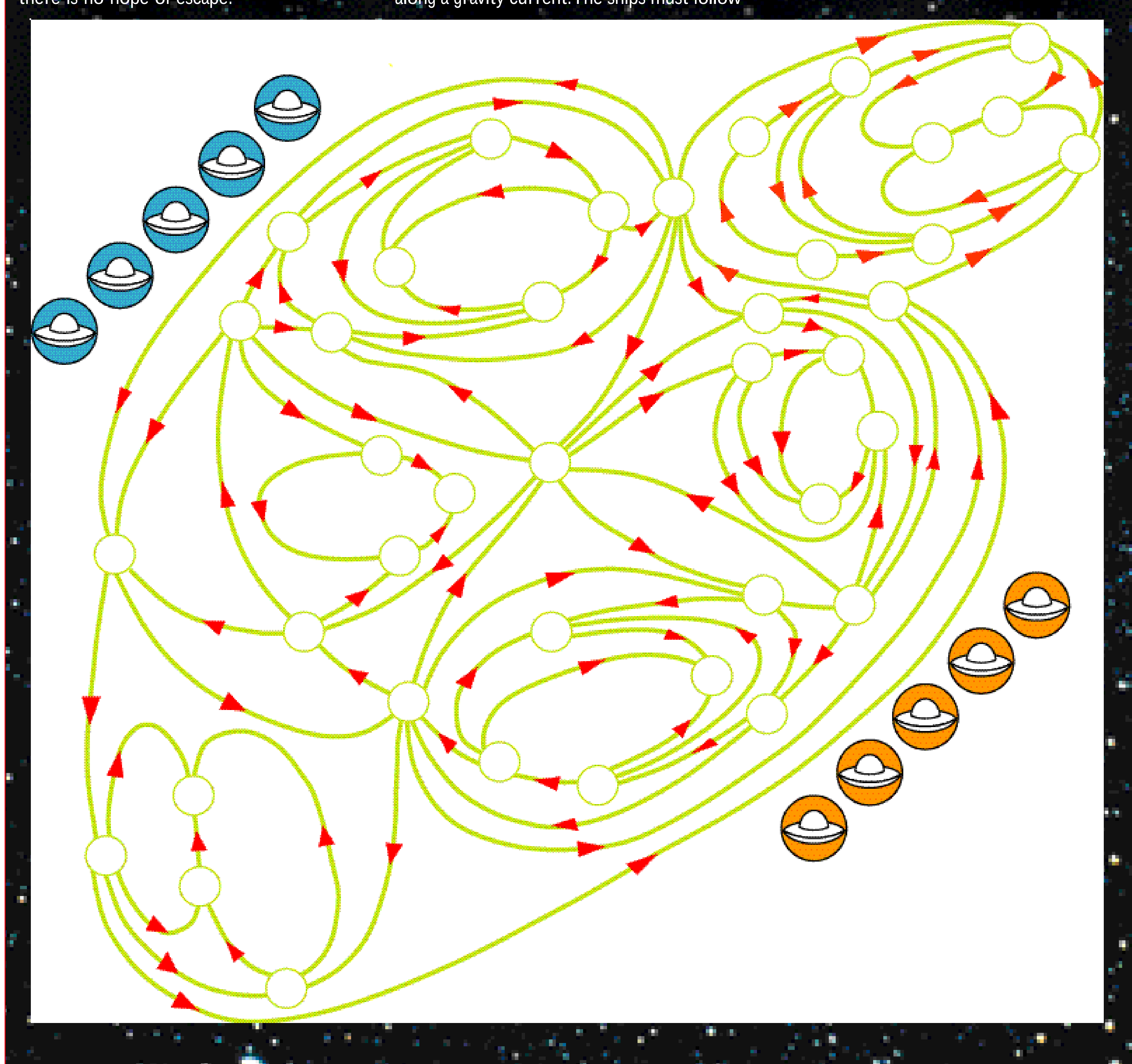
PLAYTHINK 986

DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

WAR OF THE PLANETS

You are the leader of an alien society fighting hostile invaders. The many planets in your star system are linked by gravity currents. You need to keep your spaceships moving from one planet to another and avoid falling into a gravity well from which there is no hope of escape.

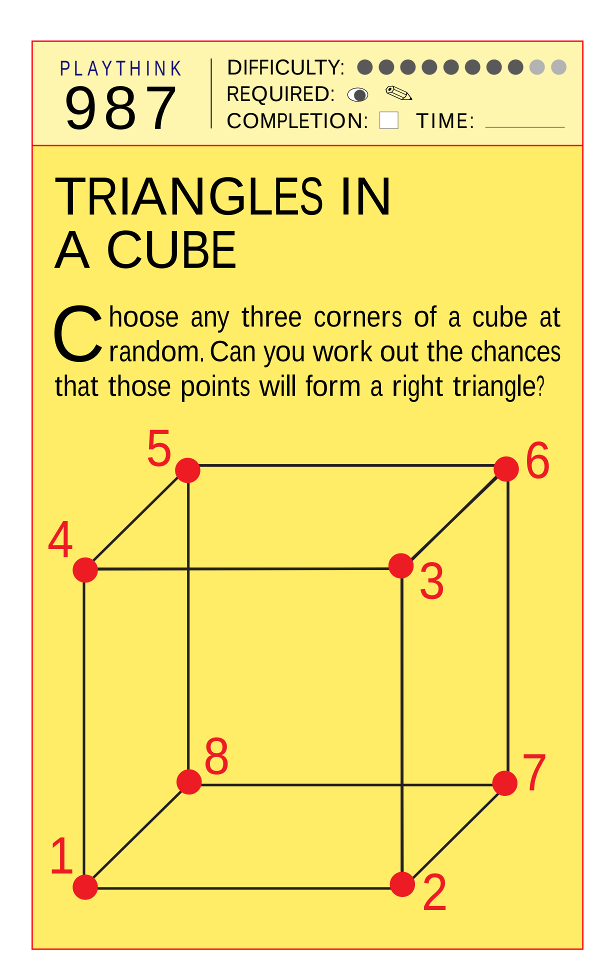
That's the scenario for this two-person game. Each player gets six spaceships and places them, in turn, on any planet—though a planet can hold just one spaceship at a time. After all the spaceships have been placed, players take turns moving their ships along a gravity current. The ships must follow the direction of the arrows; if all the currents point toward the planet, the ship cannot leave. The last player who can make a move is the winner.

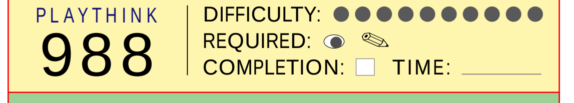


PLAYTHINK 987 | DIFFICULTY: ●●●●●●●●○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

TRIANGLES IN A CUBE

Choose any three corners of a cube at random. Can you work out the chances that those points will form a right triangle?

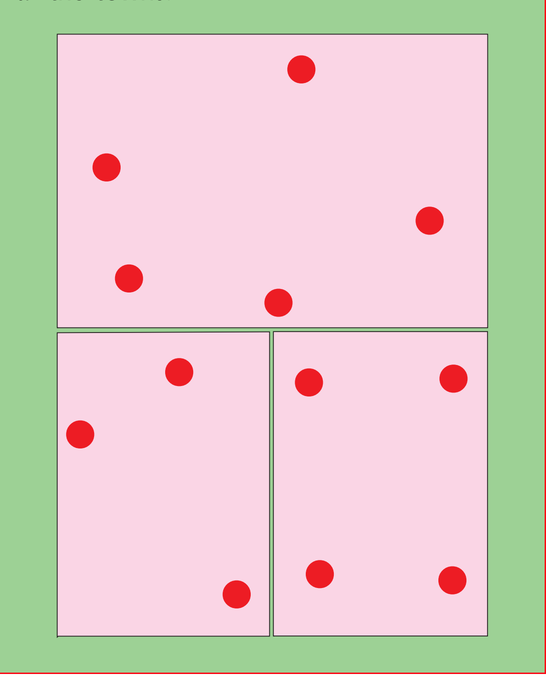




PLAYTHINK 988 | DIFFICULTY: ●●●●●●●●●● | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

MINIMAL ROUTES

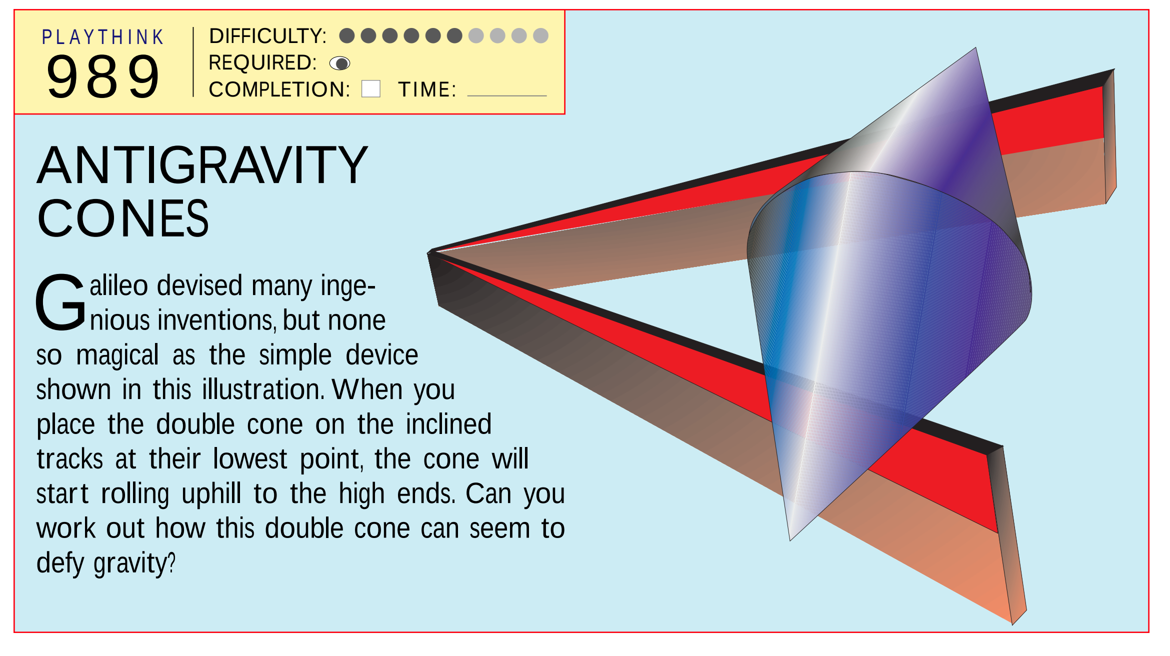
Three, four and five towns are represented by red points on the three maps shown here. For each map, can you draw the shortest possible road system that links all the towns?



PLAYTHINK 989 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

ANTIGRAVITY CONES

Galileo devised many ingenious inventions, but none so magical as the simple device shown in this illustration. When you place the double cone on the inclined tracks at their lowest point, the cone will start rolling uphill to the high ends. Can you work out how this double cone can seem to defy gravity?



PLAYTHINK 990 | DIFFICULTY: ●●●●●●●○○○ | REQUIRED: 👁 ✎ | COMPLETION: ☐ TIME: ____

MINIMAL WEIGHTS

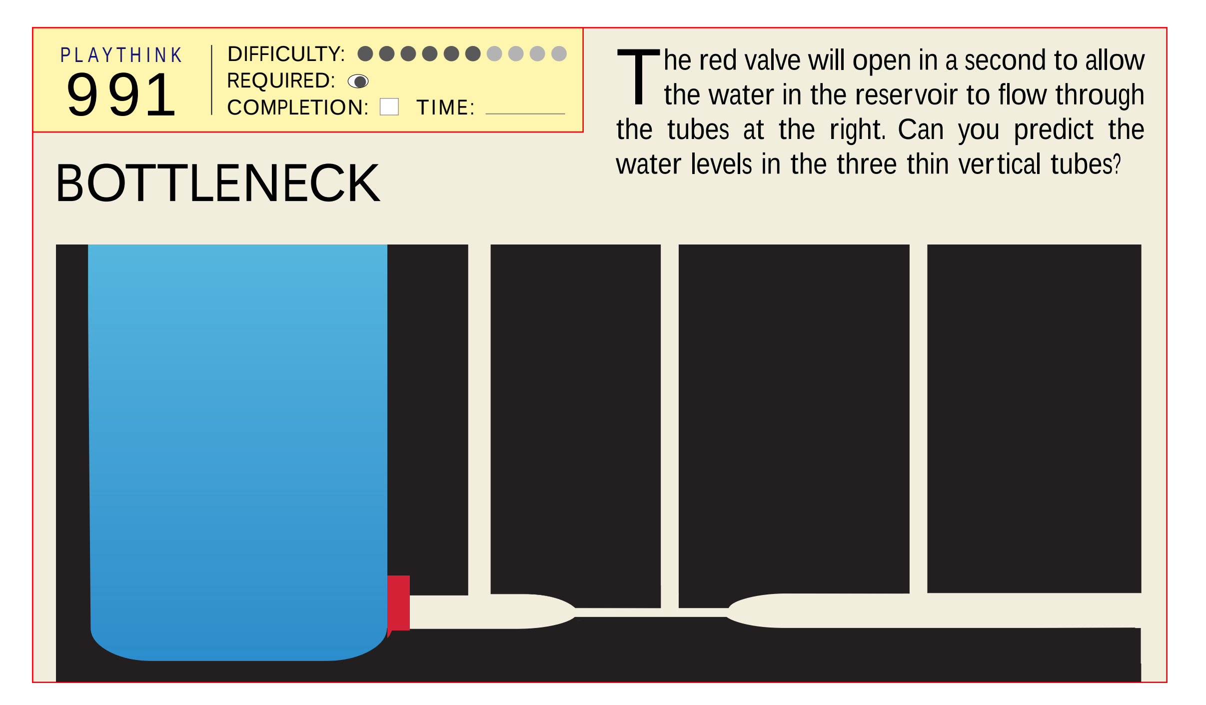
Suppose you need to have the ability to weigh any integral weight from 1 to 40 grams on a scale like the one shown here. Can you work out the minimum number of weights you must have?



PLAYTHINK 991 | DIFFICULTY: ●●●●●●○○○○ | REQUIRED: 👁 | COMPLETION: ☐ TIME: ____

BOTTLENECK

The red valve will open in a second to allow the water in the reservoir to flow through the tubes at the right. Can you predict the water levels in the three thin vertical tubes?

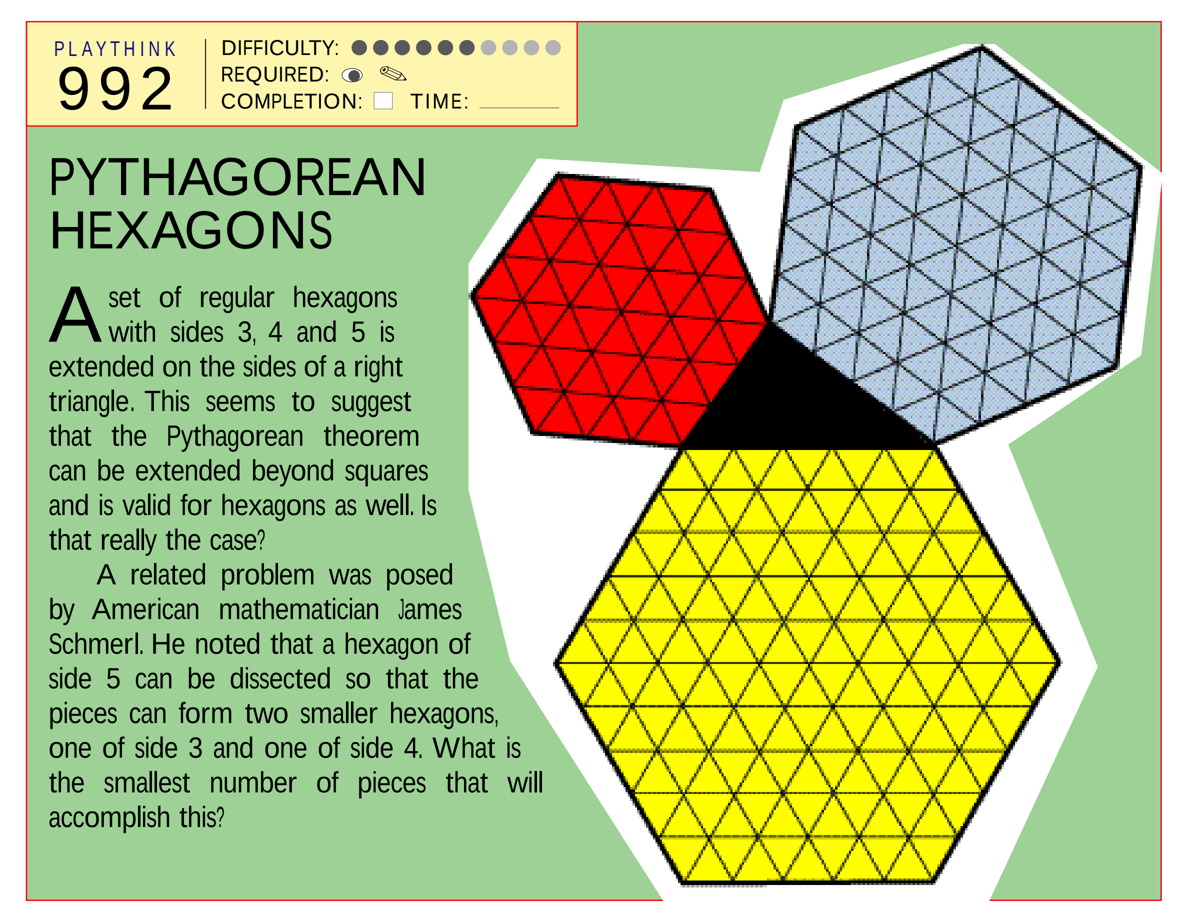


PLAYTHINK 992
DIFFICULTY: ●●●●●●○○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

PYTHAGOREAN HEXAGONS

A set of regular hexagons with sides 3, 4 and 5 is extended on the sides of a right triangle. This seems to suggest that the Pythagorean theorem can be extended beyond squares and is valid for hexagons as well. Is that really the case?

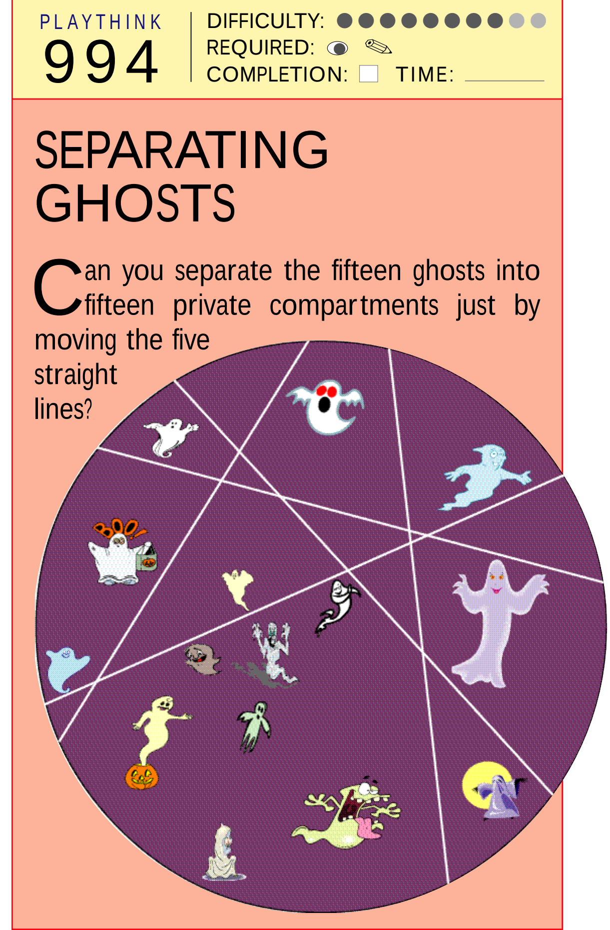
A related problem was posed by American mathematician James Schmerl. He noted that a hexagon of side 5 can be dissected so that the pieces can form two smaller hexagons, one of side 3 and one of side 4. What is the smallest number of pieces that will accomplish this?



PLAYTHINK 994
DIFFICULTY: ●●●●●●●●○○
REQUIRED:
COMPLETION: ☐ TIME: ______

SEPARATING GHOSTS

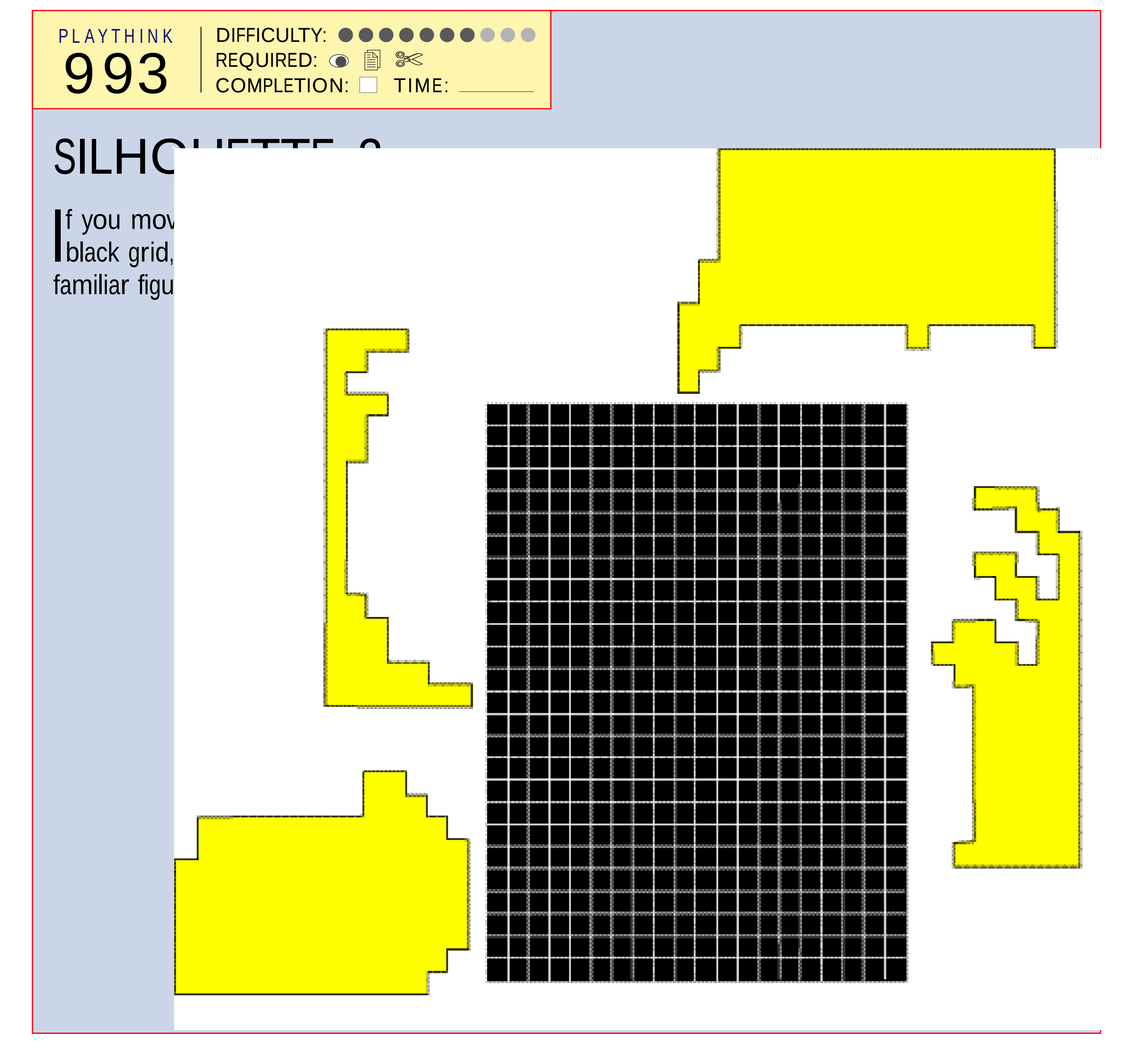
Can you separate the fifteen ghosts into fifteen private compartments just by moving the five straight lines?



PLAYTHINK 993
DIFFICULTY: ●●●●●●●○○○
REQUIRED:
COMPLETION: ☐ TIME: ______

SILHO

If you mov
black grid,
familiar figu



PLAYTHINK 995
DIFFICULTY: ●●●●●●●●●○
REQUIRED:
COMPLETION: ☐ TIME: ______

BIRD NEST

Seven birds live in a nest. They are very organized and send three birds out each day in search of food. After seven days every pair of birds will have been in exactly one of the daily foraging missions. For example, on the first day birds 1, 2 and 3 go out; that means the pairs 1-2, 1-3 and 2-3.

Can you work out how every pair can



PLAYTHINK 996
DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

LINKED TUBES

Several tubes of various shapes are linked in such a way that liquid can pass from one to another. The network of tubes is connected to a reservoir of water, at left. If the reservoir is opened and the water flows into the tubes, can you work out what the water level will be in each tube?

PLAYTHINK 997
DIFFICULTY: ●●●●●●○○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

PROGRESSING SQUARES

Start with a small square of side 1. Employ the diagonal of that square as the side of a second square. Employ the diagonal of the second square as the side of a third square. Continue in this manner to create an infinite progression of squares.

Without measuring, can you work out what the length of the sides of the eleventh square in the series will be?

1 3 5 7

PLAYTHINK 998
DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁 ✎
COMPLETION: ☐ TIME: ____

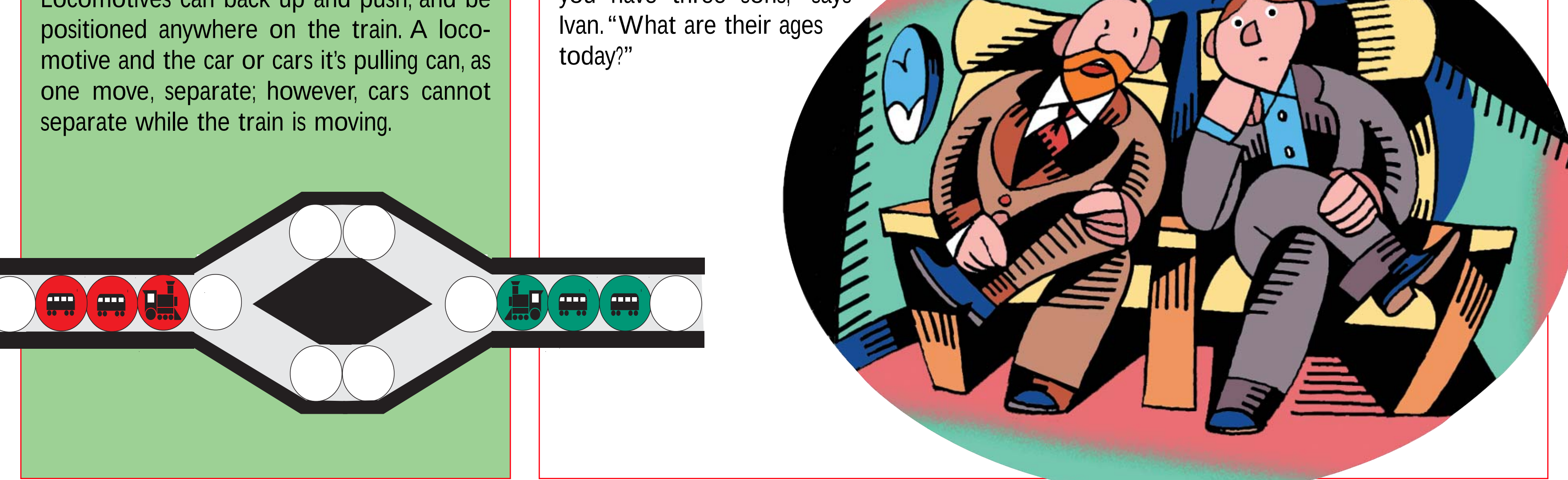
THE TOSS OF THE DIE

If you toss a die six times, what are the chances that all six faces will turn up?

PLAYTHINK 999
DIFFICULTY: ●●●●●●●●○○
REQUIRED: 👁 ✎ 📄 ✂
COMPLETION: ☐ TIME: ____

MOVING TRAINS

Two trains meet at a switch, each needing to pass the other. No locomotive or carriage may stop on the diagonal section of tracks, and only two carriages or a locomotive and a carriage may park on either side of the switch. Employing only the locomotives to move the carriages, how many moves will it take you to get the red train to exchange places with the green train? Locomotives can back up and push, and be positioned anywhere on the train. A locomotive and the car or cars it's pulling can, as one move, separate; however, cars cannot separate while the train is moving.



PLAYTHINK 1000
DIFFICULTY: ●●●●●●●○○○
REQUIRED: 👁
COMPLETION: ☐ TIME: ____

THE LAST PUZZLE

This last challenge was very carefully selected. The puzzle is a classic that contains the best elements of recreational mathematics. The solution requires thinking, concentration, creativity, logic, insight and attention to the smallest detail. Enjoy!

Two Russian mathematicians meet on a plane.

"If I remember correctly, you have three sons," says Ivan. "What are their ages today?"

"The product of their ages is thirty-six," says Igor, "and the sum of their ages is exactly today's date."

"I'm sorry, Igor," Ivan says after a minute, "but that doesn't tell me the ages of your boys."

"Oh, I forgot to tell you, my youngest son has red hair."

"Ah, now it's clear," Ivan says. "I now know exactly how old your three sons are."

How did Ivan figure out the ages?

SOLUTIONS

CHAPTER 1 SOLUTIONS

1 The Roman numeral for seven (VII) can be made by cutting the Roman numeral for twelve (XII) in half horizontally.



2 The solution given in the *sangaku* tablet is as follows: Imagine that the perpendicular line is drawn separately from the line specified in the puzzle. If they are in fact different lines, then they would start at the center of the blue circle and run to different points on the diameter. As is the case with most of the surviving *sangaku* puzzles, the proof of the theorem is not given, making them difficult (if not impossible!) for us to understand. Take heart. I have included this example only as a means of explaining the inspiration behind my book; all other PlayThinks have answers. I promise.

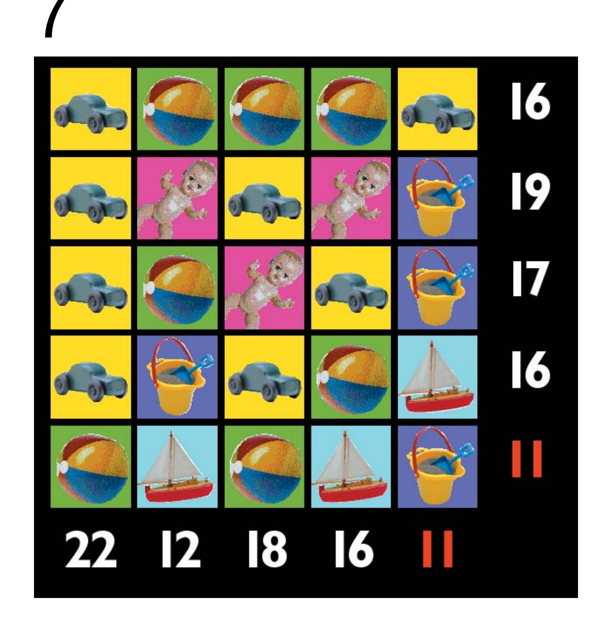
3 16,807 measures of flour. That's 7 x 7 x 7 x 7 x 7. This puzzle, which comes from the ancient Egyptian "Rhind Papyrus," was written by the scribe Ahmes in 1850 B.C. Perhaps the world's oldest puzzle, it has inspired a great many variations over the thousands of years since its creation.

4 The frames are exactly identical. Because the frames are three-dimensional, they can be arranged in a nontransitive way so that A is inside B, B is inside C, and C is inside A.

5 Door number 5 is the right answer. In many instances people choose a door that is more square than the original. That is because the background figure often influences one's perception of the door's shape.

6 The egg. The riddle does not specify that the eggs in question are chicken eggs and, according to paleontologists, reptiles and dinosaurs existed long before birds and chickens. Fossilized eggs dating back one hundred million years have been uncovered. Thus it can be said that eggs came before chickens.

7



8 The answer is four. The outlines of the four sets of rectangles are shown next to their corresponding squares.



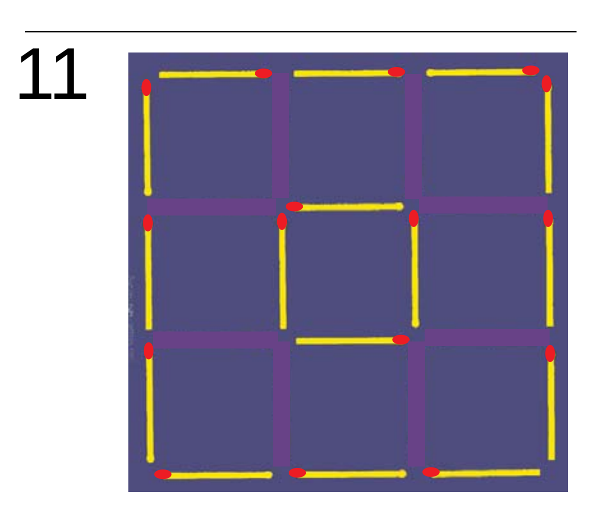
9 The frowning clown is the thirteenth clown from the right in the second row.

The human perceptual system is designed to detect an item that stands out as different without a systematic search. This principle is used in the design of instrument panels: under normal conditions, in which all indicators point in the same direction, any change is easily spotted.

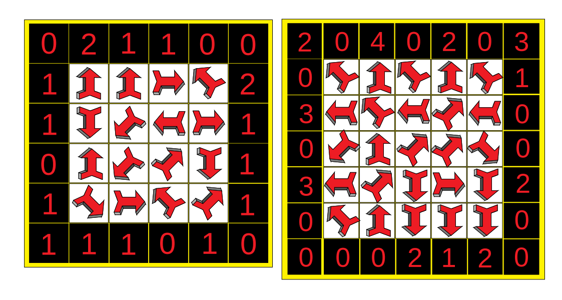
10 The winning sequence is yellow, orange, red, pink, violet, light green, dark green, light blue and dark blue.

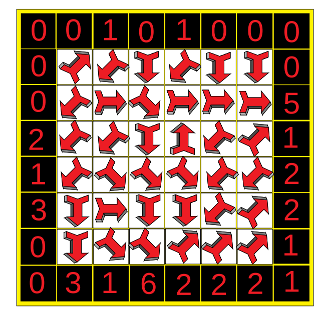
This puzzle was created in much the same way that animated cartoons are drawn. Many elements of the scene were painted on transparent cells, then stacked on top of one another in the correct order to create the illusion of one seamless drawing.

11



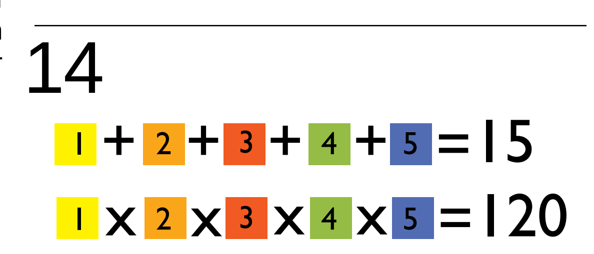
12 One of the many possible solutions for each puzzle.





13 The choices offer identical odds. But in a psychological experiment, about four in ten people preferred the single draw and held to this view even when the other choice was altered to provide fifty draws from the box of 100.

14



15 The number 2,520 is obviously divisible by 5 and 10. But since all five of the numbers are single-digit, 10 is excluded. So the third number must be 5.

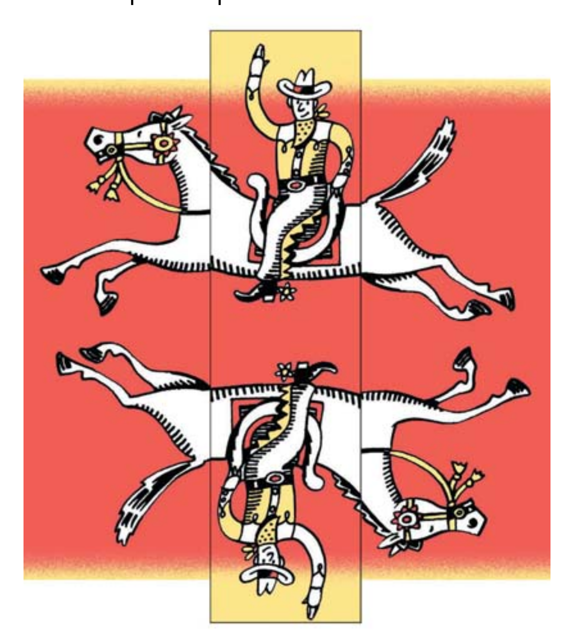
Adding the known numbers (8 + 1 + 5) gives us 14. Since 30 – 14 = 16, the total of the remaining two numbers must be 16.

Multiplying the known numbers (8 x 1 x 5) gives us 40. Since 2,520 / 40 = 63, the product of the two remaining numbers must be 63.

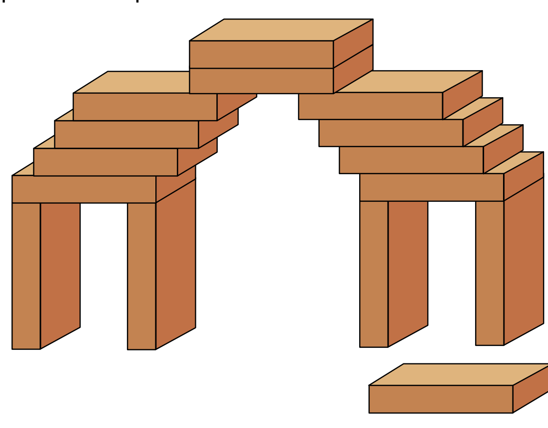
Only 9 and 7 can be added to make 16 and multiplied together to make 63.

So the answer is 5, 7 and 9.

16 This puzzle is another that often creates a conceptual block. But, as you can see, the solution is quite simple.



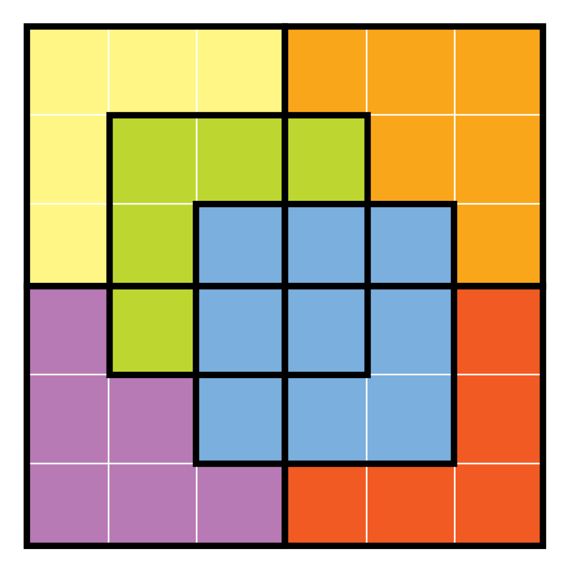
17 The key to constructing the bridge is to set up two dominoes as temporary supports, as shown in the illustration below. When enough dominoes have been placed to give the structure its overall stability, the supports can be removed and placed on top.



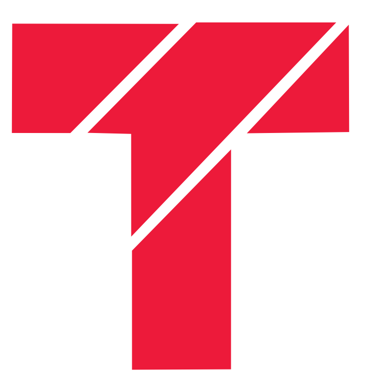
18 If the drawer contained socks, then you would need to select only four to get a matching pair. But gloves have an attribute that socks don't: handedness. It is not enough to have two gloves that are the same color—they must be of complementary handedness. So to ensure that you have one pair of gloves, you must select one more than the number of gloves of one-handedness, or twelve. Assuming that you can distinguish in the dark between right- and left-handed gloves, you may need to select only eleven.



19 The outlines of the six overlapping squares for one six-by-six square, six three-by-three squares, three two-by-two squares and eight one-by-one squares—eighteen squares altogether.

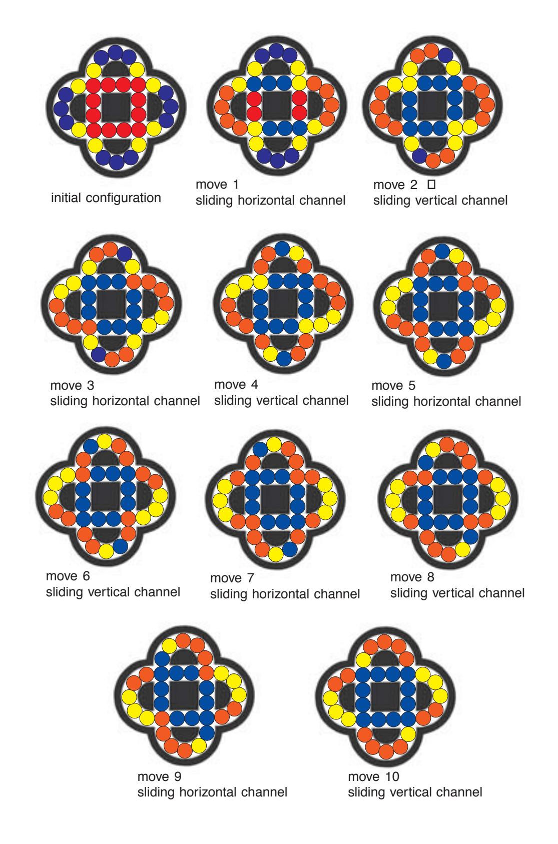


20 Sam Loyd, widely acclaimed as the greatest American puzzle inventor, created the classic T-Puzzle. In terms of elegance and simplicity, the T-Puzzle has never been surpassed. It is deceptively simple because it has few pieces. But it is a good example of a problem that looks easy at first but possesses elements that often lead to a conceptual block. Once the block is in place, it is often impossible to find the solution even if the pieces are cut out and handled.



The solution may come eventually, in a flash of inspiration. Such a moment of insight—the "Aha!" experience—is usually accompanied by a feeling of achievement for one's own act of creative thinking.

21 A three-move solution (with thanks to Joe DeVicentis). All the moves are clockwise.



22 There are sixteen possible combinations of choices for the firing of the four lasers. Four combinations will form a closed energy field around the man:

left, left, left and left
left, right, left and right
right, left, right and left
right, right, right and right

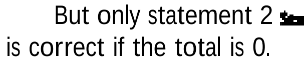
The probability of success, then, is one in four.

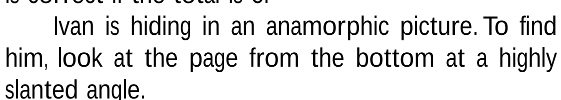
23 The best way to work out the solution is with a diagram such as the one at the right.

As you can see, both statement 1 (Gerry's) and statement 3 (Anitta's) can be true, *so the number cannot be more than 100.*

Statement 3 and statement 2 (George's) can be true if the total is between 100 and 1.

But only statement 2 is correct if the total is 0.





Ivan is hiding in an anamorphic picture. To find him, look at the page from the bottom at a highly slanted angle.

24 If the treasure were buried on the orange island, then all the statements would be false. And if the treasure were buried on the purple island, then all the statements would be true. But if the treasure were buried on the yellow island, then only the statement for the purple island would be false. Therefore, the treasure is on the yellow island.

25 If the horses were not circling, there would be seven factorial (7!, or 5,040) possible arrangements. But because they form a circle, each arrangement is identical to six others that could be formed by declaring one horse or another is the "first" horse in the circle. That means the answer is 7!/7 or 6! (6x 5 x 4 x 3 x 2 x 1). Six factorial is still a large number: 720.

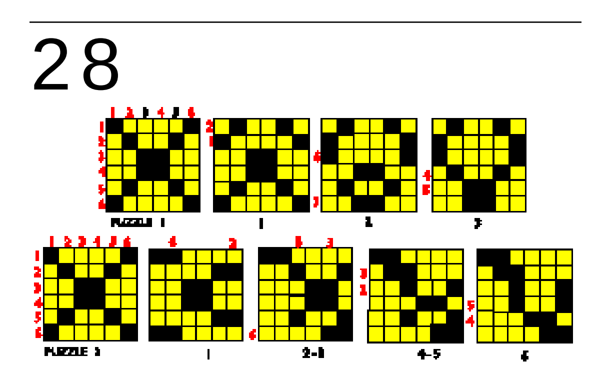
26 The trick here is looking at the way the books are lined up. The bookworm eats through only the front cover of volume 1, all of volumes 2, 3 and 4, and only the back cover of volume 5. The total distance is 19 centimeters.

27 The first step you must take to solve the problem is to find the number of combinations of three colors you can make from five colors. Plugging the values into a general formula for the number of combinations gives you:

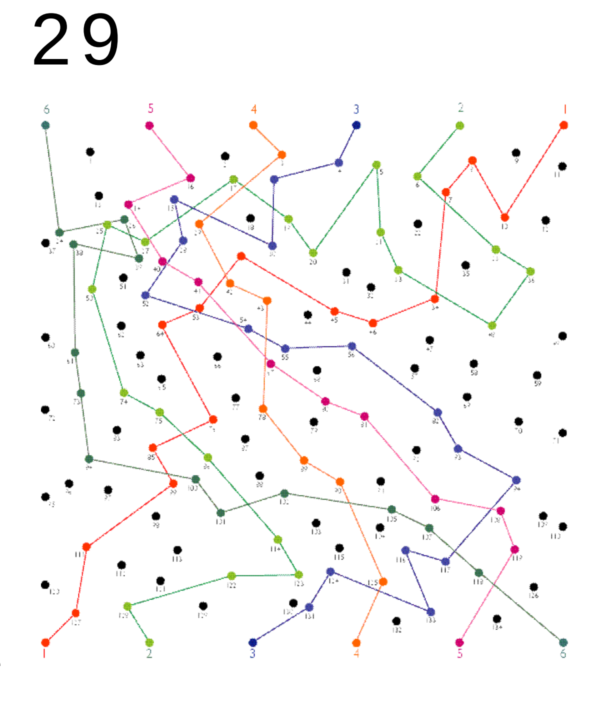
$$5!/(3! \times (5-3)!) = (5 \times 4 \times 3 \times 2 \times 1) / (3 \times 2 \times 1 \times (2 \times 1)) = 120/12 = 10$$

That result tells us there are ten possible combinations of three colors out of five. But the number of combinations tells us nothing about the order in which the colors are placed on the mask. The different orders in which the three colors can be painted on the mask is 3!(3 x 2 x 1), or six for each color combination. That means there is a total of sixty possible ways the mask could be painted using three colors out of five.

28



29

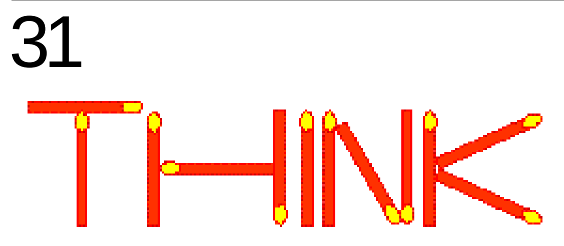


30 Your friend is wrong. Because the odds for each coin is independent of the others, there are in fact two possible outcomes for a single coin, four possible outcomes for two coins and eight possible outcomes for three coins:

1	2	3
H	H	H
H	H	T
H	T	H
H	T	T
T	H	H
T	H	T
T	T	H
T	T	T

In only two tosses out of eight will the coins land all heads or all tails.

31



32 The message is hidden as an anamorphic image. If you hold the page at a very slanted angle, you will be able to read it: HELLO.

33

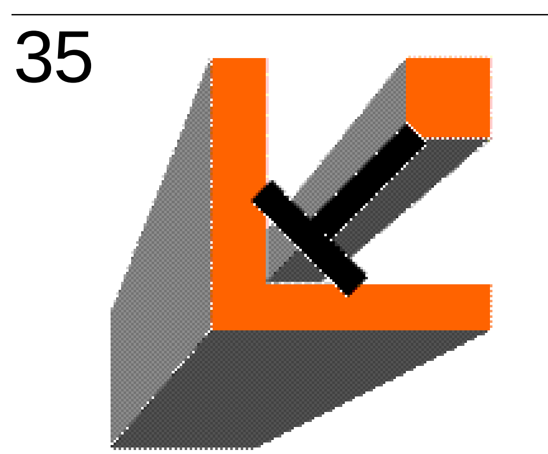
2 + 2 = 4
2 + 3 = 5
5 – 2 = 3
6 – 3 = 3

34 The message is a statement that mathematical concepts are understood, as well as an attempt to convey logic.

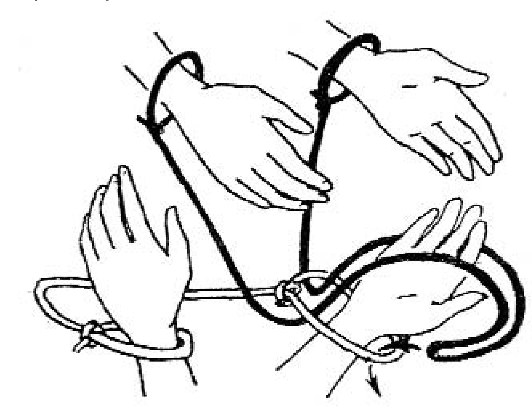
1 + 2 = 3 ➛ true
2 + 2 = 4 ➛ true
3 + 2 = 4 ➛ false

The triangle stands for "plus"; the diamond stands for "equals"; the pentagon stands for "true"; and the hexagon stands for "false."

35

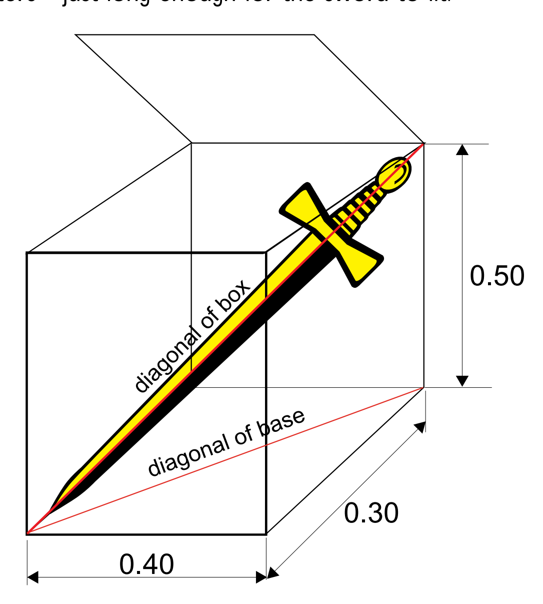


36 The hostages can easily separate themselves. One of the hostages grabs his rope with both hands so that a loose, untwisted loop is made in his rope on the other side of his partner's rope. Then he tucks the loop through the circle of rope around his partner's wrist; as you'll soon discover, it's only possible to keep the rope untwisted by moving toward one wrist, not the other. Next, he moves the loop up toward his partner's fingers. When the first hostage then passes the loop over his partner's hand and tucks the loop back through the rope, they are free.

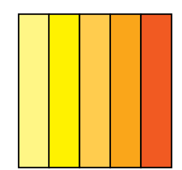


37 $6 + \frac{6}{6} = 7$

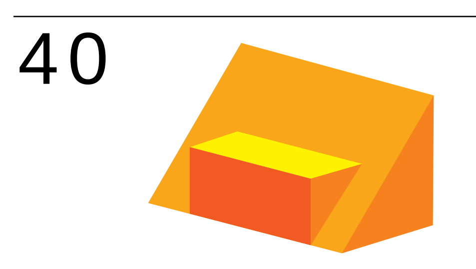
38 The solution uses the Pythagorean theorem (the square of the length of the hypotenuse of a right triangle equals the sum of the squares of the lengths of the other two sides) to calculate the length from the lower front left-hand corner of the chest to the upper back right-hand corner. First the diagonal of the base is determined to be 50 centimeters; then that length and the height of the chest can be used to calculate the maximum length through the box. That turns out to be 70.7 centimeters—just long enough for the sword to fit!



39 The solution is so obvious that many people overlook it.

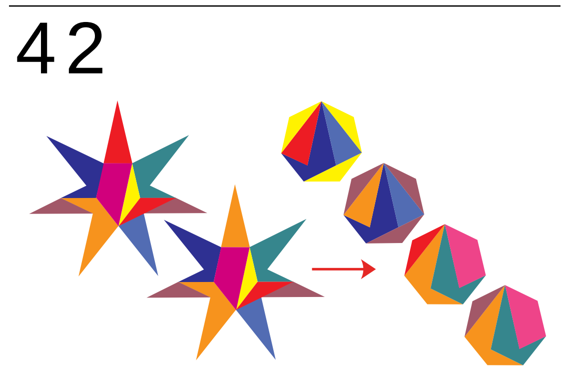


40



41 The answer is a man. A man crawls on all fours in the morning of his life (when he is a baby), walks upright in middle age, and uses a cane in old age.

42

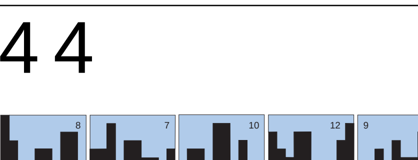


43 "With two statements there are four possible combinations of truth or falsehood:

true/true
true/false
false/true
false/false

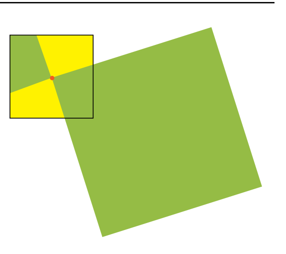
The first combination can't be right because at least one of the statements is false. The second and third can't be right either because if one of the statements is false, it's impossible for the other to be true. The only logically consistent possibility is that they both lied. That means Mister Ladybug has the yellow dots and Miss Ladybug has the red dots.

44

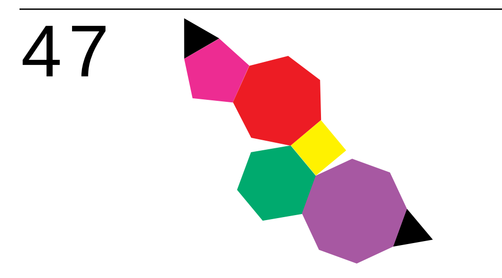


45 It can't be done. If you start drawing a line outside the black closed line and cross it an odd number of times, you will end up inside the black line. To close the new line, you must intersect the black line, making an even number of intersections. Not only are nine intersections impossible; all odd numbers of intersections are impossible.

46 The larger rug covers exactly 25 percent of the smaller one. The proof of this is shown in the diagram at right.



47

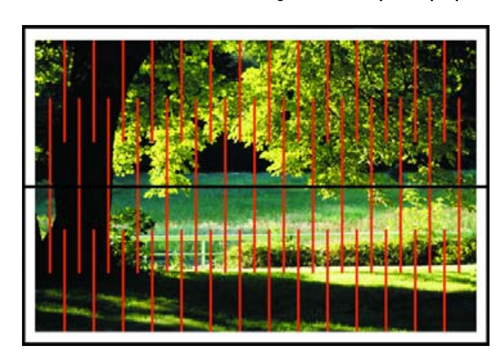


48 The best-case scenario—that the two missing socks make a pair, leaving you with four matched pairs—can happen in only five different ways. If the socks can be labeled A1, A2, B1, B2, C1, C2, D1, D2, E1 and E2, then the best-case scenario occurs only when the missing socks are A1-A2, B1-B2, C1-C2, D1-D2 or E1-E2.

The worst-case scenario—that the two missing socks do not make a pair, leaving you with only three matched pairs and two orphan socks—occurs when the missing socks are A1-B1, A1-B2, A2-B1, A2-B2, A1-C1, A1-C2, A2-C1, A2-C2, A1-D1, A1-D2, A2-D1, A2-D2, A1-E1, A1-E2, A2-E1, A2-E2, B1-C1, B1-C2, B2-C1, B2-C2, B1-D1, B1-D2, B2-D1, B2-D2, B1-E1, B1-E2, B2-E1, B2-E2, C1-D1, C1-D2, C2-D1, C2-D2, C1-E1, C1-E2, C2-E1, C2-E2, D1-E1, D1-E2, D2-E1, D2-E2.

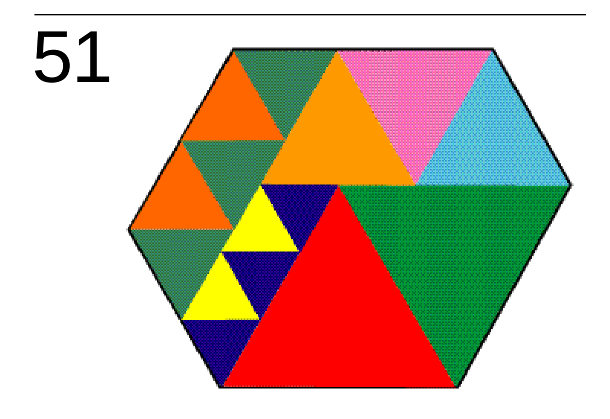
That's forty different ways to get the worst-case scenario. As you can see, the worst-case scenario is eight times more likely to occur than the best-case scenario.

49 Mark the card as shown, fold in half along the horizontal line and cut along the red lines. The result will be a long, thin loop of paper.



50 The total number of possible permutations in a seven-digit phone number is seven factorial (7!, or 7 x 6 x 5 x 4 x 3 x 2 x 1), which equals 5,040. So the probability of any given combination being the right phone number is 1 in 5,040, or about .02 percent. For a complete discussion of factorials, see "Combinations and Permutations," p. 140.

51



52

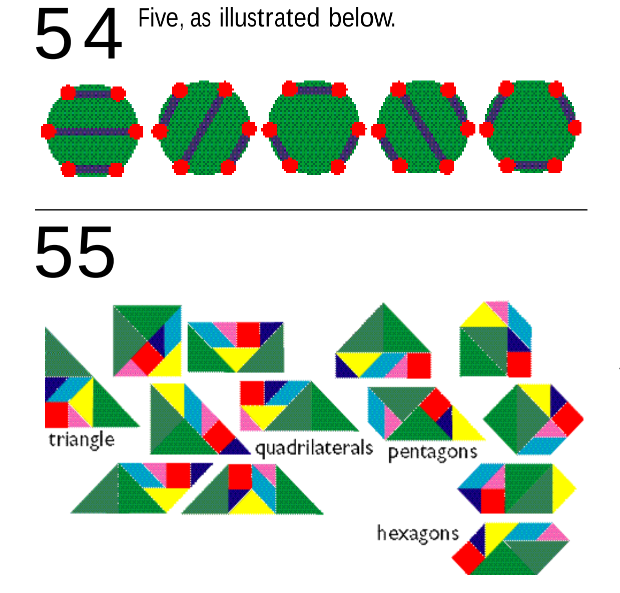
53

There were six people at the meeting. Each person shook hands five times, but that makes for fifteen handshakes, not thirty, since each shake was shared by two people.

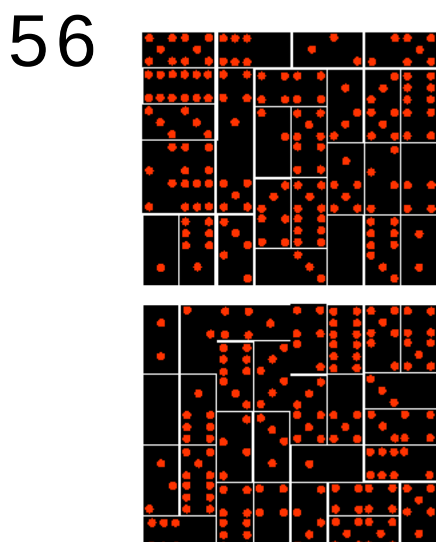
54

Five, as illustrated below.

55



56



57

Change the word *man* to *person*. Otherwise, it is possible that the man has a wife and many daughters and that one of them knocked on the door.

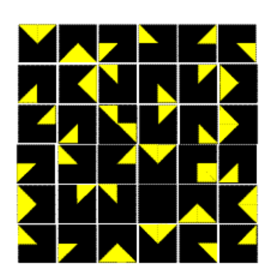
58

The maximum number of attempts can be found by adding:

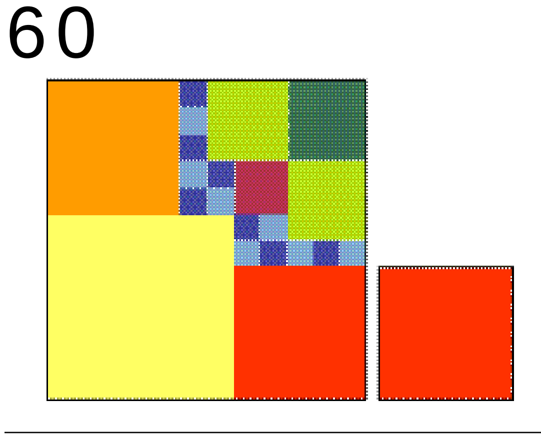
8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 36 attempts

59

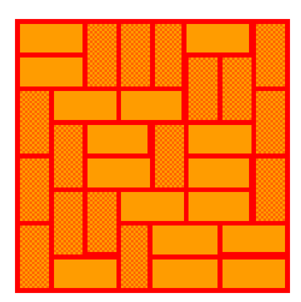
In each row the yellow wedges add up to make a complete square.



60



61



62

Pentahex 2 is not part of the honeycomb.

63

Since the nine bananas, nine oranges and nine apples in the three fruit baskets together cost $4.05, the cost of one of each fruit should cost just one-ninth of that, or $0.45. (There's no need to know that apples are 10 cents each, bananas are 20 cents each and oranges are 15 cents each.)

64

There were seven people at the reunion: a man and his wife, their three children (two girls and a boy) and the man's mother and father.

Without the stipulation that both halves of the relationships were present, there could be as few as four people; after all, one man can simultaneously be a father, a grandfather, a son, a brother and a father-in-law.

65

Based on her remark, we know that Miss Blue's dress is either pink or green. Since the woman who replied to her was wearing a green dress, that means Miss Blue must be wearing pink. That leaves the blue dress for Miss Green and the green dress for Miss Pink.

66

You can write four numbers:

a. $2^{2^2} = 2^4 = 16$, the smallest number
b. 222
c. $22^2 = 484$
d. $2^{22} = 4{,}194{,}304$, the largest number

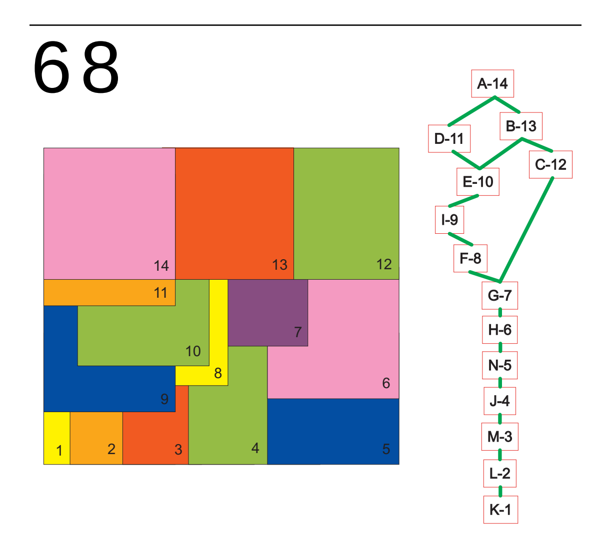
Using powers is an efficient way to write very large or very small numbers. Raising a number to a power simply means multiplying it by itself as many times as is indicated by the power. So:

$$2^{22} = 2 \times 2 = 4{,}194{,}304$$

67

If you shake one of the coins out of the bank labeled "15¢," you can figure out how to correctly label all the banks. Since you know that the bank is mislabeled, it cannot hold 15 cents—instead, the bank contains either two dimes or two nickels. The coin that drops out will tell you what the other coin is. Say the answer is two dimes; that leaves you with three nickels and a dime between the two remaining banks, one labeled "20¢" and one labeled "10¢." Since the bank labeled "10¢" cannot have two nickels in it—because it is mislabeled—it must contain a nickel and a dime, and the other bank must have the two nickels.

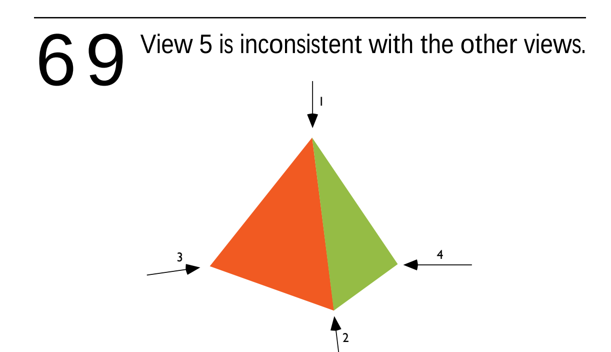
68



The relative layering of D-11 and B-13 cannot be determined. Similarly, the position of C-12 can actually be anywhere from 8 to 12. See chart.

69

View 5 is inconsistent with the other views.



70

Fold 3 is impossible. In general, it is not possible to fold the strip so that stamps that touch only at the corners will appear next to each other in the stack.

71

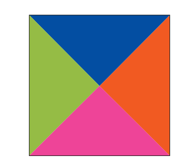
Card number 3 is not found in the colored pattern.

72

Each letter has been shifted one place down alphabetically. A becomes B, B becomes C and so on. The secret message is ONE THOUSAND PLAYTHINKS.

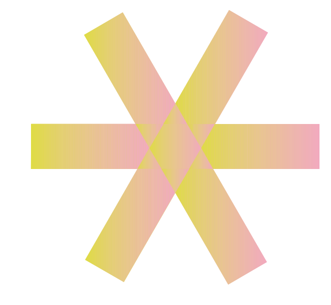
73



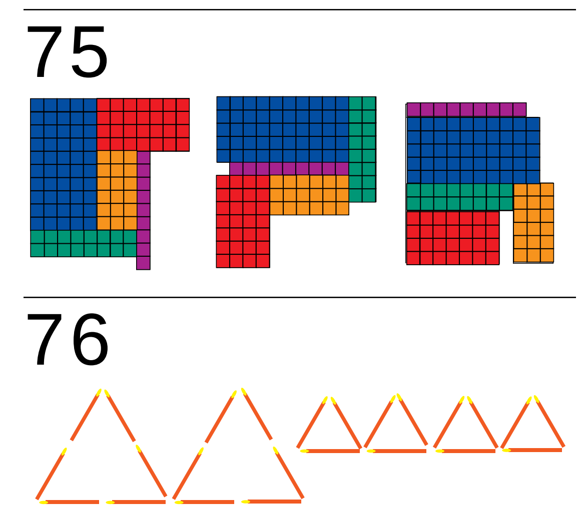


74

When overlapped, the strips can create a six-pointed star.

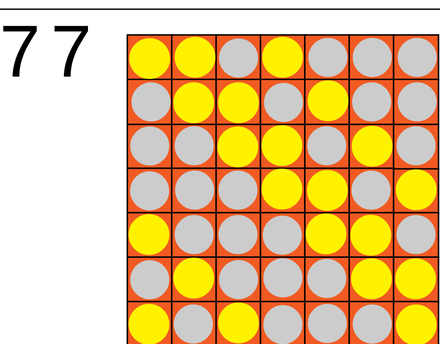


75



76

77



78

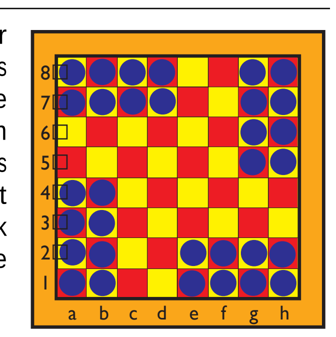
The missing animal is the donkey. The pattern is made of six animals running across the five-by-four grid. Each time the pattern is repeated, the first animal in the series is omitted. If each animal were a number, the series would read: 12345623456345645656.

79

The odd one out is the third in the first row.

80

The answer is yes. As pictured, it is possible to place a maximum of thirty-two knights on the board so that each piece can attack one and only one other piece.



81

One in six. Three hats can be distributed among three people in six different ways: ABC, ACB, BAC, BCA, CAB, CBA.

82

In every second row some of the letters differ from those in the row above it. Those letters spell out the message:

"There is only one good—knowledge—and one evil—ignorance."

—SOCRATES

83

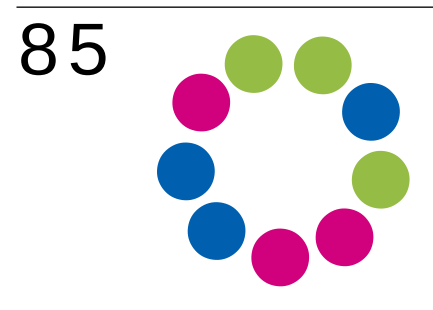
The probability of the coin landing on a corner is about 50 percent. You can verify this by tossing the coin onto the board many times. In general, the probability that a coin will cover a corner can be calculated by dividing the area of the coin by the area of a single square of the game board.

84

The five numbers between 1 and 100 that have twelve factors:

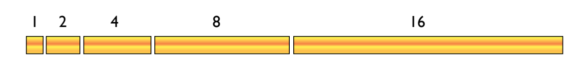
60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
84: 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84
90: 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90
96: 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96

85

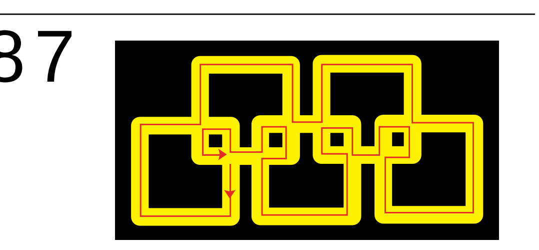


86

Four cuts, as shown, will be sufficient. Notice that the lengths are equal to the place numbers in the base 2, or binary, number system.



87



88

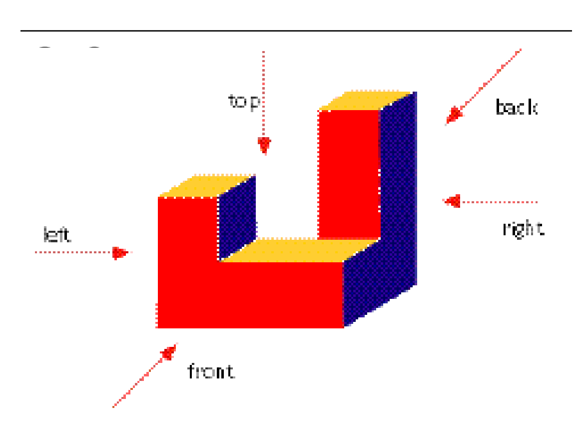
The sequence of colors the guide shouted was:

red, blue, blue, blue, blue, red.

The tourists all met in the central cave. Note that even a tourist who started in the central cave would wind up back there at the end of the sequence.

The two roads emanating from each point of the pentagonal labyrinth represent a type of mathematical problem—a strictly bifurcated directed graph. This particular PlayThink, which is based on the "road coloring problem" of graph theory that mathematicians such as R. L. Adler, L. W. Goodwin, B. Weiss, J. L. O'Brien and J. Friedman have tackled so intensively is like other problems of this sort, still unsolved in general. When mathematicians say "in general," they mean they don't have a ready formula for solving every problem of that sort; instead, the answers are found through trial and error.

CHAPTER 2 SOLUTIONS



90

The sixteen views are combined correctly in the table:

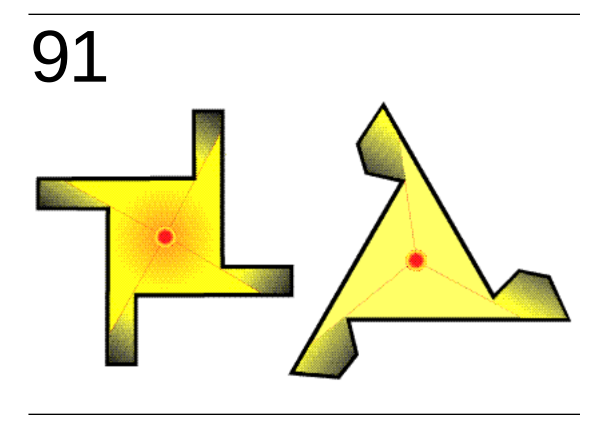
A/15	E/10	I/14	M/13
B/11	F/12	J/7	N/1
C/8	G/16	K/9	O/4
D/6	H/3	L/2	P/5

The multiview problems combine spatial awareness with logic—the ability to visualize in three-dimensional views.

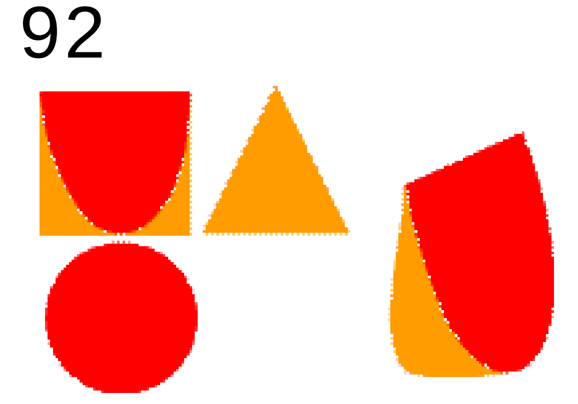
In fact, the overhead views and front views given correspond fairly well to what architects call a plan and front elevation. The plan represents the shape as laid out horizontally on the ground; the elevation is a front view that is derived exactly and immediately from the dimensieons of the plan.

Other elevations derived by architects in the same way are those of the remaining sides of the building, each seen as a direct face-on-view, with no perspective.

91

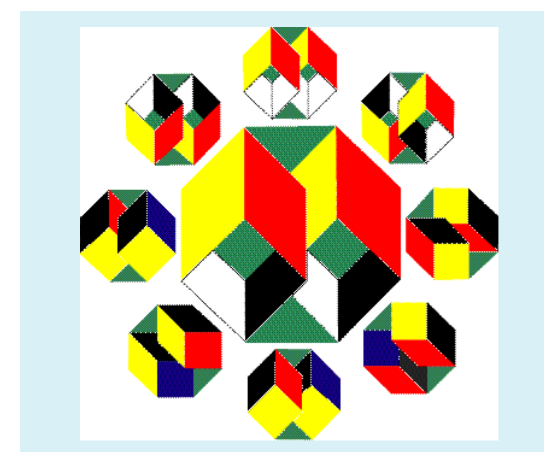


92

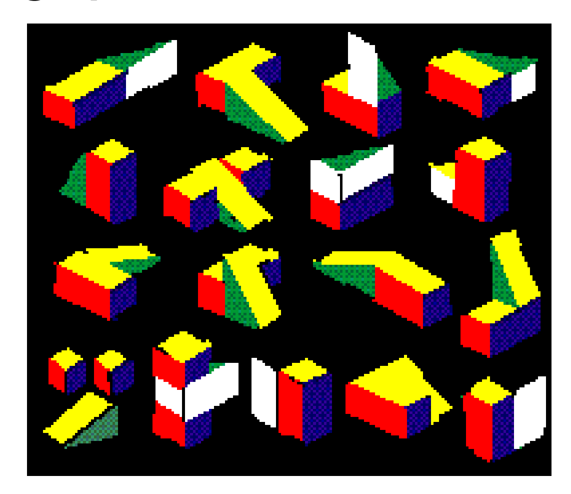


93

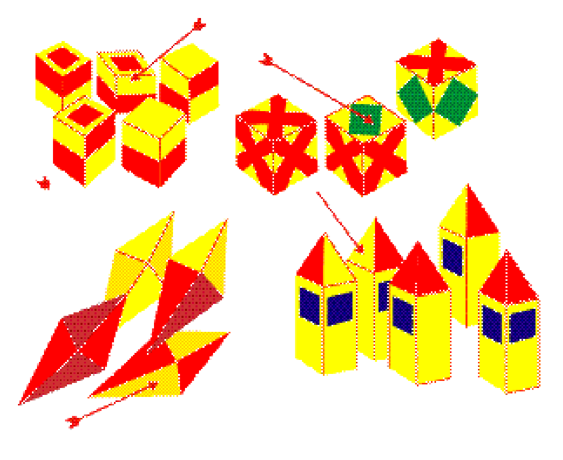
The large central figure was also traced along the lines of the center grid.



94



95

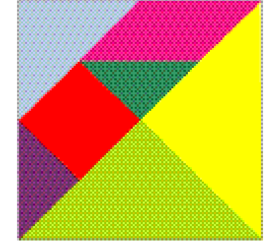


96



97

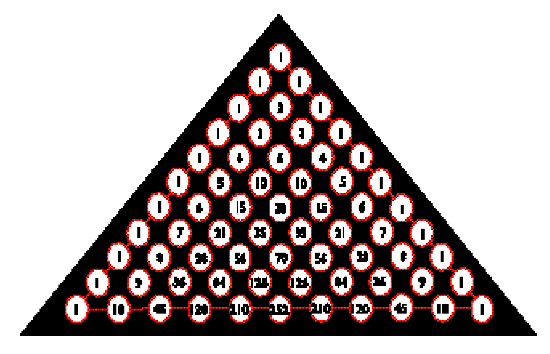
As seen from the point marked in red, the sculpture resolves into a perfect square divided into seven colored pieces that fit together seamlessly.



The sculpture was designed using the laws of perspective and the rules for perspective design, all to demonstrate the importance of point of view when observing a three-dimensional object.

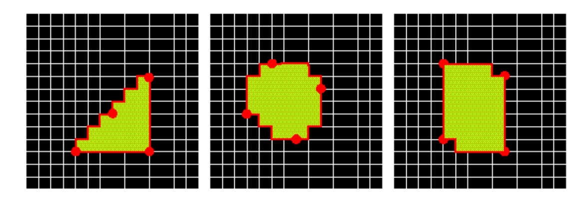
98

Each number is the sum of the two numbers immediately above it. This mathematical tree is called Pascal's triangle.



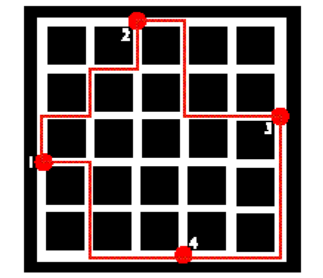
99

There can be many shapes of squares in taxicab geometry. Shown below are several squares that are six blocks on a side.



100

In the geometry of Gridlock City, the shortest route that links all four points is twenty blocks long. And there are 10,000 different routes that you could take that are that short.

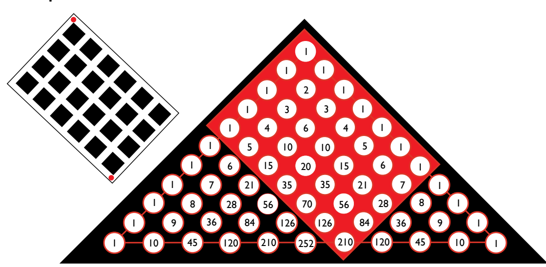


101

There is generally more than one shortest path between two points in a place like Gridlock City. For example, to go to a point halfway around the block, you can move clockwise or counterclockwise—both paths are equally short.

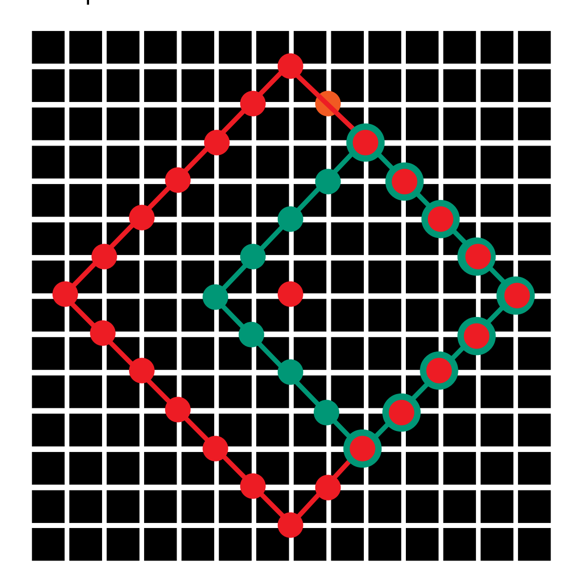
To work out the number of paths of minimum length to each intersection in the grid, you begin by marking the starting point with a 1, which represents the fact that standing still is the shortest path to where you started. The shortest path to the corner is a straight line, so mark each of the nearest corners with a 1 as well. But as was mentioned above, there are two equally short paths to the corner opposite the starting point, so mark that point with a 2. If you carefully fill in the grid and then tilt the grid a bit as shown, you should see part of the famous Pascal's triangle (PlayThink 98).

As seen in the image below, when the plan of Gridlock City is superimposed on Pascal's triangle, point B is located at the point marked 210. Thus, there are 210 equally short paths between point A and point B.



102

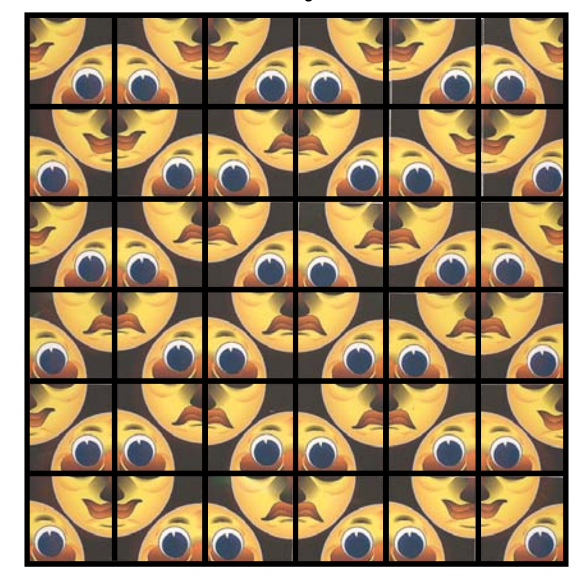
In taxicab geometry circles are squares. The circleof 1 kilometer radius is shown in red, with an intersecting circle with radius ⅔ kilometer (but a center that is two blocks to the east) shown in green. The two circles intersect at nine different points.



Although Euclidean geometry states that any two intersecting circles can have at most two points in common, taxicab geometry allows circles to intersect at any number of points. The larger the squares, the more points they can have in common.

103

A configuration with nine frowning faces and four smiling faces.



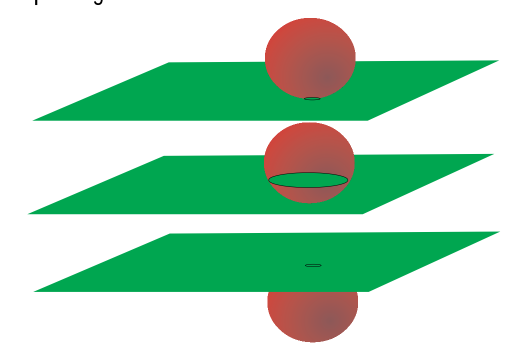
105

In Flatland a law was passed requiring women to constantly twist and turn. In that way they would always be visible.

106

The Flatlanders would not be able to sense the approach of the ball until it intersected with the plane of their world. Those at a distance would see a point appear from nowhere, and that point would grow into a circle that would eventually reach the size of the ball itself. Then the circle would begin to diminish, shrinking to a point and eventually disappearing.

For some Flatlanders the event would be a catastrophe: if they were at the point where the ball intersected their world, they would be lifted off their world into the mysterious "third" dimension. Indeed, if someone from our dimension wanted to take objects from Flatland, they would encounter few problems. Even the most secure vault in Flatland is simply a two-dimensional box with heavy walls. Someone from our dimension could reach into that vault and take anything without disturbing the walls or picking the lock.

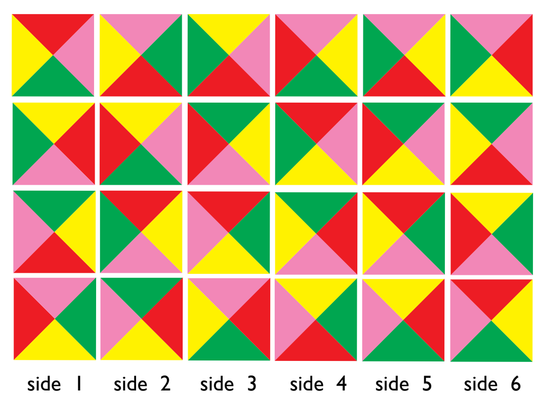


107

The answer sounds trivial—you can bring them together by closing the book—but some physicists speculate that if our three-dimensional space could be made as malleable as the pages of a book, folding space itself may be the means by which humans can travel from star to star.

108

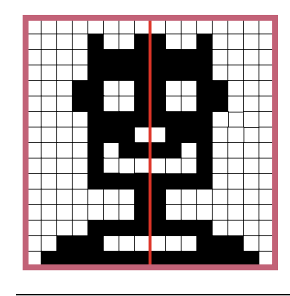
A cube sitting on one side can face in four different directions. A cube has six sides. So four directions per side multiplied by six sides gives a total of twenty-four possible orientations.

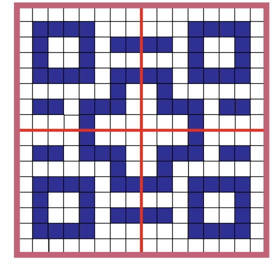


109

There are sixty different ways a dodecahedron can be placed on a table.

110

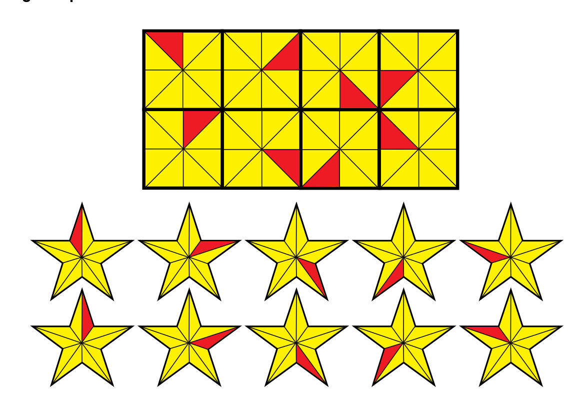




111

The square can undergo eight transformations; the star can undergo ten.

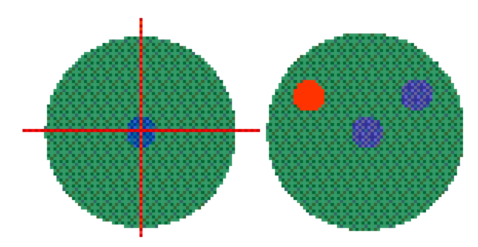
The operations involved are called symmetries. When speaking of symmetry, a mathematician is talking about the way of transforming an object so that it preserves its shape. The object can be rotated or flipped about an axis; the set of transformations of that sort for a specific object is called the symmetry group.



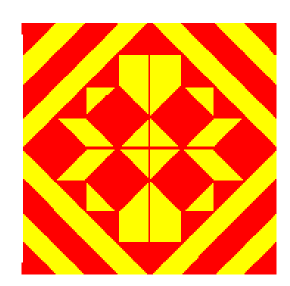
112

The player making the first move can always win by following these instructions: Place the first coin on the exact center of the table. After that move, always respond to the opponent's move with a symmetric move, which will always be possible.

Since the first player's moves are always safe, he or she can't lose. The second player will eventually run out of safe moves.

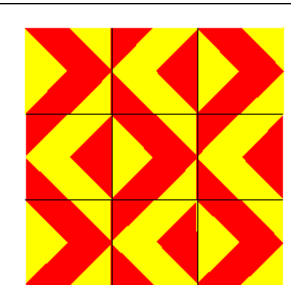


113



114

The last two tiles don't follow the rule.



115

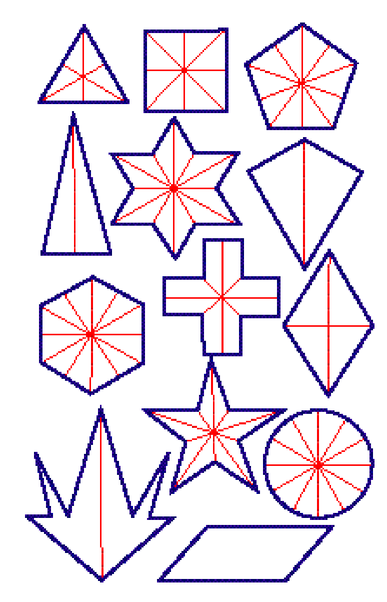
isosceles triangle			2
scalene triangle			1
equilateral triangle			6
square			8
greek cross			8
rhombus			4
parallelogram			2

116

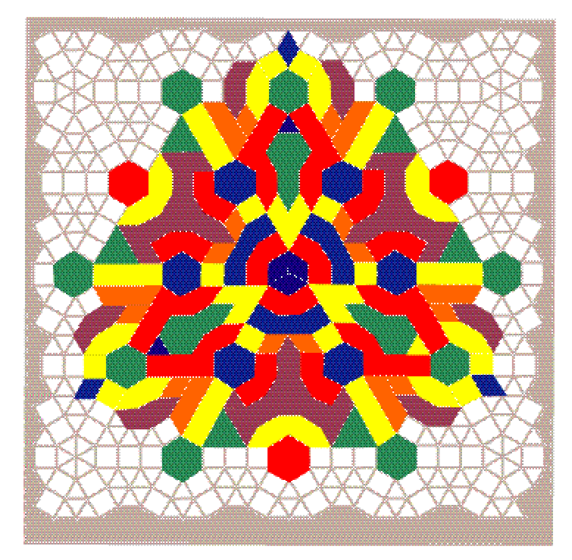
The red letters are the capital letters in the alphabet that have only vertical symmetry. The blue letters are the capitals that have only horizontal symmetry.

117

The parallelogram has no symmetry axes. The circle has an infinite number of symmetry axes.



118

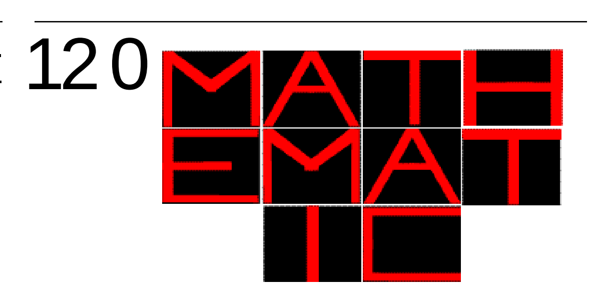


119

The symmetries of the capital letters can be categorized as follows:

1. Letters that are symmetrical only in the vertical plane: A, M, T, U, V, W, Y
2. Letters that are symmetrical only in the horizontal plane: B, C, D, E, K
3. Letters symmetrical in both the vertical and horizontal planes: H, I, O, X
4. Letters that possess only rotational symmetry: N, S, Z
5. Asymmetric letters: F, G, J, L, P, Q, R

120



121

The blue letters are the capital letters that have both horizontal and vertical symmetry. The red letters are asymmetrical.

122

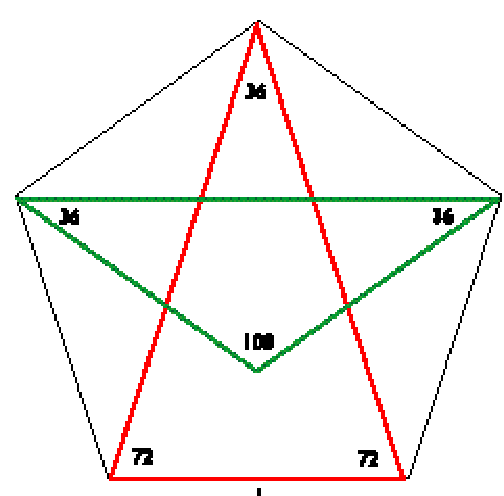
The red letters are asymmetrical. The blue letters have a twofold rotational symmetry. Although some shapes—and letters—have no bilateral symmetry, they still possess rotational symmetry.

123

The first face in the first row, the second face in the seond row and the third face in the third row are impossible to re-create.

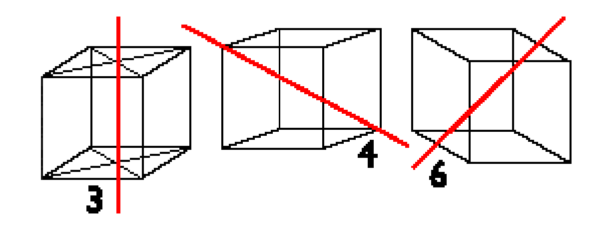
124

The ancient Greeks proved that the pentagram comprised two golden triangles with sides equal to the golden ratio—approximately 1.618—often symbolized by the Greek letter ø.



125

The cube has three fourfold axes of rotational symmetry, four threefold axes and six twofold axes. In general, having a certain number-fold of rotational axes means that if you rotate the object through part of a full rotation equal to the inverse of that number (for example, one-third rotation for a three-fold axis), you get a figure identical to the original.



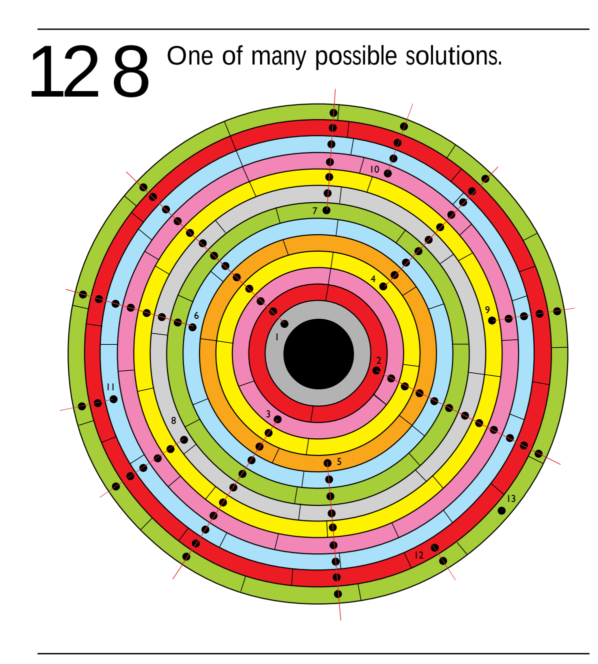
126 At the end of the game, only one green square remains on top of the board.

I hope that you anticipated from the outset that the triangles, circles and semicircles would fall through the square holes. I also hope you noticed that in some cases differences in orientation would prevent triangular and semicircular shapes from falling through similarly shaped holes.

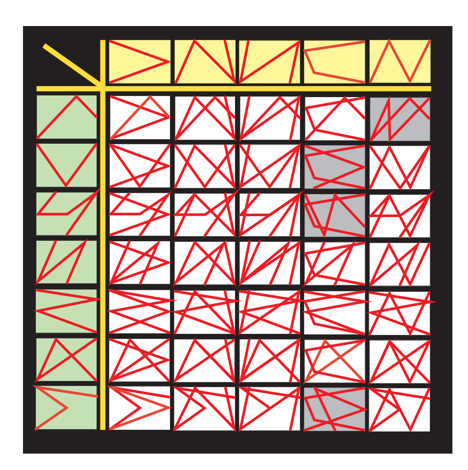
Number of shapes initially on top	16	16	16	16
SHAPES				
Number of shapes initially falling through	6	4	8	12
Number of shapes falling through after 1/4 turn clockwise	5	6	7	0
Number of shapes falling through after 1/2 turn clockwise	3	1	1	3
Number of shapes falling through after 3/4 turn clockwise	1	5	0	1
Number of shapes falling through after one full turn clockwise	1	0	0	0
Number of shapes staying on top	1	0	0	0

CHAPTER 3 SOLUTIONS

128 One of many possible solutions.

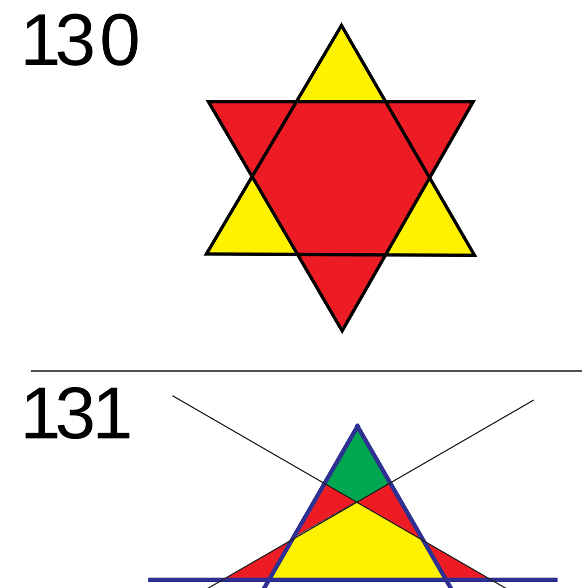


129



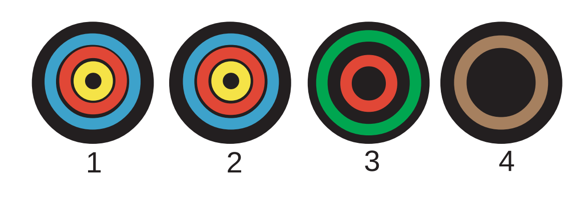
130

131



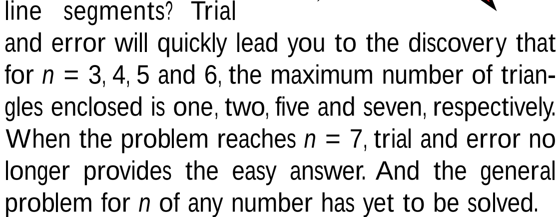
132 The sets of straight lines will blur into concentric circles of different sizes. That baffling result is due to an optical illusion. You no doubt have seen such an illusion before, but you may not know why it works. Don't feel bad: even scientists who study human perception are not sure why straight lines can be perceived as circles.

The most important element of the illusion is something you don't really see—the center point around which the rest of the disc revolves. The distance from that center point to the middle of the line will give you the approximate radius of the circle you see when the turntable begins its motion.



133 A solution with seven triangles is illustrated.

In general, what is the largest number of nonoverlapping triangles that can be enclosed by n straight line segments? Trial and error will quickly lead you to the discovery that for $n = 3, 4, 5$ and 6, the maximum number of triangles enclosed is one, two, five and seven, respectively. When the problem reaches $n = 7$, trial and error no longer provides the easy answer. And the general problem for n of any number has yet to be solved.



134 A simple closed curve is one that does not cross itself. A loop of string that follows that rule can always be stretched into a circle; likewise, a circle of string can be pulled to form a loop. But with a loop or a circle, there is always an inside and an outside.

One way to determine whether a point is on the inside or the outside is to carefully shade in all the interior spaces of the loop. But that is time-consuming. A short and elegant solution is to draw a line connecting the point to an area clearly outside the loop and count the number of times the line crosses a curve. If it crosses an odd number of times, the point is inside the loop; if it crosses an even number of times, it is outside the loop.

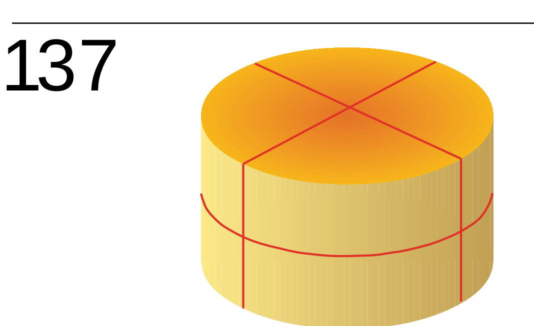
This rule is known as the Jordan curve theorem.

135 You can check with your own randomly drawn lines, but the intersections will always align. That surprising result is known as Pappus's theorem.

136 There is only one convex polygon: the hexagon in the lower right-hand corner.

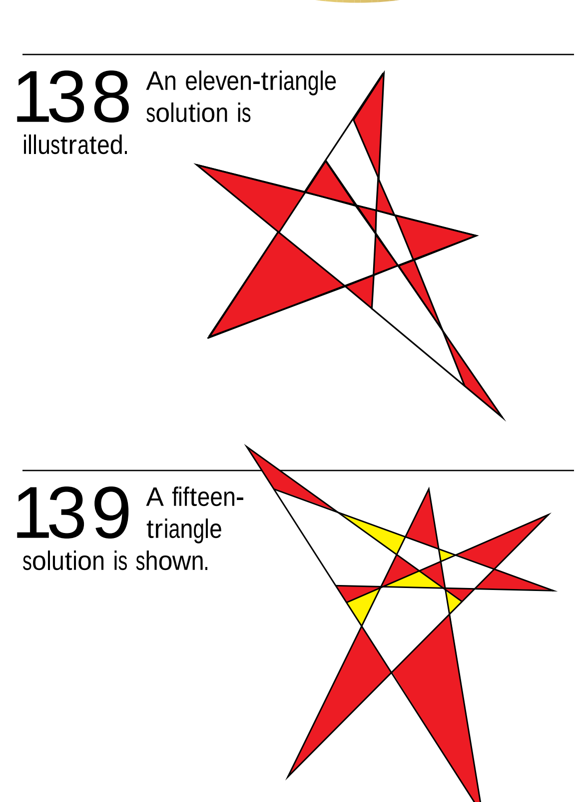
The figure-eight-shaped polygon is different from all the other objects because its lines intersect.

137



138 An eleven-triangle solution is illustrated.

139 A fifteen-triangle solution is shown.



140

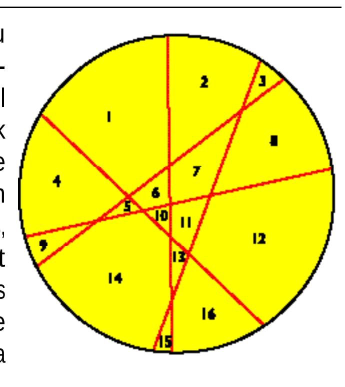
The illustration above shows four cuts dividing a cake into eleven pieces. As a general rule, try to place each new cut across all the previous cuts. In that way every nth cut creates n new pieces.

Lines	Pieces	Total
0	1	1
1	1 + 1	2
2	2 + 2	4
3	4 + 3	7
4	7 + 4	11
5	11 + 5	16
and so on		

The general principle can be written as a general formula for n number of cuts: $(n\,(n+1))/2 + 1$.

141

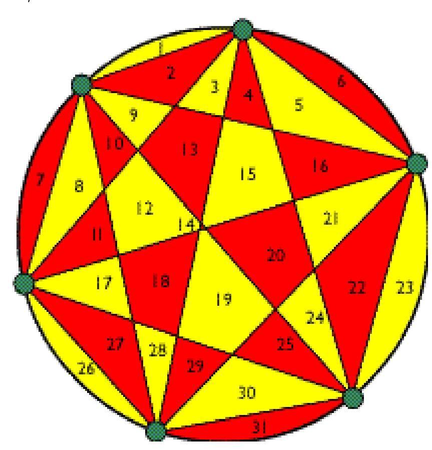
If you understand the general rule (see PlayThink 140), this should be easy: if four cuts can make eleven pieces, then a fifth cut across the previous four will make five new pieces, for a total of sixteen.



142

In spite of the answers for the smaller numbers of points, the answer for six points is thirty-one, not thirty-two.

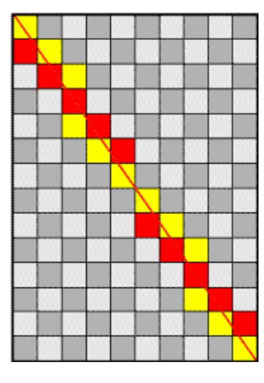
This is a beautiful example of why guessing at an answer is not the best way to solve a problem. The sequence of the partitions created for the series from zero points to nine points is 1, 2, 4, 8, 16, 31, 57, 99, 163, 256.



143

In general, the number of compartments crossed by the laser equals the sum of the two sides of the box minus the greatest common divisor of those two numbers. In this instance:

10 + 14 – 2 = 22.



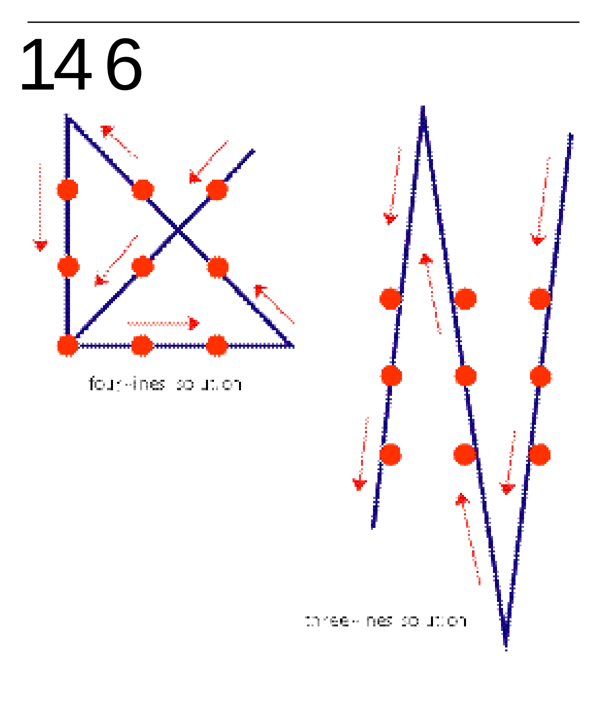
144

145

Minimizing the number of intersections is easy: make all the lines parallel. Maximizing the number of intersections is much more difficult. Two lines can meet at only one point; three lines at exactly three points; four lines at six and so on. A little trial and error with drinking straws, pencil and paper or computer graphics will lead you to the maximum solution. All you need to do is avoid making any line parallel to another—eventually every line will intersect with every other line.

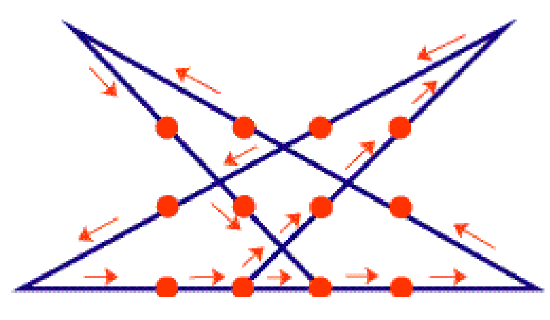
So for five lines there is a maximum of ten intersections.

146

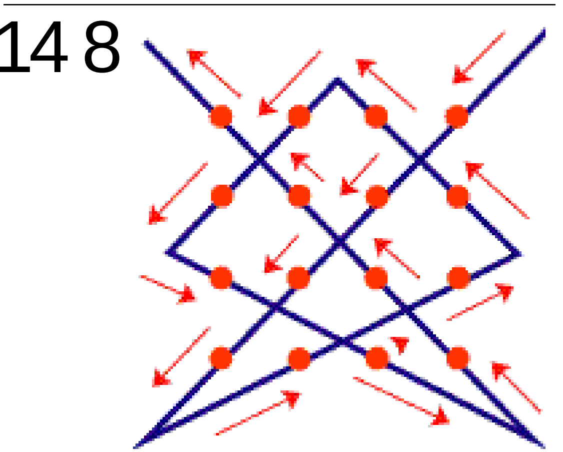


147

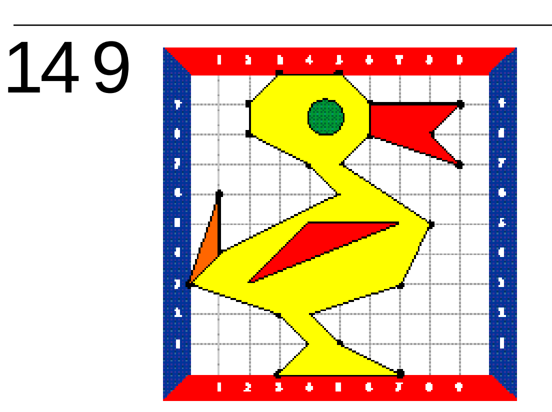
Once gained, a valuable insight can be generalized. If you have solved the problem of the nine points, the answer to problems involving greater numbers of points should come easily. For this problem five lines are needed.



148



149



150

Most people think that three is the best answer. But if the three trees surround a steep hill or valley, then a fourth tree can be planted at the top of the hill or at the bottom of the valley, forming a tetrahedron. A tetrahedron is a three-dimensional shape made of four equilateral triangles, and therefore all four of its points are equidistant.



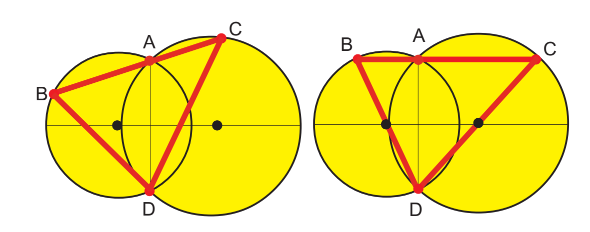
151

Since Fido is tied to a tree, he can reach anywhere within a ten-foot radius of the tree. His bowl is five feet from the tree, on the opposite side from where Fido started.

152

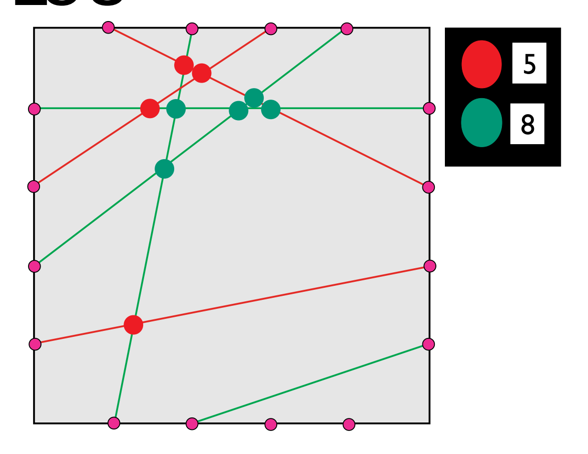
Begin by creating a triangle that connects points *B* and *C* to point *D*. You will find as you move points *B* and *C* around—careful to make sure that line *BC* always runs through point *A*—that the angles *BDC*, *DBC* and *BCD* remain the same. That means that the way to make line *BAC* the longest is by making lines *BD* and *CD* the longest they can be. Lines *BD* and *CD* are at their longest when they are the diameters of their respective circles. It is then that line *BAC* is at its longest.

It just so happens that when *BD* and *CD* run through the diameters of the circles, line *BAC* is perpendicular to line *AD*.



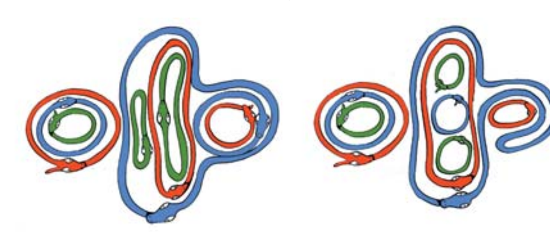
153

Sample game in which green wins.

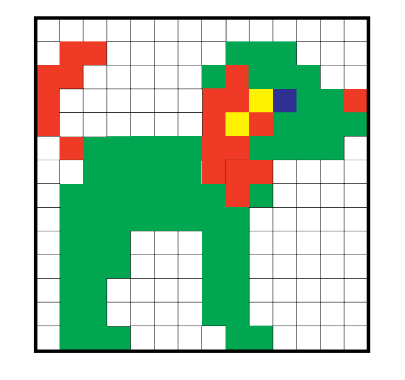


154

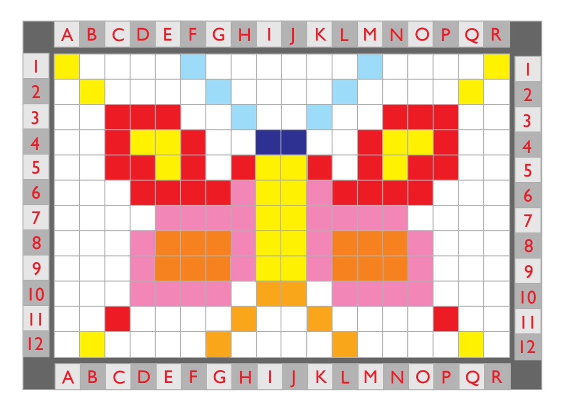
Two solutions are possible. The hidden snake is either green or blue.



155

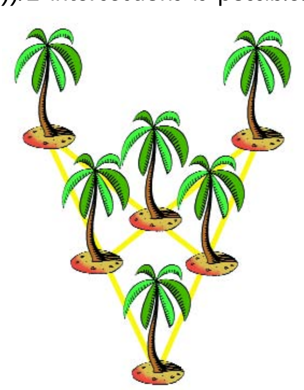


156



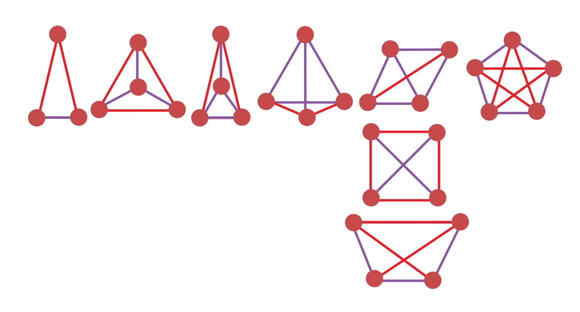
157

This is another example of a problem concerning lines, intersections and restrictions on possible configurations. With *n* lines, a maximum of (n (n – 1))/2 intersections is possible. Fewer intersections are also possible, down to (n – 1) intersections, a case in which all but one of the lines run parallel.

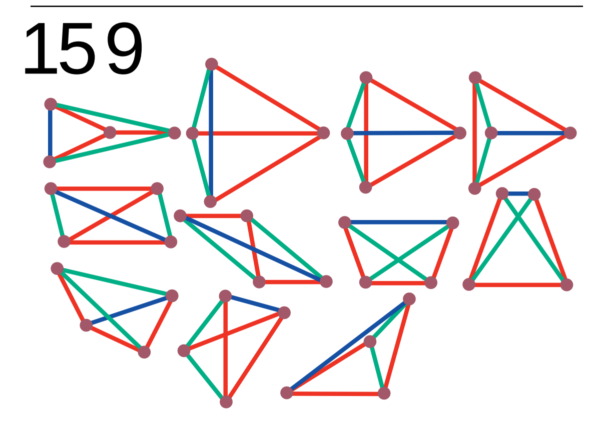


158

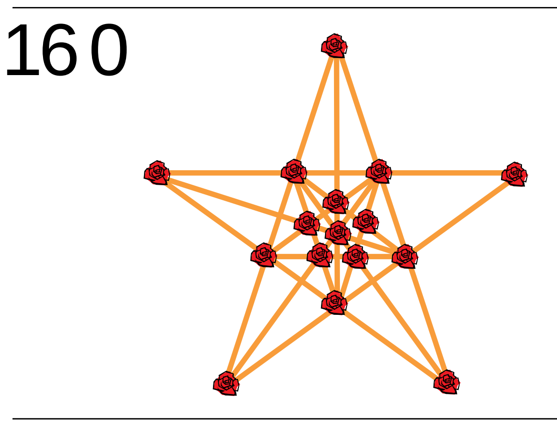
There are exactly eight two-distance sets; all eight are illustrated here. In each figure the red lines have one length, and the blue lines have another.



159



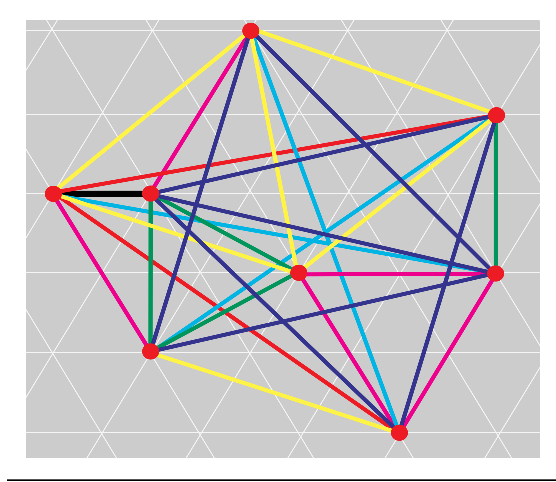
160



161

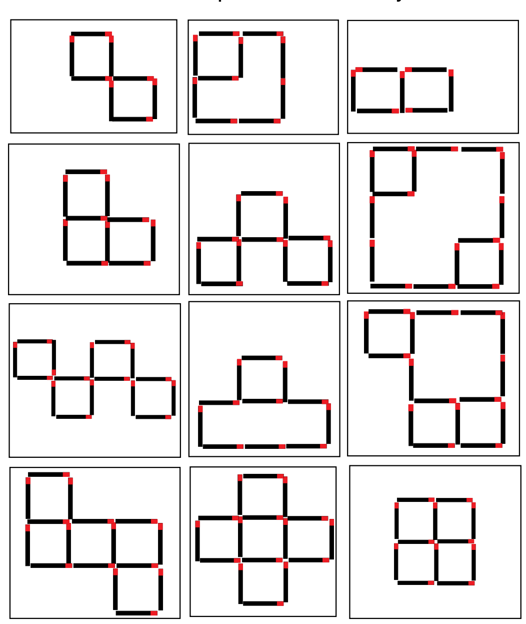
The Hungarian mathematician Ilona Palåsti discovered the eight-point multidistance graph in 1989. It is the largest set known: an eight-point, seven-distance set.

Distance 1 is black; 2, red; 3, cyan; 4, green; 5, magenta; 6, yellow; 7, blue.

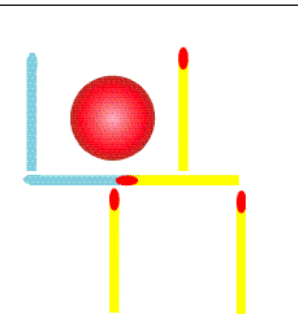


162

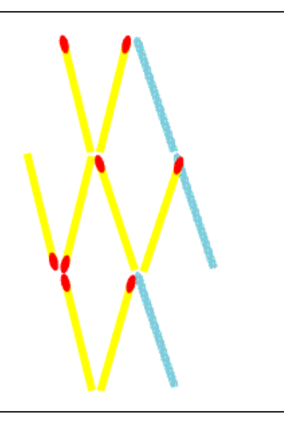
The key to solving this sort of puzzle is to visualize the answer before you pick up the first match. Some answers involve squares of different sizes; some overlap; many have common sides. But if you have trouble imagining the answer, the trial-and-error approach will help you work toward a solution and better understand the principles behind the puzzle. Once you have mastered these games, you can devise more complex versions on your own.



163

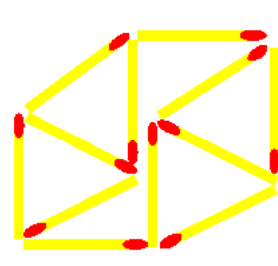


164



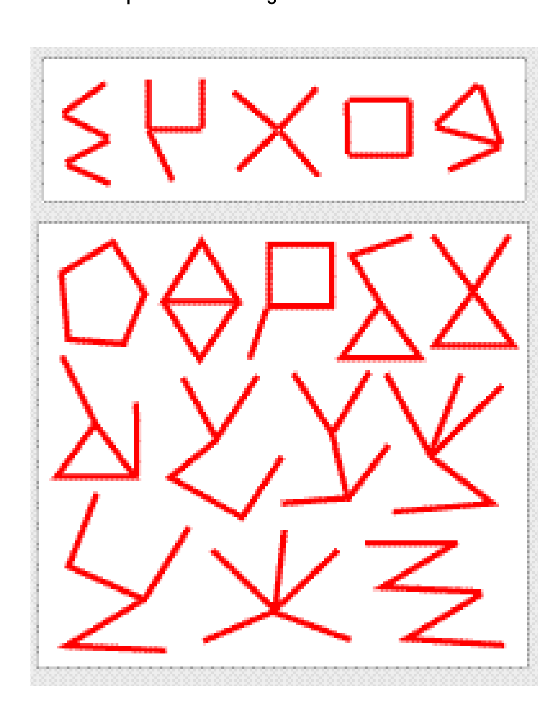
165

The solution shown here requires twelve matches meeting at eight points in the plane. A triangular pyramid with six matches meeting at four points would do it in space.



166

With four matchsticks five configurations are possible. With five matchsticks there are twelve possible configurations.



167



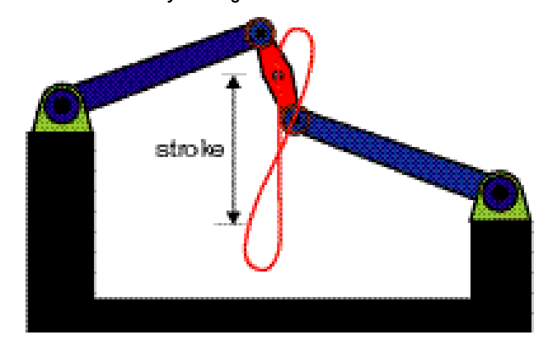
168

As you can see, area and perimeter have little bearing on each other. The way the area and angles vary as other elements of the shape change introduces the concept of function, which you will see more of later.

	CONSTANT	CHANGE
AREA	NO	YES
PERIMETER	YES	NO
SIDES	YES	NO
ANGLES	NO	YES

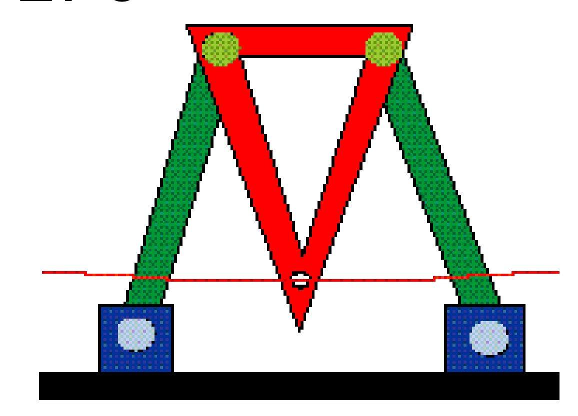
169

The linkage illustrated below is a schematic representation of the famous Watt's linkage, which draws a figure-eight-shaped curve. Part of that curve—called Bernoulli's lemniscate—is a nearly straight line.

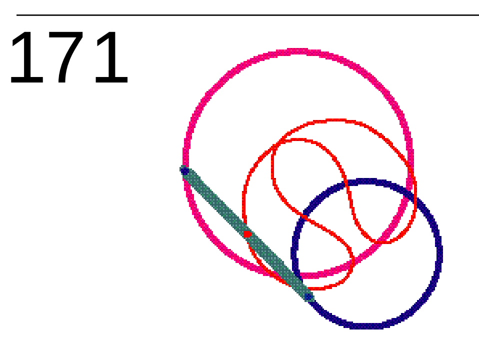


170

The path is approximately a straight line.

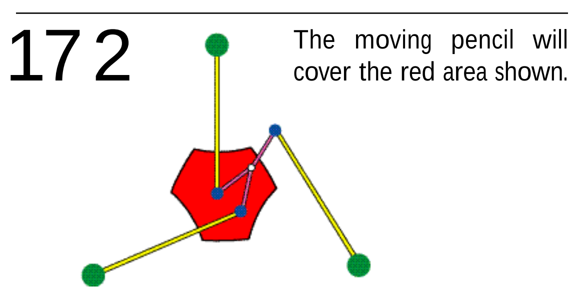


171

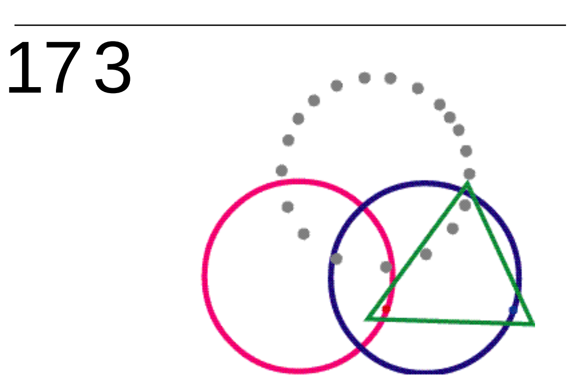


172

The moving pencil will cover the red area shown.



173



CHAPTER 4 SOLUTIONS

174

The only way to get to the ladybug's friend is through the red flower at the top of the diagram, so red must represent up.

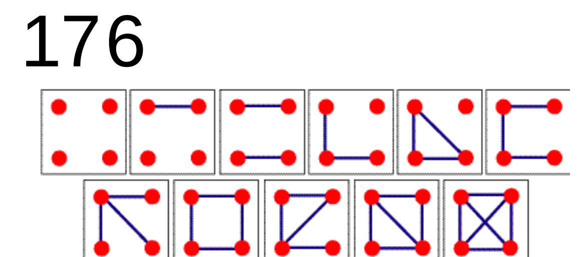
Purple cannot be up, and if it were down, the ladybug's first move would be off the diagram. If purple meant left, then the ladybug would move to a yellow flower and the only allowable direction for pink would be right—a never-ending loop! So purple must represent right.

After figuring out that, it is easy to tell that blue represents left and yellow represents down.

175

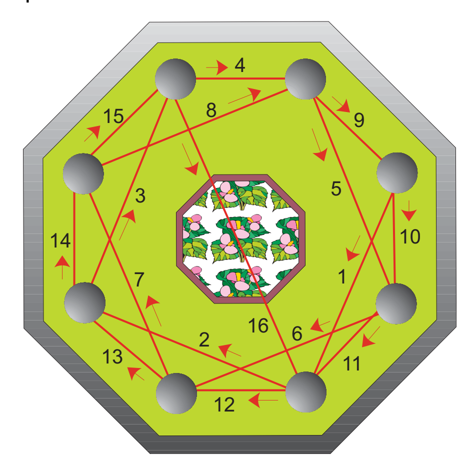
A pirate with one peg leg pushed a two-wheeled cart. The pirate's dog walked beside him.

176



177

There are twenty allowable paths between the pillars, but no matter how many different ways I ran, I could never get past seventeen legs. If the courtyard had had seven pillars or nine pillars, I would have been able to reach the mathematical maximum. As it was, I later discovered through my study of topology, the full complement of paths is impossible to achieve in a courtyard with eight pillars.



178

If you have trouble solving this problem, it may be because it is often difficult to visualize the hidden edges and corners of a solid figure. It may be useful to draw a topologically equivalent two-dimensional diagram of the three-dimensional solid. Such a diagram makes each edge and corner visible and enables you to see the relationships between them.

Our solution is drawn on that sort of diagram.

179

When he solved the problem of the Seven Bridges of Königsberg (see page 71), Leonhard Euler discovered the general rule for tackling this class of puzzle. The secret is to count the number of paths leading from each point of intersection, or junction. If more than two junctions possess an odd number of paths, the pattern is impossible to trace.

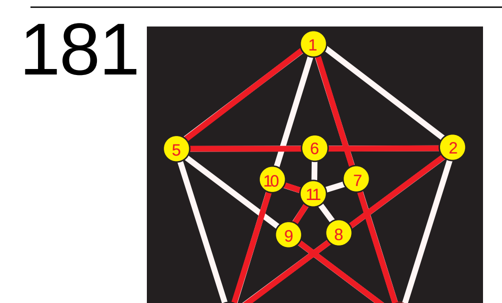
In this instance, paths 4 and 5 are impossible.

If there are exactly two junctions that have an odd number of paths, the problem may be solved, but only if you begin and end at those two junctions. Path 7 has this property; to fully trace it, you must start at one of the lower corners and finish at the other.

180

There are ten allowable routes.

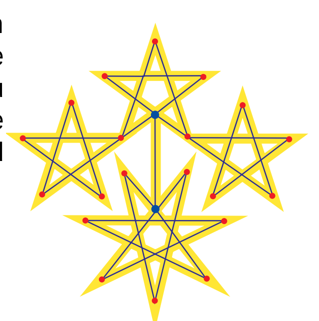
181



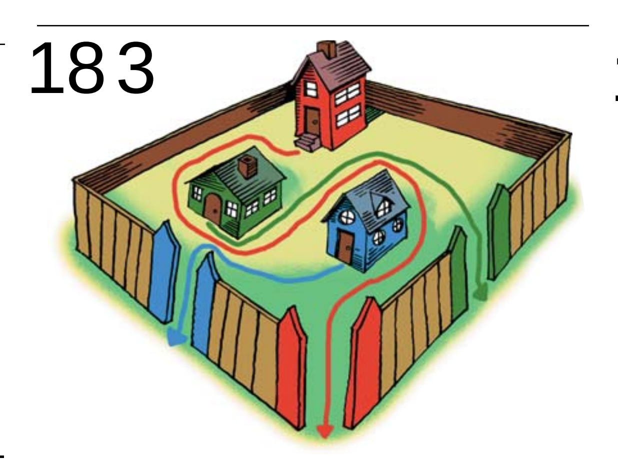
The Hamiltonian Circuit: 1-5-6-2-8-4-10-11-9-3-7-1

182

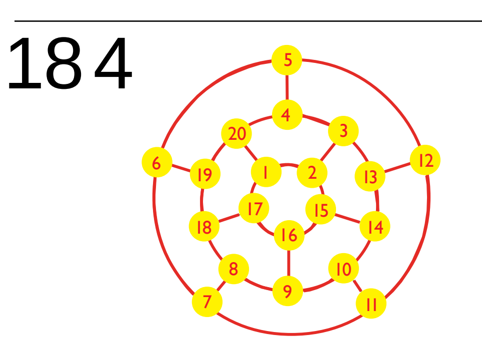
You can trace the figure, but only if you start at one of the blue points and end at the other.



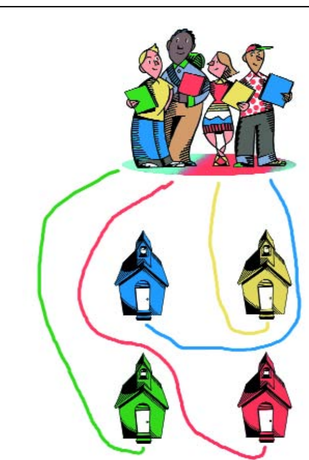
183



184



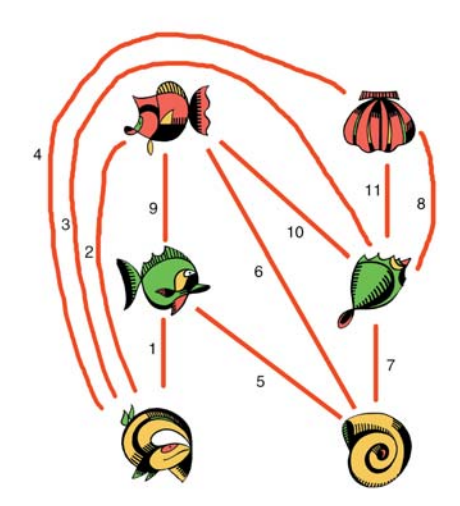
185



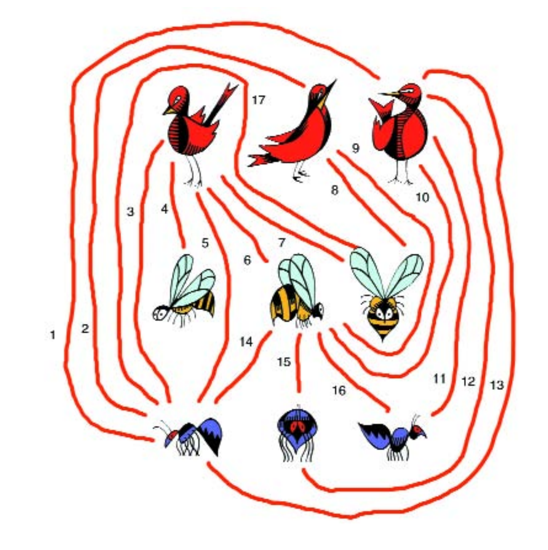
186

It is, in fact, impossible to connect each house to each utility without at least one pair of lines intersecting. Or, as one ingenious solution has it, tunneling under some of the houses.

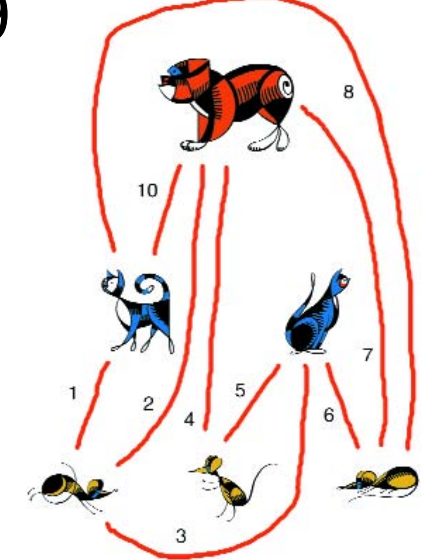
187



188

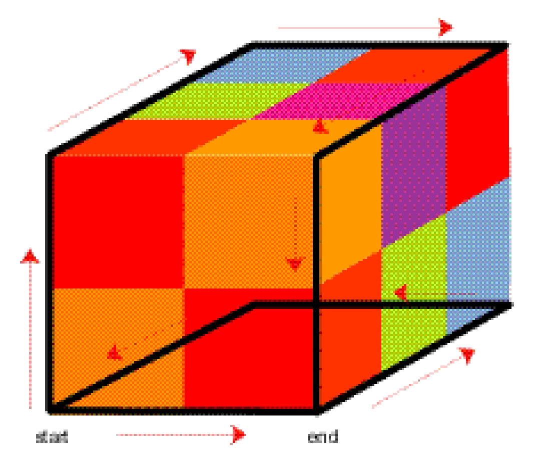


189

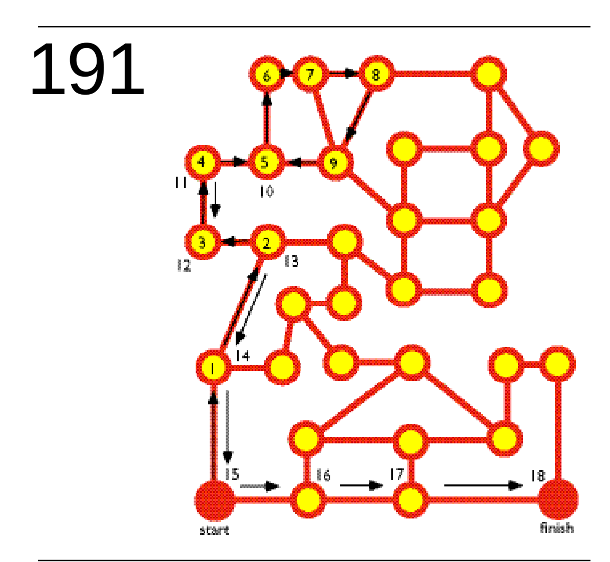


190

The worm can crawl 22 centimeters, as shown below.

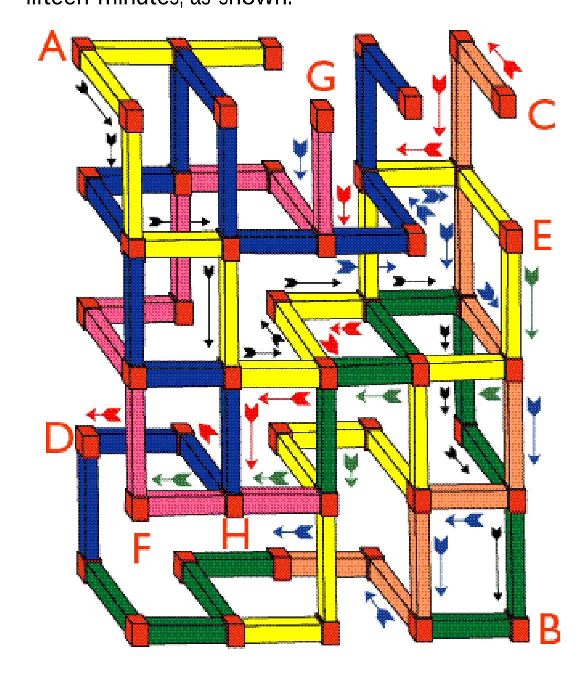


191



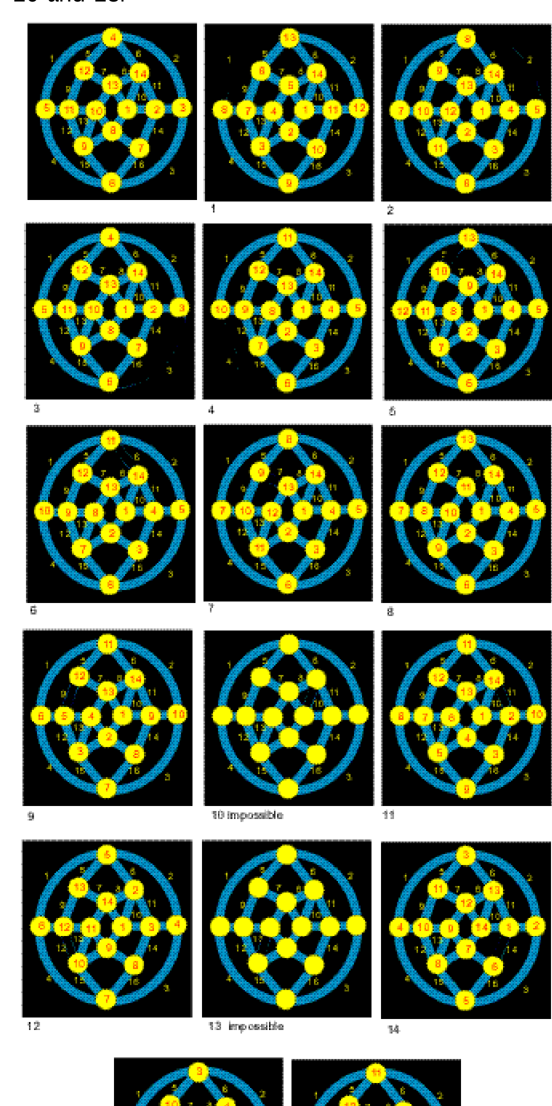
192

To travel from A to B will take thirteen minutes; from C to D also thirteen minutes; from E to F nine minutes; and from G to H fifteen minutes, as shown.



193

The two guideways that, when missing, make the puzzle impossible are numbers 10 and 13.

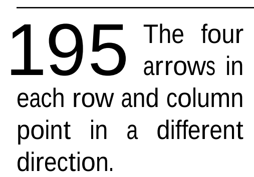


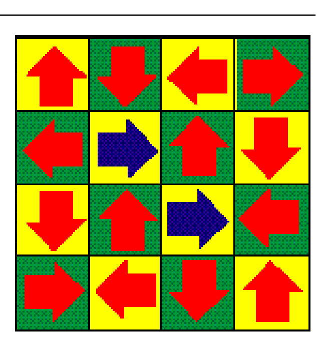
194

Puzzle master Sam Loyd first published his Mars puzzle in *Our Puzzle Magazine* in 1907; ten thousand readers wrote in saying they had tried to solve the problem and found "there is no possible way." Those ten thousand readers had solved the puzzle—THERE IS NO POSSIBLE WAY.

195

The four arrows in each row and column point in a different direction.

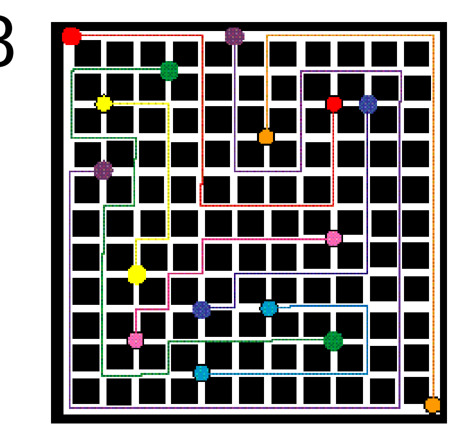




196

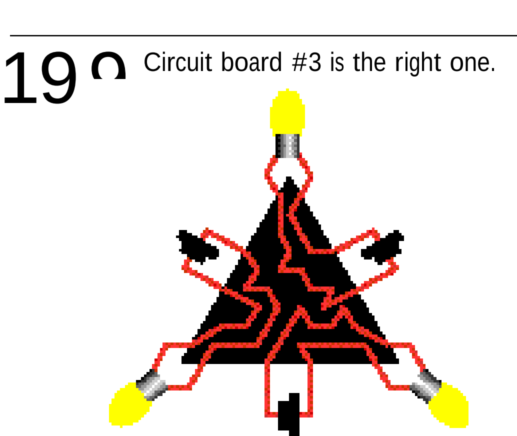
197

198



199

Circuit board #3 is the right one.

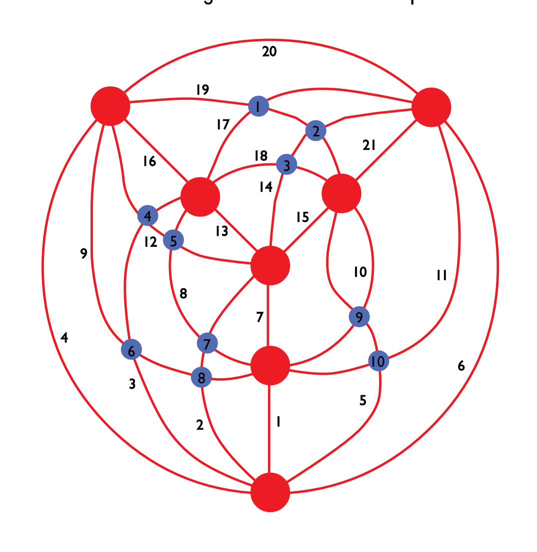


200

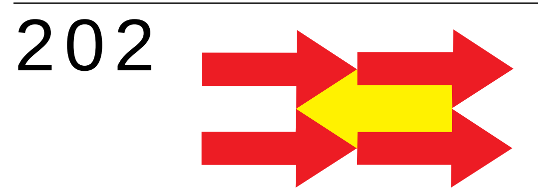
Before taking into account the symmetries of the cube, you can place the arrows in 4,096 (4^6) different ways. But eliminating configurations that are symmetrical duplicates leaves you with just 192 different ways to label the cube with arrows.

201

Line number 5 can be moved so that it crosses only one other line. That makes for nine intersections, which is the fewest possible when interconnecting seven different points. Mathematicians have their own way of saying that: The minimal crossing number for seven points is nine.

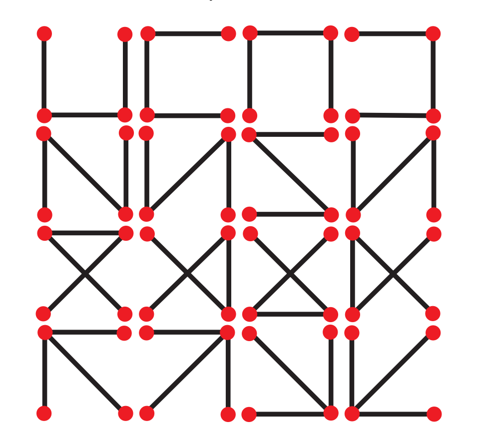


202



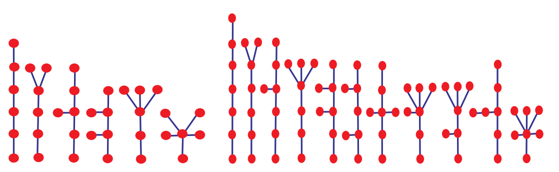
203

The sixteen tree graphs that connect the four points are illustrated below.



204

The topologically distinct trees for six and seven points are illustrated below.



206

TREE GAME CARD SETS:

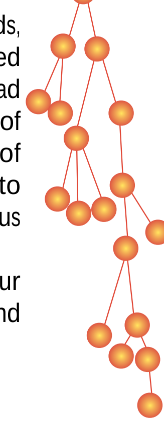
1-20-35-61
2-5-32-56
3-29-47-75
4-17-24-25
6-8-21-59
7-11-30-31
9-36-41-45
10-22-38-49
12-19-26-63
13-27-50-55
14-39-43-48
16-42-46-62
18-23-53-64
28-34-40-58
33-44-51-60
15-37-52-54

207

One solution is shown. There are many others, but all will have eighteen branches.

In the case of strings and beads, the correct answer can be suspended in the manner shown, and each bead will hang from exactly one piece of string. Therefore, the number of strings or lines or branches is equal to the number of beads or points, minus one.

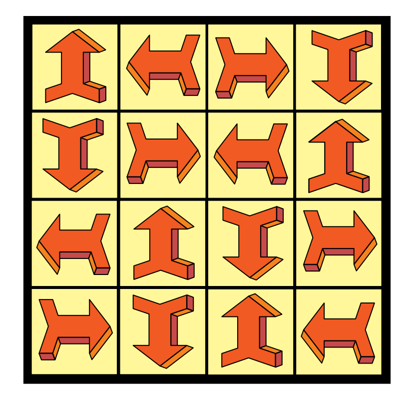
No matter how you draw your tree, that is both the maximum and the minimum number of lines.



208

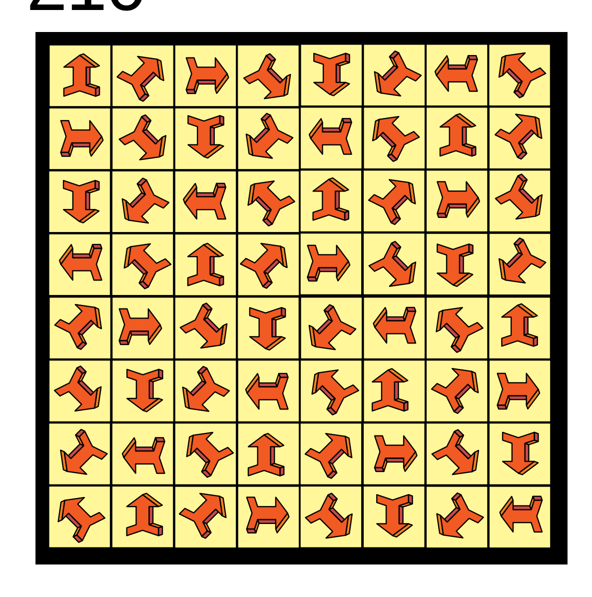
Each row and column contains arrows that point in the eight main directions.

209



210

One of many possible solutions.



211

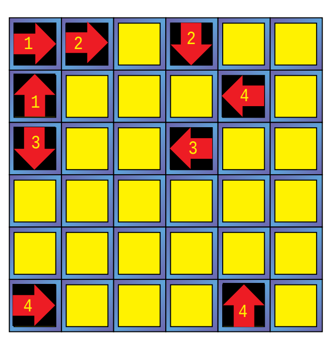
The route is 5, 1, 2, 4, 3.

212

One route is 6, 1, 5, 3, 2, 4; another one is 6, 1, 4, 5, 3, 2.

213

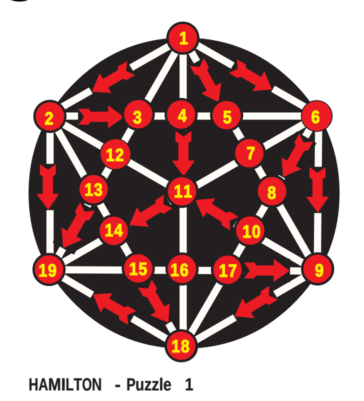
There are many solutions, including the one shown here.

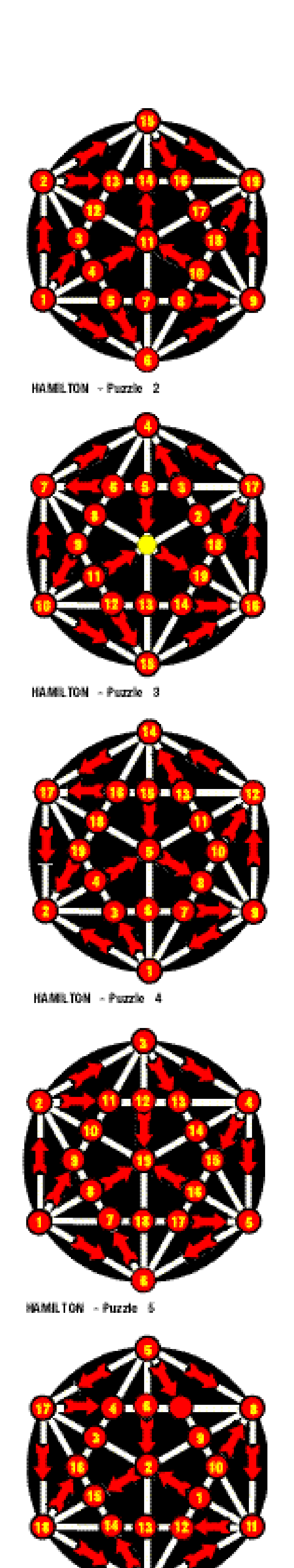


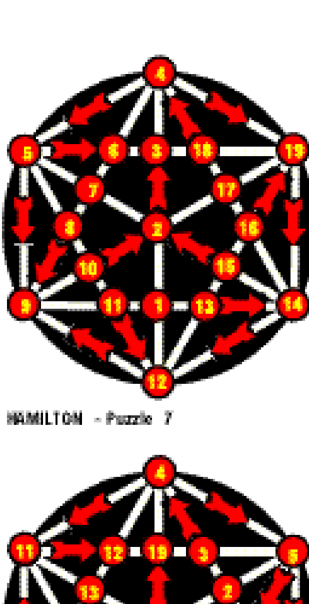
214

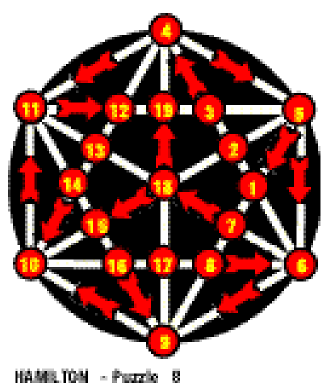
No matter how the arrows are placed, there will always be a path that interconnects the six cities. This is because this is a complete digraph puzzle.

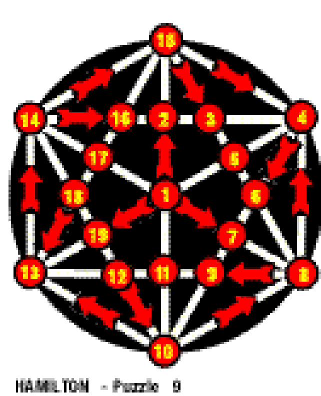
215









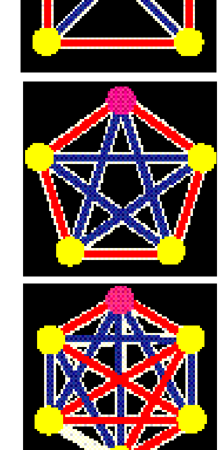


216

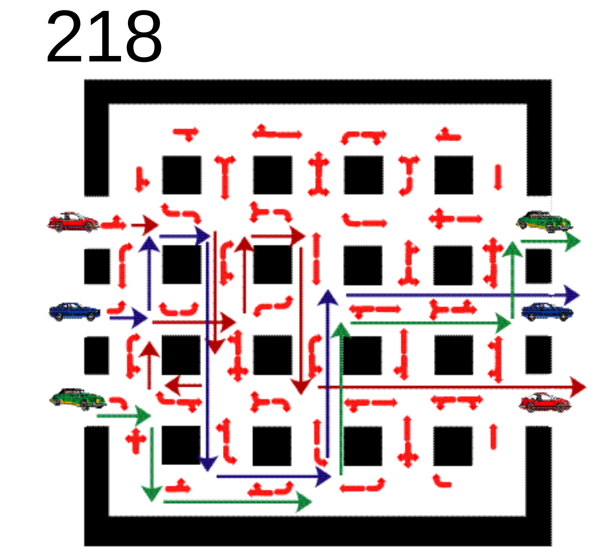
You can avoid love or hate triangles in groups of four or five, as shown at right. For any three points there are always two different types of relationships represented.

For the group of six, however, there is no way to avoid having a love triangle or a hate triangle. As you can see, no matter which color you choose for the uncolored line, you will be forced to create either a blue triangle or a red one.

This is one of the applications of Ramsey theory. There are quite a few others.



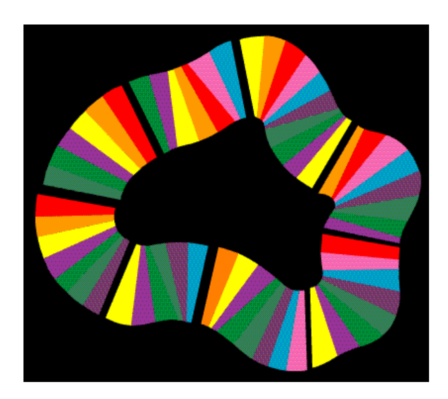
218



CHAPTER 5 SOLUTIONS

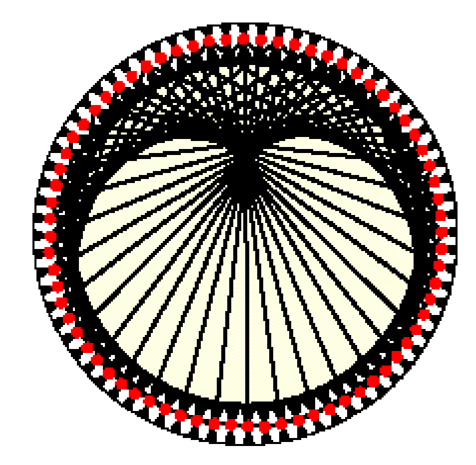
219

The secret of the puzzle is in the color sequence: yellow, orange, red, pink, blue, violet, light green, dark green, purple. The sequence moves in a clockwise direction. The pieces are put together so that the next piece continues the sequence where the last piece left off.

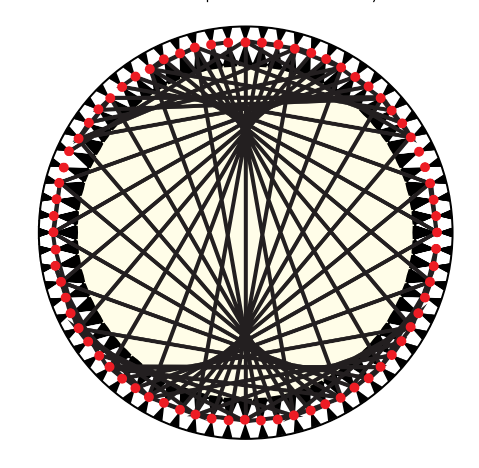


220

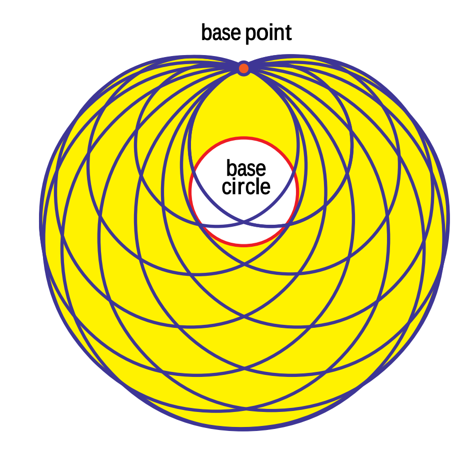
The pattern that emerges is a 1:2 web, though it is also known by a grander name: cardioid, or heart curve.



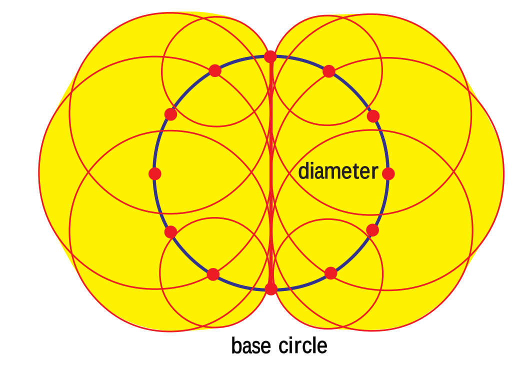
221
This pattern is the 1:3 web, also known as the nephroid, or kidney curve.



222
The pattern will be a cardioid.

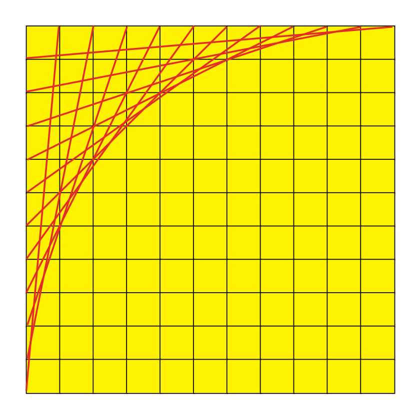


223
The pattern that emerges is a nephroid.



224
Center
Circumference
Radius
Diameter
Tangent
Arc
Chord
Segment
Semicircle
Sector
Quadrant

225
The path formed when one object chases another object that moves in a predetermined way has a special name: the pursuit curve, or tractrix.



226
The red fluid completely fills the circle of radius a/2, but it only partly fills the square. By observation we can see that the fluid fills an area equal to $3 + \frac{1}{7}(\frac{a}{2})^2$, from which it follows that π equals $3 + \frac{1}{7}$.

The area of the triangle is $(a^2)/2$.

227



228
1. Round manhole covers cannot fall through their round holes accidentally. Square or other polygonal covers can.

2. Heavy round covers can be rolled into position, while other shapes would have to be carried.

3. Round covers can cover holes no matter how they are oriented vis-à-vis the hole. Square covers fit only when they are positioned in one of four orientations.

229
As the rollers are moved forward, their point of contact with the weight moves backward at a rate of 1 meter a turn. But the rollers are also in contact with the ground and moving forward in comparison with it at a rate of 1 meter a turn. Together, that means that the weight moves forward in relation to the ground at a rate of 2 meters per full turn.

230
The circumference of every circle is approximately $3 + \frac{1}{7}$ times the diameter. That number is famously known as π and is the best known of the irrational numbers—numbers that continue an infinite number of decimal places without repeating.

$$\pi = 3.14159265358\ldots$$

Most students learn about the number π when a teacher simply gives its definition. Here you discovered it yourself, much as the ancient Greeks did thousands of years ago.

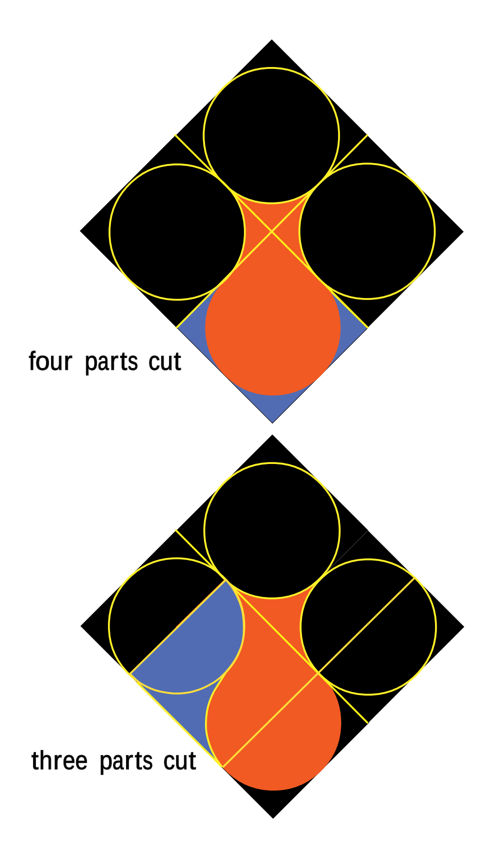
231
The combined areas of the two red crescents—which are the areas of the two small semicircles not covered by the large black semicircle—equal the area of the right triangle itself.

Although the circle itself cannot be squared, other figures bounded by circular arcs can be. That fact arouses false hope in those who would still like to square the circle.



232
The area of the red parts is slightly more than 1.3 times greater than that of the black areas. The black areas seem larger because of an optical illusion.

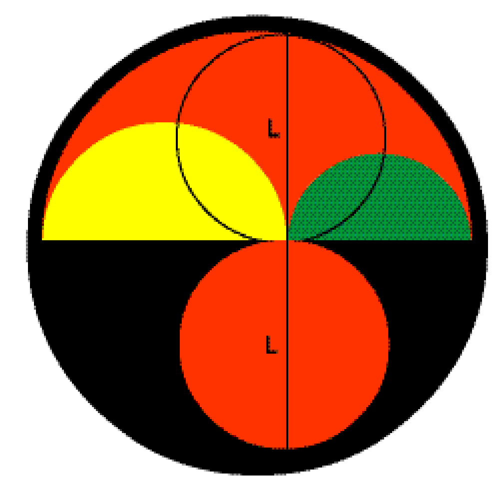
233



234

The area of the sickle is equal to the area of a circle that has a diameter of *L*.

The famous Greek scientist Archimedes first solved this problem, which now bears his name.



235

Rather than count all the lines, you can calculate the total. Fourteen lines emanate from each point, and 14 x 15 = 210. But since each line is shared by two points, the actual number of lines is half that, or 105.

According to Euler's Problem (PlayThink 179), it is possible to trace the design in one continuous line.

Mystic roses may be considered to be all the diagonals and sides of a regular polygon of a given number of sides.

236

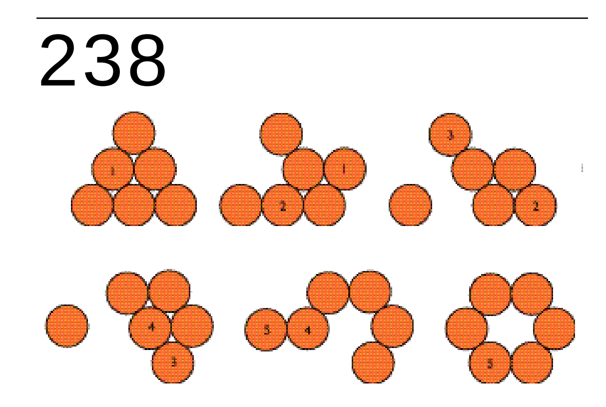
The area of the circle is πr^2.

237

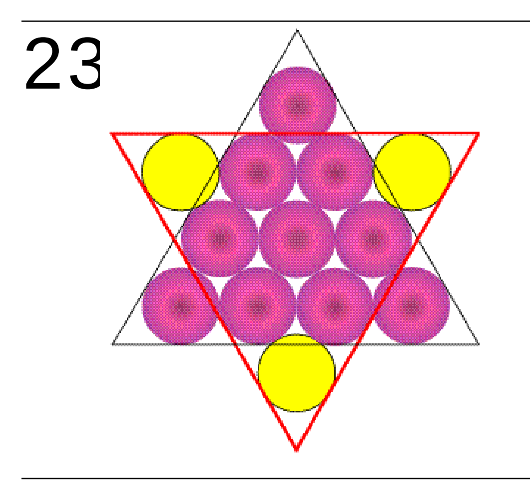
Triangle 1 provides the largest total area of circles.

This puzzle is a special case of the famous Malfatti's problem. In 1803, Gian Francesco Malfatti, an Italian mathematician, asked for the three largest cylinders (in volume) that could be placed in a prism.

238



23

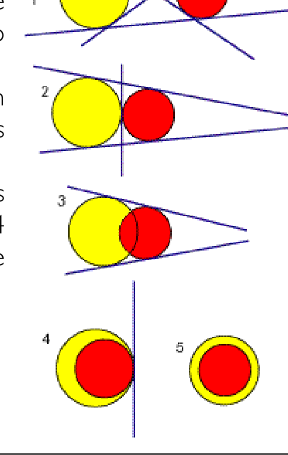


240

There are basically five ways to arrange two circles on a plane.

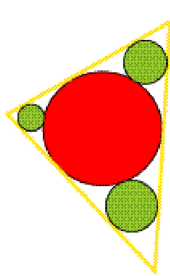
There are ten common tangents, as illustrated at right.

Yes. If the circles were identical, cases 4 and 5 would not be possible.



241

The three yellow circles will grow so large that they become, in the limit, the sides of a triangle. The red circle will become an inscribed circle in that triangle.

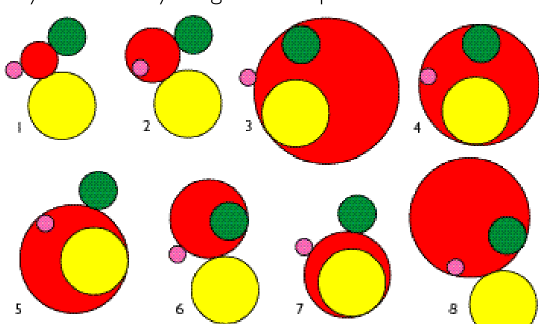


242

Surprisingly enough, there are only eight different ways that three circles may touch a fourth on a plane. All eight are illustrated below.

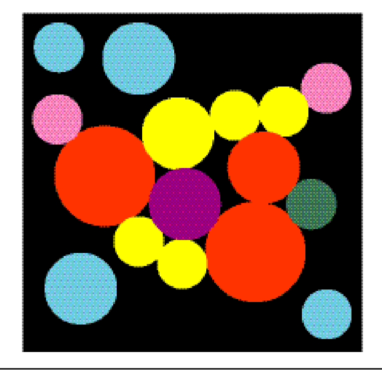
For the general case, take three circles and move them together so that they are mutually tangent; then, in the space between the three, draw a circle that touches all of them. In that way four circles can be mutually tangent. You may also draw a circle around all three that is mutually tangent.

Four is also the maximum number of circles that may be mutually tangent on a plane.



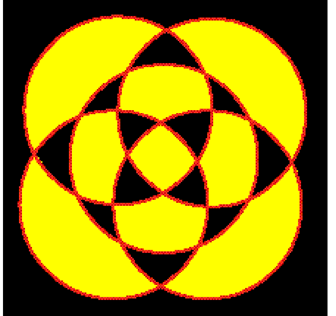
243

The color of the circle depends upon the number of circles it touches.



244

Five intersecting circles will divide a plane into twenty-two regions, as shown.



Euler's formula for polyhedra (see below) is also valid for this sort of connected graph; simply imagine the polyhedra distorted and flattened on a plane.

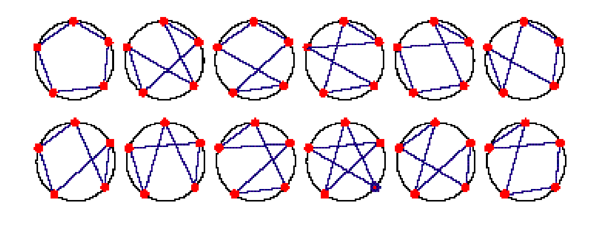
A circle can intersect another circle at two points. For each circle, that makes for $2(n-1)$ points of intersection. Counting all the circles and dividing by 2 (since each point was counted twice) gives $n(n-1)$ points of intersection, or vertices. Each circle is also divided into $2(n-1)$ segments, giving a total of $2n(n-1)$ edges.

Euler's formula gives:

$$\begin{aligned} \text{Regions} &= \text{Edges} - \text{Vertices} + 2, \text{ so} \\ &= 2n(n-1) - n(n-1) + 2 \\ &= n^2 - (n - 2) \text{ regions} \end{aligned}$$

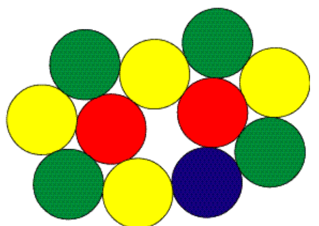
245

There are twelve polygons possible with these five points. Only two of the polygons are regular; the rest can be divided into two groups—essentially, two shapes in five different orientations each.

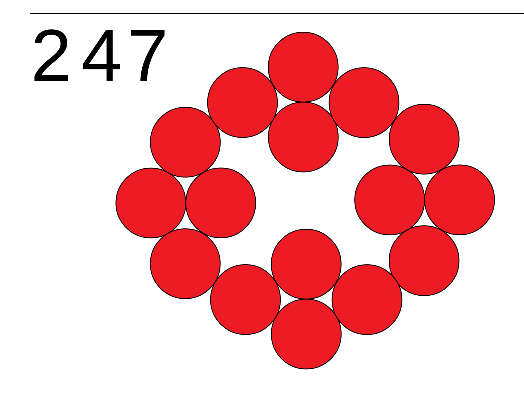


246

It takes only eleven circles, as illustrated, to form a configuration that requires four colors. No matter how you arrange the colors, a fourth will be needed where the blue chip is located.

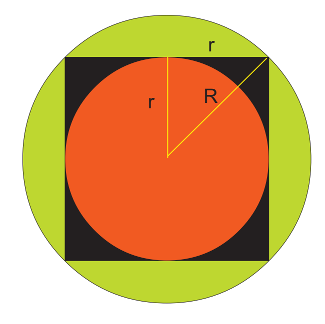


247



248

The bigger circle has twice the area of the smaller one. Simply put, a diagonal running from the center of the square is equal to the square root of two times the distance from the center of the square to the middle of one side. That diagonal is the radius for the large square; the distance to the middle of a side is the radius of the other. Since the area of a circle is proportional to the square of the radius, the larger circle has twice the area of the smaller one.

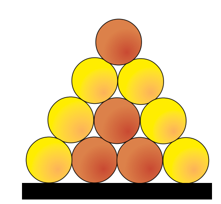


249

You will see a perfect circle.

250

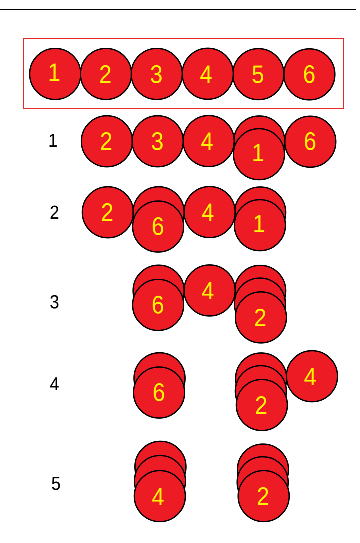
One of many solutions.



251

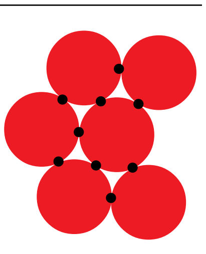
The top roller will always remain exactly above the other, regardless of their size.

252



253

Six identical circles, as shown.

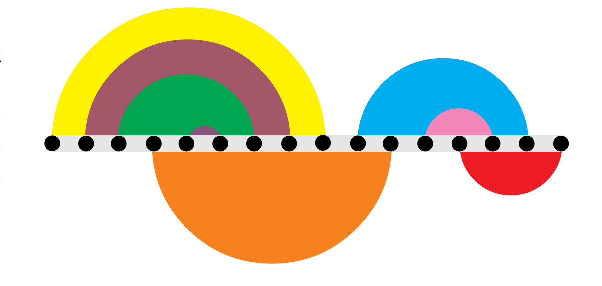


254

Red circle's diameter = $1/2$
Yellow circle's diameter = $1/4$
Green circle's diameter = $1/4\,(2 - \sqrt{2})$, or about $1/6$

255

One of many solutions.



256

The perimeter of the rosette is exactly equal to the circumference of the larger circle. This is true no matter how the circles in the rosette are arranged (as long as they all pass through the same point) or how many circles there are in the rosette.

257

Every triangle has this property. The nine-point circle is half the size of the circumcircle (the circle that passes through all three vertices of the triangle), and its center is halfway between the center of the circumcenter and the orthocenter.

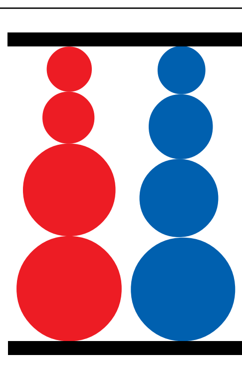
Charles-Julien Brianchon and Jean-Victor Poncelet first published this theorem in 1821, though the Englishman Benjamin Bevan proposed an equivalent problem in 1804.

258

Because of the great size of the sphere, there is quite a bit of safe space where the wall of the tunnel meets the floor. If he squeezes himself into that space, he can let the stone roll past him and escape.

259

No matter how you triangulate the polygon, the sum of the diameters of the circles will always be the same. To verify this, simply measure the circles and add the diameters together.



260

The common chords of three intersecting circles will always pass through a single point.

261

The three intersections of the tangent lines will always lie on a straight line. Imagine that the circles are three spheres of unequal size upon a flat plane. The lines between the circles are lines of perspective, which converge on the horizon.

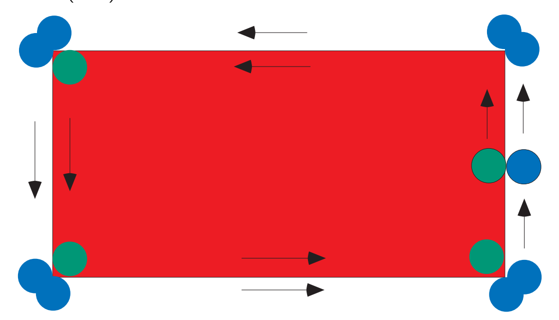
262

The optimal solution is
Turn 1: 1, 2, 3, 4, 5
Turn 2: 2, 3, 4, 5, 6
Turn 3: 2, 3, 4, 5, 7

263

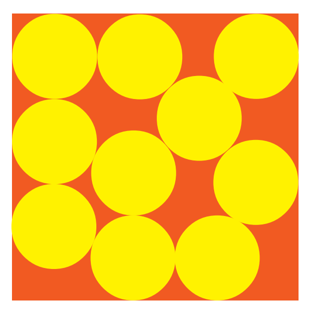
As a circle rolls a distance equal to its circumference, it makes one complete revolution. The perimeter of the rectangle is 12 circumferences. That means that the outside circle will make 12 revolutions as it rolls along the rectangle's sides; it will also make a quarter turn at each corner. So the outside circle will make a total of 13 revolutions.

The inside circle travels a distance equal to 12 circumferences minus 8 radii. Each radius is the circumference divided by 2π. That makes the total travel $12 - (4/\pi)$, or about 10.7 revolutions.



264

This is the best solution, proven by Michael Mallard and Charles Payton in 1990. In cases of packing circles into squares, mathematicians have found that as the size of the circles decreases, the density of circle to square approaches .9069. That is the limit obtained for the familiar tight packing of circles so that their centers form a lattice of equilateral triangles.



265

266

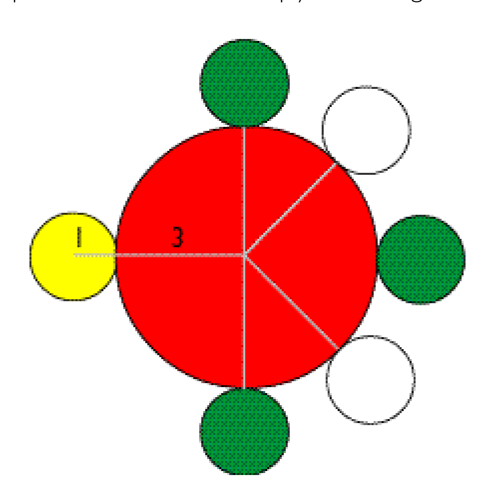
In an earlier puzzle we saw that one coin rolling over another rotates twice as much as one might have anticipated. In this instance the coin rolls through two full circumferences (a third of a circumference for each coin), so it makes four revolutions.

And it will once again face left.

267

The small circle travels on a path that is three times longer than its circumference; if it were a straight line, it would make three revolutions. But because it is rolling over the surface of a circle, the smaller circle "gains" an extra revolution. This would be true even if the smaller circle did not roll but simply kept the same point of contact as it slid along the circumference of the larger circle—the circle would make one complete revolution without rolling at all. The circle makes a total of four revolutions.

The concept of the revolution is a mind trap in this puzzle; a revolution is simply a 360-degree turn.



268

The intuitive answer is that the coin will be upside down because the coin has rolled along an edge equal to half its circumference. But if you try the puzzle experimentally, you will find that the coin rotates twice as much. It winds up facing left in the orientation it started with.

269

The density of the rectangular packing is $\pi/4$, or about 78 percent. The density for the hexagonal packing is $\pi/(2 \times \sqrt{3})$, or about 90.7 percent. The hexagonal packing is the most efficient of all possible packings.

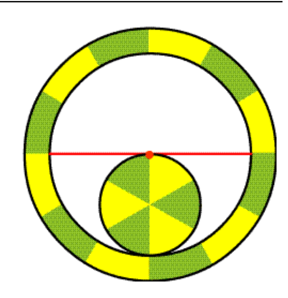
270

It is possible for one sphere to simultaneously touch twelve other spheres of the same size: six spheres around the equator and three around each pole. This is the maximum number of spheres that can "kiss" at one time. Therefore, the number of spheres that may be packed into a sphere three times as large is thirteen.

The number of identical spheres that can touch a single sphere of the same size is called the kissing number. Problems involving kissing numbers are related to many important fields, including error-correcting codes—codes employed to send messages over noisy electric channels.

271

Curiously enough, the point will trace a straight line—the diameter of the larger circle.



272

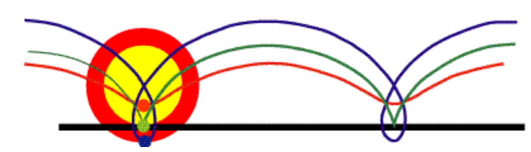
The plane is 50 kilometers from the North Pole. During its eastbound leg, the plane remained a constant distance from the pole.

273

Just take the first or last coin from the vertical row and place it on top of the coin in the middle.

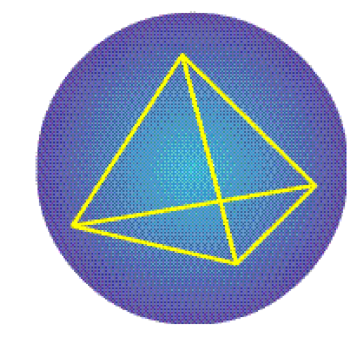
274

The three points trace examples of a family of curves called orthocycloids. The point on the circumference traces a cycloid. The point on the inside traces a curtate cycloid. And the point on the flange traces a prolate cycloid.



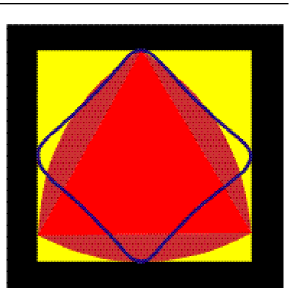
275

Imagine that the four cuts have created a tetrahedron in the interior of the sphere. Based on that tetrahedron, the sphere has been divided into the following regions: four at the vertices, six at the edges, four at the faces of the tetrahedron, and the tetrahedron itself. The total is fifteen regions.



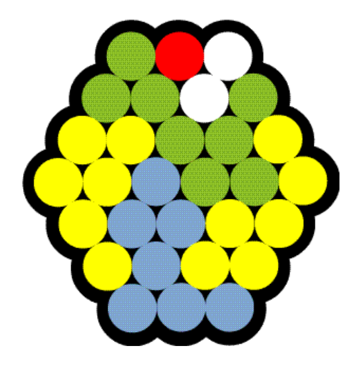
276

The point will trace a near-perfect square. This property was exploited in the invention of a tool that drills square holes.



277

The final board layout for one solution reached in fifty moves.



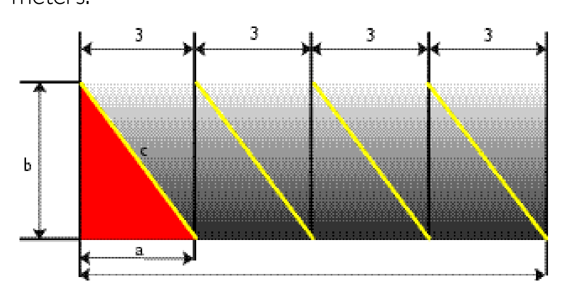
278

Imagine that you can split the cylinder and lay it flat, as shown. According to the Pythagorean theorem:

$$c^2 = a^2 + b^2 = 9 + 16 = 25 \text{ meters}$$

$$c = 5 \text{ meters}$$

Thus, length of the rope is 4 × 5 meters, or 20 meters.



279

The intuitive answer is that since 2 meters is inconsequential compared to the circumference of the earth, the belt would hardly budge. But in this case intuition is wrong.

A little analysis shows why. The circumference of the earth is 2π times its radius, and the length of the belt is 2π times both the radius of the earth and the height that the belt is pulled off the surface. If the difference between those two lengths is 2 meters, then:

$$2\pi(r + x) - 2\pi r = 2 \text{ meters}$$

$$2\pi r + 2\pi x - 2\pi r = 2\pi x = 2 \text{ meters}$$

$$x = 1/\pi \text{ meters, or about .33 meters}$$

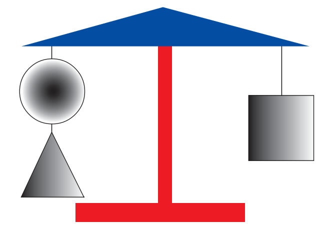
The same answer would hold for an "earth" of any size, even one the size of a tennis ball.

280

The sphere and the cylinder have the same surface area: $4\pi r^2$.

281

The volume of the cylinder is exactly equal to the volume of the sphere plus the volume of the cone. This is a fundamental theorem on which the determination of the volume of the sphere depended. Archimedes considered it one of his greatest triumphs.

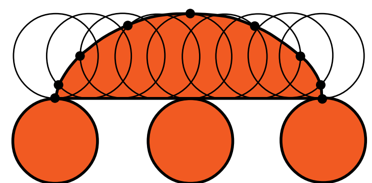


The ratio of a cone, sphere and cylinder of the same height and radius is quite elegant:

1:2:3

282

The area of the cycloid is three times that of the generating circle. That answer shocked mathematicians when it was first discovered.

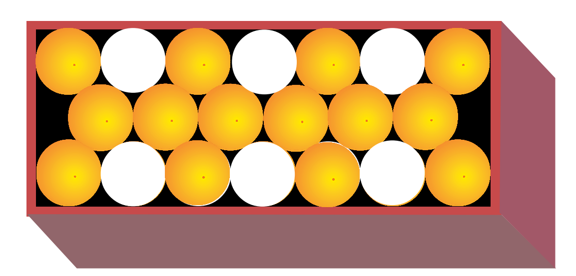


The length of the cycloid arch, from cusp to cusp, is four times the diameter of the circle, which was also an unexpected result. Mathematicians were certain that it would be an irrational number, much like the circumference of the circle. As a curve, the cycloid is much more complex than the circle, so it is little wonder that the discovery that its length is so simple would be surprising.

In 1664, Evangelista Torricelli, a student of Galileo, wrote the first treatise on the cycloid.

283

Six spheres can be removed, as shown.



284

The shortest path—the straight line—is not the quickest. Instead, the ball that rolls on the cycloidal track will be the first to arrive. Amazingly, the cycloidal path is the longest of the four.

The cycloid is called the curve of quickest descent, or brachitochrone. The ball descending the cycloid reaches a high speed early in its descent and uses that speed to race ahead of the others.

285

All except the last curve will turn like a circle.



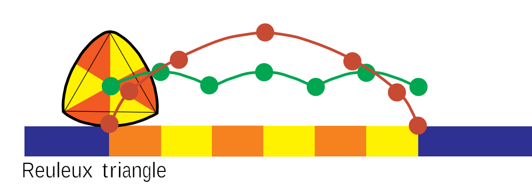
286

circle

triangle

square

hexagon



287

The problem lies in drawing the swords from the scabbards. It is impossible for the warrior with the wavy sword to pull it out of its scabbard. The other swords will go in and out of their scabbards, although the helical sword must be "unscrewed," a time-consuming act that would leave its owner at a bit of a disadvantage.

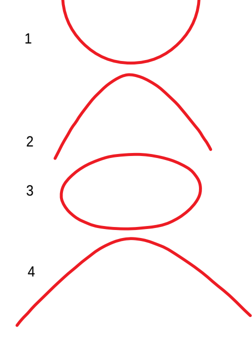
288

1. A cut parallel to the base makes a circle.

2. A cut parallel to a line generating the cone makes a parabola.

3. A cut inclined to the axis at an angle greater than the semivertical of the cone makes an ellipse.

4. A cut inclined to the axis at an angle less than the semivertical of the cone makes a hyperbola.

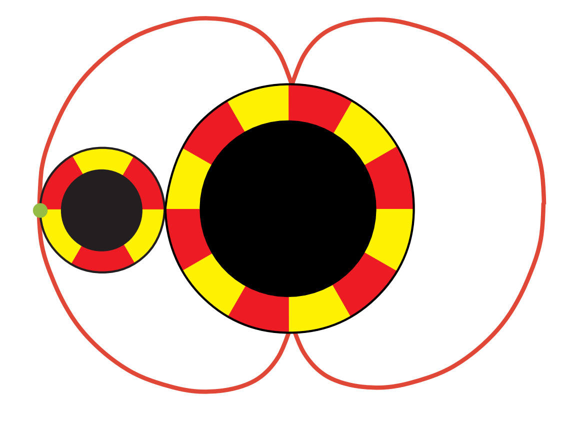


289

An ellipse is formed by slanted cuts through cones or cylinders. The man can make such a cut by picking up his water glass and tilting it. The surface of the water will form a perfect ellipse.

290

The resultant curve will be a nephroid, or kidney curve.

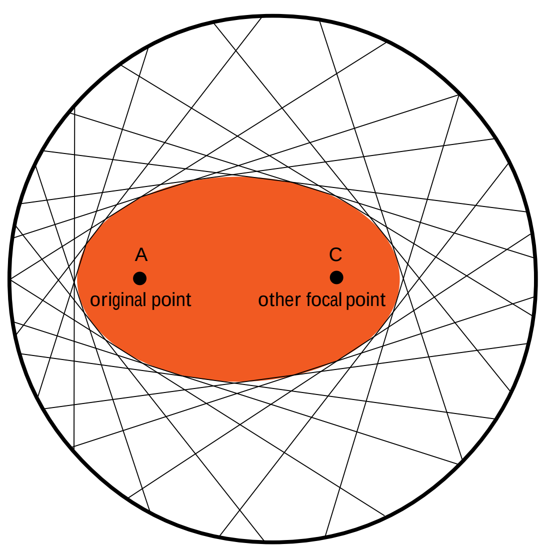


291

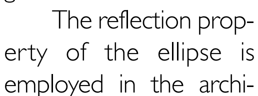
Mark a point on the circle, then fold the circle along any line so that the edge of the circle just touches the point. Make a crease in that fold line. Repeat the process of folding and creasing. Before too many folds, you will begin to see an ellipse surrounded by all the fold lines.

The fold lines are tangents to the ellipse and form an envelope around it. With other circles of paper, investigate what happens to the ellipse as you move the mark nearer the center of the circle.

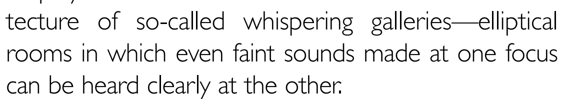
In the illustration below, the points marked A and C are the foci of the ellipse.



292 No matter where you hit a ball placed at one focus of the ellipse, it will always travel to the other focus, which is where the pocket is (as long as you don't strike the obstacle). On the other hand, if the ball sits between the two focal points, striking the ball off the ellipse will send it on a path that never gets closer to a focus.



The reflection property of the ellipse is employed in the architecture of so-called whispering galleries—elliptical rooms in which even faint sounds made at one focus can be heard clearly at the other.

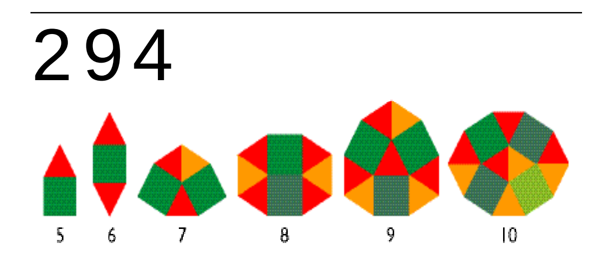




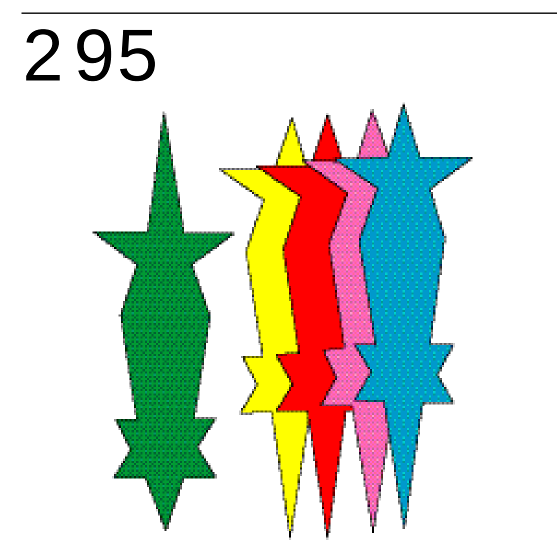
CHAPTER 6 SOLUTIONS

293 The pink shape is the only polygon shown that is not regular; not all of its sides and angles are identical.

294



295



296 The missing number is 5. The sum of the numbers on the convex angles of each polygon is five times greater than the sum of the numbers on the concave angles.

297 The mechanism cannot make the pentagonal profile.

298 The result will always be 1. The operation you worked out,

points – sides + regions = 1

is Euler's formula. It is an important mathematical relationship and a beautiful example of simplicity amid complexity.

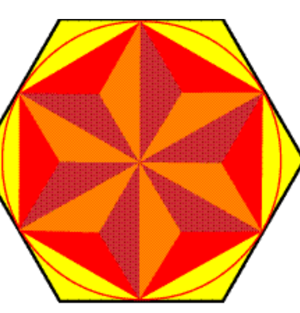
299 The only square with a perimeter equal to its area has four sides of 4 units. The only rectangle with that property measures 3 by 6 units.

300 One might imagine that the shrinking circles would eventually approach a size of 0. But, surprisingly, the limit is somewhat farther out. It takes very advanced mathematics to produce the result, but the ultimate solution is that the radii of the shrinking circles approach a limit approximately 1/8.7 that of the radius of the first circle, or about .115 units.

301 Both areas are identical. The total area of the small triangles is equal to that of the big triangle. And the overlapping parts in white decrease both equally.

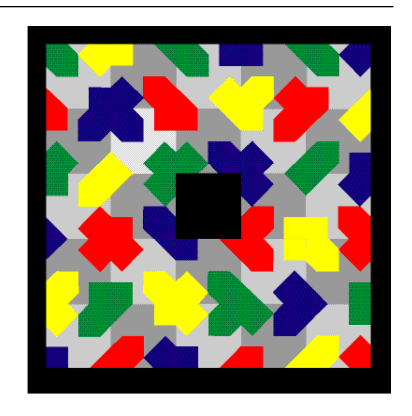
302 The key to understanding this problem is realizing that for a rectangle divided by a diagonal, the area on one side of the diagonal is equal to that of the other side. For a one-by-two rectangle, that means each side has an area of 1 square unit. There are nine squares divided in this way; that means 4.5 square units are enclosed by the band and 4.5 are outside the band. Add the 4.5 to the three squares completely enclosed by the band to get the total area, which is 7.5 units squared.

303 The first step to solving the puzzle is turning the inner hexagon so that its corners touch the outer hexagon. Then divide the inner hexagon into six equilateral triangles, and each of those equilateral triangles into three identical isosceles triangles. It is clear that the six areas of the outer hexagon uncovered by the inner hexagon are equal in size to those isosceles triangles. From that, it is easy to see that the outer hexagon has an area of 4 square units.

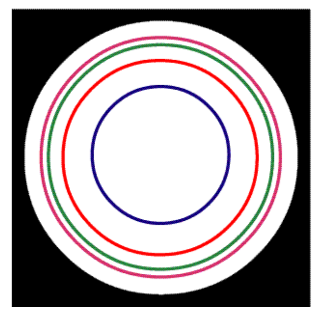


304 There are exactly fifteen identical overlapping equilateral triangles. If you counted the triangles formed by the overlap, there would be a total of twenty-eight.

305



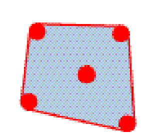
306 If you inscribe a number of different regular polygons on a circle, each shape will envelop a circle bearing a definite ratio to the original circle. For example, a triangle will envelop a circle 50 percent the size of the original; a square, 71 percent; a pentagon, 82 percent, and a hexagon, 87.5 percent.

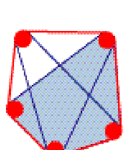


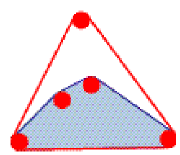
The astronomer Johannes Kepler was fascinated with the idea of inscribing regular polygons and three-dimensional polyhedrons in circles and spheres. Kepler believed the results might somehow lead to a better understanding of the arrangement of the planets in our solar system; ultimately, no relationship could be found.

307 Yes. This mysterious and completely unexpected fact was discovered by English mathematician Frank Morley in 1899—which is why it's called Morley's triangle.

308 Four points are not enough—imagine a triangle with an interior point. It takes five points to guarantee a convex quadrilateral. This fact was demonstrated by the Erdös-Szekeres theorem in 1935. If you surround the five points with a rubber band, there are three possible outcomes:







1. The band forms a convex quadrilateral with the fifth point tucked inside.

2. The band forms a pentagon; connecting two of the vertices will result in a convex quadrilateral.

3. The band forms a triangle with two points inside. Connect the two interior points with a line—on one side of the line, there will be one vertex of the triangle; on the other side, two vertices. Connect those two vertices to the interior points to make a convex quadrilateral.

309

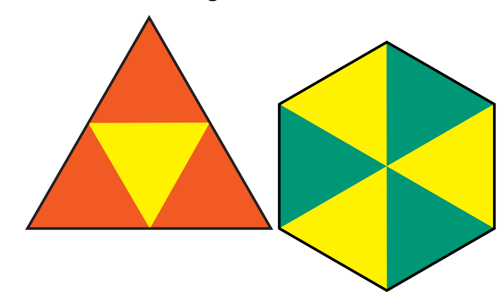
A total of 213 goats can graze in the orchard.

Mathematics provides a quick and elegant way to solve this kind of problem, which involves lattice polygons. In 1898 the Czech mathematician Georg Pick discovered a simple method for finding the area of a polygon whose vertices lie upon the points of a square grid plane: simply count the number of lattice points inside the polygon, then count the number of lattice points on the boundary, including the vertices. The area is then equal to the number of enclosed points, plus half the boundary points, minus 1. This is called Pick's theorem.

In this problem there are 115 enclosed lattice points and 198 boundary points, so 115 + (198/2) – 1 = 213.

310

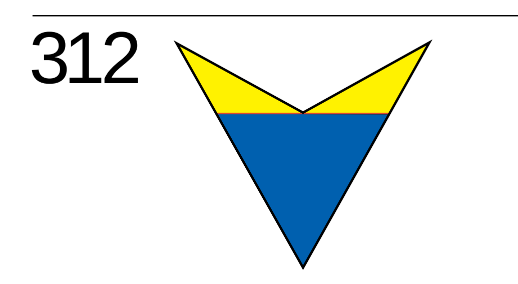
The area of the triangle to the area of the hexagon is 2:3.



311

1. 36 triangles
2. 52 triangles
3. 36 triangles and 13 squares
4. 22 squares
5. 76 triangles
6. 9 triangles and 6 squares
7. 15 regular hexagons
8. 29 squares
9. 31 squares

312



313

There are twenty-seven triangles.

In general, the number of triangles of different sizes in a triangular grid follows the sequence 1, 5, 13, 27, 48, 78, 118, etc., for triangles of increasing size. For triangles with an even number of levels, the general formula is

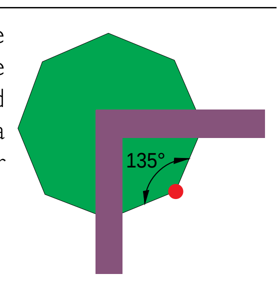
$$\frac{n(n + 2)(2n + 1)}{8}$$

For odd numbers of levels, the general formula is

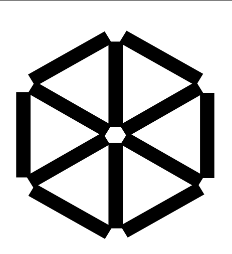
$$\frac{n(n + 2)(2n + 1) - 1}{8}$$

314

To enclose the most area, the panels should be opened at 135 degrees. The area is one-quarter of a regular octagon.

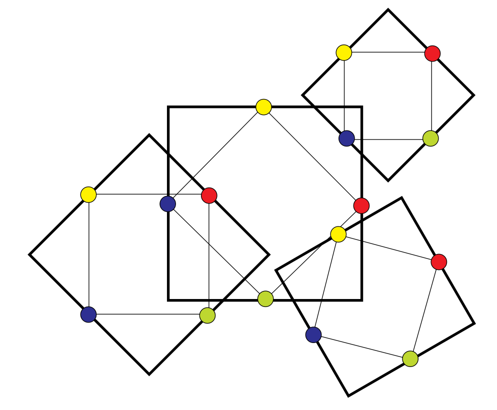


315

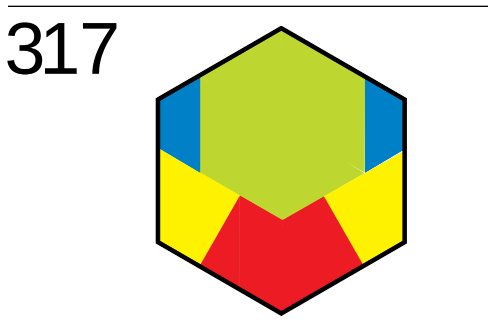


316

The midpoints and the reconstructed squares are shown here.



317



318

319

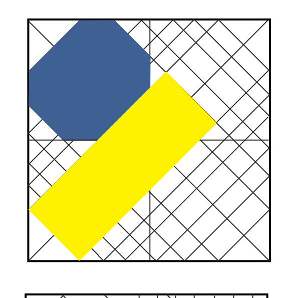
320

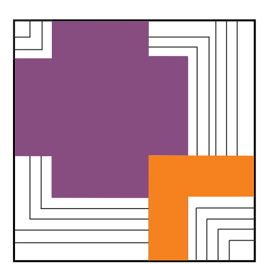
The order is circle, pentagon, square and triangle. The circle is the polygon that encloses the most area per unit of perimeter. It is the most economical shape for a fence or enclosure.

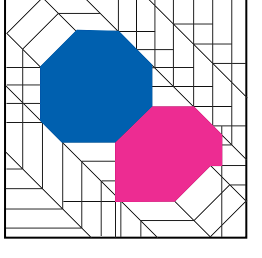
321

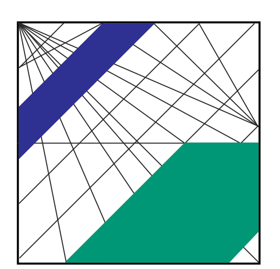
The answer is triangle, square, pentagon and circle. The circle, of course, has the shortest perimeter to enclose a given area.

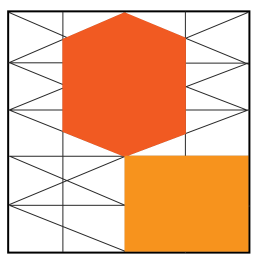
322

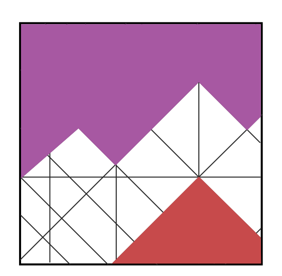








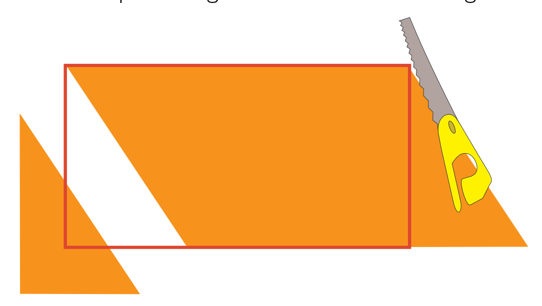




One common form of camouflage found in nature blurs the outlines of an object so it simply fades into the background. But another kind, featured in this problem, involves the deliberate creation of a pattern that distracts the eye; in that way the shape of the object is somewhat less obvious. Redundant lines attract the attention of the eye with their regularity and angularity. Also, a number of shapes within the pattern are misleading: they are close to but not identical to those that are sought.

323

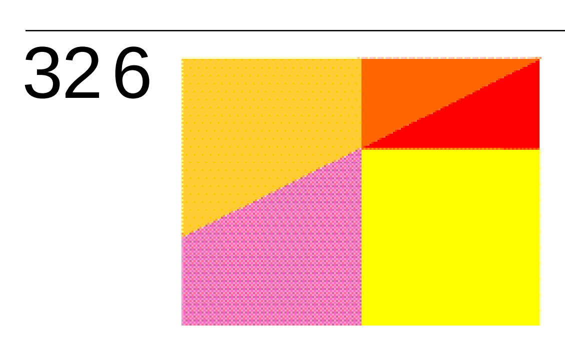
It takes just one cut, as shown. Join the triangle created by the cut to the other end of the parallelogram to form the rectangle.



324

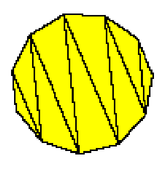
325

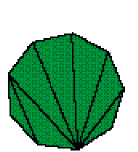
326

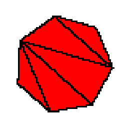


327

In general, a convex polygon of *n* sides needs n – 3 diagonals to triangulate it, and those diagonals create n – 2 triangles. Thus, for the heptagon, four diagonals make five triangles; the nonagon needs six diagonals to make seven triangles; the undecagon employs eight diagonals to make nine triangles.

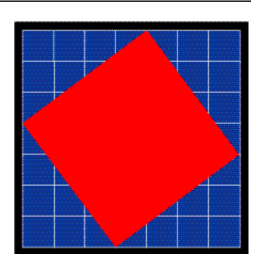






328

A five-by-five square can be so inscribed on the seven-by-seven square, as shown.

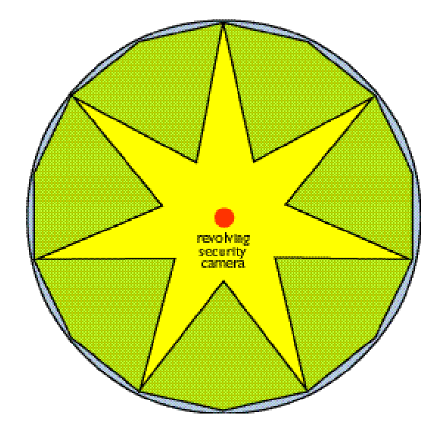


329

To create a triangle from three strips, it is necessary that the sum of the lengths of any two sides be greater than the length of the third side. The green and blue sets of strips do not follow this rule and thus cannot form triangles.

330

A simple solution would be walls that form a polygon of fourteen sides. Another solution, one requiring the least amount of floor area, would be to make the walls form a seven-pointed star.



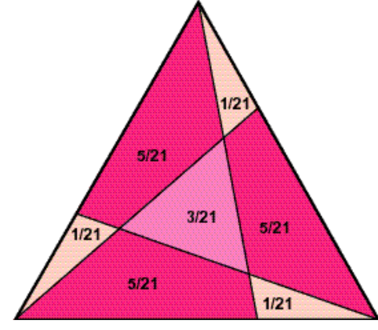
331

The minimum solution, discovered by the late Andrei Khodulyov, uses fifteen additional linkages, to be added as shown.

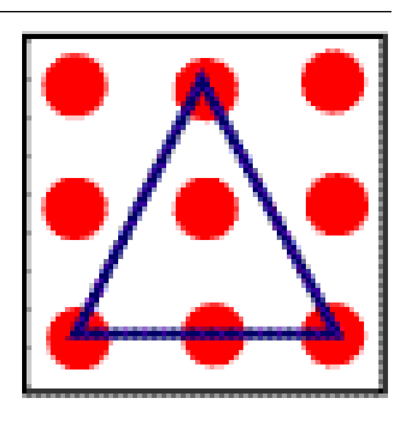


332

Each trisecting line divides the triangle into $^1/_3$ or $^7/_{21}$, which is again divided into three parts, which simple observation tells us can be only $^1/_{21}$, $^5/_{21}$ and $^1/_{21}$. It follows that the central triangle is $^3/_{21}$.



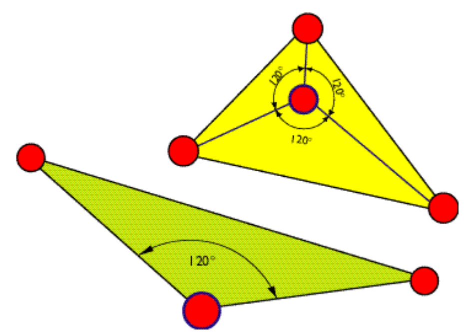
333



334

The two problems are related, since the three villages—no matter how they are arranged—can be said to be the vertices of a triangle. For the triangles and the villages, the most economical path will have three arms that meet at a point that is at the minimum total distance from the villages, or vertices.

In the case of a triangle in which all three angles are less than 120 degrees, the paths will be straight lines that meet at a point where they form angles of exactly 120 degrees, as illustrated above. For a triangle in which one angle is 120 degrees or more, the minimum path will pass through that vertex.

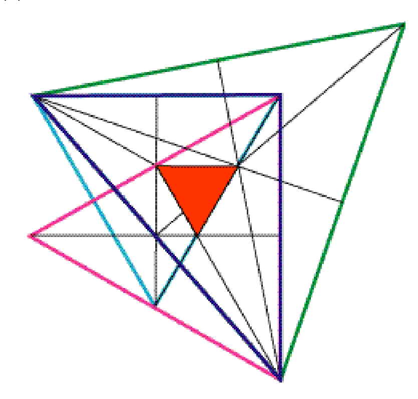


335

This will work with every triangle. In the example shown, the triangles are constructed inward; as you can see, the centers still form an equilateral triangle that has the same center as the original triangle. Interestingly, the difference in the area of the three constructed triangles is equal to the area of the original triangle.

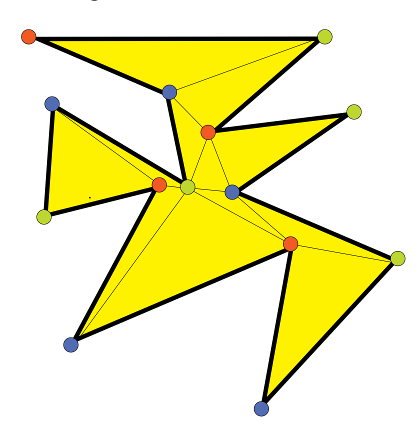
336

Curiously enough, the line from the vertex that bisects the median will divide the opposite side in a 2:1 ratio.



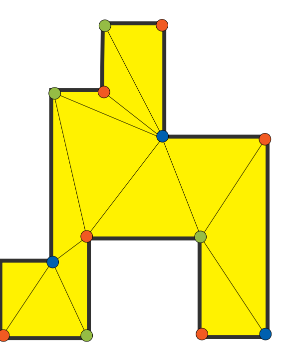
337

Four cameras are sufficient (see the red dots in the diagram). There are many ways to arrange them.



338

The three blue points are the solution.

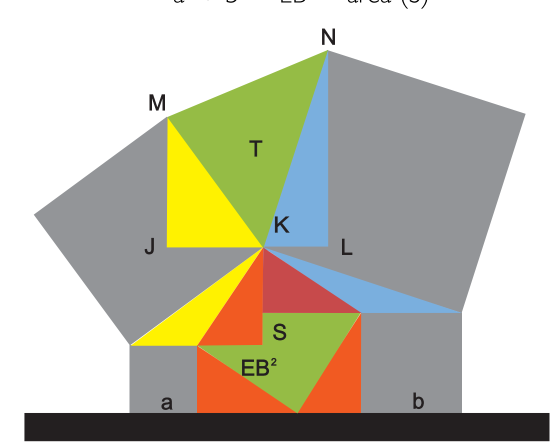


The question of how many cameras are needed to always cover every point on the floor was first posed by Victor Klee in 1973. Within a few days Rutgers University mathematician Vasek Chvátal had proved that if the shape of the floor has *n* vertices, then there are n/3 positions from which it is possible to view the whole gallery. The question became known as the "Chvátal Art Gallery Problem" until Bowdoin College mathematician Steve Frisk used his ingenious triangulation proof to figure out the exact placement of the cameras.

Here's what you do: Triangulate the layout and color the vertices of each triangle in three colors. The same three colors should be used for each triangle, and the same color should be used for every vertex that occupies a given point. The cameras should be placed at the points that have the color that appears the fewest times.

339

area (T) = area (JLNM) – area (JKM) – area (KLN)
= (a + b)(2a + 2b)/2 – ab – ab
= $a^2 + b^2 = EB^2$ = area (S)



340

341

342

No. The divisions of Cake 1 and Cake 3 are equal, but the red group gets bigger slices from Cake 2.

If the number of chords (or cuts through the cake) is even and equal to four or more, the areas (or pieces of cake) are always equal.

If the number of chords is odd or less than four, the areas will not be equal—unless the chords go through the center of the circle, as they do in Cake 1.

This puzzle was inspired by the "Pizza Problem," which was discovered by L. J. Upton in 1968, and proved by Larry Carter and Stan Wagon in 1994.

343

The order is yellow, orange, red, pink, violet, light green, dark green, light blue, dark blue and lime. The order is also that of an increasing number of sides, from the triangle with three to the dodecagon with twelve.



345

Miraculously, yes!

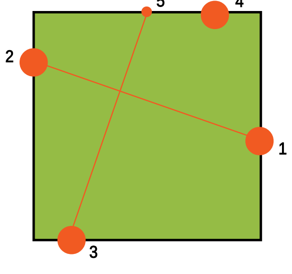
This gem of geometry is known as von Auble's theorem, and it will work with nonconvex quadrilaterals and even quadrilaterals in which three or four corners are colinear.

346

In a quadrilateral the length of each side is always less than the sum of the other three sides. Thus, the blue set of strips, of lengths 2, 3, 3 and 8, cannot form a quadrilateral.

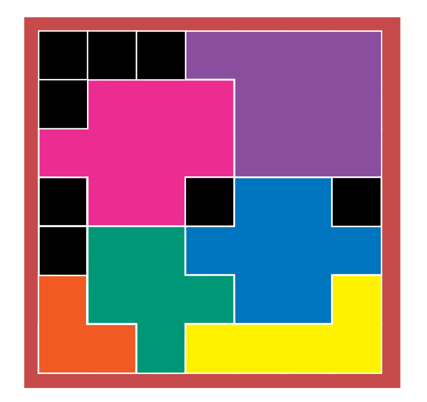
347

The solution begins with drawing a line between two points, such as the one between points 1 and 2. Then draw a line from point 3 that is both equal in length and perpendicular to the line between 1 and 2. The endpoint of that line, marked as 5, is obviously on the line of the square.



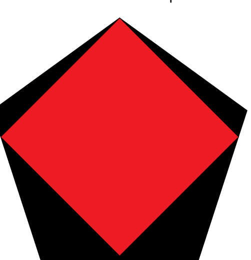
Draw a line through points 4 and 5, and draw a parallel line through point 3. To complete the square, draw lines perpendicular to these lines through points 1 and 2. The four lines will intersect to form a square.

348



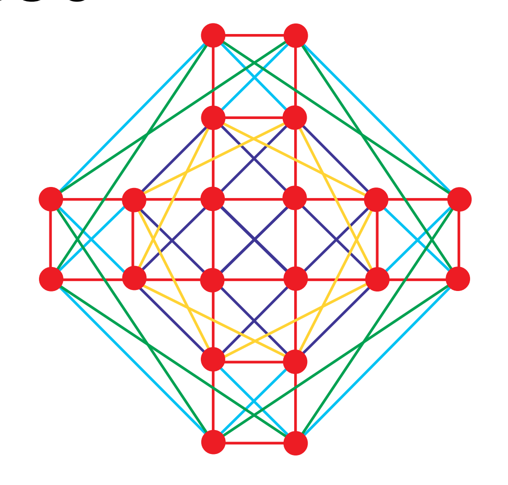
349

Given a pentagon with sides of 1 unit, the square shown in the problem has a side larger than 1.0605. But the square shown here, which is a solution published by Fitch Cheney in the *Journal of Recreational Mathematics* in 1970, has a side greater than 1.0673.



350

Twenty-one squares are possible.



351

1. $1\frac{1}{2}$ square units
2. $2\frac{1}{2}$ square units
3. 1 square unit
4. 2 square units
5. 3 square units
6. $2\frac{1}{2}$ square units
7. $4\frac{1}{2}$ square units
8. $6\frac{1}{2}$ square units
9. $5\frac{1}{2}$ square units
10. 2 square units
11. 5 square units
12. 18 square units
13. $5\frac{1}{2}$ square units
14. 7 square units
15. 7 square units
16. $7\frac{1}{2}$ square units

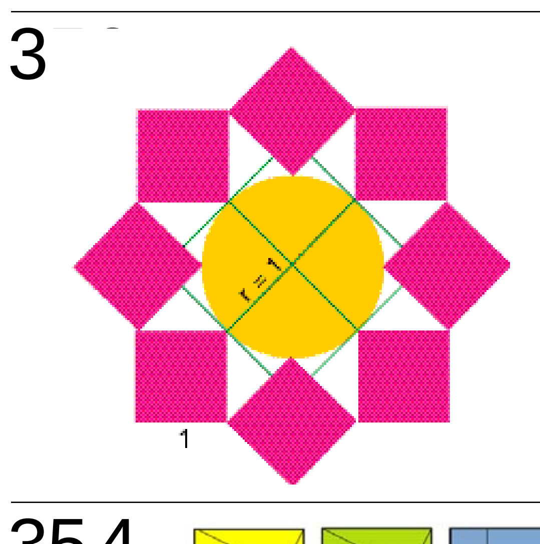
352

There is a total of 204 squares of various sizes:

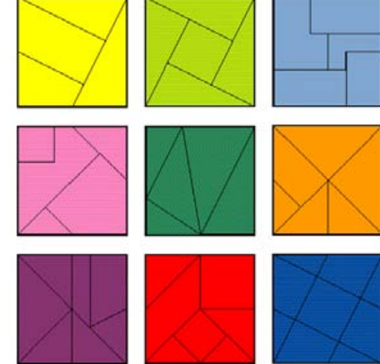
1 square unit—64
4 square units—49
9 square units—36
16 square units—25
25 square units—16
36 square units—9
49 square units—4
64 square units—1

The total number of different squares on a square matrix with *n* units on a side is simply the sum of the squares of the first *n* integers.

3



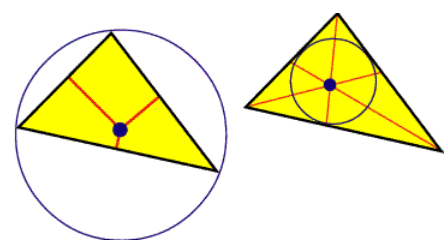
354



355

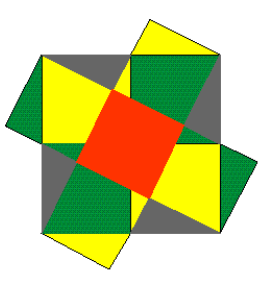
To find the incenter, bisect the three angles of the triangle, as shown.

To find the circumcenter, bisect each side with a perpendicular line.



356

The figure can be rearranged into five identical squares, as shown. Therefore, the red square has one-fifth the area of the original figure.



CHAPTER 7 SOLUTIONS

357

You can make three actual words. The first letter can be any one of the three; the second can be either of the two remaining letters; the third is the letter left over: 3 × 2 × 1 = 6 possible words. The possibilities are

OWN, ONW, NOW, NWO, WON, WNO.

For *n* different letters, numbers or objects, the number of possible arrangements can be calculated:

$$n \times (n - 1) \times (n - 2) \times \ldots 3 \times 2 \times 1$$

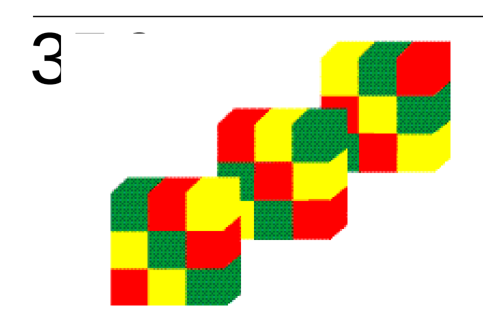
This number is called n factorial and is abbreviated as n!.

358

If rotationally similar configurations had been allowed, the answer would have been 7!, or 5,040. But because any of those 5,040 arrangements is the same as 6 others through rotation, the total number of different diadems is 7!/7, which equals 6!, or 720.

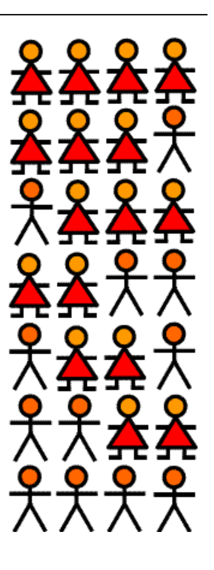
Had we outlawed similar configurations available by flipping the diadem, the answer would have been half of 720, or 360.

3

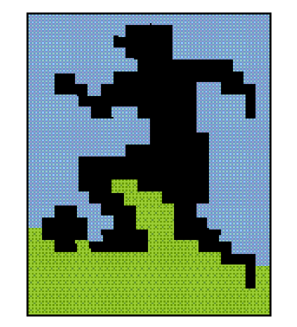


360

With four children per group, there are six different permutations in which every girl sits next to another girl, as shown. There is also the possible arrangement of four boys and no girls.



361



362

Two solutions.

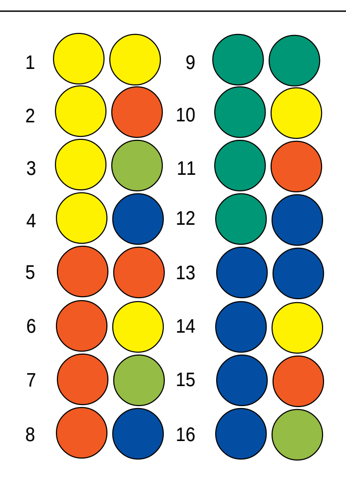
363

There are eight symbols. Horizontally row by row, the sequence is 1; 1-2; 1-2-3; 1-2-3-4; 1-2-3-4-5; and on so up to 1-2-3-4-5-6-7-8. At that point the pattern repeats.



364

There are sixteen possible pairs.



365

There are just six different arrangements for the three objects. There are three different possibilities for the leftmost fruit. For each fruit, there are two different possibilities for what goes in the middle; and for each left and middle, there is just one possibility for what goes on the right.

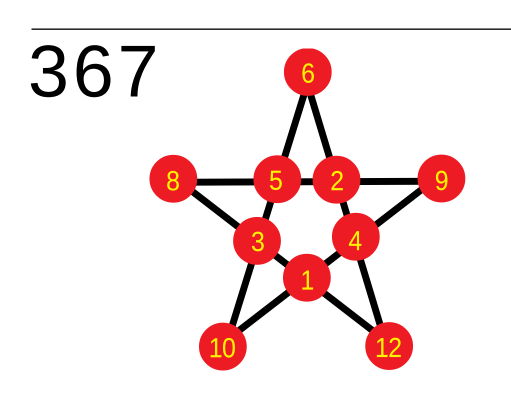
$$3 \times 2 \times 1 = 6$$

The operation from one arrangement to another is called a permutation.

366

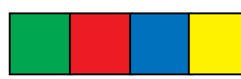
In 5,040 different ways.

367



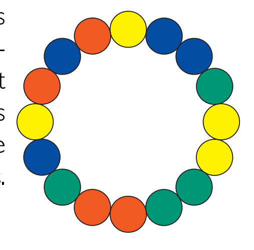
368

Once you cut out the strips, you will find that there are only twelve unique patterns.



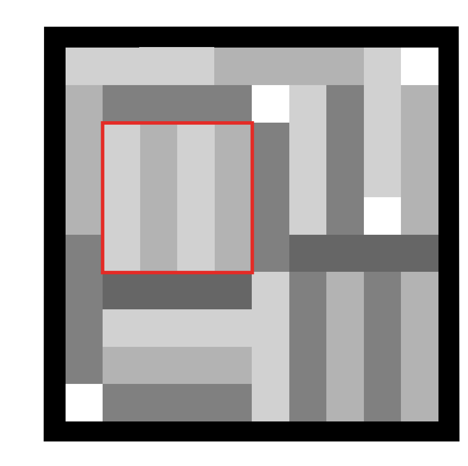
369

Mathematicians call this a universal cycle for 2-sequences. It exists for any number of colors or objects; the cycle is the square of the number of colors.



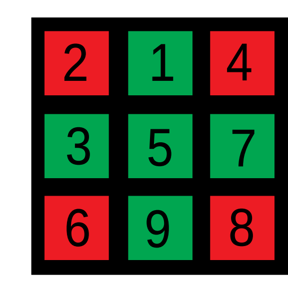
370

Note that the four strips in the square outlined in red and the square below it can be placed horizontally or vertically.

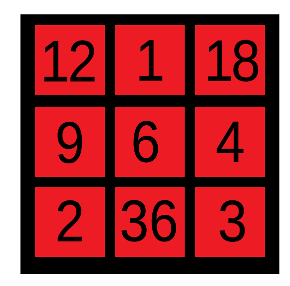


371

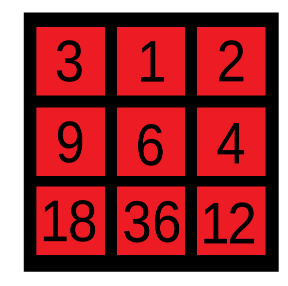
One of many ways.



372

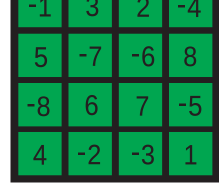


373

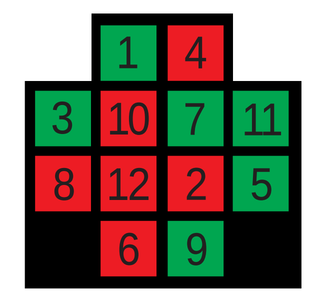


374

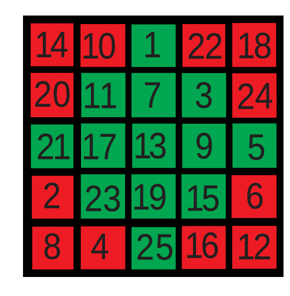
One of many solutions is shown. This one was obtained by taking the "Magic Square of Dürer" (PlayThink 377) and subtracting 17 from every value greater than 8.



375

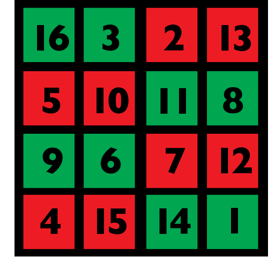


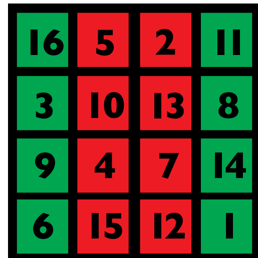
376



377

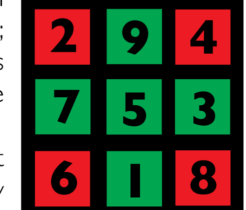
There are two solutions, shown below. In Dürer's "diabolic" magic squares, there are many sets of numbers that add up to the magic constant. Look, for instance, at the two-by-two square in the top left-hand corner: 16, 3, 5 and 10 add up to 34.





378

The sum of all nine digits is 45; distributed across three rows or columns, that means the "magic constant" is 15.



In general, the constant for magic squares with any number of rows and columns can be found without adding the digits. For any number of rows *n*, the magic constant is

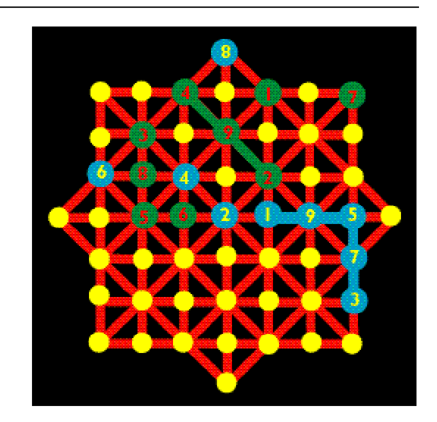
$$(n^3 + n)/2$$

To solve the Lo-Shu, you should first realize that there are eight possible triads of digits that add up to 15:

9-5-1; 9-4-2; 8-6-1; 8-5-2; 8-4-3; 7-6-2; 7-5-3; 6-5-4

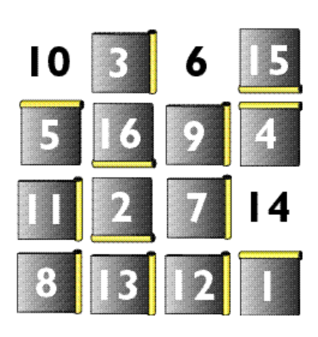
The digit in the center of the square appears in four lines (a column, a row and both horizontals). Since 5 is the only digit that appears in four triads, it must be the center digit. Since 9 appears in only two triads, it must go into the middle of a row or column, which is completed with 1 to make the 9-5-1 triad. Similarly, 3 and 7 are in only two triads, so they must be in the middle of a row or column. The remaining four numbers can fit in only one way—an elegant proof of the uniqueness of the Lo-Shu solution.

379

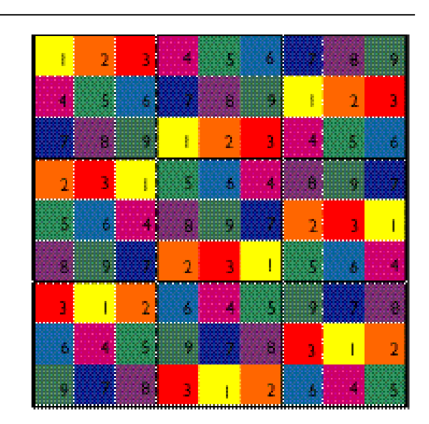


380

Only the flipped hinges are shown.



381

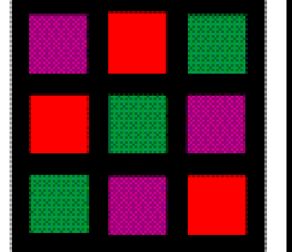


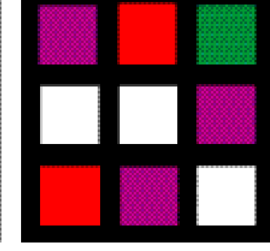
382

There are fifteen different combinations. For each monkey you could choose any of the three donkeys, so there are three possible pairs. Since this is true with each of the five monkeys, that leads to fifteen possible pairs.

383

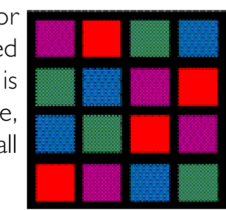
It is not always possible to complete magic color squares or diagonal color squares. In many cases one can find only the best solution—the one that fits the most tiles onto the grid.





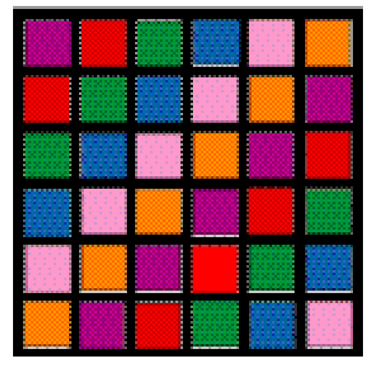
384

A magic color square extended to the two main diagonals is shown here. It is impossible, however, to extend it to all diagonals.

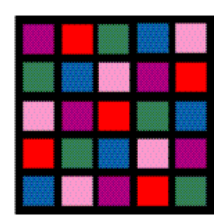


385

Diagonal magic color squares of order 6 are impossible

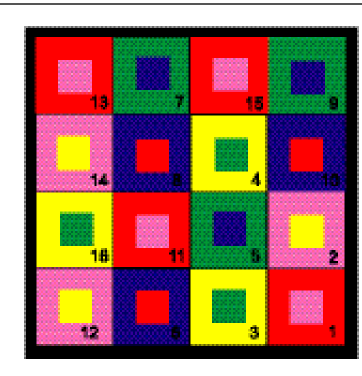


386

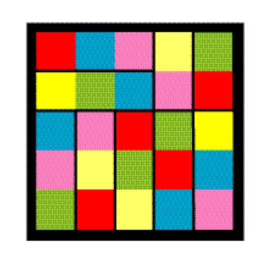


387

A full diagonal magic color square, with the rule extended across all the smaller diagonals as well as the two main ones, is shown here. In general, complete magic color squares are possible only when the order is not divisible by 2 or 3.



388



389

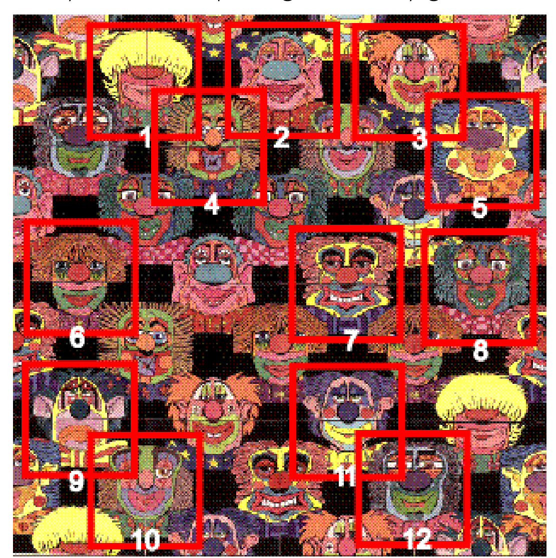


391

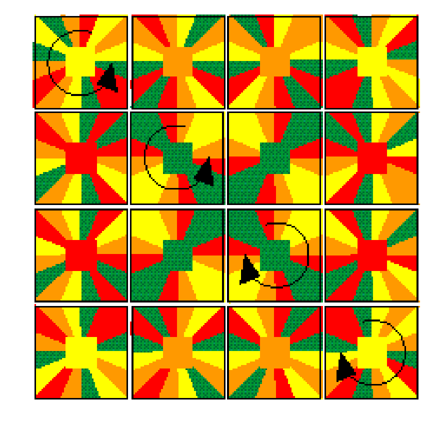
There are twelve different clowns, as identified below.

The clowns numbered 1, 2, 3, 6, 7, 8, 9 and 11 each appear three times; the rest show up only twice.

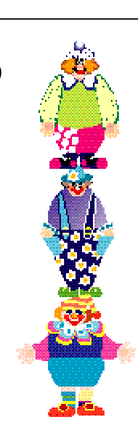
There are thirty-two complete clowns, but only twenty-four can be put together at any given time.



392

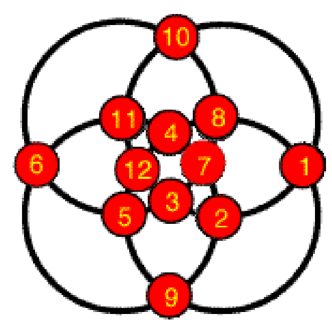


393

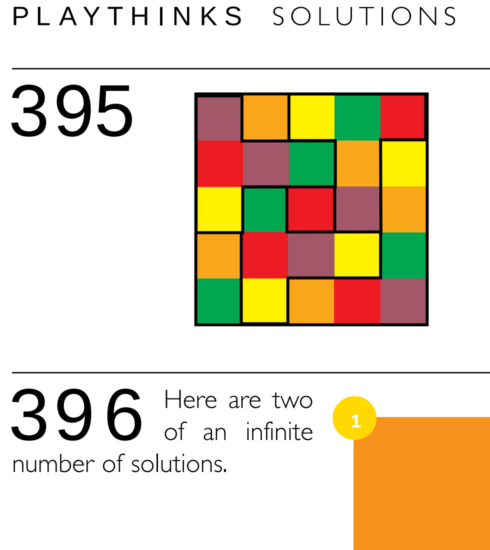


It is the only combination they haven't tried.

39

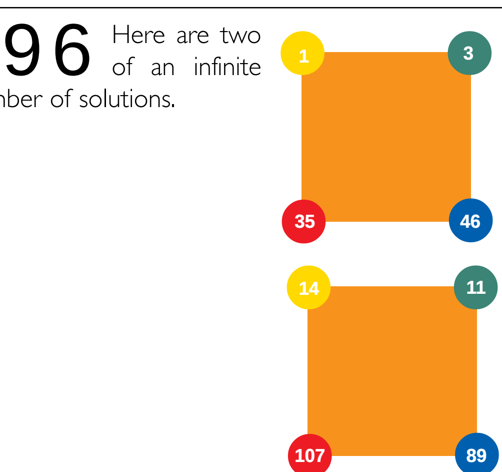


395

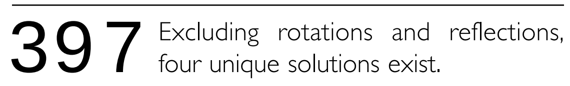


396

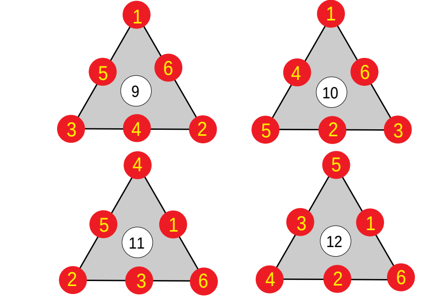
Here are two of an infinite number of solutions.



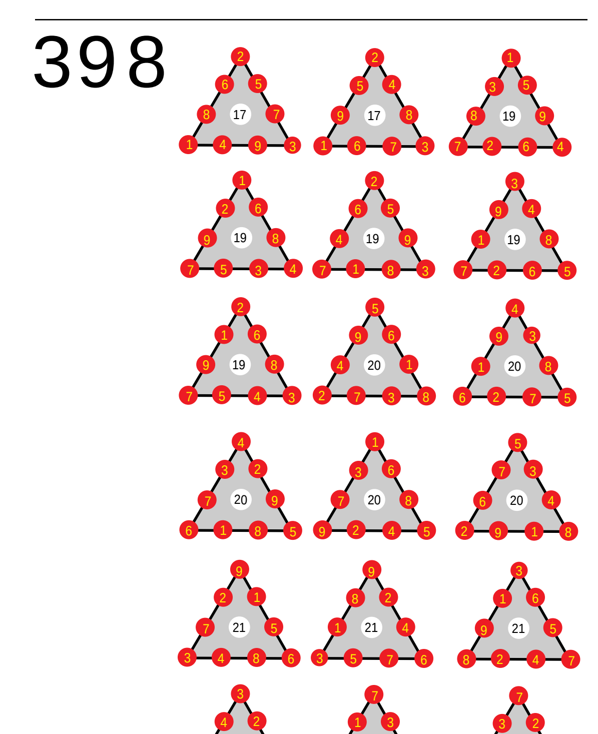
397



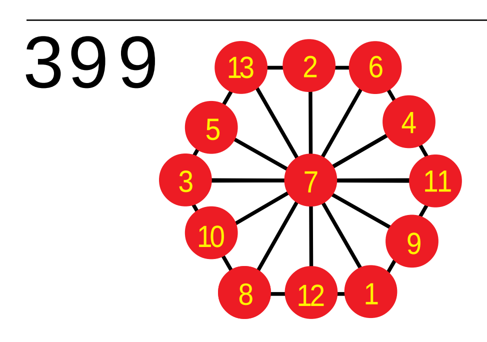
Excluding rotations and reflections, four unique solutions exist.



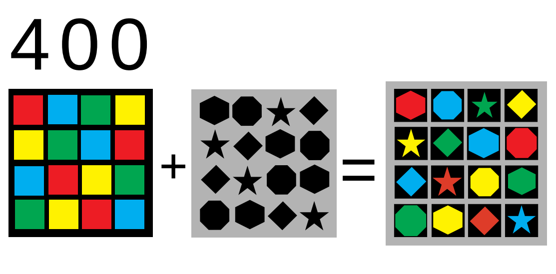
398



399

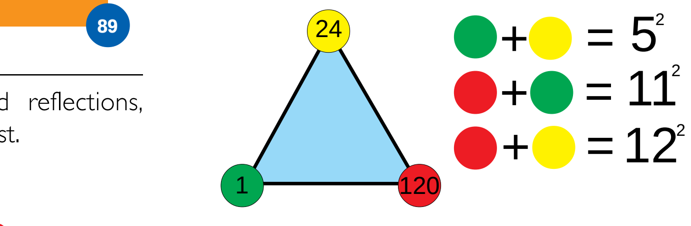


400

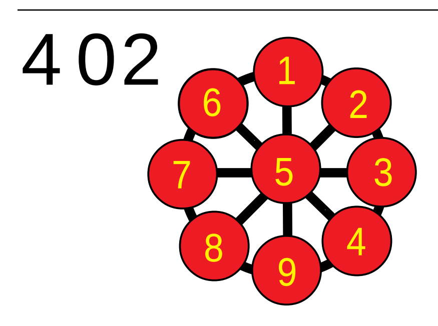


401

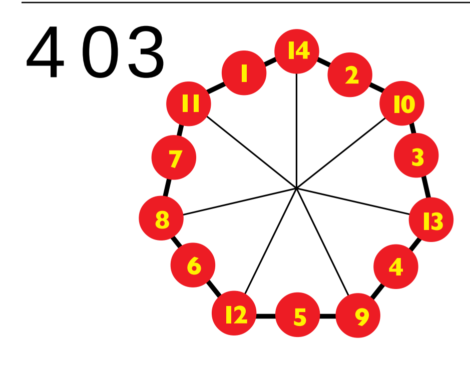
$n = 5; k = 11; p = 12$



402



403

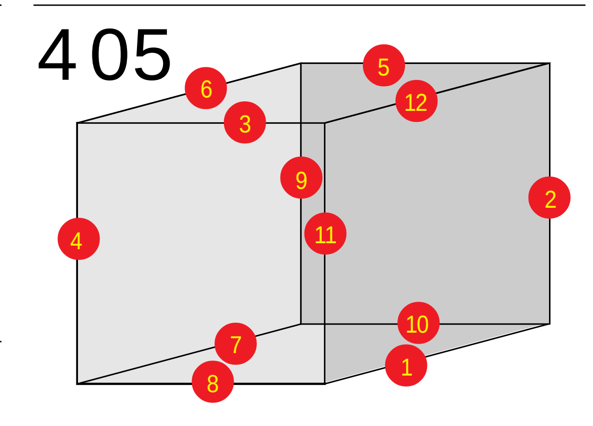


404

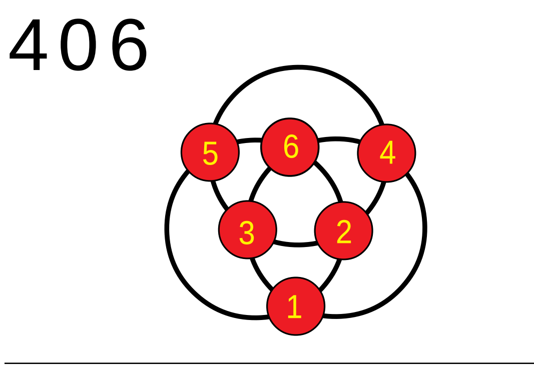
No square and no alien appears more than once in any row, column or diagonal. The middle alien with the red background will complete the pattern.



405

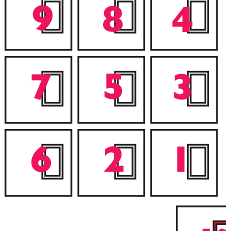


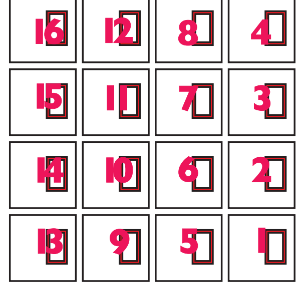
406



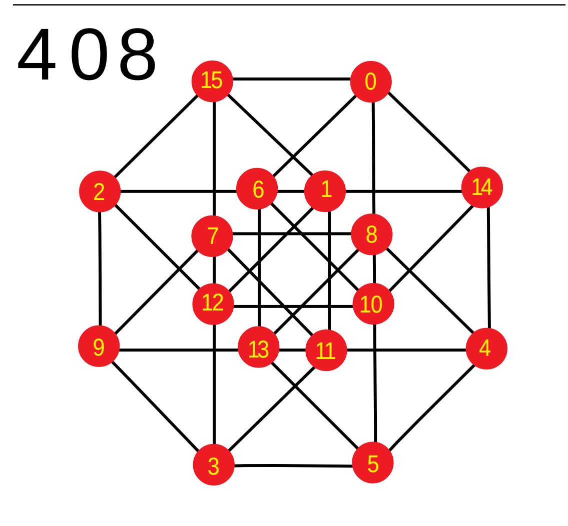
407

Two solutions are shown; many are possible.





408

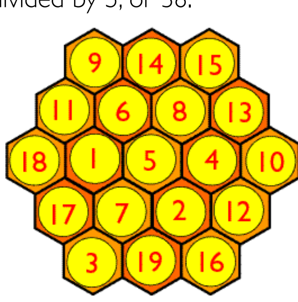


409

410

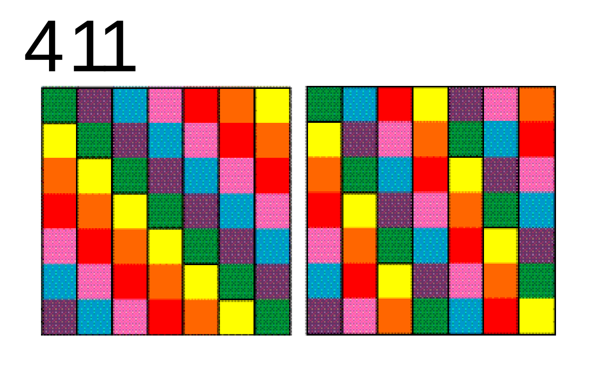
The sum of all nineteen numbers is 190, which is divisible by 5, and there are five parallel rows in each direction. Thus, the magic constant is 190 divided by 5, or 38.

In general, it is possible to arrange a set of positive integers from 1 to *n* in a hexagonal honeycomb array of *n* cells so that every straight row has a constant sum—that is, a magic constant.



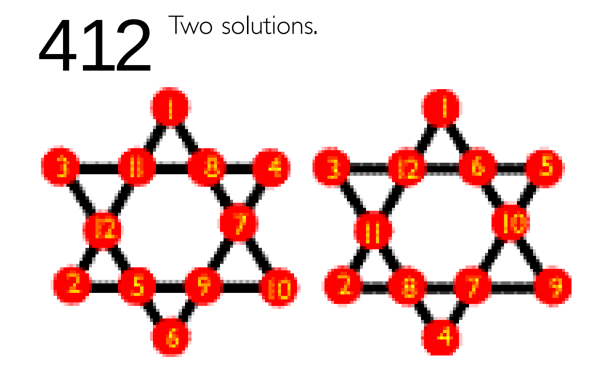
As we see illustrated above, an order 3 magic hexagon is possible. But an order 2 hexagon, one that has seven cells, is impossible. The sum of the numbers from one to seven is 28, and 28 divided by three (the number of rows in each direction) is not an integer. Likewise, magic hexagons of order 4 and order 5 are also impossible. In fact, an extremely complicated proof has shown that no magic hexagon of a size greater than order 3 is possible. What is even more astonishing is that the magic hexagon shown above, which was discovered in 1910, is the only possible solution.

411



412

Two solutions.

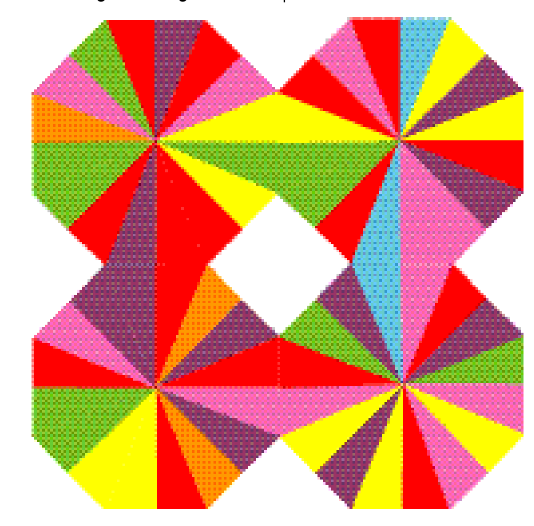


413

Top right octagon: one quarter turn counterclockwise

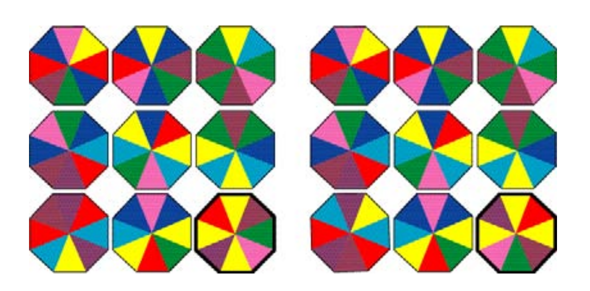
Bottom left octagon: one half turn

Bottom right octagon: one quarter turn clockwise



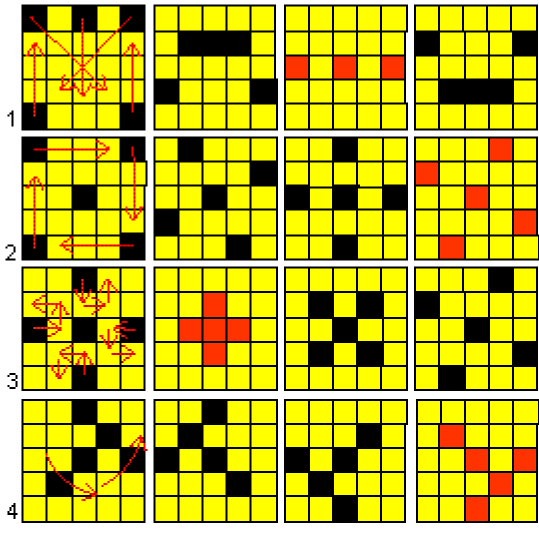
414

The octagons in the lower left and right can each be positioned in two ways, making for four solutions.

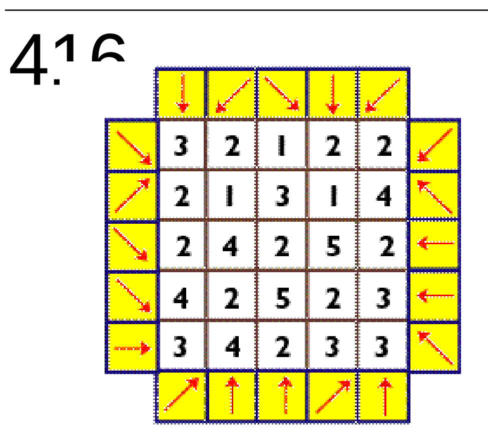


415

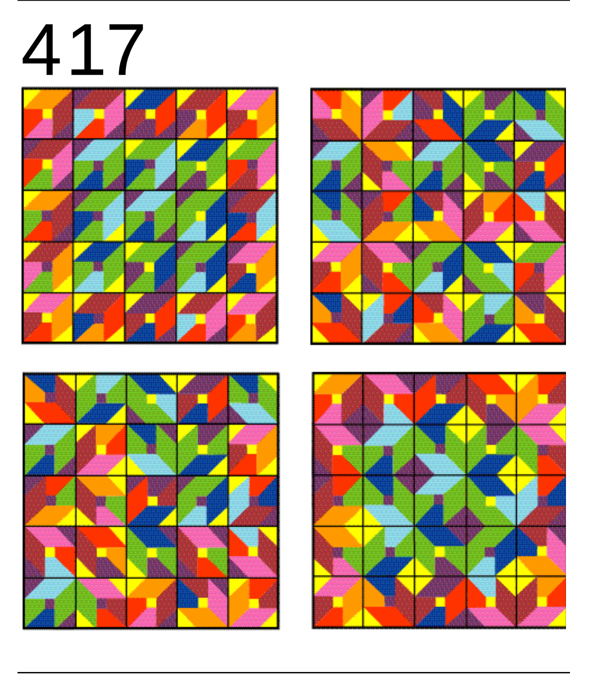
The arrows show the motion of the squares in each pattern. The missing pattern from each sequence is shown in red.



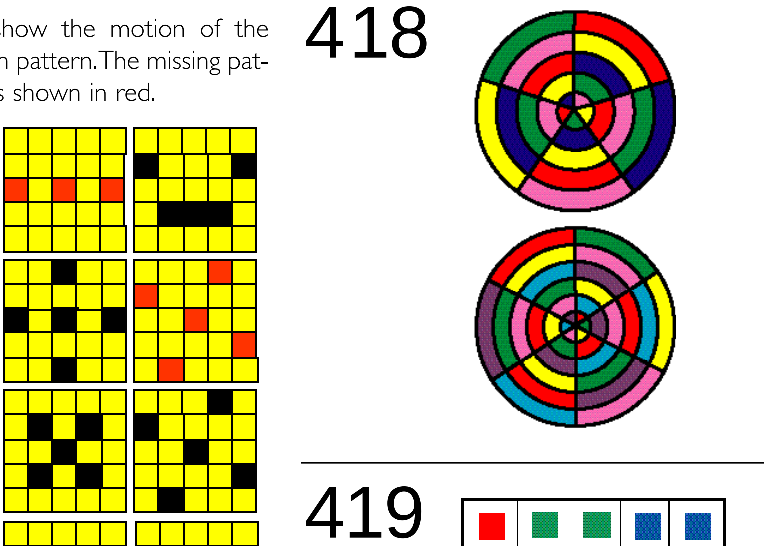
416



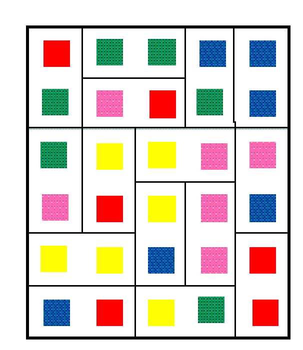
417



418

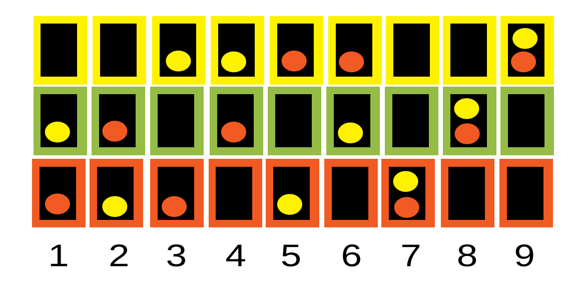


419

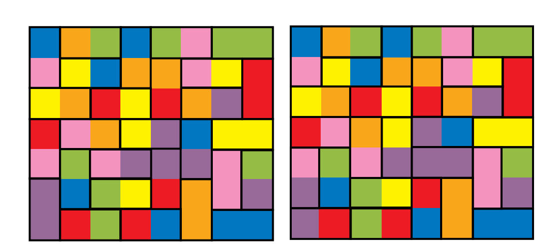


420 A simple proof shows why it is impossible: Each domino has a red square and a yellow square, so the number of squares of each color must be equal. But the chessboard was truncated in such a way that there are thirty-two red squares and only thirty yellow ones.

421 There are four different ways to distribute two desserts on two plates, as illustrated below.



422 The yellow dot stands for the pineapple; the red dot, for the apple. There are nine different ways to distribute them among the three bowls, as shown.

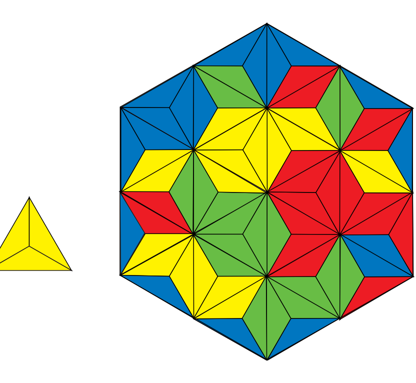


423 Two slightly different solutions are shown.

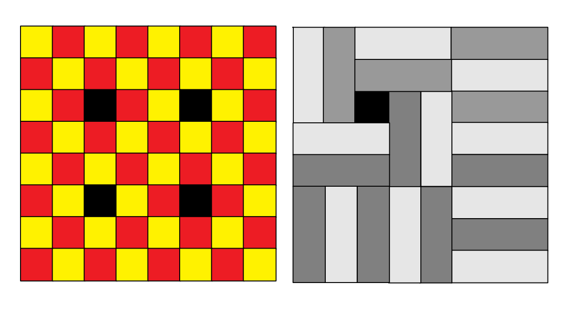
424



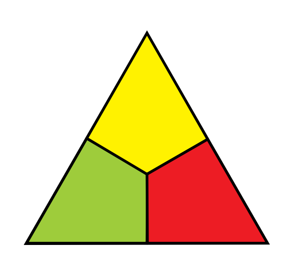
425 The missing triangle is yellow in all three segments.



426 It is possible, but only if the monomino covers one of the squares, shown in black.

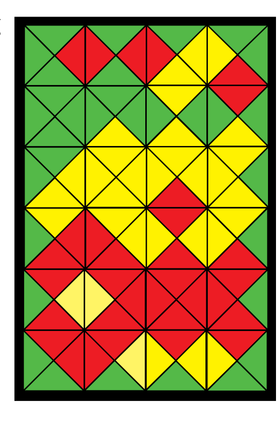


427



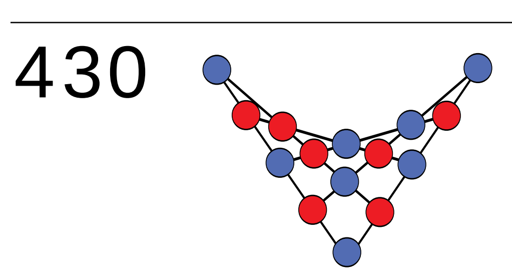
428 The missing square is all yellow.

There are many possible arrangements of the squares. One is shown here.

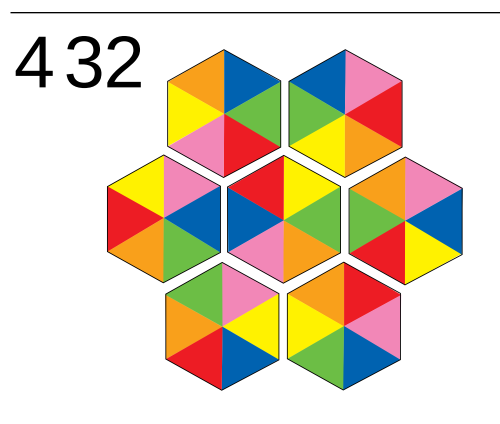


429 The zigzag runs through the greens.

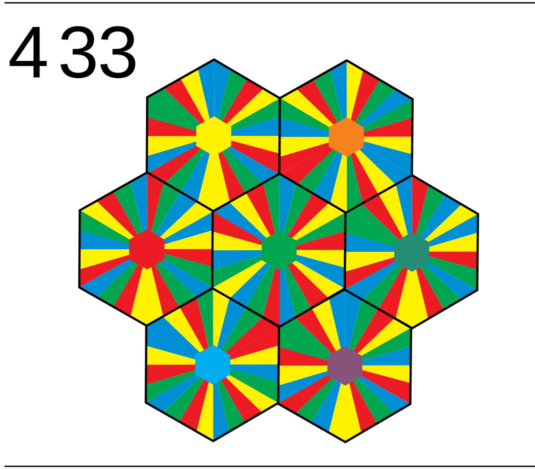
430



432



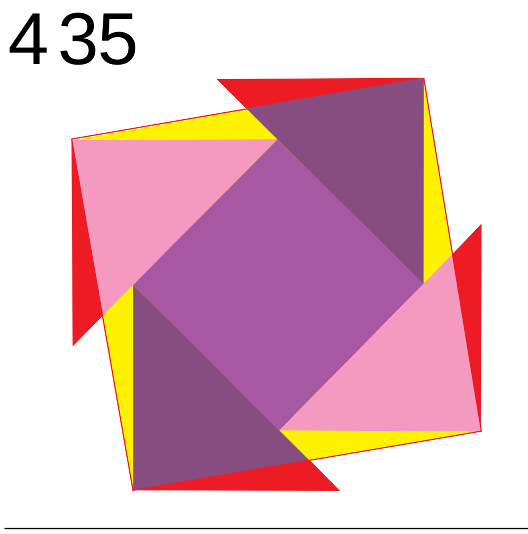
433



434 Card 3 is not found in the pattern.

CHAPTER 8 SOLUTIONS

435

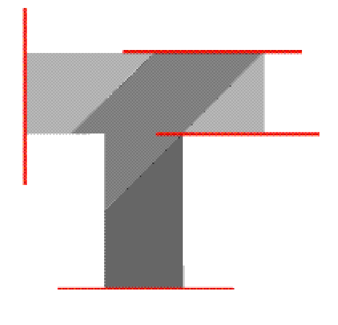


436

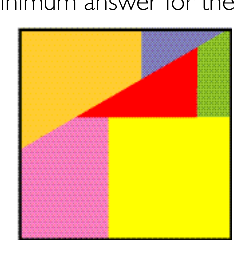


437 One of many solutions.

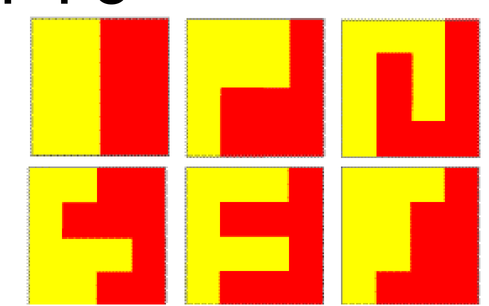
438 The result is the solution to the classic "T-Puzzle" (PlayThink 20).



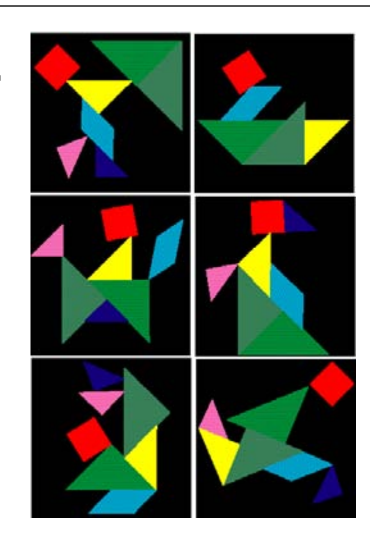
439 Although the minimum answer for the three squares problem involves making just five pieces, no one has yet found it. This solution, using six pieces, is the current record.



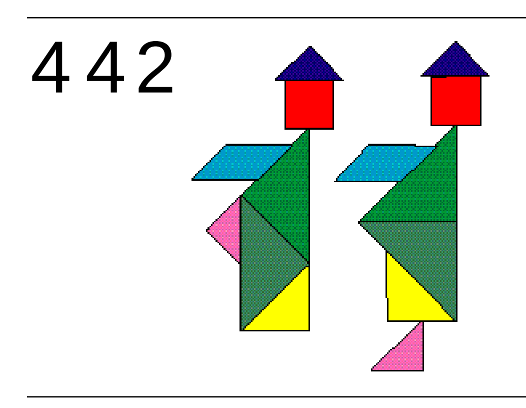
440



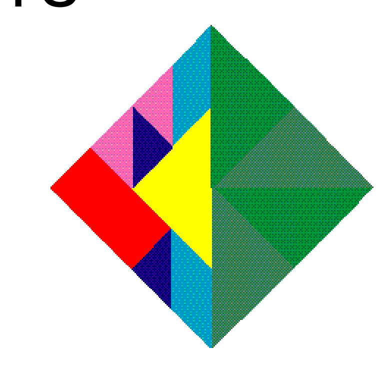
441



442

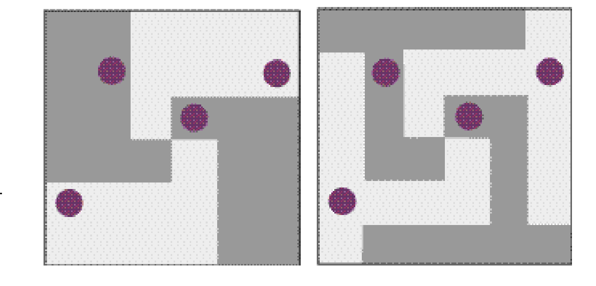


443 One of many possible solutions.

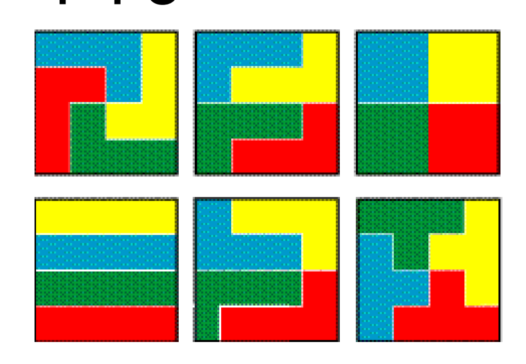


44

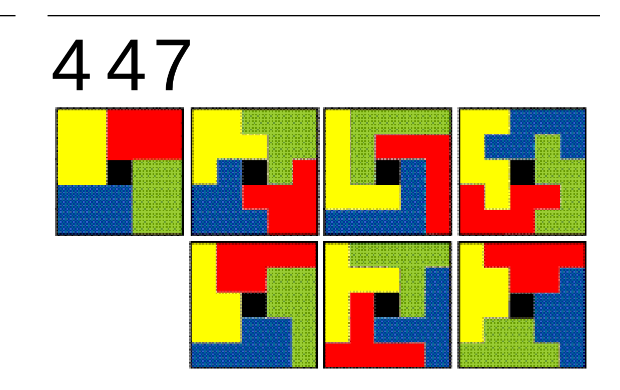
445



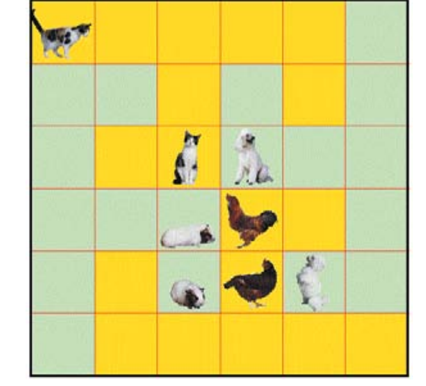
446



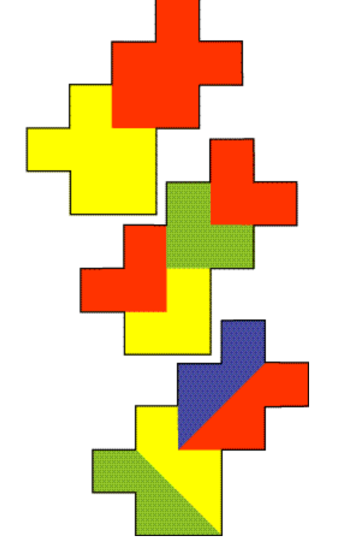
447



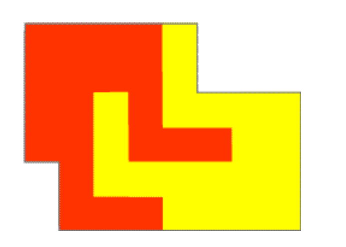
448



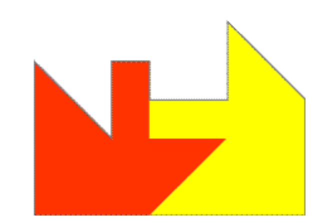
449



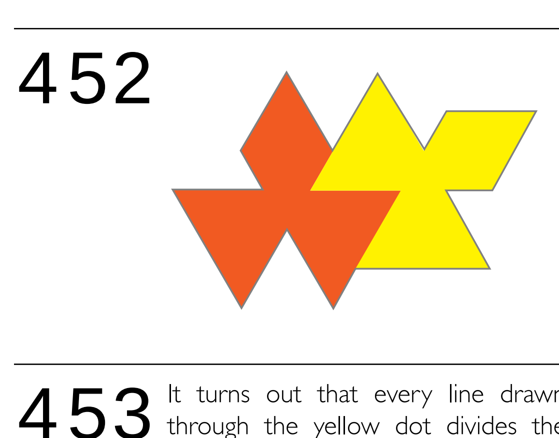
450



451



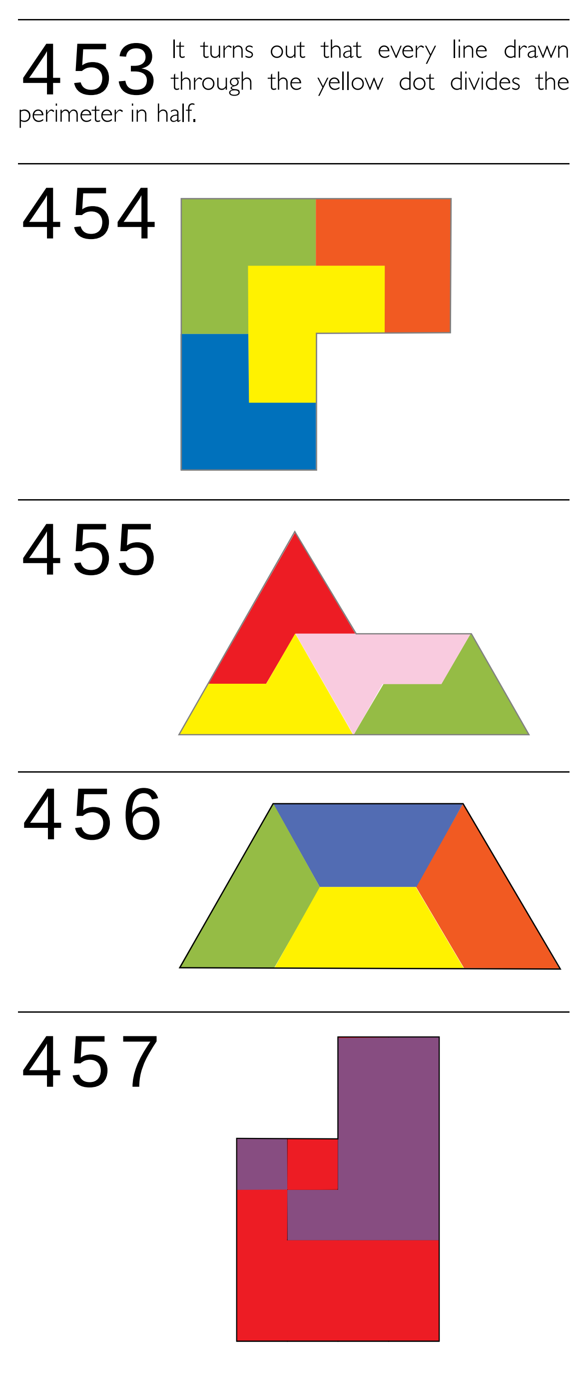
452



453

It turns out that every line drawn through the yellow dot divides the perimeter in half.

454



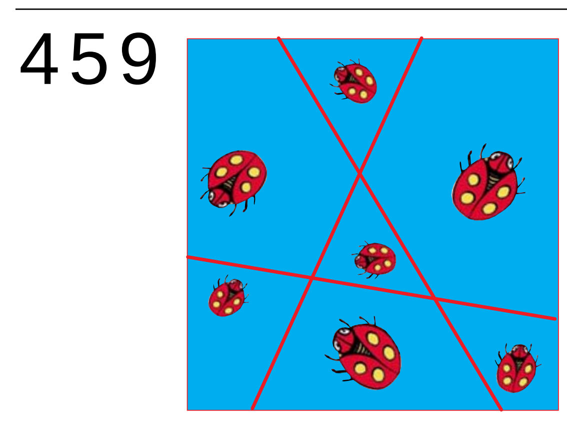
455

456

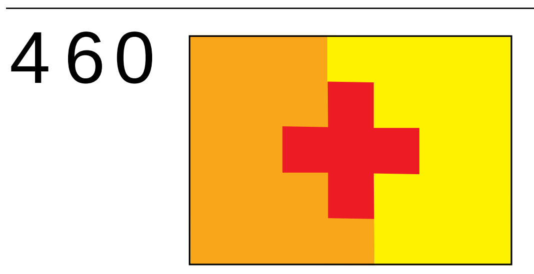
457

458

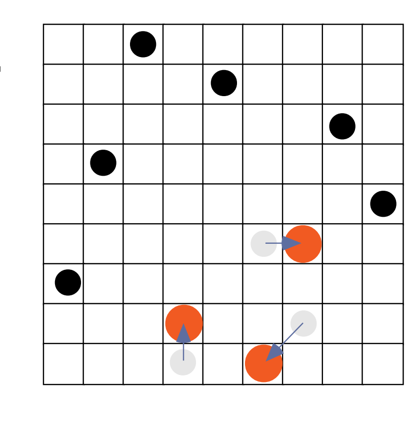
459



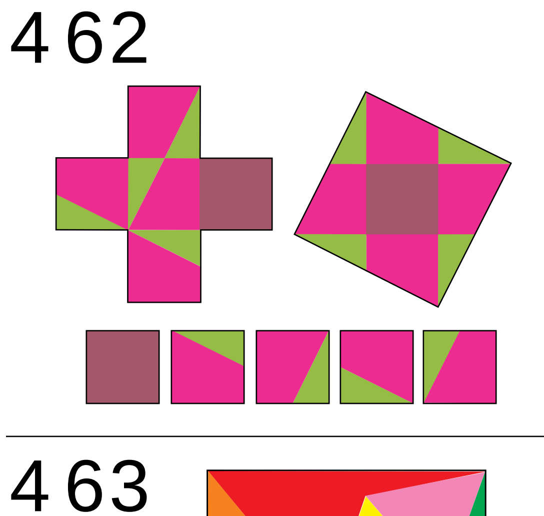
460



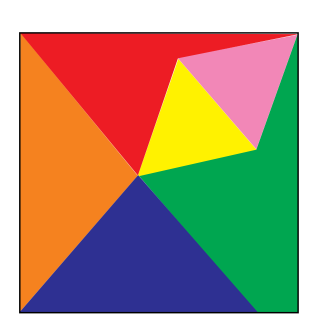
461



462

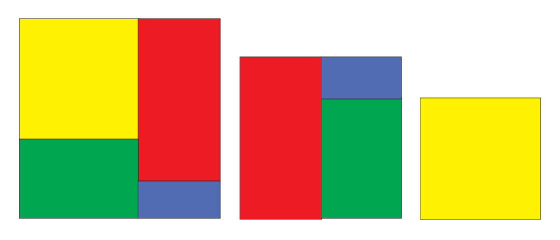


463



464

You can make the two squares from four pieces: one for the smaller square and three for the larger one.



465

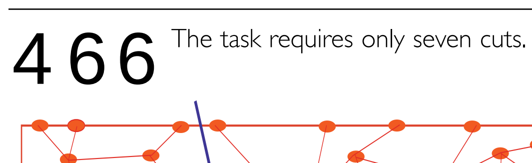
The pieces form a chain—when they are swung in one direction, they form an equilateral triangle; when swung in the opposite direction, they form a square.

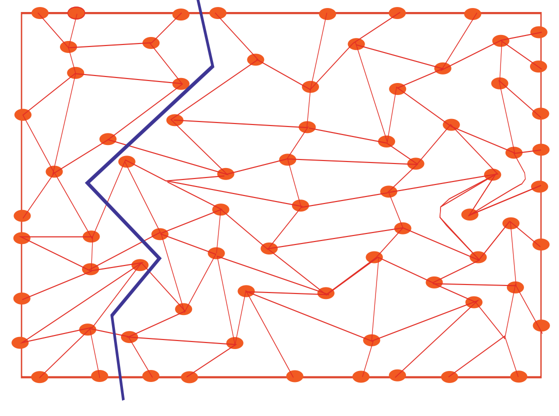


The inventor of this gem of recreational mathematics was Henry Ernest Dudeney, England's most accomplished puzzle maker. Born in 1857, Dudeney was extremely successful with dissections and set many records. This dissection, however, is his most famous discovery.

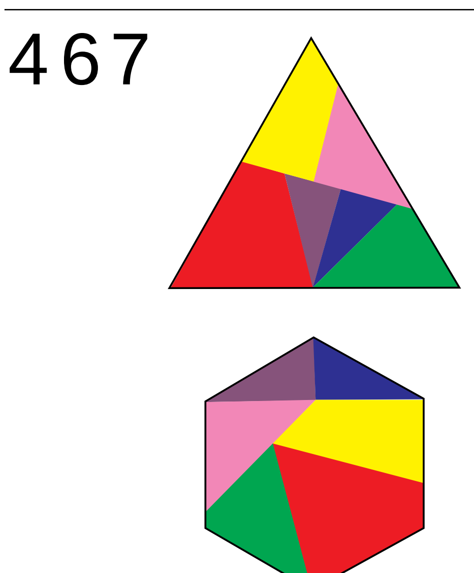
466

The task requires only seven cuts.

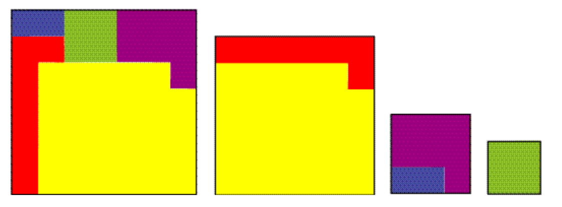




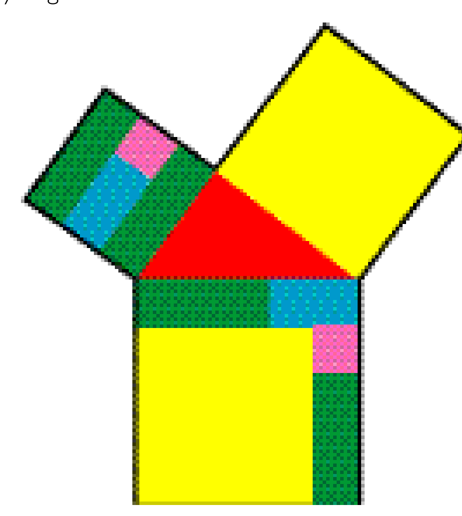
467



468 You can make the three squares from just five pieces.

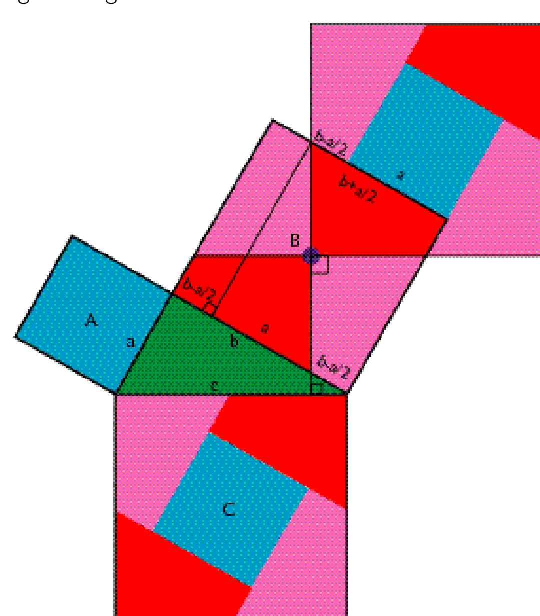


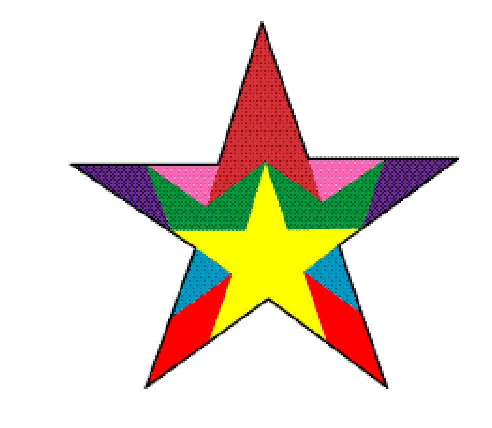
469 Congratulations! You have demonstrated the truth behind the Pythagorean theorem.

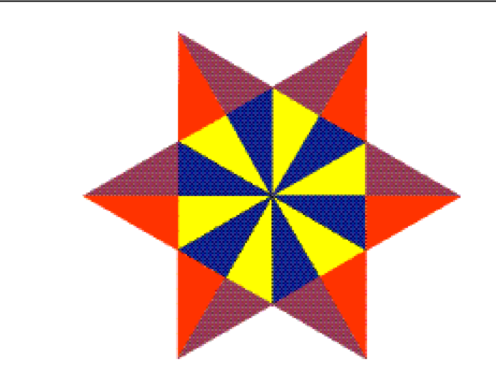


470 This puzzle is derived from one of the most beautiful proofs of the Pythagorean theorem, discovered by Henry Perigal (1801–1898). His proof involved dropping a perpendicular from the center of square B to the line c, and constructing a line parallel to line c through the center of B.

The four pieces of that dissection, plus square A can be rearranged to make square C, as shown. This construction works for every set of squares about a right triangle.

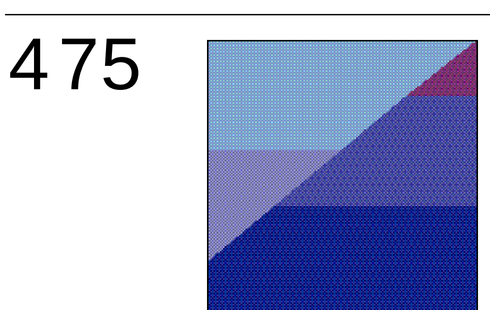




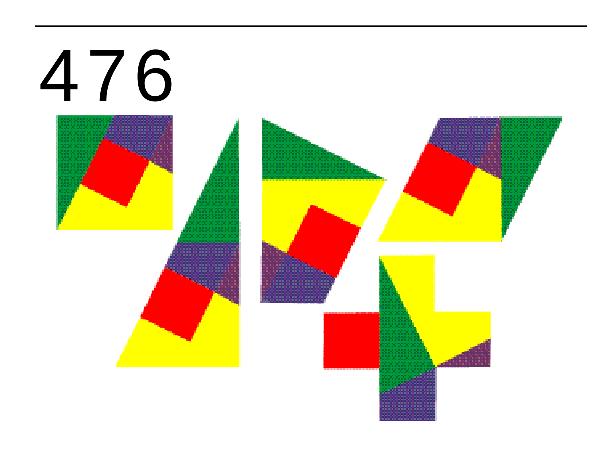




475

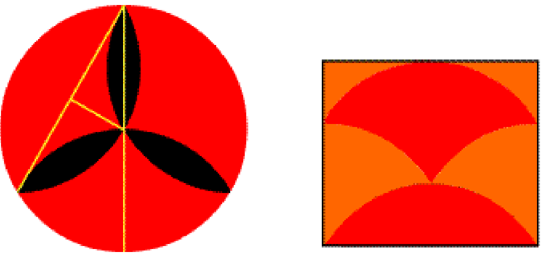


476

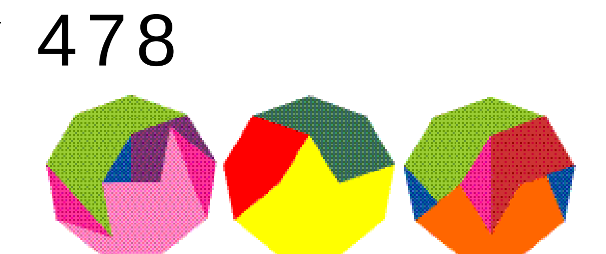




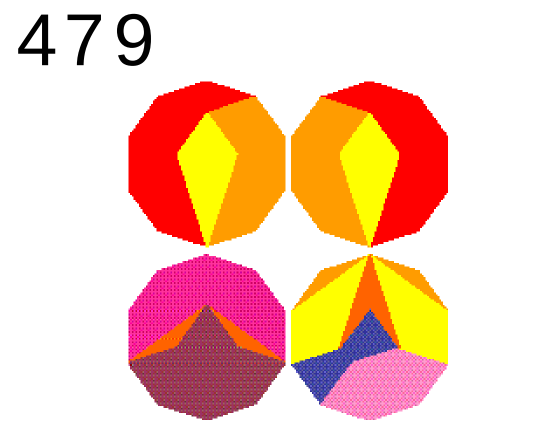
477 The solution involves six pieces.



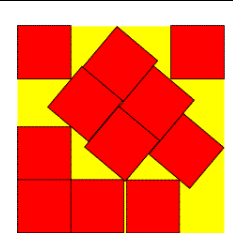
478



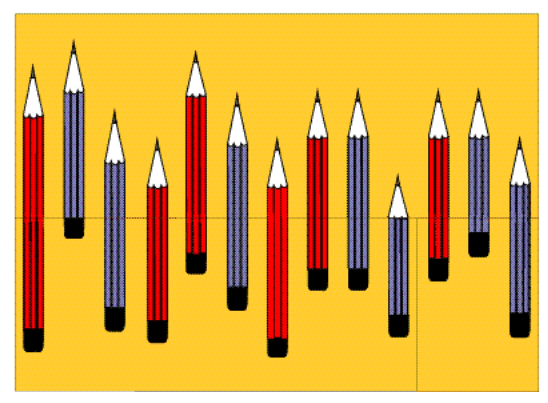
479



480 The German mathematician Walter Trump found the solution shown here. Some of the red squares are inclined by 40.18 degrees.



481 When you swap the lower parts of the drawing, you have six red pencils and seven blue pencils. A close examination will reveal which pencil changed color.



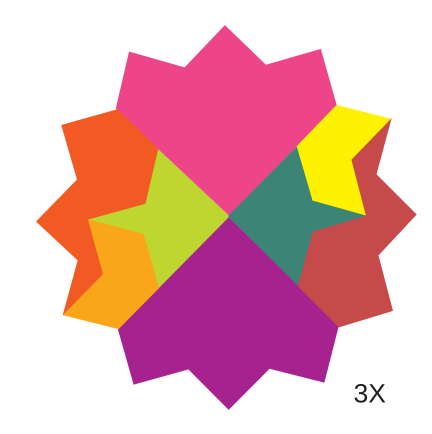
482

Each star contains the same arrangement of pieces.



483

Each star contains the same arrangement of pieces.



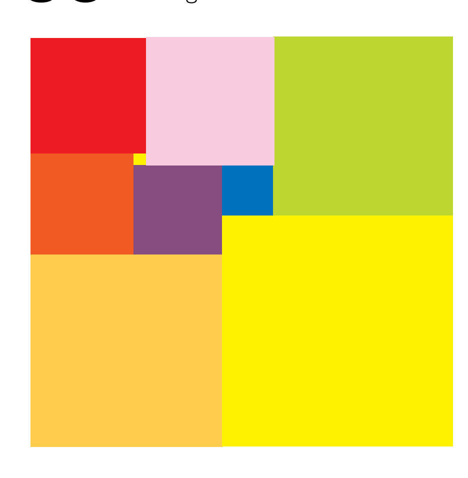
484

Area 1—1.5 units
Area 2—4.5 units
Area 3—1.5 units
Area 4—2.5 units
Area 5—2.5 units
Area 6—3 units
Area 7—4 units
Area 8—15.5 units

Since the area not taken up by the red triangle totals 19.5 units, it is larger.

485

The thirty-two-by-thirty-three perfect rectangle is the smallest known.



486

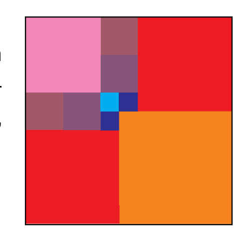
Surprisingly, in spite of the coincidence that the sum of the squares and the area of the big square are both 4,900, there is no known solution in which all twenty-four squares can be placed on the larger square without overlap. The best solutions known to date can fit only twenty-three of the twenty-four squares; in each instance it is the seven-by-seven square that must be left out. One such solution is illustrated here.

Although there are other sets of consecutive squares that add up to a square number, none is less than the sequence from one to twenty-four.



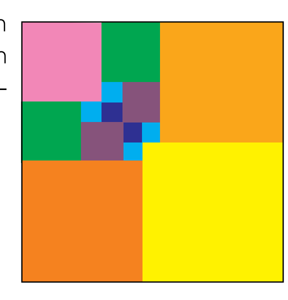
487

The minimal solution for an eleven-by-eleven square consists of eleven smaller squares, shown at right.



The minimal solution for a twelve-by-twelve square consists of four smaller squares, each six-by-six.

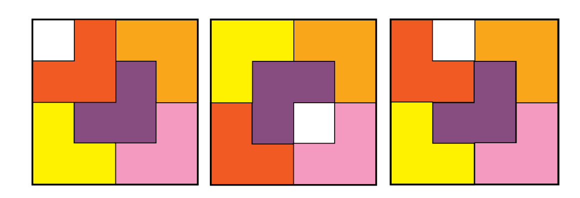
The minimal solution for a thirteen-by-thirteen square consists of fourteen smaller squares.



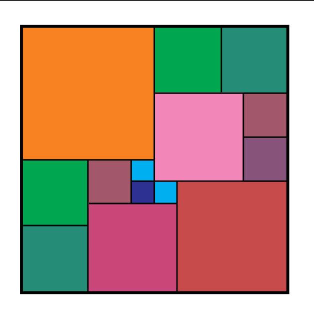
The minimal solution for a fourteen-by-fourteen square consists of four smaller squares, each seven-by-seven. I hope you've caught on to the pattern for even-sided squares.

488

The covered square can be anywhere on the board. Three sample arrangements are shown below; through reflection and rotation, one of the arrangements will uncover any given square.

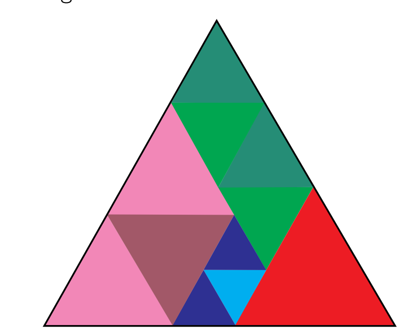


489

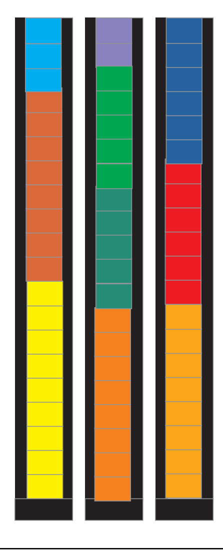


490

It takes eleven smaller triangles to completely cover the eleven-by-eleven triangle. One solution is shown.

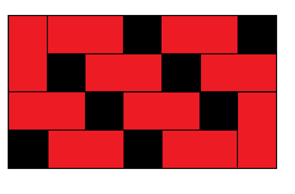


491

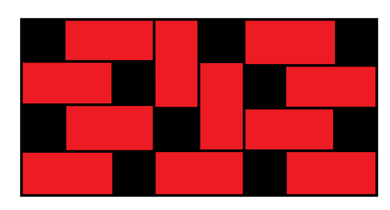


492

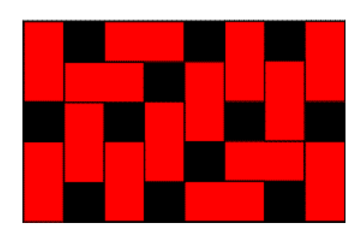
The maximum number of holes that can be made on a board cannot exceed the number of dominoes. In fact, if the length of one side of the board is evenly divisible by three, then the maximum number of holes is the product of the two sides, divided by three.



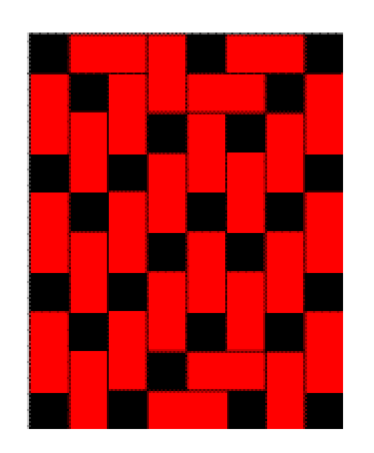
493



494



495

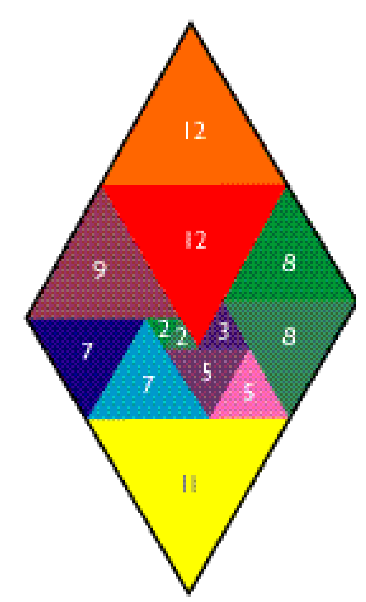


497

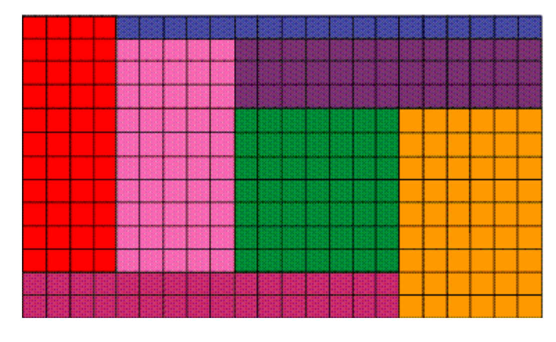
In the first and third examples, about three-quarters of the triangle is covered. In the other two, much less is covered.

498

This solution has the fewest number of triangles, with thirteen.

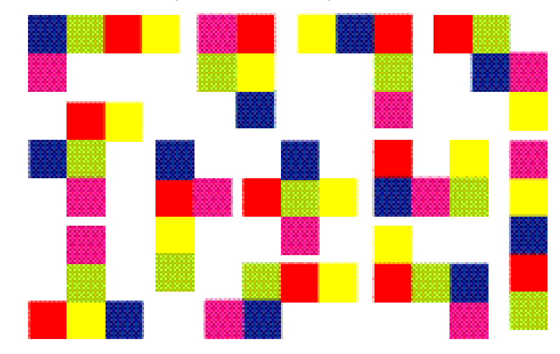


499

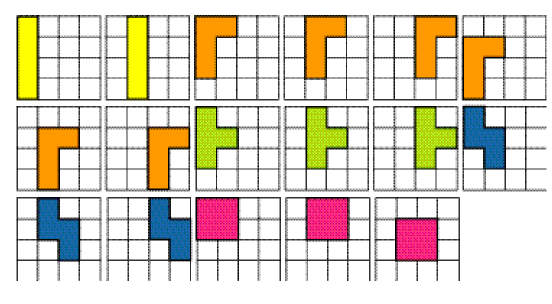


500

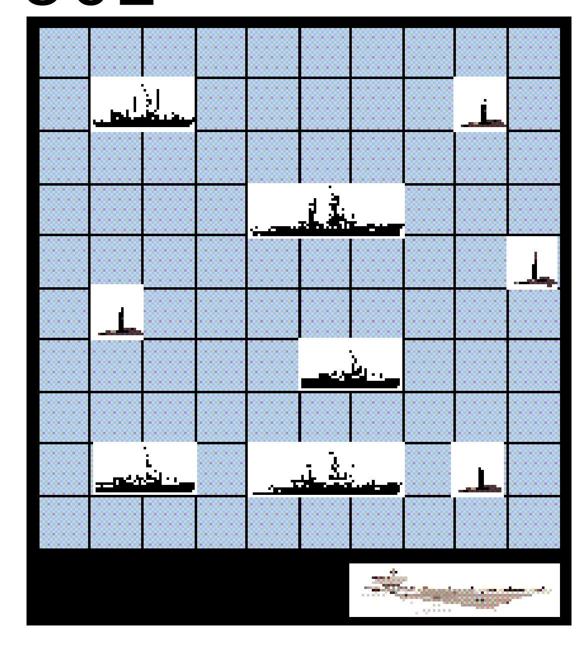
The twelve different ways of joining five identical squares are shown above. Such shapes are called pentominoes.



501



502

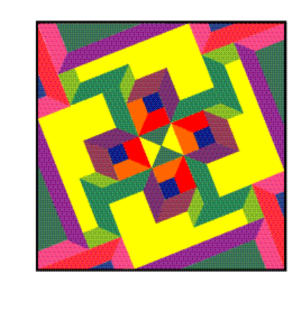


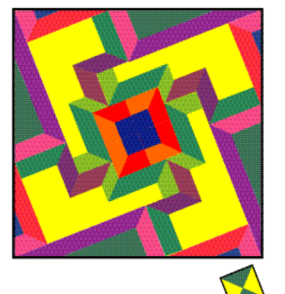
503

The two squares seem to be identical, but since 2 minus 1 does *not* equal 2, it's obvious that the second square must be smaller, though not by much. The missing area, equal to that of the small square that was removed, is spread so thinly around the remaining pieces that it is almost impossible to notice its absence.

By the way, the secret to putting together the smaller square is to swap the two triangles along every edge of the square. After you do that, the arrangement of the rest of the pieces is fairly obvious.

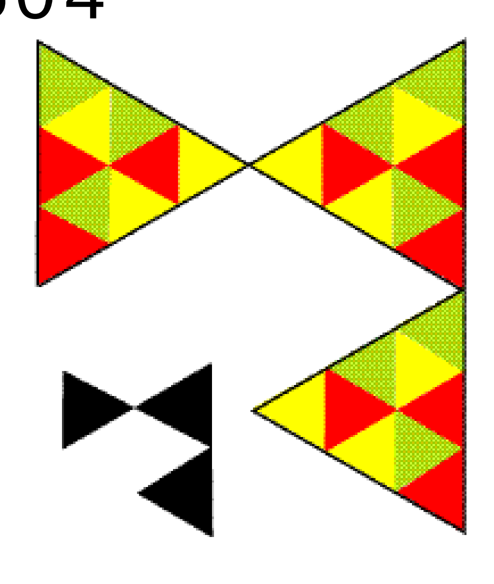
One thing to learn from such vanishing puzzles is that, in order to fool the eye and the mind, you have to be very subtle. Although humans are adept at spotting differences, they can easily overlook very small changes that are deftly hidden.





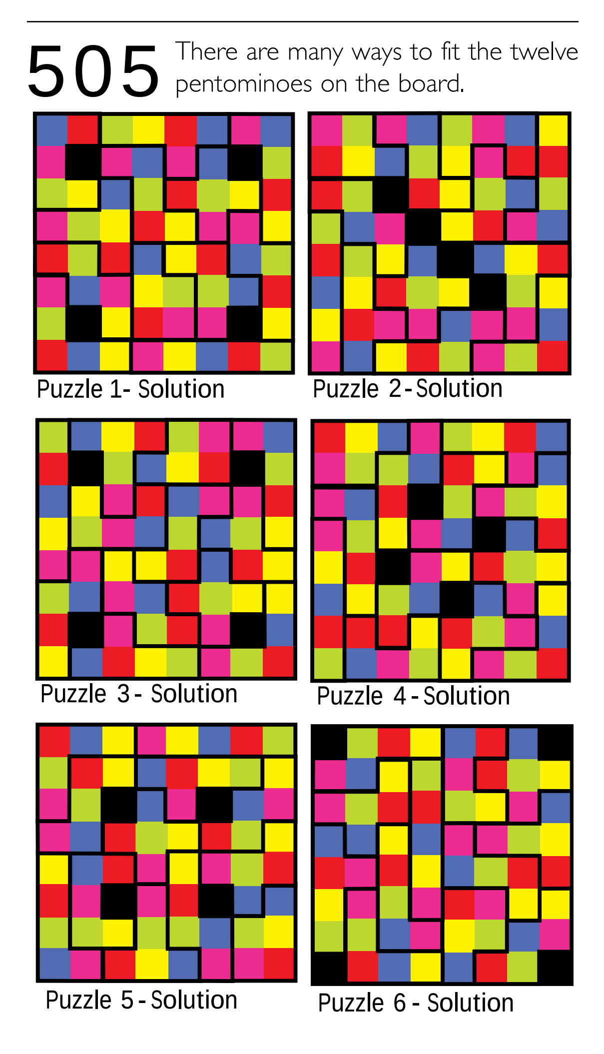
504

Nine small figures, as shown.



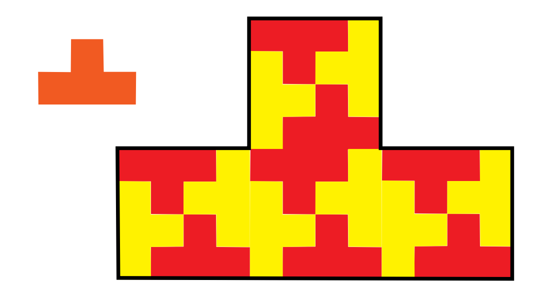
505

There are many ways to fit the twelve pentominoes on the board.



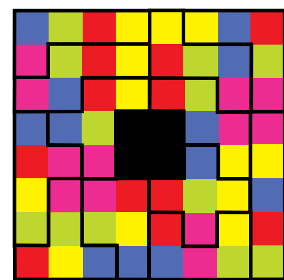
507

The large replica holds sixteen smaller T-tiles.



508

In this, one of many solutions, a yellow pentomino has been formed at the top.

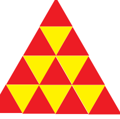


509

One of the most counterintuitive facts in geometry is that only three regular polygons—the equilateral triangle, the square and the regular hexagon—are capable of tessellating a plane.

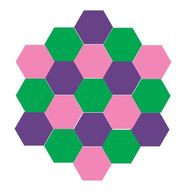
There is a beautiful logic behind the rarity of regular tessellations. At every point in which the vertices of the tessellating polygons meet, the sum of the angles of those vertices must equal 360 degrees. The only regular polygons that can tessellate, then, are the ones whose angles are factors of 360.

Six equilateral triangles, each with angles of 60 degrees, can meet at a point—and so they can tessellate.



Four squares, each with angles of 90 degrees, can meet at a point—and so squares can tessellate.

Pentagons have internal angles of 108 degrees—not a factor of 360—and so pentagons cannot tessellate.

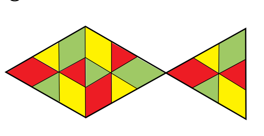


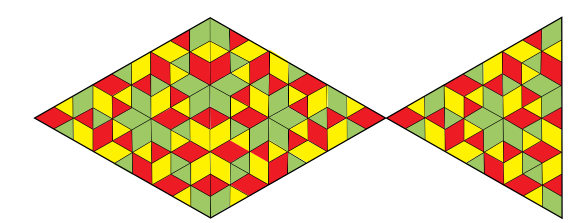
Three hexagons, each with angles of 120 degrees, can meet at a point, and so hexagons can tessellate.

As you can see, the next whole number that can meet at a point is 2—making for 180 degrees on each side. That's not a tessellation—it is a bisection. Therefore, only an equilateral triangle, a square and a regular hexagon are capable of tessellating a plane.

510

The medium-sized fish can hold nine smaller ones, and the large fish can hold nine medium-sized fish. That means all eighty-one small fish can fit inside the large fish. But even that fish should not rest easy, because there is always an even bigger fish somewhere!





CHAPTER 9 SOLUTIONS

511

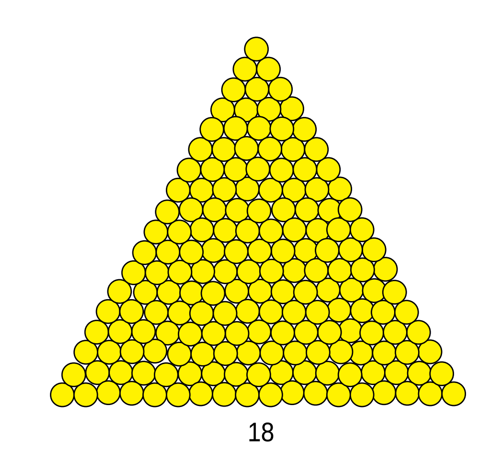
The numbered objects of the tetraktys can be arranged in (10!)/(2 × 3), or 604,800, different ways.

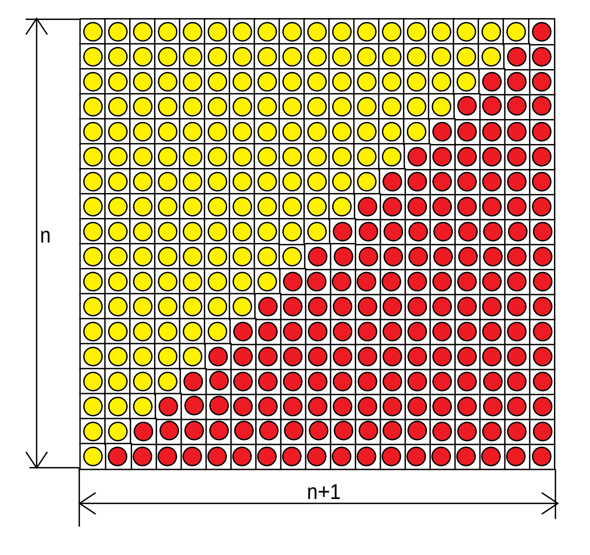
512

Triangular numbers are the sum of any number of consecutive positive integers, beginning with 1. The fourth triangular number, 10, equals 1 + 2 + 3 + 4.

Babylonian cuneiform tablets show that the formula for deriving triangular numbers has been known since antiquity. For any number *n*, its triangular number can be calculated as n(n + 1)/2.

To find the eighteenth natural number, then, simply find 18(18 + 1)/2, which is 171.



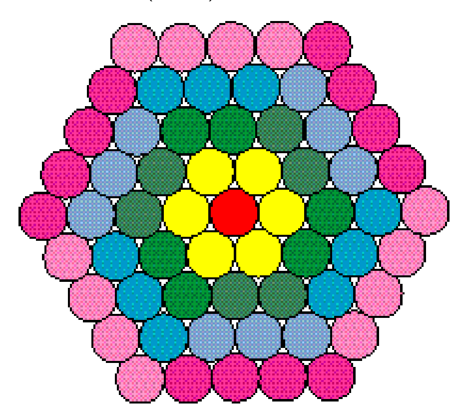


513

$99 + {}^{99}/_{99} = 100$

514

Each successive ring has 6(n – 1) elements. That means the next hexagonal number is 37 + 6(5 – 1) = 61.



515

Squares are formed by the sum of the series of odd numbers, beginning with 1.

$1^2 = 1$

$2^2 = 1 + 3 = 4$

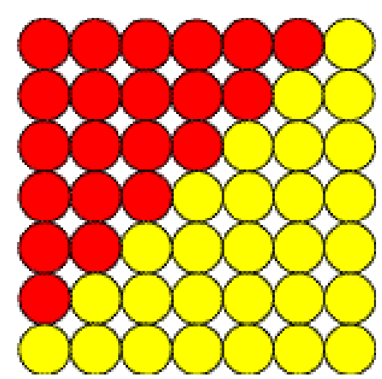
$3^2 = 1 + 3 + 5 = 9$

$4^2 = 1 + 3 + 5 + 7 = 16$ and so on

The seventh square would be $7^2 = 1 + 3 + 5 + 7 + 9 + 11 + 13 = 49$.

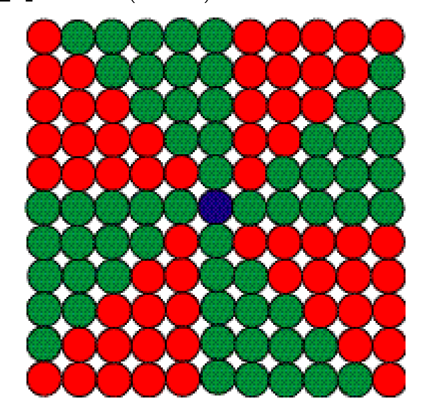
516

The sixth and seventh triangular numbers, 21 and 28, add together to make 49.



517

It is the fifth triangular number: 15.
(15 × 8) + 1 = 121



518

The tetrahedral series can be expressed by the formula n(n + 1)(n + 2)/6. That gives a series 1, 4, 10, 20, 35, 56, 84 . . .

The square pyramidal series can be expressed in the formula n(n+1)(2n+1)/6. That gives a sequence of 1, 5, 14, 30, 55, 91, 140 . . .

519

You can use the principle of one-to-one correspondence to find the answer without counting. Simply mark off pairs of sheep—one right-facing, one left-facing—until no more of one type remain.

520

The numbers are 1, 3, 9 and 27. This problem is a good exercise in getting the maximum work from a minimal number of elements.

1	=	1	9+3-1	=	11
3-1	=	2	9+3	=	12
3	=	3	9+3+1	=	13
3+1	=	4	27-9-3-1	=	14
9-3-1	=	5	27-9-3	=	15
9-3	=	6	27-9-3+1	=	16
9-3+1	=	7	27-9-1	=	17
9-1	=	8	27-9	=	18
9	=	9	27-9+1	=	19
9+1	=	10	27-9+3-1	=	20

27-9+3	=	21	27+3+1	=	31
27-9+3+1	=	22	27+9-3-1	=	32
27-3-1	=	23	27+9-3	=	33
27-3	=	24	27+9-3+1	=	34
27-3+1	=	25	27+9-1	=	35
27-1	=	26	27+9	=	36
27	=	27	27+9+1	=	37
27+1	=	28	27+9+3-1	=	38
27+3-1	=	29	27+9+3	=	39
27+3	=	30	27+9+3+1	=	40

521

Gauss realized that the series 1 + 2 + 3 + 4 . . . 97 + 98 + 99 + 100 could be written as:

1 + 100 + 2 + 99 + 3 + 98 + 4 + 97 . . .

or 101 times 50, to get the total, 5,050.

This trick works for any sum of sequential integers. Indeed, the general formula is simply n(n + 1)/2, which is the equation for triangular numbers.

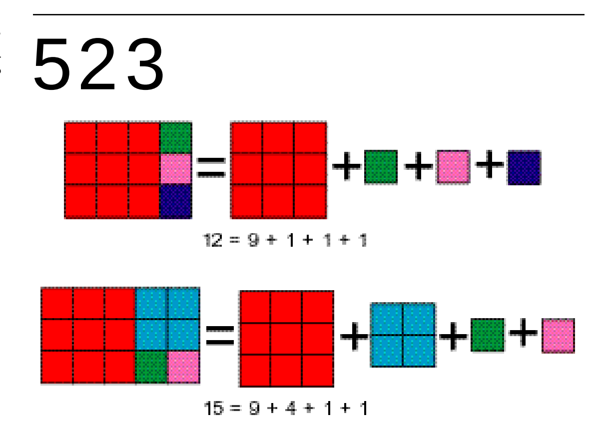
This problem is a beautiful illustration of the importance of understanding the patterns underlying ordinary routines. If you grasp what a question is really asking, you can avoid a lot of drudgery in answering it.

522

The concept of mathematical equivalence, or isomorphism, is the key to winning this game. Think back to the famous Lo-Shu magic square (PlayThink 377): the square is filled with the numbers from 1 through 9, and each row, column and main diagonal adds up to 15. As you can see, then, maneuvering to mark off three numbers that total 15 is the equivalent to playing tic-tac-toe.

The best strategy, then, is to keep that fact in mind—perhaps even to memorize the Lo-Shu magic square—and attack and defend as if you were playing tic-tac-toe. The best first move, for instance, is to color in 5.

523



524

With the Pythagorean theorem, it is quite simple to calculate the length of the hypotenuse: $1^2 + 1^2 = c^2 = 2$; so $c = \sqrt{2}$.

But finding a rational number that equals $\sqrt{2}$ is impossible. The Pythagorean disciple Hippasus first showed that the diagonal of a square with rational sides does not have a rational length. Numbers like $\sqrt{2}$, $\sqrt{3}$ and so on, which cannot be expressed by the fraction of two whole numbers, are now known as irrational numbers. Although this discovery shook the foundations of Greek mathematics, the study of length later became a bridge between geometry and algebra. Attempts to measure the properties of curves eventually gave rise to calculus.

525

4 – 1 + 2 × 3 + 5 = 20

526

Two odd numbers added together result in an even number. But that means that the sum of an odd number of odd numbers will always be odd. So five odd numbers cannot add up to 100. But six odd numbers can; the numbers 1, 3, 45, 27, 13 and 11 are just one set of odd numbers that total 100.

527

Just five. The same pickers who can pick five apples in five seconds can pick sixty apples in sixty seconds; they average an apple a second.

528

The second perfect number is 28, the sum of 1, 2, 4, 7 and 14.

Students of the Bible have noted that the first two perfect numbers are embedded in the structure of the universe. After all, God created the universe in six days, and the moon circles the earth every twenty-eight days.

The third perfect number is 496.

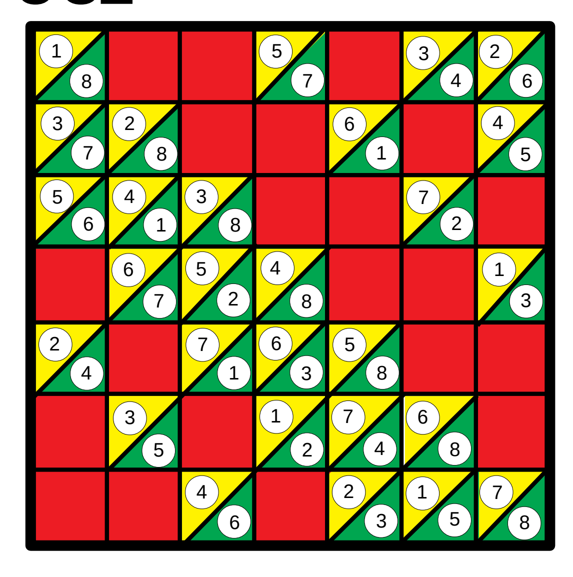
No one knows whether the supply of perfect numbers is inexhaustible, nor do we know whether there are any odd perfect numbers. That question has been vexing mathematicians since the time of Pythagoras.

529

The unique solution for four pairs of blocks is shown here.

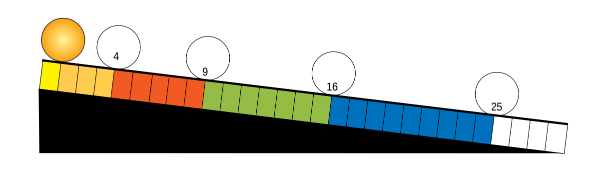
The Scottish mathematician C. Dudley Langford first laid out the general form of this problem in the 1950s after watching his son play with colored blocks. It turns out that the problem has a solution only if the number of pairs of blocks is a multiple of 4, or is 1 less than a multiple of 4.

531

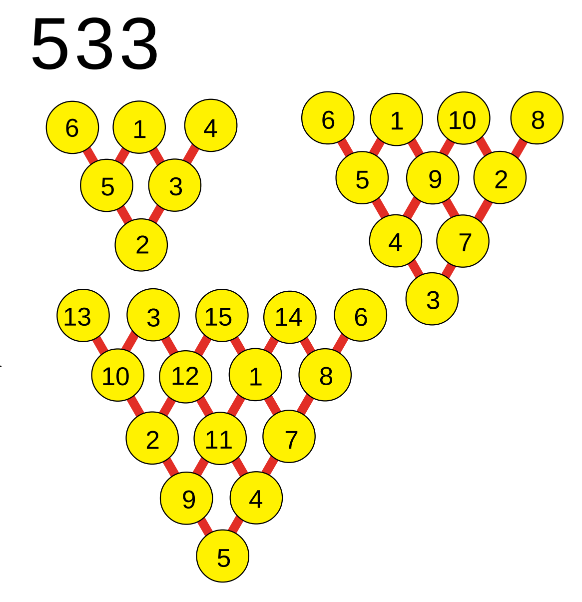


532

Regardless of how steeply the plane is inclined, a ball rolling for two seconds will always travel four times as far as it does after one second, and after three seconds, it will go nine times as far. The pattern becomes quite obvious: if the ball goes one unit after one second, then for every n number of seconds, the ball will travel n^2 units.



533



534

Twenty ladybugs.

535

Swap the 8 and the 9, then turn the 9 upside-down so it reads as a 6. Both columns will then total 18.

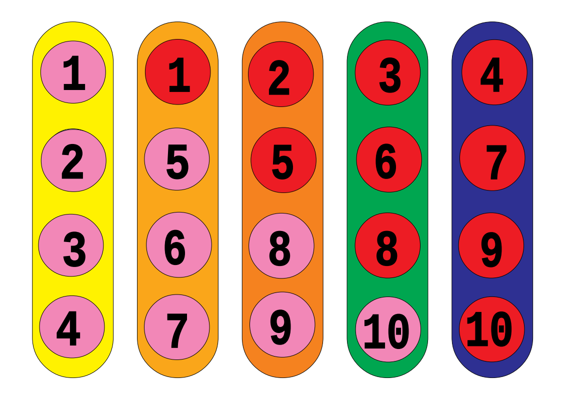
536

Since there are seven pages before page 8, there must be seven pages after page 21. The newspaper has twenty-eight pages.

537

One of many possible solutions.

538



539

540

There are many examples: 243 + 675 = 918; 341 + 586 = 927; 154 + 782 = 936; 317 + 628 = 945; 216 + 738 = 954 . . . and so on.

541

The ten digits can be permutated in 10!, or 3,628,800, ways. But since all the ways that start with 0 must be dropped, the actual number is 362,880 lower, for a total of 3,265,920.

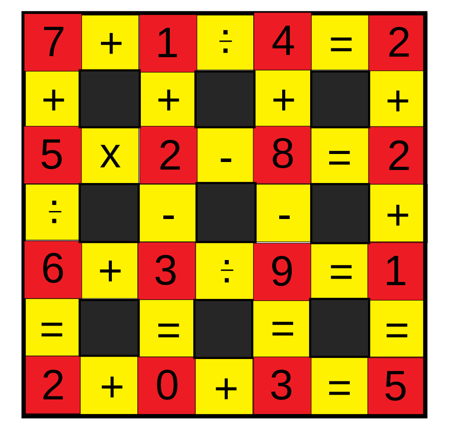
542

Four neighboring cells form a diamond in which A x D – B x C = 1.

543

One, of course, is the smallest number of persistence. Twenty-five is the smallest number that has a persistence of 2; 39 is the smallest that has a persistence of 3; and 77 is the smallest that has a persistence of 4.

544



545

Each number is the sum of its neighbors immediately above, to the left and to the upper left diagonal. Following this rule, the missing number is 63.

546

0 = 4 – 4
0 = 4 – 4
1 = 4 ÷ 4
2 = (4 + 4)/4
3 = 4 – (4/4)
4 = 4
5 = 4 + (4/4)
6 = ((4 + 4)/4) + 4
7 = (44/4) – 4
8 = 4 + 4
9 = 4 + 4 + (4/4)
10 = (44 – 4)/4

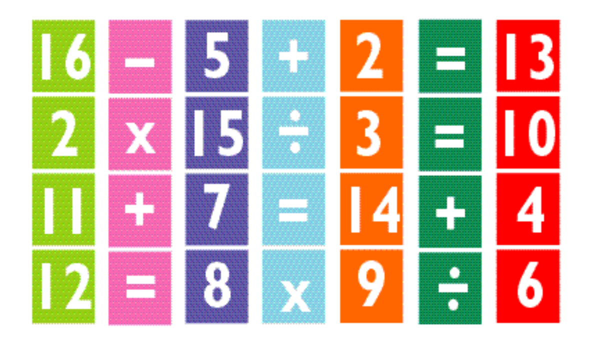
547

20 = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 13
20 = 1 + 1 + 1 + 1 + 1 + 1 + 3 + 11
20 = 1 + 1 + 1 + 1 + 1 + 1 + 5 + 9
20 = 1 + 1 + 1 + 1 + 1 + 3 + 3 + 9
20 = 1 + 1 + 1 + 1 + 1 + 1 + 7 + 7
20 = 1 + 1 + 1 + 1 + 1 + 3 + 5 + 7
20 = 1 + 1 + 1 + 1 + 3 + 3 + 3 + 7
20 = 1 + 1 + 1 + 1 + 1 + 5 + 5 + 5
20 = 1 + 1 + 1 + 3 + 3 + 3 + 5 + 5
20 = 1 + 1 + 1 + 3 + 3 + 3 + 3 + 5
20 = 1 + 1 + 3 + 3 + 3 + 3 + 3 + 3

548

Six darts:
17 + 17 + 17 +17 + 16 + 16 = 100

549



550

Surprisingly, both sums are 1,083,676,269.

551

The next four numbers are 21, 34, 55 and 89.

Each number is the sum of the two numbers preceding it. As the sequence continues, the ratio of successive terms approaches the famous golden ratio, 1:1.6180037.

552

The first digit can be any digit from 1 to 9; the second can be any of those digits except the consecutive one. That makes for eighty-one nonconsecutive two-digit numbers.

553

This problem has been around for a long time, and mathematicians have found several answers. Our solution is just one of them.

1+2+3-4+5+6+78+9=100

554

2,520 = 5 x 7 x 8 x 9

555

Most people who approach this problem see each number as the difference between the two numbers feeding into it. But that can't account for the 7, since 21 – 13 = 8.

Instead, examine the individual digits of the numbers feeding into each circle. You will find that 9, 9, 7 and 2 add up to 27, and that 4, 5, 2 and 7 total 18. Thus, the missing number can be found by adding 3, 6, 2 and 1. The missing number is 12.

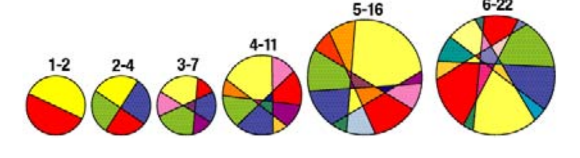
556

312211

Each term is a description of the number preceding it: "11" means there's one 1; "21" means there are two 1s; "1211" means there is one 2 and one 1; "111221" means there is one 1, one 2 and two 1s.

557

The numbers make up the cake-cutting sequence—the maximum number of pieces that can be made from a given number of straight cuts through a plane. As a general rule, each *n*th cut will make *n* number of new pieces. Thus, for the sixth cut, the number of pieces will be 16 + 6, or 22.



558

The sequence is based on the principle of persistence, in which the digits of a number are multiplied together to get another number; the function is carried out until only a one-digit number remains.

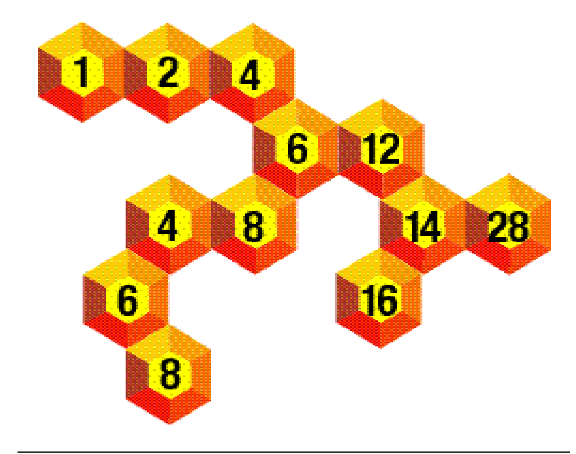
Thus, the last number in the sequence is 8.

559

The answer is 20 years old because 210 is the twentieth triangular number, equal to the sum of all the numbers from 1 to 20.

560

The numbers double as they run from left to right horizontally; the numbers go up by 2 as they run from top to bottom diagonally.



561

The possible answers are 52 and 25, 63 and 36, 74 and 47, 85 and 58, or 96 and 69. But the ages that match up with how long my friend has been practicing magic are 74 and 47.

562

IOTOIO

563

17 x 4 = 68 + 25 = 93

$$\begin{array}{r} 17 \\ \times\ 4 \\ \hline \end{array}$$
= 68 + 25 = 93

564

Add 40 to both.

170	30
+40	+40
210	70

$\frac{Y}{Z} = \frac{210}{70}$ X=40

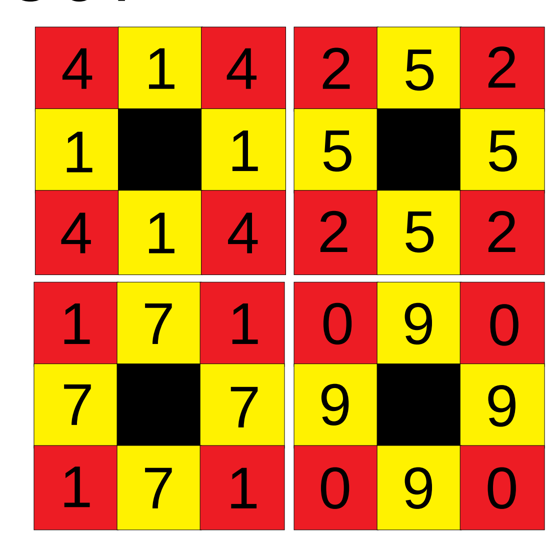
565

$2^6 - 63 = 1$

566

The puzzle begins with one hundred separate pieces and ends with one complete cluster. Since each move reduces the number of pieces or clusters by one, only ninety-nine moves are needed.

567



568



569

In a single-elimination tournament, one team is knocked out in each match. So if there are fifty-eight teams and one champion, then fifty-seven teams must be eliminated over the course of the tournament. Therefore, fifty-seven matches must be played.

The principle of identifying a one-to-one correspondence between two sets crops up in probability theory, enumeration and everyday problem solving.

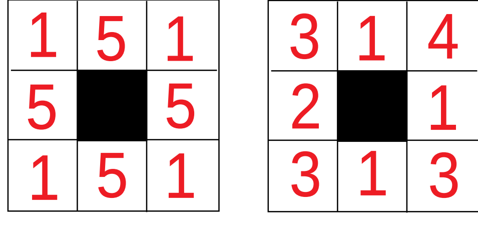
570

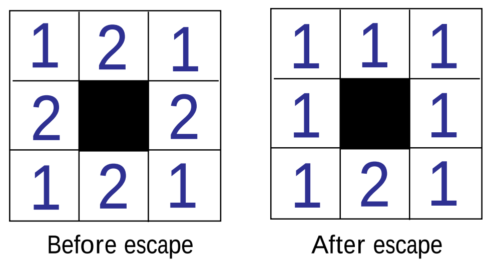
Yes, there *is* enough information to work this out. If there are even two red flowers, then it will be possible to pick a pair without one being purple. So there is only one red flower, and the rest are purple.

571

There can't be even two red flowers, or else it will be possible to pick two reds and a yellow and not have any purple flowers out of three. Similar logic dictates that there can't be more than one purple or one yellow flower. Therefore, there are only three flowers in the entire garden.

572





573

There were twenty-three emus and twelve camels.

574

I saw twenty-two two-legged birds and fourteen four-legged beasts.

575

Yes. There is a unique solution; you simply have to remember that every leg is counted—stool legs, chair legs and people legs!

Thus, for every occupied stool, there are five legs (three stool legs and two people legs). And every occupied chair counts for six legs. So 5 × (number of stools) + 6 × (number of chairs) = 39.

From that it is easy to work out that there are three stools, four chairs and seven people.

576

Yes.

577

To solve this problem, you have to work out the number of possible pairs for the nine friends. In mathematical language the problem involves a "Steiner triple system of order nine." But in more simple terms, for any given friend, four separate dinners are necessary to see all eight cohorts.

Day 1—Kate, David, Lucy
Day 2—Emily, Jane, Theo
Day 3—Mary, James, John
Day 4—Kate, Emily, Mary
Day 5—David, Jane, James
Day 6—Lucy, Theo, John
Day 7—Kate, Jane, John
Day 8—Lucy, Jane, Mary
Day 9—David, Theo, Mary
Day 10—Lucy, Emily, James
Day 11—Kate, Theo, James
Day 12—David, Emily, John

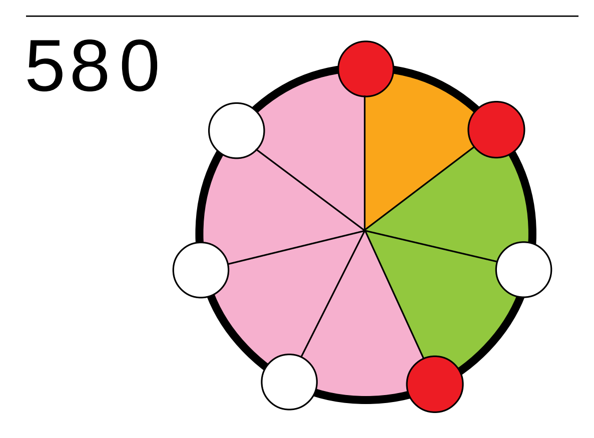
578

Since the mother cat has two lives left, the kittens must divide up the remaining twenty-three. That means there are two possible answers: seven kittens (one has five lives left and six have three lives) or five kittens (one with three lives and four with five lives).

579

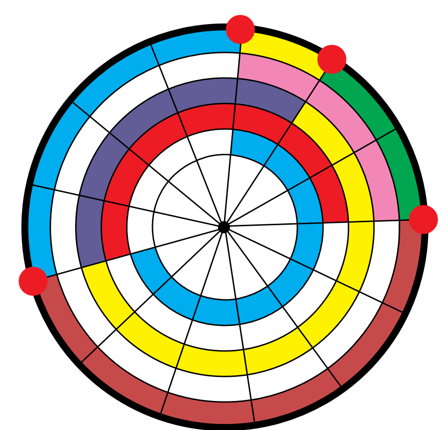
There are 9 one-digit numbers, 90 two-digit numbers and 900 three-digit numbers, for a total of 2,889 digits. That leaves 40 more digits, or 10 four-digit numbers: 1,000 to 1,009. The book must have 1,009 pages.

580



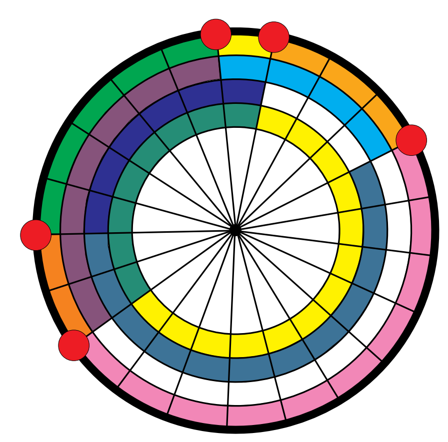
581

The four points on the circle must be distributed in one of two ways: 1-2-6-4 or 1-3-2-7.



582

For five points on a circle to represent twenty-one different units of length, they must be distributed with the spacings 1-3-10-2-5.

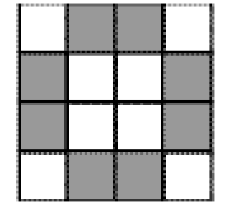


584

Day 1—4-5-2	7-1-9	6-8-3
Day 2—7-8-5	4-3-1	6-9-2
Day 3—8-1-2	4-7-6	9-3-5
Day 4—1-5-7	3-2-8	9-4-6
Day 5—8-4-1	5-6-2	3-7-9
Day 6—7-2-4	8-9-5	1-6-3

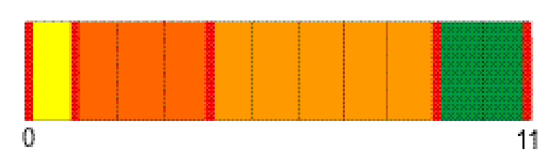
585

The secret is to look at the eight coins in the shaded areas. With any given move, either two or none of those coins will turn over. That means if the number of Jekylls is even, the configuration can be solved; if the number of Jekylls is odd, it cannot.



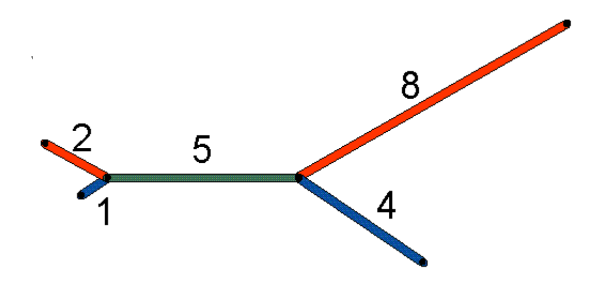
586

Minimal length rulers, invented by Solomon W. Golomb, can only be "perfect" up to length 6. All higher length rulers are "imperfect," since some distances occur more than once or don't occur at all. Using an 11-unit ruler, it is impossible to place the markers so that a 6-unit distance is measured.

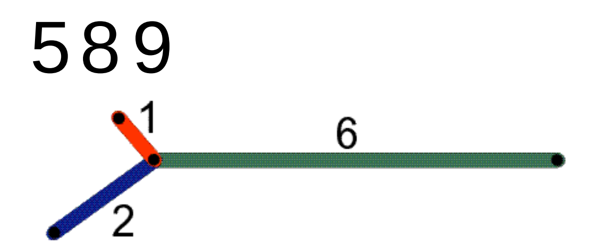


587

There were fifteen ladybugs:
3 + 5 + 6 + 1 = 15



589



590

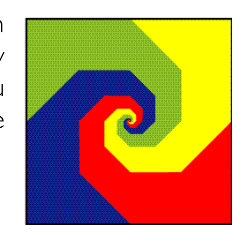
When you double the linear measurements of a two-dimensional object, its area increases by a factor of 4 (2^2). Similarly, doubling the linear measurements of a three-dimensional object increases the volume by a factor of 8 (2^3). Assuming that the density of that volume remained constant, your weight would also increase by a factor of 8. Or, to find your new weight, simply multiply your present weight by 8.

591

The red triangles occupy an area that is roughly one-third that of the square.

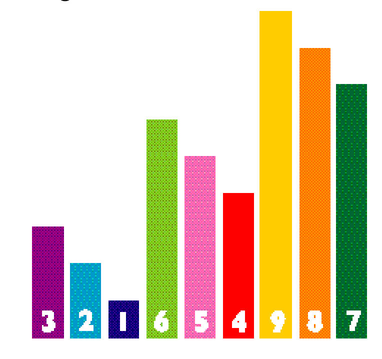
592

The red arm occupies exactly one-fourth of the square. You can divide the entire square into four such spiral arms.



593

There are some eighty-four different solutions. The one illustrated below involves lengths of 3, 2 ,1, 6, 5, 4, 9, 8, 7.



594

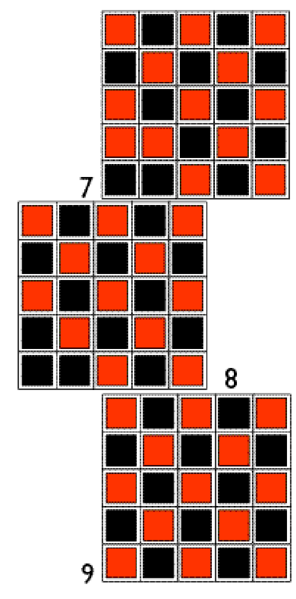
Any arbitrary sequence of ten numbers or lengths will always have an increasing or decreasing sequence of at least four members. Although nine lengths can be arranged in such a manner, the tenth member will complete either an ascending or descending run, no matter where it is placed.

595

The beaker was half full after 39 minutes.

596

You might want to experiment with other initial configurations to try to find out if the outcome will always be a checkerboard pattern. But a word of warning: The answer has never been proved.

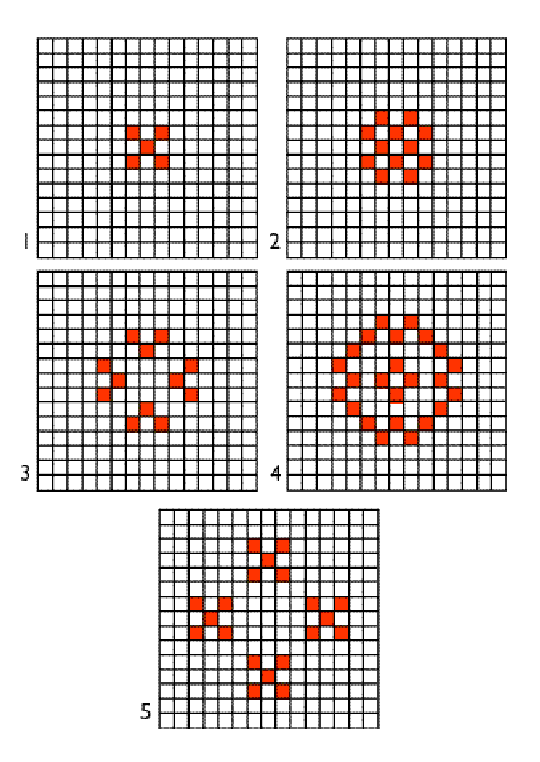


597

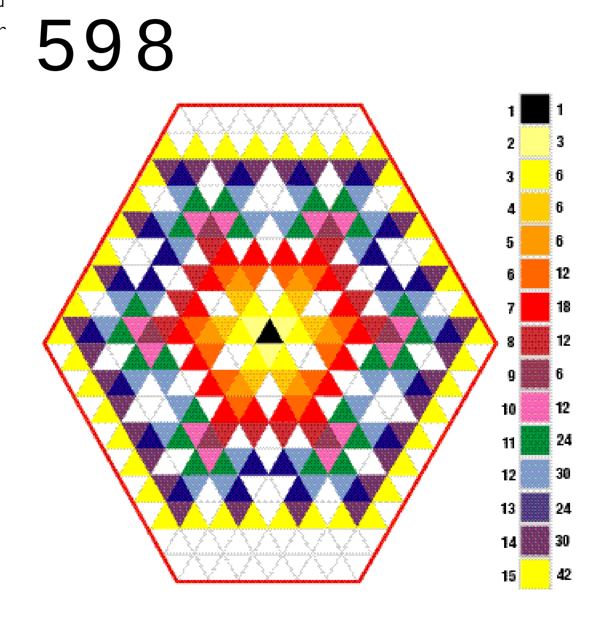
The initial configuration of five red, or live, cells transforms over five generations to four identical copies, as shown below.

This system, called a cellular automaton, has a fascinating property: virtually any starting configuration will, after a few generations, replicate into four, sixteen and sixty-four copies of itself. It is remarkable that a system so simple can possess a property as "lifelike" as self-replication.

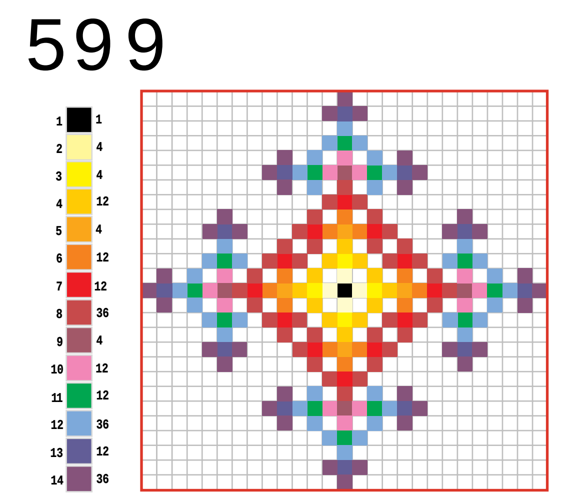
Edward Fredkin of MIT created this self-replicating system in 1960. The Game of Life, invented by Princeton mathematician John Horton Conway, is a subtler cellular automaton that works on similar principles. In it, whether a given square "lives" or "dies" depends upon the number of "live" squares around it. Finding configurations that will live, grow and even replicate is an interesting mathematical problem.



598

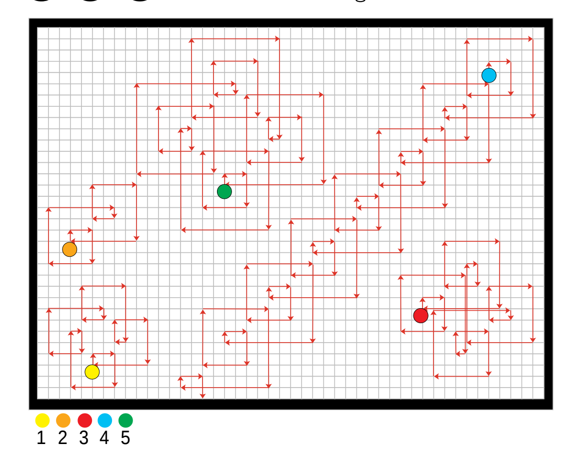


599



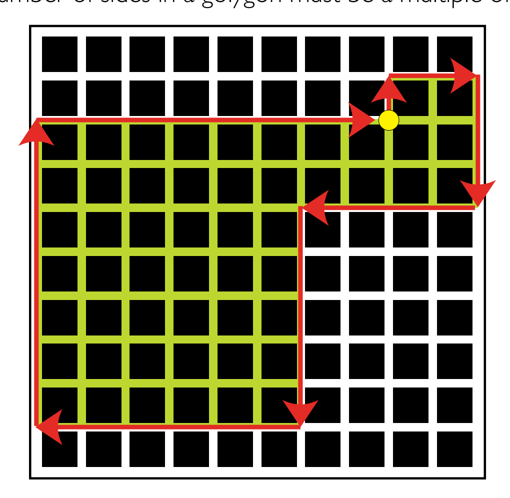
600

The ladybugs return in games 1, 2, 3 and 5 but not in game 4.



601

Sallows found this eight-sided golygon to be the simplest possible; it has the interesting ability to tessellate in the plane. The next simplest golygon has sixteen sides; there are twenty-eight variations of it. Martin Gardner proved that the number of sides in a golygon must be a multiple of 8.



602

Although many properties of primes remain unproven, one famous proof has demonstrated that there is always a prime between every integer greater than 1 and that integer's double.

603

None of the 362,880 numbers will be prime.

In each case the sum of their digits is 45, which is divisible by 9. And any number that has digits adding up to a multiple of 9 is itself a multiple of 9. This simple divisibility check shows why none of the numbers can be prime.

604

The limit on the area approaches about 1.6 times that of the original triangle. Amazingly, the curve will not extend beyond a circle that circumscribes that triangle.

As for the perimeter, say each side of the initial triangle is 1 unit in length, for a total of 3 units of perimeter. The polygon that replaces the triangle after one generation consists of twelve sides, each one-third the length of the original sides, for a total of 4 units. Every successive step sees the perimeter increasing by the same factor of $^4/_3$. Thus, there is no ultimate boundary on the perimeter—if you take infinite steps, you will have an infinite perimeter.

The yellow in the problem shows the opposite process; it will form the "antisnowflake" curve.

605

The stack of images would approach a height twice that of the original picture, but it would never actually reach that point. The sum of $1 + {}^1/_2 + {}^1/_4 + {}^1/_8 + {}^1/_{16} \ldots$ is less than 2.

606

Examine the sum of the divisors of 220:

$$1 + 2 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 = 284$$

Now look at the divisors for 284:

$$1 + 2 + 4 + 71 + 142 = 220$$

If the sum of the divisors of a number is equal to a number whose divisors are equal to the first, the pair is said to be amicable. The smallest known pair is 220 and 284.

Pythagoras knew about amicable numbers, and Arab mathematicians investigated such pairs during the Middle Ages. Euler himself published 60 pairs, and some 5,000 pairs are known today.

Although amicable numbers have been the subject of intense study over the millennia, Nicolo Paganini, an Italian schoolboy, discovered the second smallest pair—1,184 and 1,210— in 1866. This goes to show that there are sometimes great rewards awaiting even amateur mathematicians.

607

The complete decomposition of 420 is 42 x 10 = 6 x 7 x 2 x 5 = 2 x 3 x 7 x 2 x 5.

608

Anne's neighbors can be either two boys or two girls. If they are girls, then each of them must be neighbored by another girl, since they are both next to Anne. So in the instance where Anne's neighbors are girls, the entire circle must be girls.

Since there are boys in the circle, the circle is obviously not all girls. That means Anne's neighbors must both be boys, each of whom is neighbored by Anne and another girl. This alternating pattern continues around the circle, so that the circle contains twelve boys and twelve girls.

609

The best way to avoid false moves in this game is to move the smallest disk from one column to the next and then any disk other than the smallest. Although such a recipe seems arbitrary, it ensures that there will always be one legal move. And repeating the pattern over and over will miraculously bring you to the solution. There is some deep connection between the cyclical movement of the disks and the mathematical underpinnings of this game.

For Puzzles 1 through 4, the minimum number of moves is, respectively, three, seven, fifteen and thirty-one.

For Puzzle 5, which has the restriction against placing disk 1 on disk 4, nineteen moves are required.

For Puzzle 6, which has restrictions against placing disk 1 on disk 3, and disk 2 on disk 4, the minimum number of moves required is only fifteen—the same as if there were no restrictions.

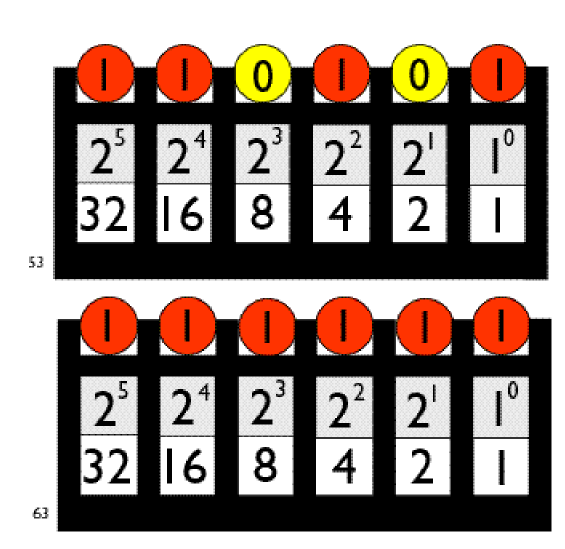
610

The answer is 24, composed of 1, 2, 3, 4, 6, 8, 12 and 24.

611

$$1 + 2 + 3 = 1 \times 2 \times 3 = 6$$

612



613

Before you turn your back, you check to see how many coins are showing heads. You know that the number of heads will increase by two, decrease by two or stay the same for every pair of coins that is turned over. Therefore, if the initial number of heads is odd, the number will remain odd, no matter how many pairs of coins are turned.

When you turn back around, you count the number of heads that are now showing. If the number is odd, as at the start (or even, as at the start), the covered coin must be a tail. If the number of heads is even for an odd start (or odd for an even start), the covered coin must be a head.

This simple trick helps demonstrate the importance of parity: the odd-even parity of this system is preserved as long as pairs of coins (not individual coins) are turned over.

614

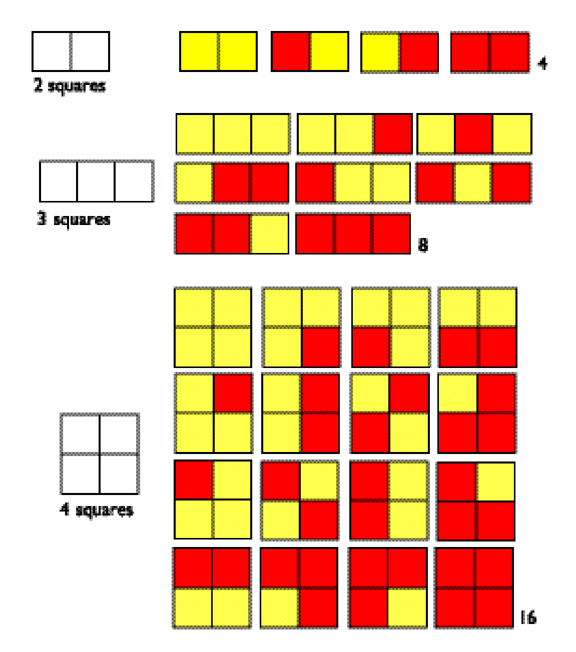
13			P		5	L		A	
4	Y		T	H	10		I		N
9		K	S		6	I			S
6	G			R	10		E		A
10		T		F	13			U	
11		N			4	A		N	D
4	E		D	U	4	C		A	T
10		I		O	11		N		

"*PlayThinks* is great fun and education."

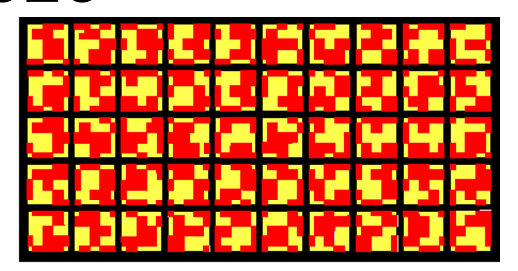
615

The filled-in two-by-two matrices are a visual representation of the numbers from 0 to 15 in the binary number system: 0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111, 1000, 1001, 1010, 1011, 1100, 1101, 1110 and 1111.

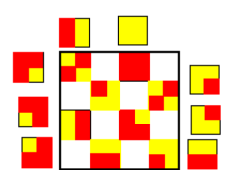
But are the sixteen tiles really different? On close inspection you may notice that there are only six different tiles, three of which are present in four different orientations.



616

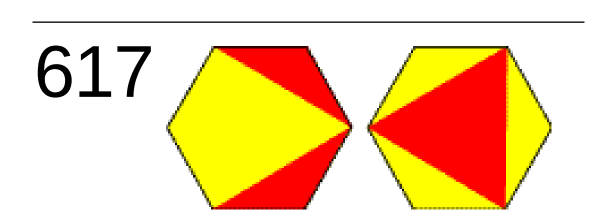


Shown above are fifty solutions to the Q-Bits color-matching game. Rotations, reflections and color reversals are not considered different.



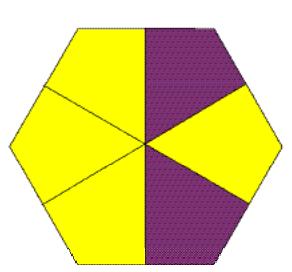
The shortest possible two-person game can end in eight moves, and there can be a great number of solutions, one shown. It can be seen that none of the remaining eight tiles can be fitted in the board.

617



618

1	100	26	89
2	72	27	95
3	90	28	69
4	59	29	93
5	94	30	63
6	77	31	96
7	86	32	91
8	85	33	73
9	80	34	81
10	51	35	78
11	58	36	76
12	68	37	99
13	92	38	74
14	53	39	79
15	84	40	83
16	62	41	82
17	98	42	87
18	67	43	64
19	97	44	55
20	52	45	57
21	71	46	54
22	61	47	88
23	75	48	70
24	56	49	60
25	66	50	65



620

Try as they might, they will fail. That is because turning over two glasses at a time changes the number of upright glasses by two or by zero. And although the number of upright glasses in the first setup was one, so that adding two gave you a total of three, the number of upright glasses in the second setup is zero. Changing two at a time will allow your friends to fluctuate between zero glasses and two glasses, but they will never get to three glasses. In other words, the first setup has an odd parity, while the second setup has an even parity. In both instances, turning over two glasses at a time will not change that parity.

621

The parity of the initial setup is odd, and an even number of moves will not change that. Therefore, both all-upright and all-inverted conclusions are impossible.

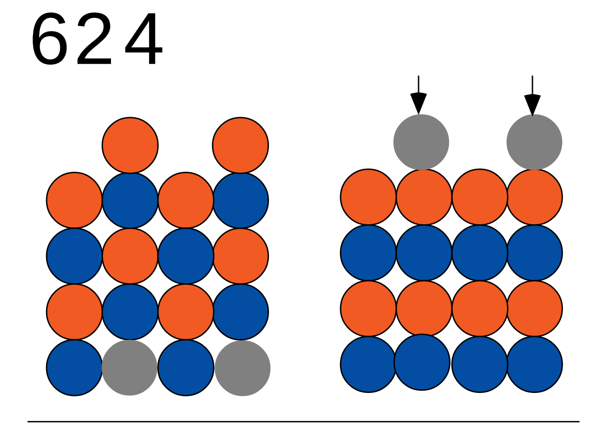
622

The thief will always stay one step ahead unless the policeman moves first and changes the parity of the game. He can do that by going around the triangular block just once, catching the thief in seven or fewer moves.

623

Hexagon 19 is the odd one out.

624



625

Since a loop of gears turns alternately clockwise and counterclockwise, an even number of gears is required for the setup to work. An odd number of gears, as in this puzzle, can't rotate at all.

626

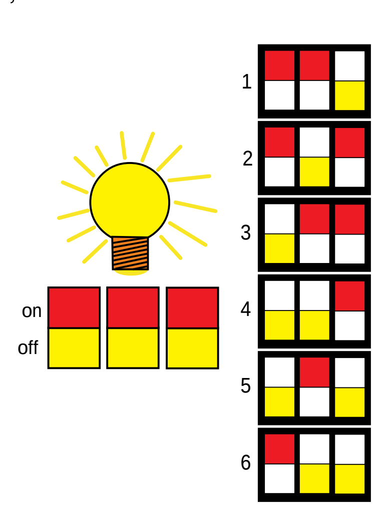
Many people claim that there is not enough information provided to solve this problem. But that is because they have taken too narrow a view.

The key is understanding what a lightbulb does: it produces not only light but heat, and it remains warm many minutes after it has been switched off.

With that in mind, you can find the solution fairly easily. First, turn on switch 1 and leave it on for several minutes so that the bulb will get good and hot. Next, turn off switch 1 and turn on switch 2, and then go quickly to the attic. If the light is on, then switch 2 works the lamp; if the bulb is dark but warm, switch 1 works the lamp. If the bulb is both dark and cold, switch 3—the one that has not been used—works the lamp.

627

Such a bet is a losing proposition. Only three out of six possible random settings allow for the light to be turned on with the flip of just one switch.

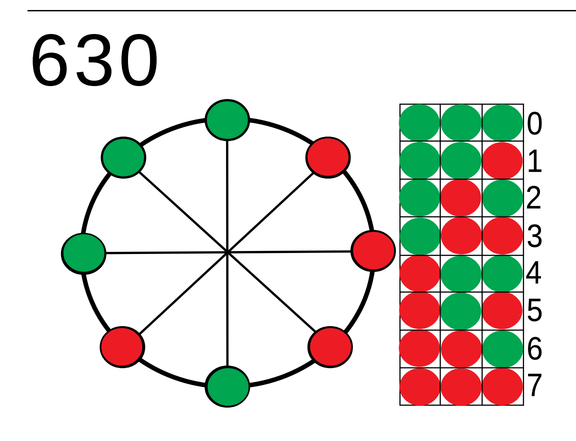


628

629

Just five moves: 1-2-3, 4-5-6, 7-8-9, 8-9-10 and 8-9-11.

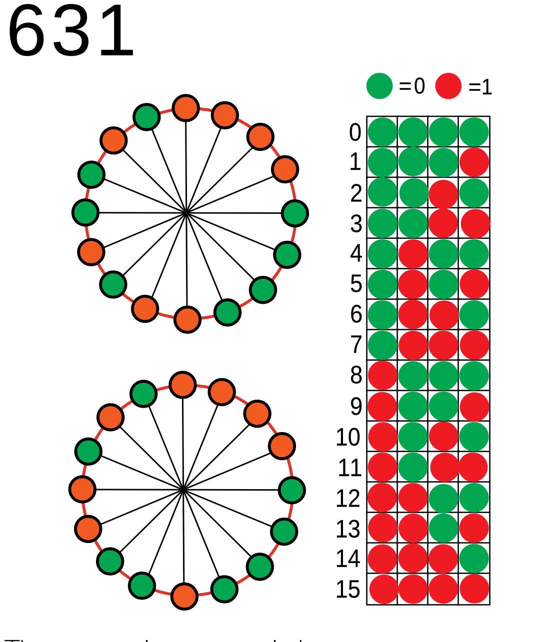
630



This solution is the only one possible.

Longer binary wheels are used to code messages in telephone transmission and radar mapping. University of California—Davis mathematician Sherman K. Stein called such binary structures memory wheels; they have also been called Ouroborean rings, a name derived from the mythological snake that ate its tail.

631

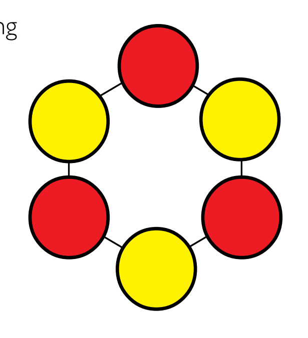


There are at least two solutions:

1-1-1-1-0-0-0-0-1-1-0-1-0-0-1-0 and
1-1-1-1-0-0-0-0-1-0-0-1-1-0-1-0.

632

The missing necklace.



633

There are three different necklaces possible. The different necklaces can be described by the number of red beads between the green ones: either none, one, or two.

CHAPTER 10 SOLUTIONS

634

You have to attack problems like this one in a systematic fashion, or the complexities will boggle your mind. The best way to visualize the variables is to draw a cell chart with, say, the positions across the top and the names down the side. Put an X in a cell that has been logically ruled out, and put a * in a cell you believe is correct.

	CHAIRMAN	DIRECTOR	SECRETARY
Gerry	X	•	X
Anita	X	X	*
Rose	*	X	X

Then work your way through the premises:

Gerry has a brother, and the secretary is an only child, so Gerry can't be the secretary.

Rose earns more than the director, and the secretary earns less than anyone, so Rose can be neither the director nor the secretary.

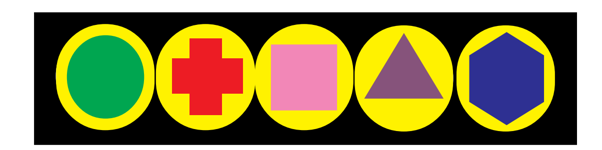
The conclusions, then, are that Anita is the secretary, Gerry is the director and Rose is the chairman.

635

The clerk forgot to mention that the parrot was deaf.

636

The first three rules eliminate 118 of the 120 possible permutations of the five disks. The last rule selects one of the remaining two possibilities.



637 The reflexive answer is that if boys and girls are equally likely, the probability that the other child is a girl is $1/2$.

But the reflex is wrong. There are four possible combinations for the Smiths' two children: boy-boy, boy-girl, girl-boy and girl-girl. One possibility (boy-boy) can be ruled out, but the other three are equally likely. Of the possibilities that remain, only one involves a second girl, so the likelihood that the Smiths have a second girl is only $1/3$.

This problem is an example of conditional probability—that is, the probability of one event given the fact that another event has occurred. The results are counterintuitive and generally misunderstood.

638 The question is, rather, *where* can you build it? Only on the North Pole.

639 Fish.

640 Green.

641 He married the sister first.

642 The note meant: "I ought to owe nothing for I ate nothing."

643 Fifty percent of the birds will be watched by one other bird, and another 25 percent will be watched by two other birds. That leaves 25 percent unwatched.

644 The first child says he is a truth teller. The statement is true if he's telling the truth and false if he is lying.

What the second child says is true no matter whether the first child is telling the truth. She is therefore a truth teller.

The truth of the third child's statement depends on the truthfulness of the first child. If the first child is lying, the third is telling the truth; if the first is telling the truth, the third is lying.

The possibilities are either (from left to right): liar–truth teller–truth teller or truth teller–truth teller–liar. Either way, two are telling the truth and one is lying.

645 The two heirs swapped horses.

647 The chances of drawing a red ball are $20/50$, or 40 percent. The chances of drawing a blue ball are $30/50$, or 60 percent.

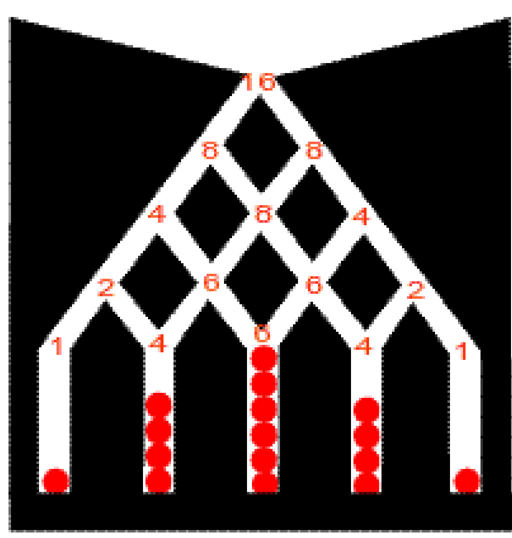
648 "Solve PlayThinks."

649 You should take the bet. The probability of at least one man getting his own hat back is almost .632.

650 86.

651 The fifth row of Pascal's triangle provides the answer. The average number of balls that reaches each juncture corresponds to a Pascal's triangle for which each successive row from the bottom is multiplied by an additional factor of 2, so that each row has the same sum.

In a full-sized probability machine, which possesses a large number of balls and branches, the distribution pattern approaches the famous Gaussian curve, also known as the bell curve.



652 You are better off fighting the brontosaurus. Although your chances of beating any stegosaur is $1/2$, beating three in series brings the probability to $1/2 \times 1/2 \times 1/2$, or $1/8$.

653 His reasoning was wrong. Sure, the chance of an unlikely event happening twice is fairly low, but the sailor's safety can't be calculated just by looking at the random nature of another shell landing in that hole. For one thing, the destination of a shell is not entirely random—the guns are being aimed, and gunners who have success with one shot may try sending another round in the same direction. For another, each time a random phenomenon occurs, the probability of the specified event happening again is exactly the same. So even if the guns were not being aimed, the spot where the shell hit is just as likely to be hit with the next round as any other spot.

654 The ladybug should start at the fifth aphid from the bumblebee—either clockwise or counterclockwise, depending on which direction she travels.



655 It took only seven trips, four to the ship and three back.

1. I took the Denebian to the spaceliner airlock and left it there.
2. I returned alone.
3. I took the Rigellian up to the spaceliner airlock.
4. I returned with the Denebian.
5. I took the Terrestrial up to the spaceliner airlock.
6. I returned alone.
7. I took the Denebian back up to the spaceliner airlock. Then all three could enter together.

656

Number 1 is Jerry, who likes chicken.
Number 2 is Ivan, who likes cakes.
Number 3 is Jill, who likes salad.
Number 4 is Anita, who likes fish.

657

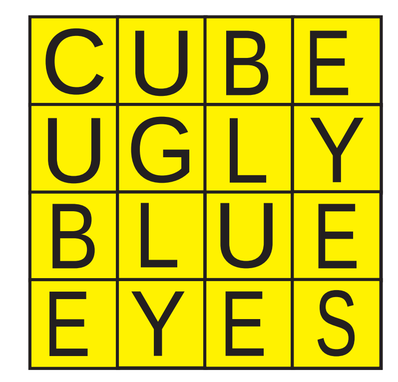
The probability that the underside matches the top is $^2/_3$. If you see heads, there are three, not two, equally possible scenarios:

1. You see the head half of a head-tail coin.
2. You see one side of a double-headed coin.
3. You see the other side of a double-headed coin.

In two of the three cases, the underside matches the top.

This result is so counterintuitive that many people refuse to believe it. If you are skeptical, try experimenting with "coins" cut out of cardboard. Keep track of your results and see if your probabilities match what I've outlined above.

658



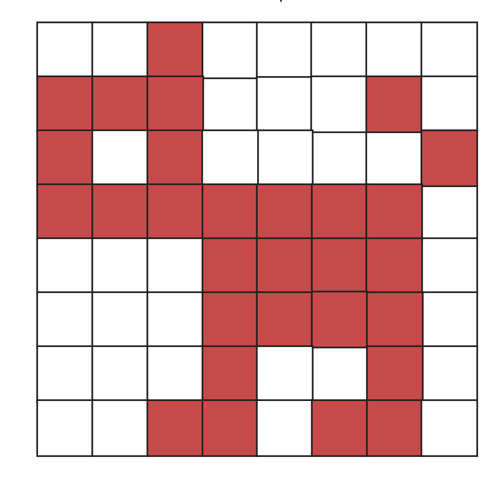
659



660

Puzzles like this depend as much on logic as they do on observation. Logic is required to sort out the visual evidence and make sure there is enough information to draw a conclusion. In this instance, even though all the information is not there, logic can help you deduce a symmetrical answer.

I want to emphasize logic here, because many particularly observant or logical people are perplexed if they are asked to solve a puzzle without all the pieces. They are often hesitant to use deduction or even intuition to come up with an answer.



661

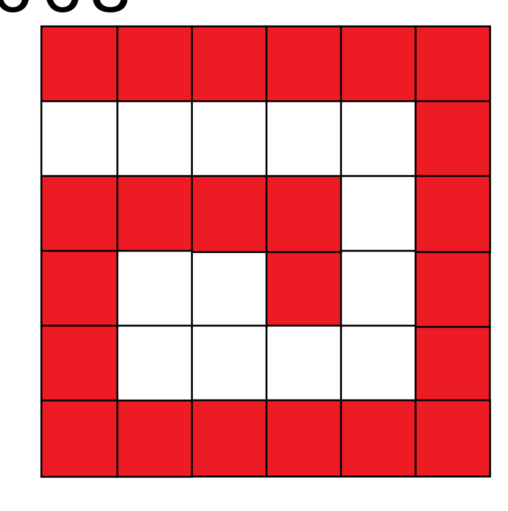
Own the casino!
The math of roulette and other casino games guarantees that, over time, the casino owners make far more money than they pay out. For every person who leaves the casino richer, there are many others who have lost considerable sums.

662

"Just between you and me" and "Split second timing."

663

A possible solution:



664

There are six possible outcomes for rolling three marbles, and in four of those instances Peter wins. So his chances of winning are $^2/_3$.

665

1. The word ***three*** is misspelled.
2. The word ***mistake*** should be plural.
3. There are only two mistakes in the sentence, which is the third mistake.

666

RANGE and ANGER.

667

If you count the letters, you will find there is 1 D, 2 I's, 3 S's, 4 C's, 5 O's, 6 V's, 7 E's and 8 R's. The secret word is DISCOVER.

668

This puzzle is somewhat related to the birthday paradox. The usual answer people give is that it should take about 100 links. But research conducted at Harvard University in Massachusetts demonstrates that any two strangers in the United States can be linked by a chain of intermediate acquaintances only five to six people long.

This problem, known as the "Small World" problem, is the basis of the popular trivia game in which one tries to link any actor to Kevin Bacon in just six steps. Both Hollywood and the world at large are examples of networks, a system with many interconnections. Chains of acquaintances have always been important, but with the revolution in travel and communication that has taken over the world in the last fifty years, people are connected through a very few steps to almost every other person on earth.

669

B \ A	2	4	5
1	L	L	L
3	W	L	L
6	W	W	W

The player with die A will, over the long run, win 55 percent of the time, as demonstrated in the chart above.

670 In the last composition the rectangle and the oval do not overlap.

671 "Take us to your leaders."

672 There were fourteen squares on the sheet, six on one side and eight on the other.

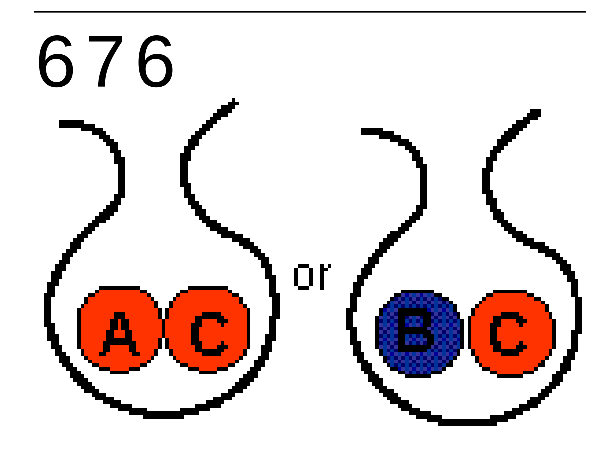
673 Only the second statement is true. Statement number 3 rules out both number 1 and number 3.

674 The passenger realized that if his face were clean, one of the other passengers would have realized that his own face was blackened by soot. Since neither of them stopped laughing, he realized his face must be sooty as well.

675

6	+	6	+	8	+	8	=	28
+		+		+		+		
6	+	6	+	6	+	6	=	24
+		+		+		+		
12	+	12	+	10	+	8	=	42
+		+		+		+		
8	+	10	+	12	+	6	=	36
32		34		36		28		

676



At first glance it seems the chances of a red ball remaining in the bag are 50 percent. But there are actually three—not two—equally possible states:

1. The initial red ball (A) was drawn, leaving the added red ball (C).
2. The added red ball (C) was taken, leaving the initial red ball (A).
3. The added red ball (C) was taken, leaving the blue ball (B).

As you can see, in two out of the three cases, a red ball remains in the bag.

In the initial draw, the chances of pulling out a red ball are 75 percent. But once the first ball is drawn, the odds change.

677 The chances are not $^{2}/_{3}$ but $^{1}/_{3}$. The reasoning is simple. Choose any card. Of the three remaining cards, only one is the same color, so the chances of picking it are just one in three.

Your friend's reasoning is incorrect because the three possibilities he or she considered are not equally likely.

678 Although Amos and Butch are sure shots, Cody's chances of survival are twice as good as the other cowboys'.

The reason is straightforward. If Amos or Butch gets the first shot, the one who gets it will eliminate the other (since they represent the greatest threat) and take his chances with Cody. Cody then has a 50 percent chance of shooting the survivor and a 50 percent chance of missing and getting shot. If Cody draws the first shot, he'd be well advised to miss, because if he actually shouts either Amos or Butch, the other could gun him down.

So Cody's chances of surviving are 50 percent.

Amos and Butch both have the same chances: If they lose the draw, they are shot in the first round; if they win the draw, one shoots the other and take his chances with Cody. Since both outcomes are equally likely, the chances for either Amos or Butch turn out to be 0 percent plus 50 percent, divided by two—or 25 percent.

679 The formula for solving such problems has eluded mathematicians for centuries. Practical solutions are best found through simple trial and error. In the circle of thirty-six prisoners, the proper positions to plant your enemies are numbers 4, 10, 15, 20, 26 and 30.

680 In the case of our coin-flipping experiment, a surprising probability is found in Benford's law. The odds are overwhelming that at some point in a series of 200 tosses, either heads or tails will come up six or more times in a row. Most fakers don't know this and will not put such nonrandom occurrences in their fake results.

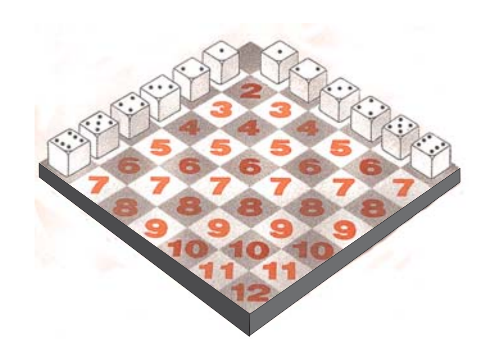
681 In calculating probability, mathematicians generally limit themselves to four possible outcomes: heads-heads, tails-tails, heads-tails and tails-head. But there is can be a fifth possible result—uncountable. For example, one coin could land on edge. Or it could be lost down a grate. Or be carried off by a bird in midflight. Perhaps mathematicians should account for such occurrences when they calculate probabilities in the future.

682

The answer is ONE WORD.

683

There are six possible even numbers that can come up—2, 4, 6, 8, 10 and 12—and only five possible odd numbers—3, 5, 7, 9 and 11. In spite of that, as the diagram shows, there are eighteen ways to throw an even number and eighteen ways to throw an odd number. So the odds of an even number are even.



684

In any given roll of a die, the odds that a 6 will *not* come up are $5/6$. Since each roll of a die is independent of the others, the chances of not rolling a 6 in a given series can be calculated as:

Two rolls: $5/6 \times 5/6 = .69$

Three rolls: $5/6 \times 5/6 \times 5/6 = .57$

Four rolls: $5/6 \times 5/6 \times 5/6 \times 5/6 = .48$

which means that more often than not, you will roll at least one 6 after four rolls.

685

Remarkably, the probability of two people sharing a birthday is about .5 in a group of just twenty-three people.

To calculate this, you have to look at the probability that everyone has a *different* birthday. For a group of two people, the probability is extremely high—about $364/365$—that they will have different birthdays. With a group of three, the probability is not as high—$363/365$—and since the group of three still contains the group of two, the two probabilities are multiplied. Continue along this track until the probability of everybody in the group having different birthdays drops below .5, which means the probability of two people sharing a birthday is now more than .5.

The probability approaches near certainty with ninety people or more.

687

In the seventeenth century Antoine Gombaud Chevalier de Méré, a French nobleman with an interest in gambling, suspected that the odds were not in his favor, so he checked his suspicions with the famous mathematicians Blaise Pascal and Pierre de Fermat, who found that the probability of rolling double 6 after twenty-four throws was $35/36$ to the twenty-fourth power, or about .49, which meant losing in the long run.

Gombaud's small request marked the birth of the science of probability.

688

Martin Gardner has presented several versions of this paradox, but the *Parade* magazine columnist Marilyn vos Savant is most famously associated with it. Her 1990 column on the subject provided the right answer but provoked thousands of letters of disbelief and accusation.

no swap				swap			
door 1	door 2	door 3	result	door 1	door 2	door 3	result
car	monkey	monkey	win	car	monkey	monkey	lose
monkey	car	monkey	lose	monkey	car	monkey	win
monkey	monkey	car	lose	monkey	monkey	car	win

chosen door

Why? Because the answer seems so wrong!

If you stick with your initial choice, your chances of winning are one in three. That is easy to understand: one car, three doors.

That means the chance of the car being behind one of the doors you did not initially choose is two in three. Of course, if you were to switch to one of the two other doors without any additional information, that $2/3$ would be divided between two doors, for a $2/3$ chance with each door—no better than if you stuck with your initial choice.

But when the host reveals the monkey behind one of the two doors, that additional information suddenly changes the odds. The host, of course, will not open a door to reveal a car. And the host's choice of doors depends greatly on your initial choice: if your door hides the car, the host can pick either remaining door; but if your door hides a monkey, the host must pick one and only one door to open. As you can see from the chart above, the possibilities for the doors you did not pick—monkey-monkey, car-monkey and monkey-car—have now been constrained to just monkey–open door, car–open door and open door—car. Therefore, if you switch, you have a $2/3$ chance of selecting a door with a car behind it.

Again, if you are not convinced, try playing a number of games with and without swapping to check the validity of this proof. Remember, this is a case of conditional probabilities—the probability of something happening given that something else already has.

689

When the letters are arranged so that the arrows follow a clockwise order, they spell out TONY BLAIR.

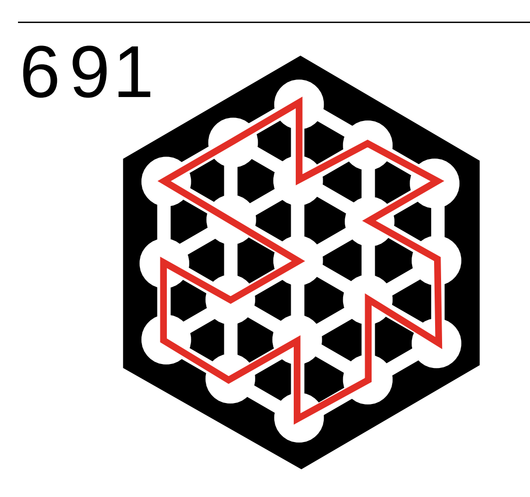
690

It isn't a fair bet, even with 3-to-2 odds.

I can ensure that my chances of winning are at least $2/3$ and are sometimes as high as $7/8$. All I have to do is let you pick your triplet. Then I pick my triplet so that it starts with the opposite of your second coin, then copies your first two. If you select HTH, I'll pick HHT and have a $2/3$ advantage. If you choose TTT, I'll pick HTT and will have a $7/8$ advantage—you can win only if the first three tosses are all tails.

CHAPTER 11 SOLUTIONS

691



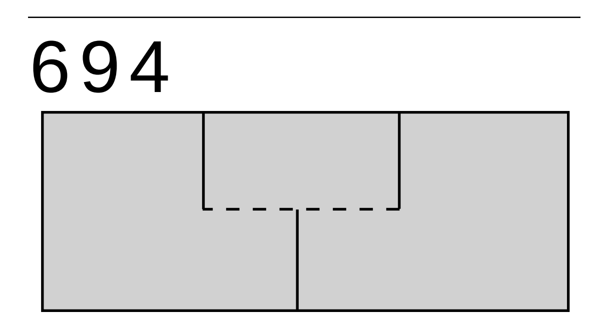
692

a-5, b-1, c-9

693

There are two different lengths—ten long and ten short. Each color comes in two lengths. The sequence that removes them all is short yellow, short orange, short red, short pink, short purple, short light green, short dark green, long light blue, short dark blue, long yellow, long orange, long red, long pink, long purple, long light green, long dark green, short light blue, long dark blue, short violet and long violet.

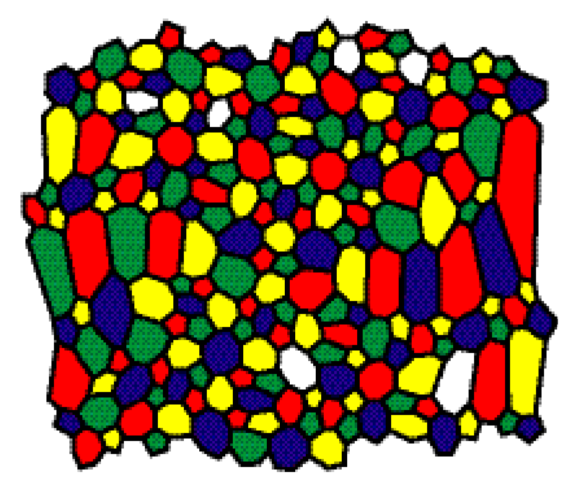
694



695

A sample game in which the map could not be fully colored is shown below. Six regions had to be left blank.

If you started with a blank map, could you do better?

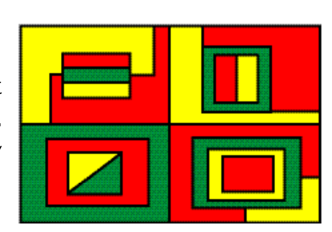


696

Only 2 and 9 are topologically identical.

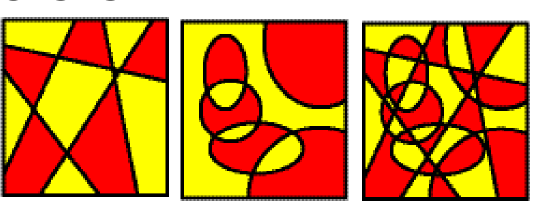
698

You will need at least three colors. One of the many possibilities is shown here.



699

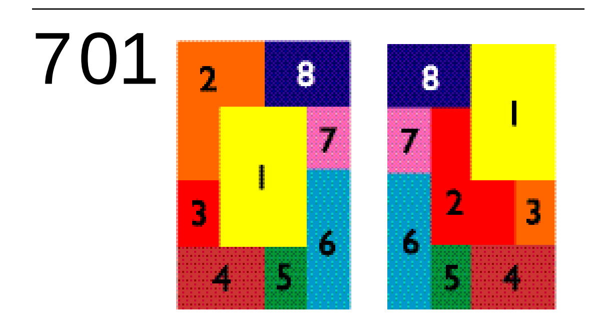
The solution illustrates the two-color theorem.



700

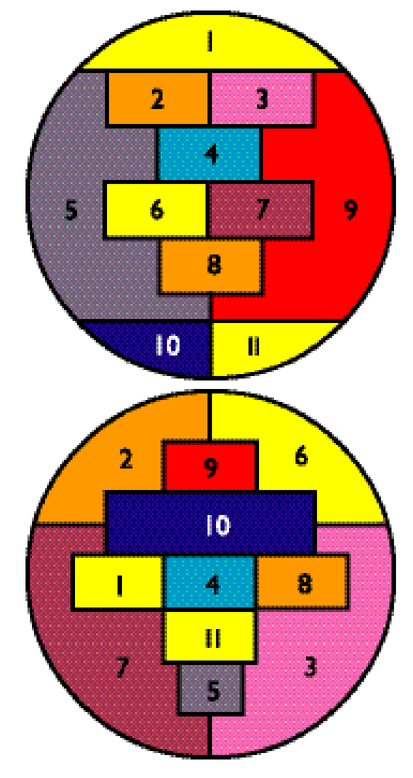
Clockwise: yellow triangle, orange pentagon, red heptagon, pink nonagon, violet square, light green hexagon, blue octagon and purple decagon.

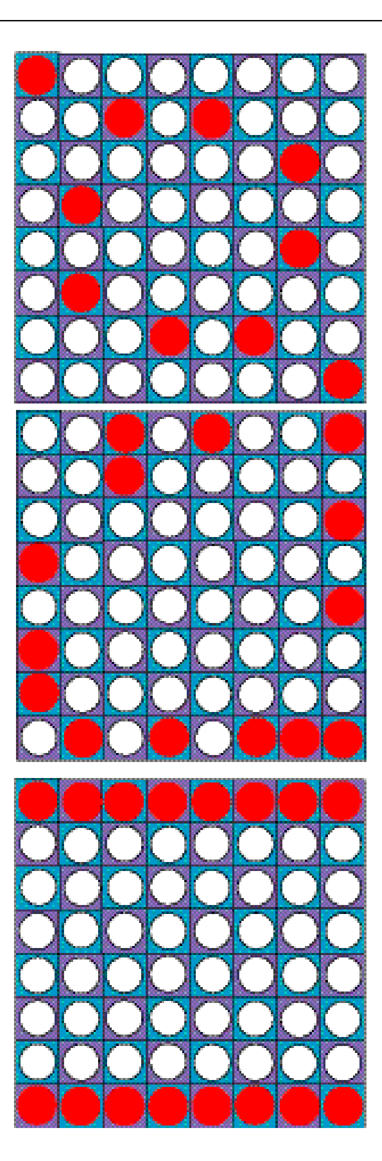
701



703

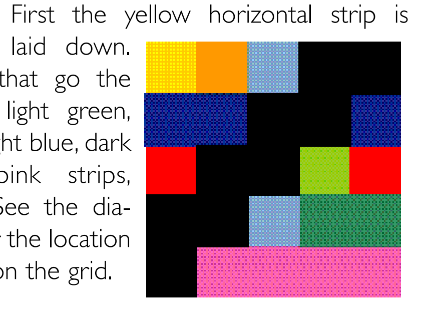
The minimum number of colors is eight, as shown.



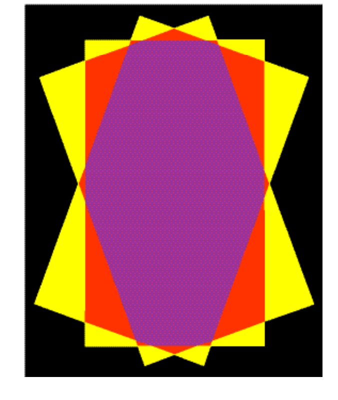


705

First the yellow horizontal strip is laid down. On top of that go the orange, red, light green, dark green, light blue, dark blue and pink strips, respectively. See the diagram here for the location of the strips on the grid.



706



707

It takes at least six moves for the eye to swap ends.

708

The strip will stay in one piece. It will be twice as long and have two complete twists.

709

The strip will break into two linked bands, one a Möbius strip of the same length and the other a band that is twice as long and has two complete twists.

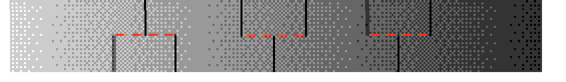
710

1. Four colors
2. Three colors
3. Two colors
4. Two colors
5. Four colors
6. Two colors
7. Two colors
8. Three colors

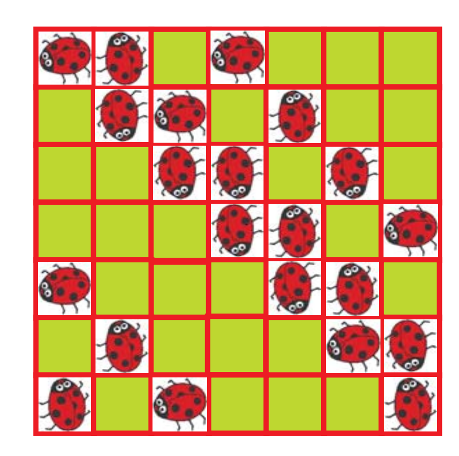
711

To make the model, make three sets of hypercard cuts, as shown, on a strip of paper and then twist the strip to create the three flaps. Glue the ends of the strip together to make the ring.

Once the glue dries, you can change the number of outside benches from one to two by turning the entire ring inside out.



712

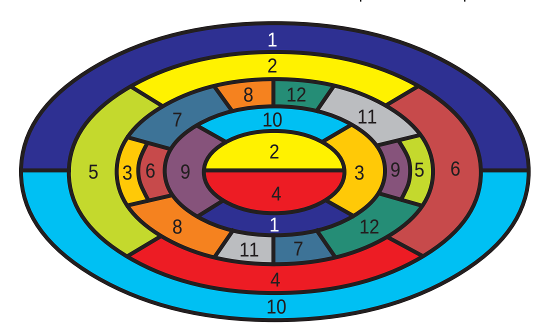


713

The quartets are 1-9-11-14; 2-3-7-13; 4-5-6-8. That leaves the pair: 10-12.

714

Each 2-pire touches each of the other eleven; therefore, twelve is the minimum number of colors needed to complete this puzzle.



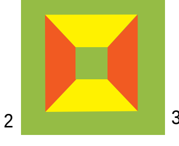
715

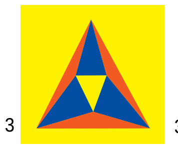
The capital *E* is topologically equivalent to *F, G, J, T* and *Y*.



716





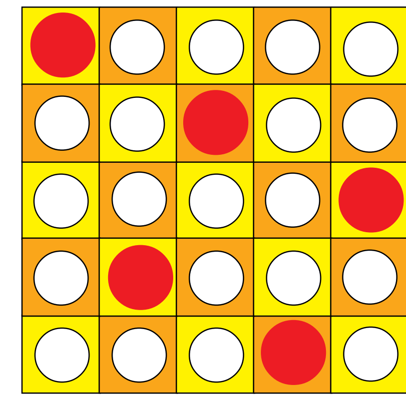






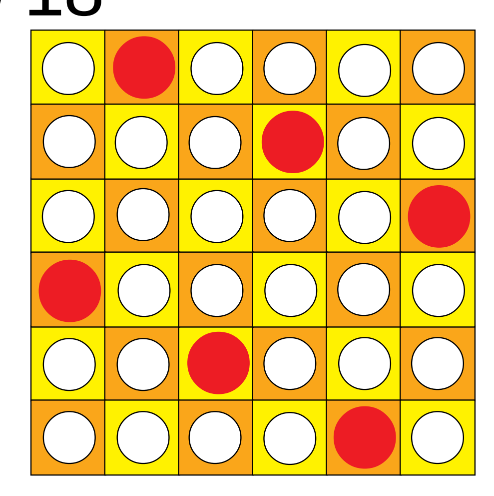
717

There are two different solutions, one of which is shown below.



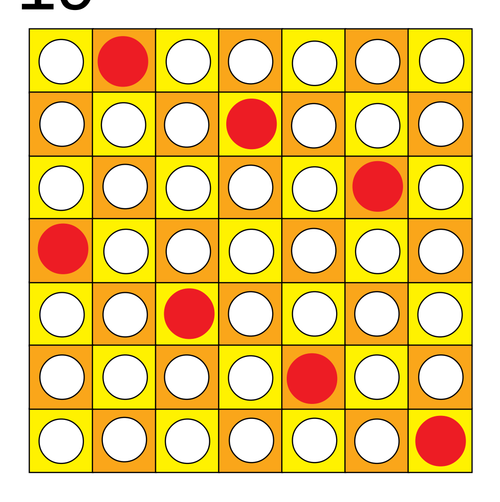
718

There is just one solution.



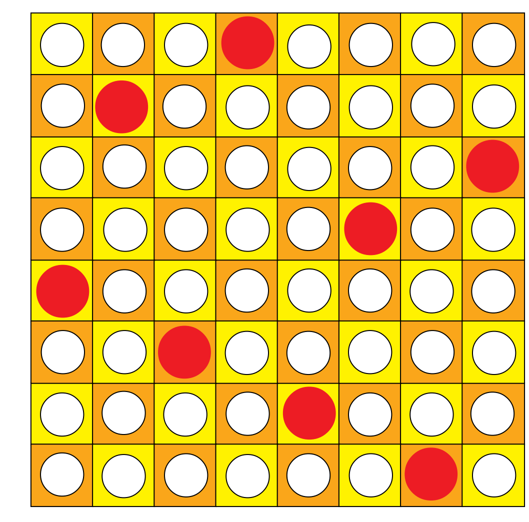
719

The unique solution is shown below.



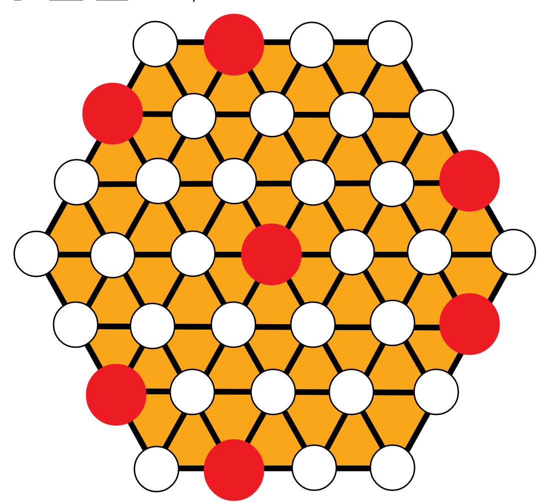
720

The diagram below shows one of the twelve different solutions, not counting rotations and reflections.

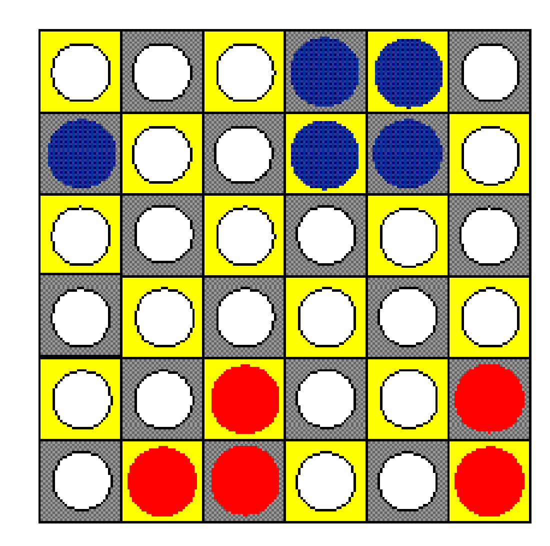


721

The illustration below shows one of four possible solutions.



722



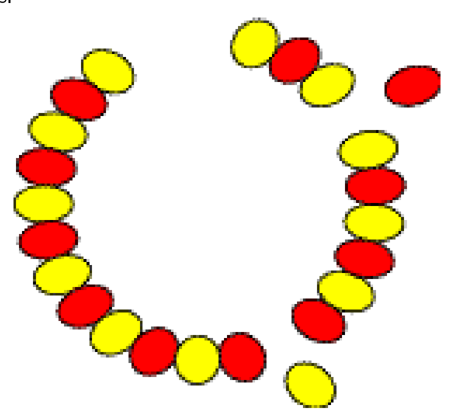
723

724

Every shape but the pentagon can be made by slicing a cube.

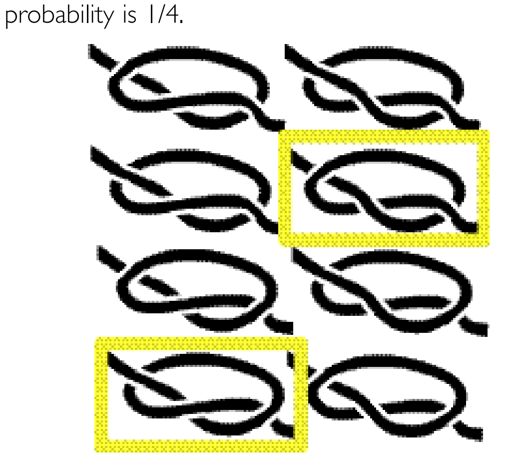
725

Two disconnections, as shown, will form five lengths of one, one, three, six and twelve beads. Combining these lengths in various ways can form necklaces of one to twenty-three beads.



726

With three points of intersection for the rope to overlap, there are eight different configurations for the loop. Only two of those will form a knot, as shown below. Therefore the probability is 1/4.



727

Only two of the loops will be knotted if the hose is pulled tight: the one at the bottom right and the one on the middle left.

728

729

The twenty-four connected cubes represent an ordinary overhand knot.

730

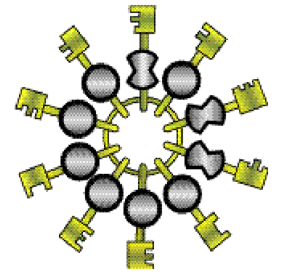
Just one cut is needed. If she cuts the fourth ring from the left, the chain will fall into four pieces—of lengths one, one, three and six links, respectively. Those pieces, alone or in combination, can cover the amounts owed for each day. For example, on day three, she can trade the two loose links for the three-link piece.

732

The keys spell out P-L-A-Y-T-H-I-N-K-S.

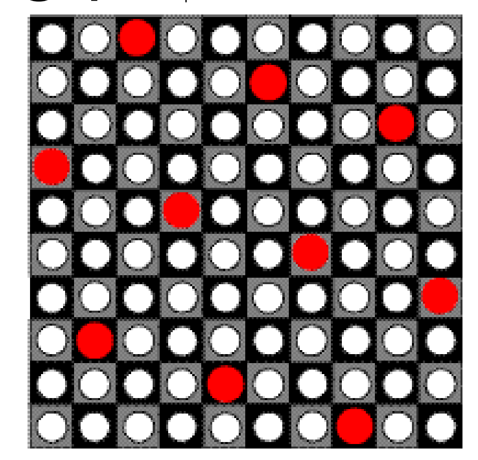
733

If you reshape all the marked keys the same way, you will need to mark three key handles. Two of the marked keys should be grouped together, and the third should be separated by one key with an unmarked handle. In that way, you can identify both the starting point—the single marked key—and the direction—toward the two marked keys—in which the memorized sequence progresses.



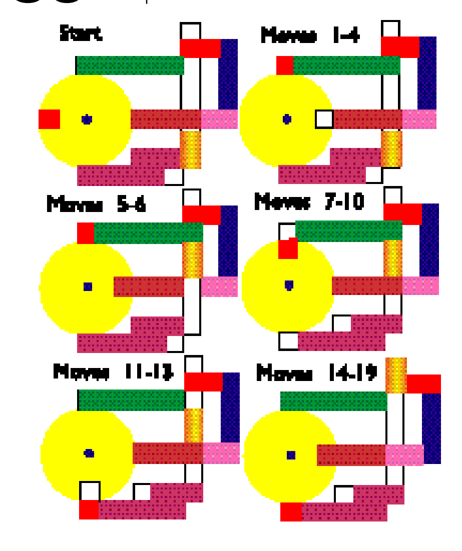
734

The solution for the ten-by-ten board is unique.



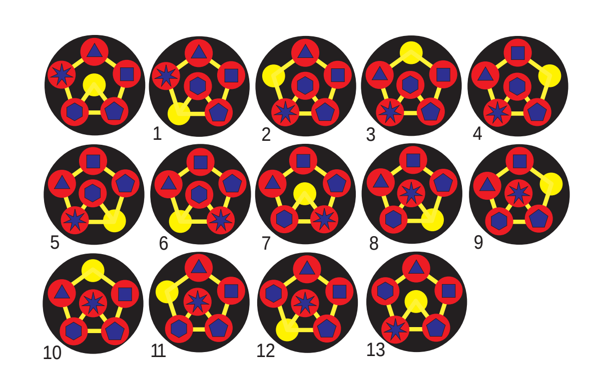
735

It will take nineteen moves to remove the piston.



736

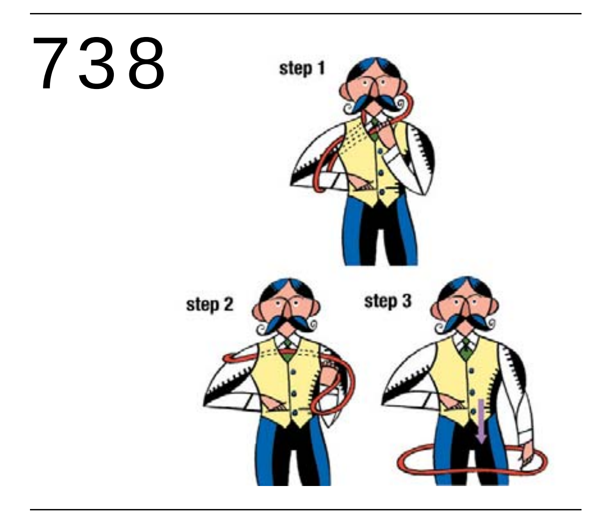
It takes thirteen moves.



737

It takes twenty moves to get all the animals to their proper cages. In general, the moves should create a cyclic order to solve the puzzle in the fewest possible moves.

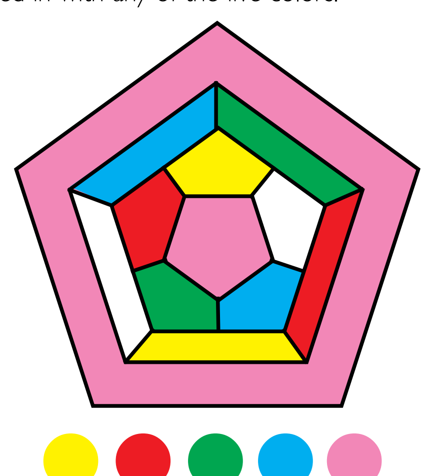
738



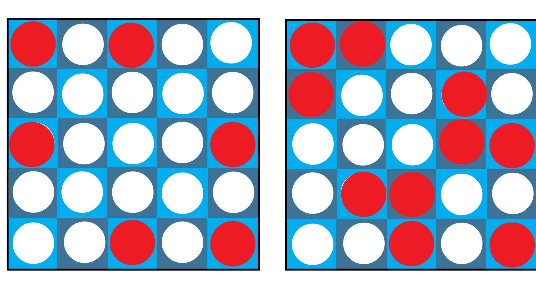
739

A winning strategy for the maximizer is to always play on the face of the distorted dodecahedron opposite the position last played by the minimizer—and to copy the color the minimizer used.

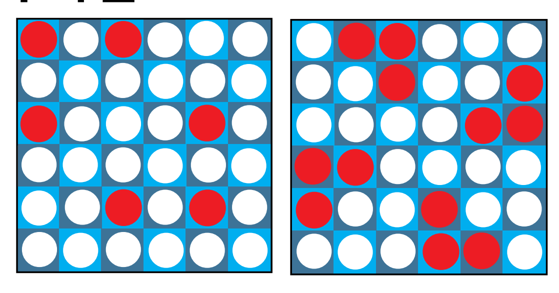
The game illustrated below began with the maximizer filling in the center pentagon and then following the outlined strategy. As you can see, this player won the game, as the last two regions cannot be filled in with any of the five colors.



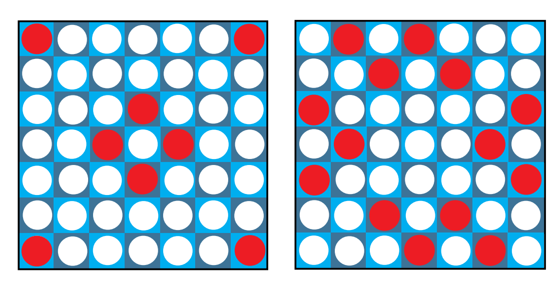
740



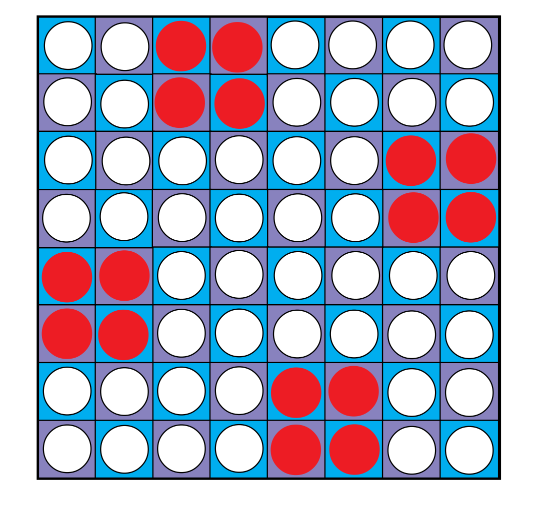
741



742

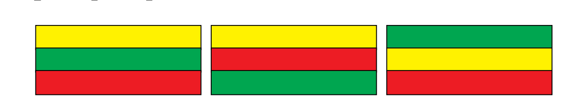


743



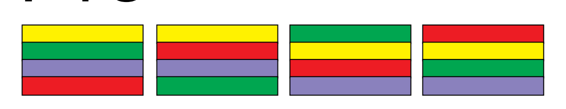
744

The three possible folds are shown.



745

The four possible folds are shown.



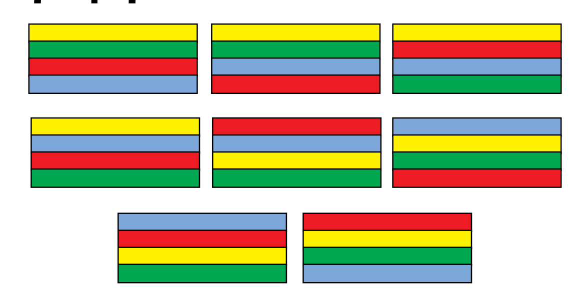
746

It turns out that it is nearly impossible to fold a sheet of newsprint in half more than eight or nine times, no matter how large or thin the sheet.

Every time you fold the sheet, you double the number of pages in the stack. One fold makes two pages, two folds make four pages; Nine folds will produce a stack of newsprint 512 pages thick—the size of a small phone book. A stack that thick prevents any additional folding.

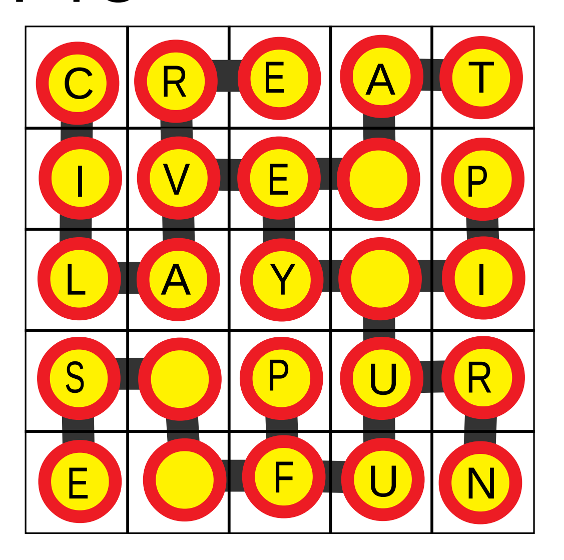
747

The eight possible folds are shown above.



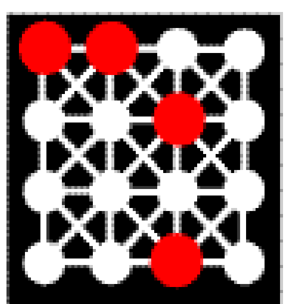
748

The message is CREATIVE PLAY IS PURE FUN.



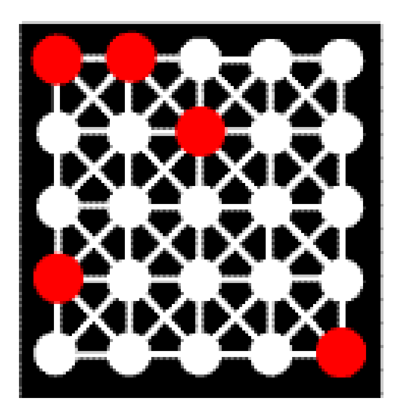
749

One of sixteen possible solutions is shown.



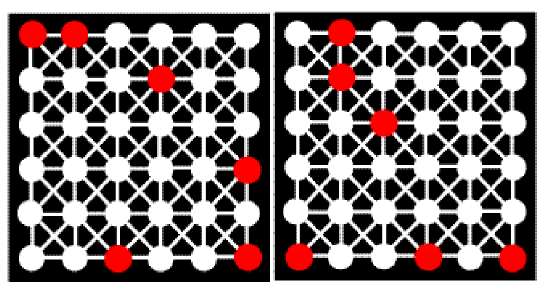
750

One of twenty-eight possible solutions is shown.



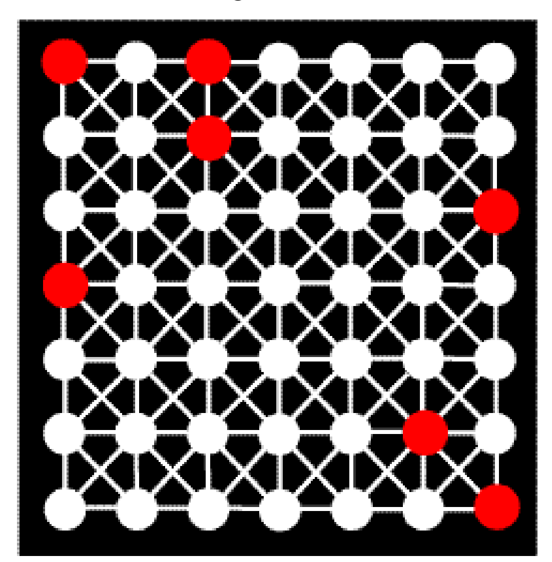
751

There are only two possible answers, both of which are shown below.



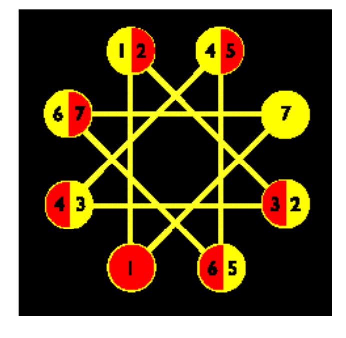
752

There is only one possible solution, shown below. No solution has been found for a matrix larger than this.



753

The key is to place each coin on a circle that is connected to the starting position of the previous coin. There will always be one pathway free, according to this strategy.

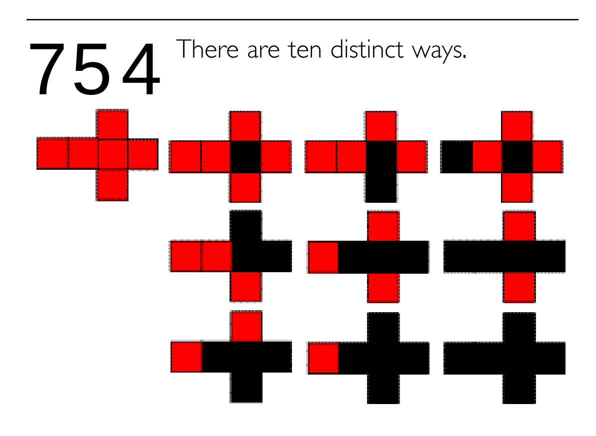


A more trial-and-error approach involves filling the star with seven coins and playing the puzzle in reverse, noting the moves. You could also imagine untangling the star to form a circle, which would enable you to visualize the solution easily.

This puzzle offers an introduction to "clock arithmetic" and finite number systems. Its star track can be described as a modulus 8 with a linking operation of +3 (or –5). That is, there are eight points spaced around a circle, and every third point is joined to form a single continuous track.

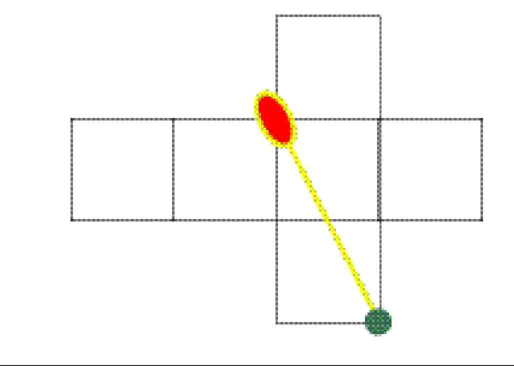
754

There are ten distinct ways.



755

The shortest route will not follow the edge of the cube. To envision the shortest route, imagine flattening out the faces of the cube, as shown below. If you draw a straight line from the ladybug to the aphid, you will see that the shortest path does not run down the edge.

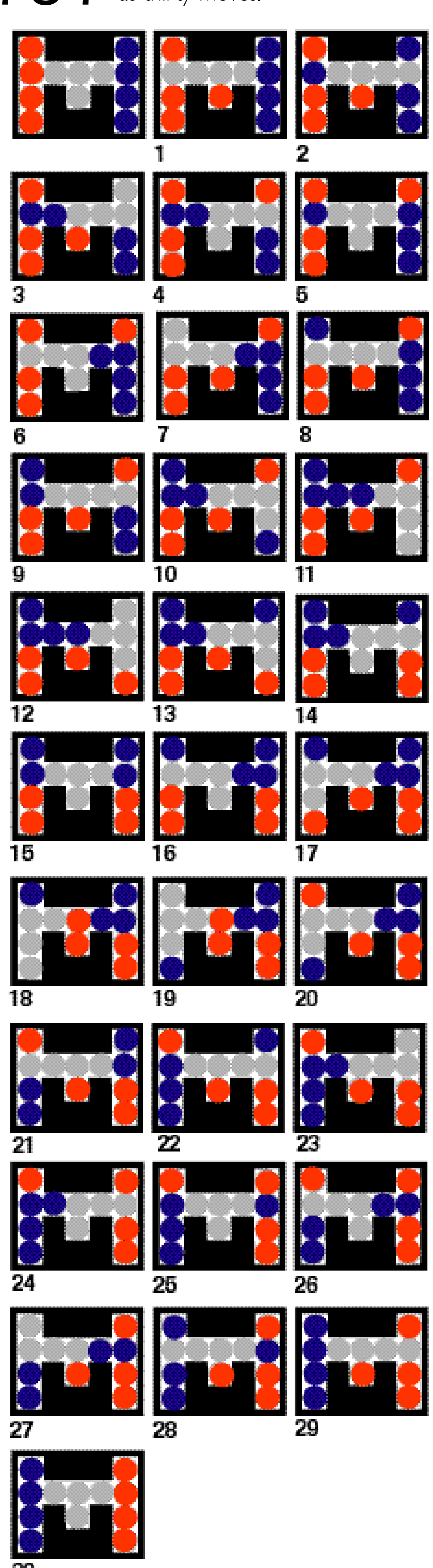


756

One of the small chains was separated into its three separate links, and then that was used to link together the other four chains.

757

The cars can be transferred in as few as thirty moves.

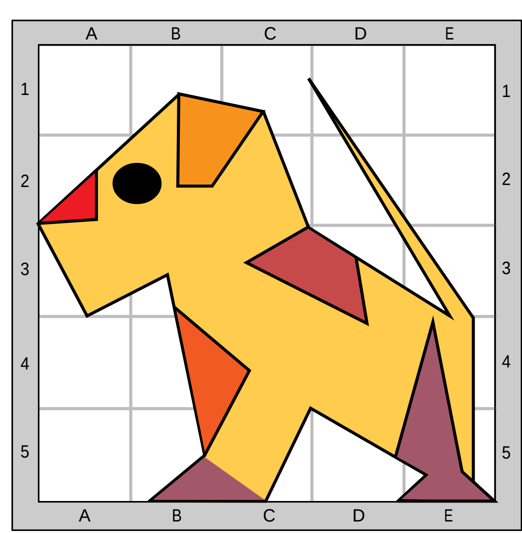


758

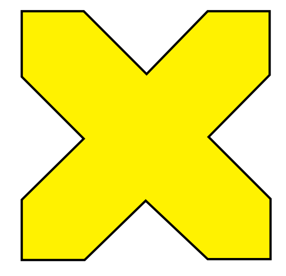
Did you notice that the numbers below each set of disks add up to 15 and 24, respectively? The sequence shows the number moves that must be made in succession by each color group (for instance, one red, two blues, three reds, three blues, three reds, two blues, one red). If you follow the sequences, you will come to the solution—which otherwise may remain elusive—in the fewest possible moves.

For example, the first puzzle can be solved by first moving the red disk into the center space, followed by two moves of the blue disks, then three moves of the red disks, and so on. Because of the restrictions on the movements of the disks, the moves will be obvious.

759



760

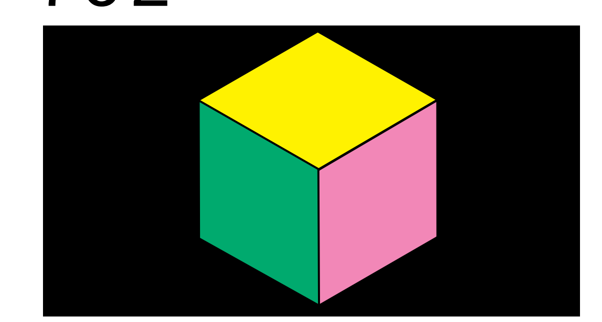


761

Yellow and orange, red and green, pink and blue.

762

Only the bottom left cube.



763

1. There are twenty-four ways to place the first cube. In any of those twenty-four ways, the second cube can be in one of four locations. And at any given location, the second cube can turn in one of twenty-four different ways. So 24 x 4 x 24 = 2,304 different ways.

2. As long as the cubes stay in the same order, the variations possible with three cubes are simply 24 x 24 x 24, or 13,824, different ways.

3. As long as the eight cubes maintain their relative position—and counting each individual turn of one face of a cube as a variation on the entire pattern—the number of variations is 24^8, or 110,075,314,176.

It is little wonder, then, that there are so many cube games on the market, or that Rubik's Cube, involving twenty-six joined cubes, has proved so difficult.

764

Harpo Marx.

765

To reveal the true form of the distorted images, simply hold the outer edge of the page about 15 centimeters from your nose and look at the page from a very slanted angle. Close one eye and everything will be clear.

766

Groucho Marx.

767

Regular tetrahedrons will not fill the space. When four pyramids are grouped together to define a larger tetrahedron, the central space is a regular octahedron.

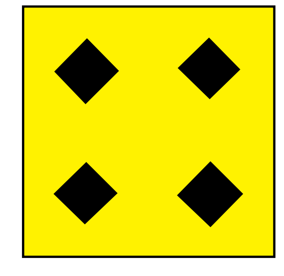
Therefore, the pyramid is made up of eleven tetrahedrons and four octahedrons.

768

As you learned from the previous puzzles, a simple parity check can determine whether one configuration can be obtained from another. Simply switch pairs of blocks until the desired pattern is achieved. If the number of swaps is even, the puzzle can be solved; if the number of swaps is odd, it is impossible.

For this puzzle, the solution is possible in thirty moves.

769

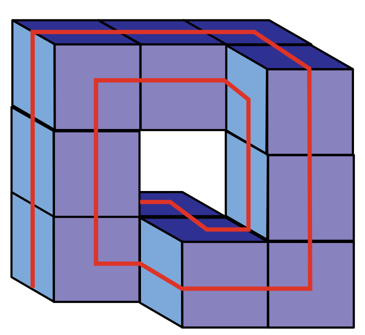


770

The minimal prismatic one-sided ring is made up of ten unit cubes.

771

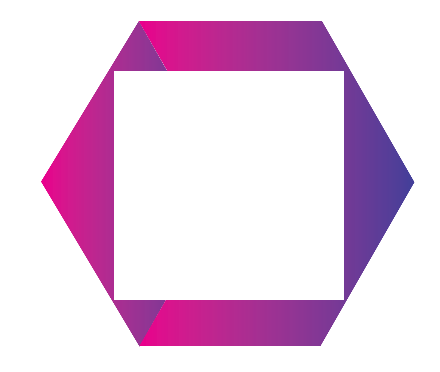
Numbers 2, 3, 4, 5 and 10 are identical. And numbers 7, 8 and 9 are identical. But number 1 and number 6 are unique.



772

If you hold a cube so that one corner points directly toward you, its edges outline a hexagon. It then becomes obvious that the cube has ample space for a square hole slightly larger than one of its faces.

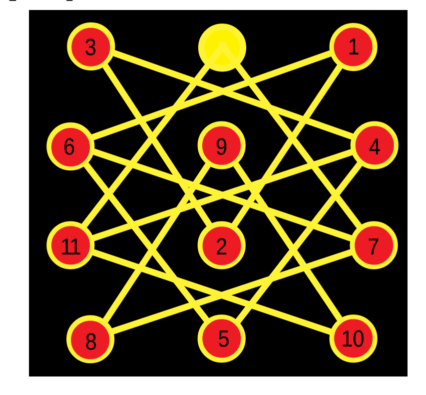
If a cube has sides of 1 unit, a square hole can be drilled through it with sides of almost 1.06 units.



773

1. Fifty-eight cubes
2. Eighteen cubes
3. Twenty cubes
4. Fifty-six cubes
5. Thirty-three cubes
6. Eighteen cubes
7. Thirty cubes
8. Forty cubes

774



775 To solve this difficult problem in a systematic fashion, you can create a table that shows the number of different cubes possible for each combination of colors.

Number of red corners: 8 7 6 5 4 3 2 1 0

Number of yellow corners: 0 1 2 3 4 5 6 7 8

Number of different cubes: 1 1 3 3 7 3 3 1 1

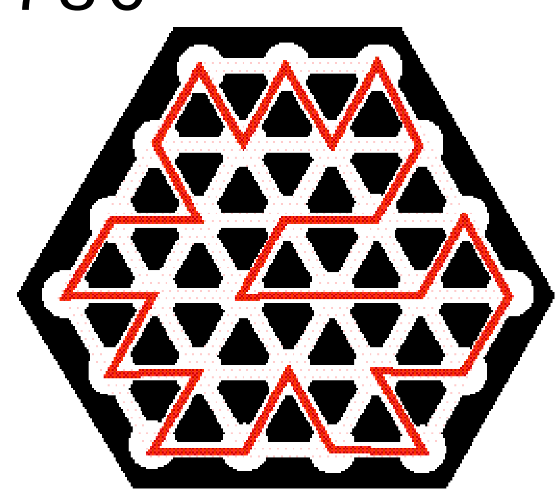
Therefore, twenty-three different cubes are possible.

776 The green loop.

777 Only the yellow, green and orange nets can be folded into a perfect cube.

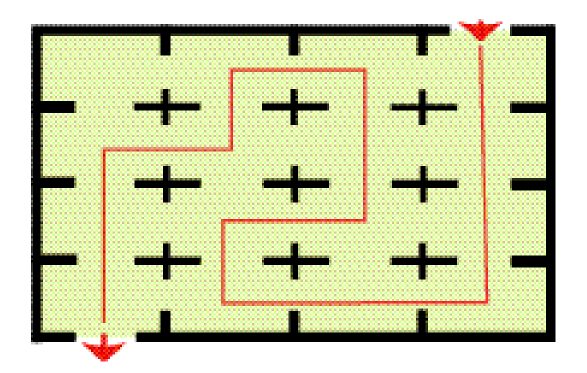
778 Orange and green, yellow and pink, blue and red.

780



781 The sliced-off tetrahedron has one-sixth the volume of the whole box.

782 No path exists. The next best answer is a path that leaves one room unvisited.



783 The famous sliding-block puzzle and the story behind it.

If you put in some effort to solving the 14-15 puzzle, you may be disappointed that you didn't find an answer. Don't be. This famous puzzle, designed by Sam Loyd, is impossible to solve. Loyd knew this when he introduced the puzzle some 120 years ago—but he offered a $1,000 reward to anyone who could devise a solution and thereby touched off an international craze. Indeed, the only other instance of such a worldwide involvement in recreational mathematics was the Rubik's cube fad in the 1980s.

Loyd's 14-15 configuration is just one of 600 billion possible arrangements of the numbered tiles, and like Loyd's, half are impossible to bring into sequential order. A simple parity check can determine whether a given configuration has a solution. Simply swap the misplaced tiles, then count the number of swaps—if the number is even, it's possible; if it's odd (as in this case), it's not.

In the language of computer science, the 15 puzzle, as it is generally known, is a model of a sequential machine. Each movement of a block is an input; each configuration of the blocks, a state.

784 By playing correctly, the second person will always win. If the first player takes one bee, the second player takes two bees on the exact opposite side of the daisy. If the first player takes two bees, the second player takes one bee, again on the opposite side of the daisy. Either way, this leaves two equal sets of bees placed symmetrically around the daisy. All the second player has to do now is keep the two patterns symmetrical for the rest of the game, and he or she will never lose.

CHAPTER 12 SOLUTIONS

785 In theory, the gravity train would work as planned. And interestingly enough, every trip would take the same amount of time—about forty-two minutes. In fact, if the earth were hollow, an object dropped through the earth would arrive on the other side in just forty-two minutes as well.

Of course, the earth isn't hollow. Friction and air resistance cannot be ignored.

786 Less than at the earth's surface. Even though you are closer to the center of the earth's mass, there is enough mass above you to cancel out the effects of some of the mass below you.

787 Your weight is a measure of the gravitational pull of the earth's mass upon your body. The closer you are to the earth's center of mass, the more strongly you will feel its pull.

Because of the earth's bulge, then, you weigh about .5 percent less at the equator than you do at the poles.

788 Yes. Weight is a relative magnitude, and your weight may change from planet to planet, but a spring scale will always be able to measure that weight—even though your weight will often be 0.

789 No. At the surface the moon's gravitational pull is only one-sixth that of the earth's, so astronauts on the moon will weigh only $\frac{1}{6}$ of what they did on the earth.

790 This thought experiment was devised by the great Albert Einstein and demonstrates his equivalence principle: The effect of being at rest in a gravitational field is the same as the effect of being at rest in an accelerated system.

If you are in an accelerating rocket as described, you will feel yourself pulled toward the floor with the same force—and watch objects fall at the same speed—as if you were in a room on the earth, though it is the floor that is actually rising up to meet the objects.

In the absence of other information, then, it is impossible to tell whether you are on the earth or in an accelerating rocket.

791 Common sense tells us that heavy objects should accelerate faster than lighter ones, but experimental science has proven this is not the case.

Newton's second law of motion shows that acceleration is directly proportional to force (weight, in this case) and inversely proportional to mass. The equation can be written as:

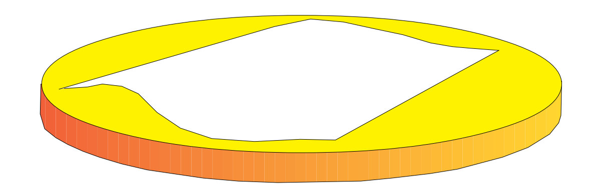
$$a = f/m$$

where a is acceleration, f is force and m is mass.

The resistance to motion due to mass is called inertia. Therefore, even though a large stone may weigh 100 times more than a small rock, it has 100 times more mass (and inertia), and so the two factors cancel out.

In general, and ignoring air resistance, the acceleration of every falling body near sea level is 32 feet per second per second.

792 To eliminate the difference in air resistance, place the slip of paper on top of the coin. Then drop the coin, giving it a slight spin to keep it horizontal as it falls. The coin and the paper should fall together.



793 Weightlessness can be achieved for up to a minute in an airplane flying a controlled parabolic course. The pilot steers the plane so it follows the path of a free fall. Because every object in the plane—including the plane—is falling at the same rate, the effect is simulated weightlessness.

794 The book underneath will probably stay in place, but the book on top will move along with the book you are pulling.

The reason is friction. The frictional force is proportional to the normal (or perpendicular) force, and the normal force is equal to the amount that an object presses down on a surface. The normal force on the book underneath the one you are pulling is equal to the weight not only of that book but of the two books on top of it as well. The friction between that book and the book on the bottom is then greater than the friction between that book and the one sliding across its top (that is, the one you are pulling), so the book will tend to stay put.

795 The biggest and heaviest apples will rise to the top.

The arrangement that is most stable is one in which the most densely packed apples are at the bottom. The smaller an object is, the more likely it is to find a space to drop into at a lower position. Therefore, in a group of mixed apples, the smaller apples can be more densely packed than the large ones and will eventually sink to the bottom.

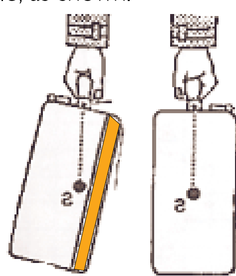
796 When I pull on the string from the bottom slowly and steadily, the top part of the string must bear both the weight of the book and the strength of the pull. The tension on it is greater than the tension on the lower half, so the top thread will break first.

If I pull with a sharp jerk, inertia comes into play. The book is little affected by the jerk at first, so the force of the jerk is not transmitted to the top string. The tension is therefore greater on the bottom thread and it breaks first.

797 Whether large or small, packed spheres will occupy about .5235 cubic meters for every cubic meter of space they are packed in. This is independent of the size of the ball, as long as the radius is small in relation to the size of the box.

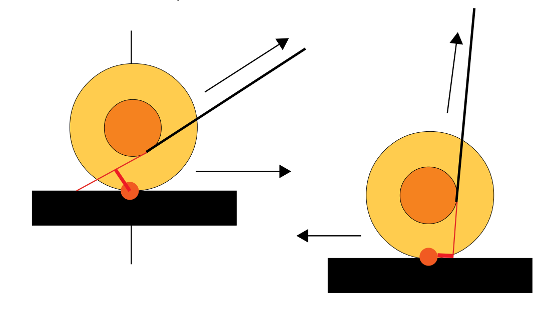
Even though each void is smaller for tightly packed small spheres, there are more voids altogether. Each box will weigh the same.

798 A false bottom filled with heavy items will noticeably affect the center of mass of a suitcase. This will cause the suitcase to hang at an odd angle, as shown.

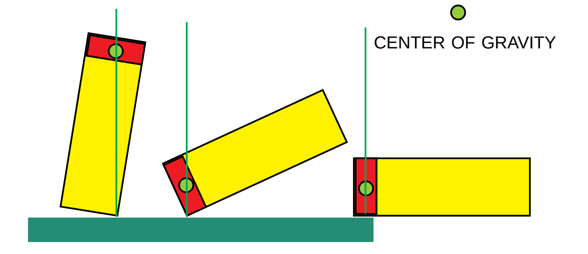


799 One weighing will do. He simply needs to place one red, two blue, three green, four yellow and five orange balls on the scale. If red is the color of the odd balls, the scale will read 1510 grams; if it's blue, 1520 grams and so on.

800 If you pull up at a sharp angle, a torque is created that turns the spool away from you. If you pull instead at a more shallow angle, the opposite torque is created, and the spool will roll toward you.

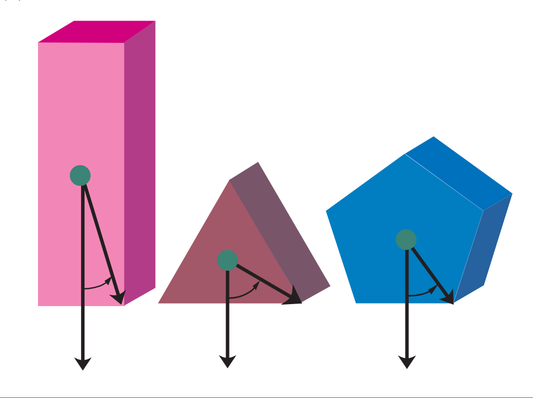


801 A very heavy weight is fixed to the red end of the box. As you can see in the diagram, such a weight greatly affects the behavior of the box.



802 The fish actually weighs 50 kilograms. Each end of the rope pulls down on the spring with the same force, so two "marlins" are being weighed in the setup shown in the illustration: the real marlin tied to one end of the rope and the restraint on the dock attached to the other end. The restraint on the dock may be an imaginary fish, but the force it exerts on the balance is real. To get an accurate weight, the fisherman should have tied the fish directly to the scale.

803 The triangular shape is the one where the angle (as measured from the center of gravity) is the greatest between the force of gravity and the point around which the shape will topple. That also means it is the most stable.

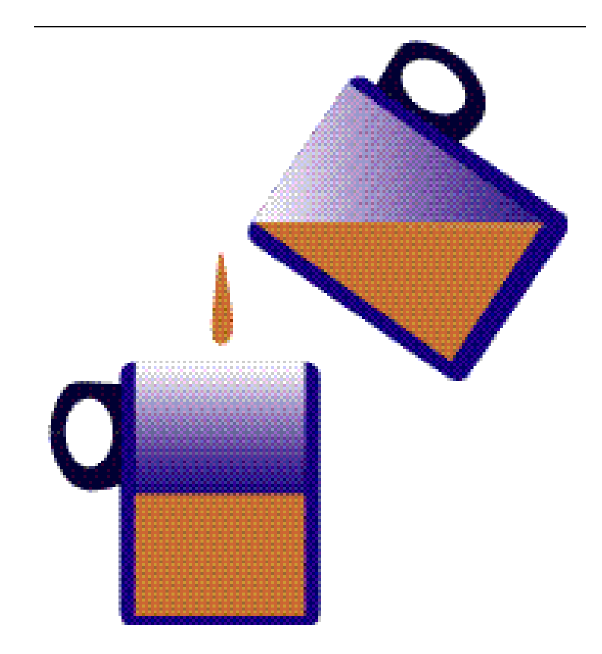


804 Friction will always keep the stick from falling. The finger that is farther from the stick's center of gravity carries a lighter load and therefore experiences less friction, so that finger moves first. As it is brought closer to the center, more and more weight is borne by that finger until the kinetic friction between the stick and the finger is greater than the static friction between the stick and the other finger. At that point the first finger stops and the second finger begins to slide. The stick will first slide over one finger, then on the other, switching back and forth until both fingers meet at the stick's center of gravity.

Starting from the middle, the finger that moves first immediately bears less weight and continues to bear less weight as it moves. There will be no alternating motion in this case.

805 If you hang weights of 50 or 100 kilograms on the hook, nothing will change and the balance will continue to read 100 kilograms. The tension in the rope lessens as more weight is placed on the hook and becomes 0 when the 100-kilogram weight is added.

When more than 100 kilograms are placed on the hook, the rope becomes slack, and the readings on the scale will equal the suspended weights. So for a 150-kilogram weight, the scale will read 150 kilograms.

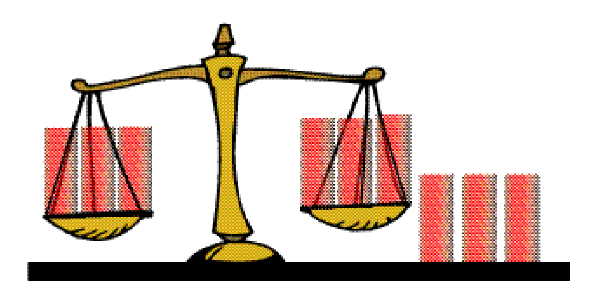


807 The weight is the same in both instances. The weight depends on the mass of the bottle and its contents, and that does not change. When flies are in flight, their weight is transmitted to the bottle by air currents, especially the downdraft generated by moving wings.

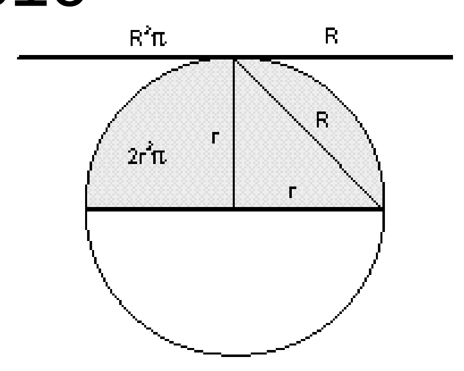
808 First, measure the diameter of the bottom of the bottle. Halve that, square the answer and multiply that number by 3.14159 to get the area of the base.

Then measure the height of the liquid, turn the bottle upside-down and measure the height of the air. Add those numbers together and multiply the sum by the base to get the volume of the entire bottle.

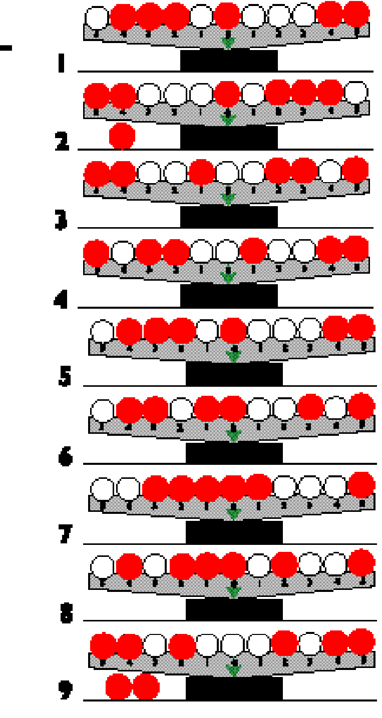
809 Weigh three parcels against three parcels. If one side is heavier than the other side, one of those three must contain the ring. If both sides are equal, the ring must be in one of the three that were not weighed. From the group of three with the ring, weigh one against another. The heavier of the two has the ring; if both are equal, the ring will be found in the unweighed parcel.



810 Both areas are identical.



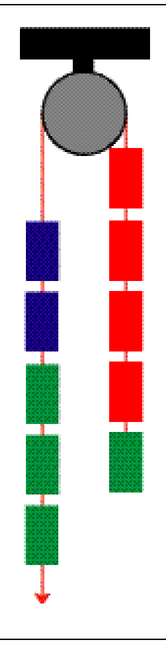
811



812

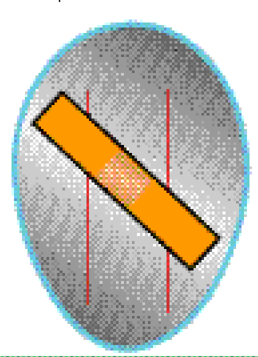
813 The rule that objects with a low center of gravity are the most stable relates to static equilibrium. Balancing a stick is a more dynamic situation, one in which the finger is constantly moving to stay under the stick's center of gravity. A long stick has a large "moment of inertia" (the property of an object to resist turning). Because of this resistance to turning, the stick's center of gravity will shift slowly, giving you time to move your finger back under the center. Short objects have smaller moments of inertia and can turn more quickly than you can respond.

814 The left side of the pulley is heavier by the difference between one red and one green weight.



815 The spout of the yellow can reaches to its rim and so may be filled completely. The green can, while taller, has a low spout and thus may be only partially filled. The yellow can will hold more.

816 The ingenious inner structure, shown below, is quite simple. A small cylinder filled with a very viscous liquid is embedded in the egg at a slanted angle. The cylinder also contains a small but heavy piston that will move very slowly through the liquid—it takes about seventy seconds for it to travel from one end of the cylinder to the other. The piston is heavy enough to throw the egg off balance except during the middle of its transit. Then, for about ten seconds, the egg can be placed in equilibrium on its pointed end.



817 Start both timers simultaneously. When the three-minute timer ends, turn it over quickly. When the four-minute timer ends, turn the three-minute timer over once again—there will be one minute's worth of sand to add to the four minutes to make a full five minutes.

818 When this paradox was first discovered, complex explanations were advanced to account for the hourglass's behavior. But its workings are quite simple.

When the cylinder is turned over, the hourglass's high center of gravity makes it topple over, and its buoyancy helps wedge the glass against the sides of the cylinder. Friction between glass and glass holds the hourglass in place until enough sand has passed to the lower compartment to drop the center of gravity. Only then will the hourglass free up and rise to the top.

819 The heaviest balls will not be slowed as rapidly by the coarse surface. Therefore the heaviest balls will collect in the compartment farthest from the chute, and the lightest will collect nearest the chute.

820 Clockwise.

821 The two bolts will not move in relation to each other.

822 Clockwise.

823 Clockwise.

824 Both racks will move up.

825 To the left.

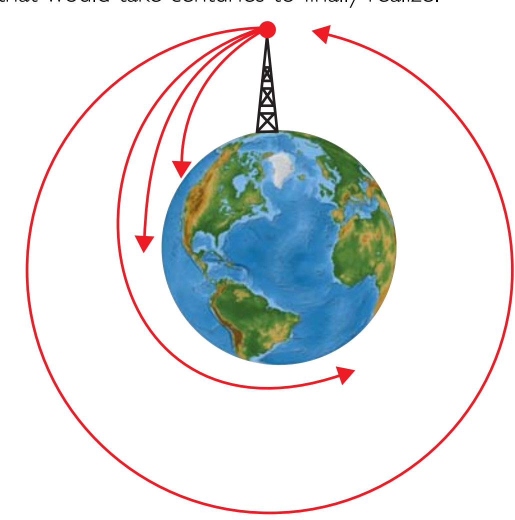
826 After $1^{1}/_{4}$ clockwise turns of the leftmost gear, the letters will spell out LEONARDO.

827 If you throw your Frisbee with real gusto, it will travel all the way around the earth without falling. Since there is no friction from the air, it will continue to orbit without any need of additional propulsion. It will become a satellite.

The moon and communication satellites circle the earth in much the same way as the planets circle the sun.

Sir Isaac Newton studied the paths taken by objects in ballistic flight and theorized that a cannonball fired parallel to the horizon with a great enough force from a great enough height could achieve a path that would match the curvature of the earth. Such a path would take objects completely around the earth, ignoring factors such as air resistance.

Newton, then, was the first to describe how an artificial satellite could be launched. It was an idea that would take centuries to finally realize.



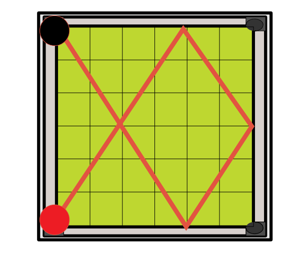
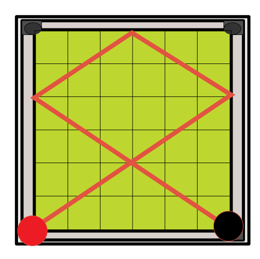
828 The vertical fall of the dart (from a straight-line trajectory) and the vertical fall of the monkey will be exactly the same. No matter what the velocity of the dart is, it will strike the monkey.

829 Many people are boggled by this puzzle and try to sum up an infinite series straight out of advanced math. But the answer is simple: it takes the joggers an hour to meet, and the fly travels 10 kilometers in an hour.

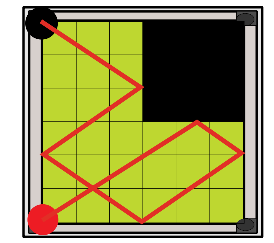
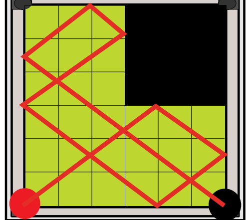
In his fascinating book, ***Time Travel and Other Math Bewilderments***, Martin Gardner tells a story about the Hungarian mathematician John von Neumann, who was asked this puzzle at a party. Neumann gave the correct answer in an instant. The person who posed the question was disappointed; he usually could count on mathematicians to overlook the obvious answer and try instead to solve the problem through the time-consuming process of summing up an infinite series.

Von Neumann was startled. "But that's how I solved it," he said.

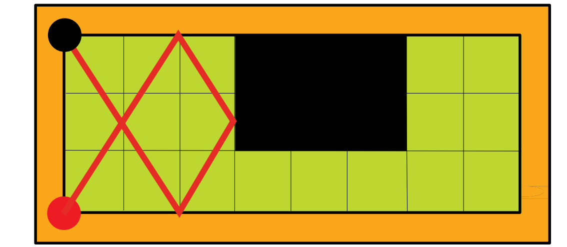
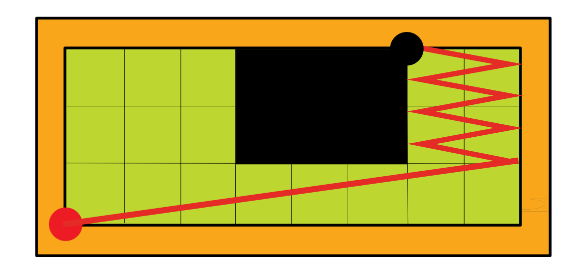
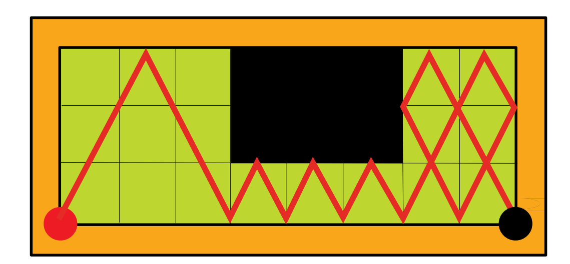
830

831

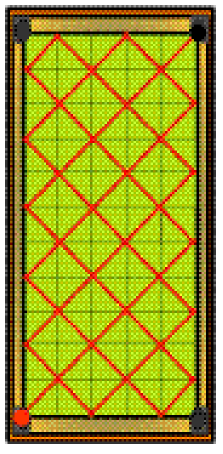



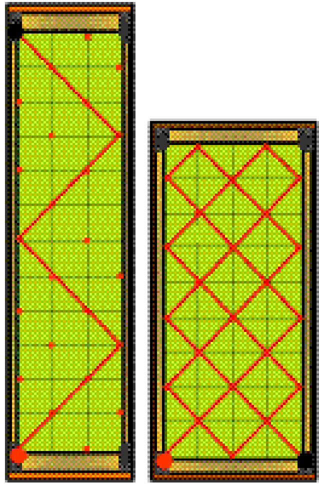
832

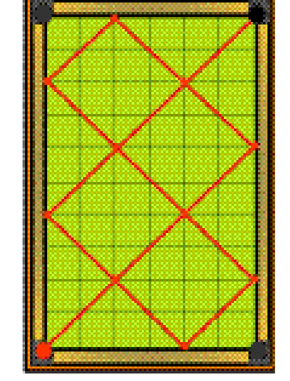




833 If we strike a ball in the corner at a 45-degree angle, it will land in one of the three other corners after a finite number of rebounds. To find out which corners, color in the starting point and every other intersection point of the unit grid. In the first three tables, only one of the other corners will be filled in—a sign that that is the corner in which the ball will eventually land. If all the pockets are filled in, double the size of the unit squares and repeat the process.

In general, if the dimensions of the table are odd-odd, the ball will end up in the opposite pocket; if the dimensions are even-odd, it will end up on the side the ball started from. If the dimensions are even-even, divide by 2 until at least one of the dimensions is odd.


odd-odd


even-odd


even-even

834 The wheel with the weight at the center will arrive first. Because the weight is at the center, it will not resist turning as much as the weight placed near the rim. That means the wheel will speed up much more quickly. But the wheel that has the weight near the outside, though it doesn't speed up as quickly, will not slow down as quickly either: it will roll longer than the other wheel.

835 The bomb will follow a parabola (trajectory 3). The vertical component is the same as a free fall (trajectory 1), but the bomb also carries a horizontal motion imparted by the airplane. Since the vertical motion is accelerating, the curve will become steeper, as in trajectory 3, rather than shallower, as in trajectory 2.

836 Mr. Smith should throw the Frisbee backward so that the dog will have to run the additional distance Mr. Smith walks while he retrieves the Frisbee.

837 The trick works. There's more than gravity working on the bucket: the falling arm of the ladder has its center of mass near the pivot point because of the heavy weight. The resultant torque causes the end of the arm to descend faster than a free fall. As long as the bucket lands in the line of the falling bowling ball, the ball will land in the bucket.

838 The frog advances 1 meter a day. After seventeen full days the frog is 3 meters from the exit. The frog escapes on the eighteenth day.

839 The balls will reach the circumference simultaneously.

Gravity is acting on any given ball in a direction in which it is free to move. The force may be resolved into two components: one parallel to the chord and one perpendicular to the chord. The force that is pulling the ball along the chord turns out to be proportional to the length of that chord. Therefore, the time of travel down one chord will be the same as that down any other.

This experiment prompted one of Galileo's most important discoveries: If balls are released simultaneously from the highest point in a vertical circle along its radial chords, all the balls will arrive at the circumference of the circle at the same time.

Galileo's demonstration proved that the time of descent along any chord from the top to the circumference is independent from its slope.

840 The bottle must be dropped from a height four times as great.

Doubling the height seems intuitively sufficient. But to double the speed, one must double the time of the fall, which means that four times the potential energy must be put into the system.

841 The bridge did not support the clown. Newton's third law of motion states that every action has an equal and opposite reaction; the clown applied a force to the rings to lift them into the air—a force that was greater than the weight of the rings. That force, plus the weight of the clown and the other ring, broke the bridge.

842 The pendulum will appear to swing in a counterclockwise three-dimensional elliptical path. If the lenses are reversed, the pendulum will appear to swing clockwise.

The illusion shows how the intensity of light influences the judgment of distance and depth. Darkened retinal images are transmitted to the brain more slowly than are bright images. This has nothing to do with the speed of light, which is constant. The image seen through the dark lens is recognized a fraction of a second later than the bright image.

When the brain gets two pictures of the pendulum in slightly different positions at the same time, it perceives them stereoscopically, creating an illusion of depth where none exists. The effect is the greatest in the middle of the swing, when the pendulum is at its fastest, because at that point the difference between the two pictures is the greatest.

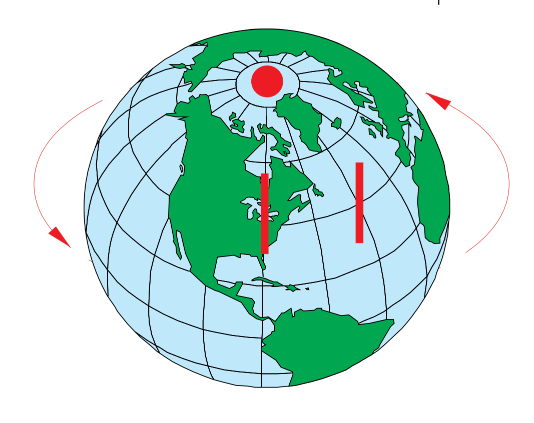
843 The small ball will rebound nearly nine times the original height.

This works because momentum and energy are conserved. When the balls hit the floor, the bottom ball reverses its velocity an instant before the top one. The small ball is moving downward with speed V and strikes a large ball moving upward with speed V after the rebound, making the two balls' relative speed 2V.

If their relative speed is 2V before the top and bottom balls collide, then their relative speed must be 2V after impact. Since the bottom ball is already moving at V, that means the top ball must be moving now at 3V. Because the top ball's velocity has tripled as a result of the impact, its maximum height after the rebound is nine times its original height.

The alignment of the ball at release is very important to achieve this full height. Releasing the balls through a tube or similar arrangement will enable you to get the maximum effect.

844 The apparent rotation of a pendulum varies with the latitude at which it is installed. Its rate at points between the poles and the equator is equal to 15 degrees per hour multiplied by the sine of the latitude. This can be explained only by the fact that the earth turns beneath the pendulum.



845 Surprisingly, the two pendulums will swing back and forth in the same period of time. This may seem counterintuitive, but the time of a pendulum's swing depends only on the length of the pendulum's arm. Whether it makes a long swing or a short one, the period will be the same.

The strange motion of the pendulum obeys certain laws:

1. The period of oscillation does not depend on the weight of the bobs.
2. The period does not depend upon the distance traveled.
3. The period of oscillation is proportional to the square root of the length of the pendulum.

The time for a pendulum to go through one cycle is $2\pi\sqrt{(L/g)}$, the length and g is the rate of acceleration due to gravity.

Since the acceleration due to gravity is the only variable besides the length, a pendulum is a simple way to measure the gravity of a planet. A 1-meter-long pendulum will complete a swing in about 1 second on earth and 2.5 seconds on the moon.

846 Surprisingly, the pendulums will not end up with the same amount of energy. Instead, energy will be periodically exchanged between them in such a way that sometimes one and sometimes the other of them stops.

As one of the pendulums is set in motion, after some time its energy will pass over to the other pendulum, which will gradually overtake the first swing. Eventually the first pendulum will be stationary. Then the whole procedure starts over again.

847 The woodpecker is a simple mechanical oscillator. The hole in the ring around the vertical rod is slightly larger than the diameter of the stick. When the woodpecker is at rest, friction keeps the ring in place on the rod. But when it moves, the ring becomes vertical at the midpoint of each oscillation. Because the ring is now not wedged in place, it slips down a bit along the rod. This slight drop gives enough of a jolt to the bird to keep it vibrating. So at each drop, potential energy is converted into movement—kinetic energy.

The oscillating woodpecker also demonstrates the basic principle of a grandfather clock: the simple escape mechanism.

848 Velocity is speed in a particular direction, so the velocity of the ball is constantly changing because its direction is constantly changing.

A change in velocity means acceleration. And the ball is accelerating toward the center of the circle. In fact, anything that moves in a circle accelerates toward the center of the circle. The acceleration changes the velocity just enough to make the ball follow the path of a circle.

If the string broke, the ball would move off in a straight line tangent to the circle at that point.

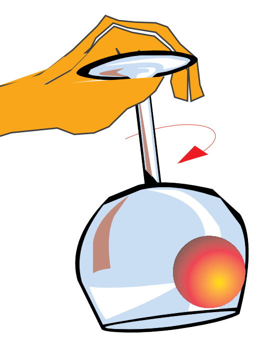
849 Pull out a ball on one end and release it, and another will pop off at the opposite end. If you pull two to the side and release them, two will pop out at the other end.

Can the balls count?

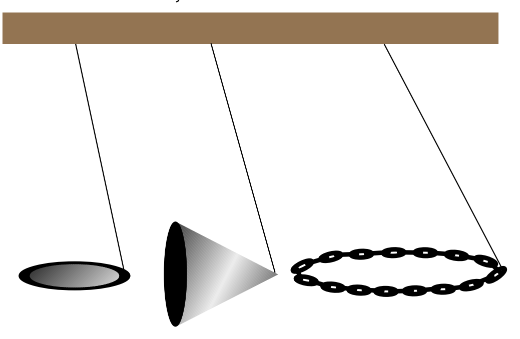
The collision between two bodies where relatively large forces act during a very short interval of time is called an impact. When the highly elastic steel balls collide, they exchange velocities. Faster than the eye can follow, the energy of the impact is passed along to each neighboring ball, and the ball on the end receives that energy and swings into the air.

The effect is the same regardless of the number of balls that are released. The toy demonstrates Newton's third law of motion: To every action there is always an equal and opposite reaction.

850 Place the glass over the marble and move it around so that the marble starts to spin around the inside of the glass. Once the marble starts rotating, it will begin to rise off the table. When the marble is spinning fast enough, you can lift the glass off the table. The marble will not drop immediately; it will continue to spin around under its own momentum.



851 Suspended bodies tend to rotate around the axes of the greatest moment of inertia (see answer to PlayThink 813). This property will make the three objects rotate as shown below.



852 The stool—and the boy—will start rotating in the opposite direction. The angular momentum is conserved by having the two opposite rotations cancel out.

853 Nothing will happen!

The response to the tire's angular momentum will try to drive the stool into the ground.

854 Pushing the handle forward with his right hand and backward with his left one will cause the wheel to tilt to the left.

As paradoxical as it may sound, to turn the stool, the boy must push upward on the right side of the handle and down on the left side. He will then feel the gyroscopic precession: the property of the axis of a spinning body to resist a tilting force by moving in a direction at right angle to that force. The bicycle wheel, which is no different from a gyroscope, resists the tilting force, and its axis begins to rotate at a right angle to what one might expect. The turn of the wheel to the left is transferred to the revolving stool with the boy.

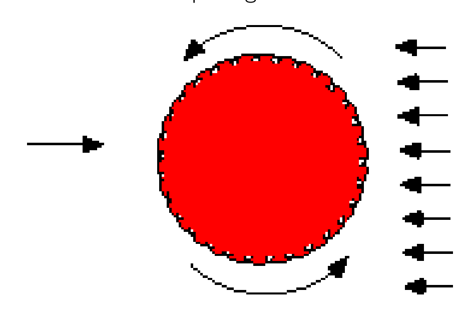
A turning wheel resists any change in speed and direction. Unless you push it in some specific way, the wheel will keep spinning in the same direction. If you turn it, it tilts. If you tilt it, it turns.

Indeed, any fast-spinning object will act like a gyroscope—bicycle and motorcycle riders often experience gyroscopic effects.

855 The centripetal force caused by the rotating cylinder is perpendicular to the wall, creating friction. When the circular acceleration is high enough, the friction force can overcome the force of gravity and prevent the carnival riders from falling when the floor is removed.

856 Golf balls always fly with backspin. The dimples trap a layer of air that spins the ball. The top layer of trapped air moves faster than the bottom layer, giving the ball greater lift. This is called Bernoulli's principle, which is also the basis of airplane flight.

A smooth golf ball would travel about half the distance that a dimpled golf ball can cover.



857 The skater will spin much faster. By bringing her arms to her chest, she decreases the moment of inertia of her body because more of her weight is now concentrated near the center. To compensate for this, there is an increase in her angular velocity. If the spin becomes too fast for her, she can stretch her arms back out to slow down.

All moving objects have energy of movement, or kinetic energy. The kinetic energy stored by something spinning depends on two things: the way its weight is distributed and how fast it spins.

Flywheels utilize this idea, though in the opposite way. They are designed to store as much energy as possible when they spin. Most of their weight, therefore, is concentrated near the rim.

858 The ball will miss the juggler and land to the right of him.

The trajectory will appear to be curved because the jugglers are in motion themselves. The ball will not even start toward the other juggler because it carries the thrower's velocity, which further deflects the ball to the right. This deflection, called the Coriolis effect, is associated with things in a turning frame of reference. There is even a slight Coriolis effect on everything that moves around us because the earth itself is turning.

Although the two jugglers see the ball curve, an outside observer will report that it went straight.

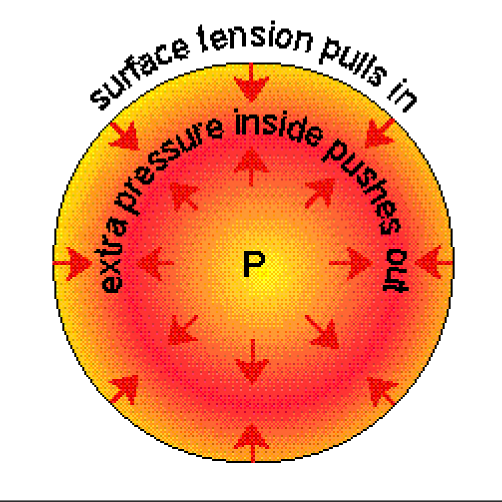
859 Every dimension of the washer will expand, so the hole will get larger too.

860 The branching pattern is more economical than the radial pattern. The branching pattern has a much shorter total length than the radial pattern, at the expense of only a slightly longer average path length. Thus, trees, blood vessels, rivers and even subway networks are all examples of branching patterns.

861 Arrangements 1 and 3 are in equilibrium.

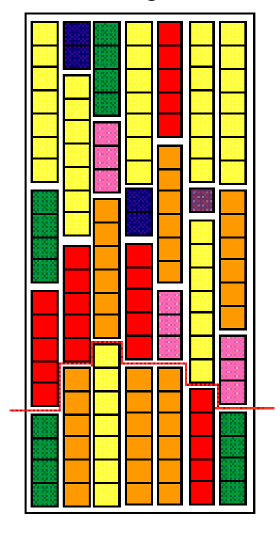
862 The pressure inside a bubble decreases with increasing size. It is inversely proportional to the radius. Thus, the smaller bubble has more internal pressure than the larger one; it will send air through the passageway and into the larger bubble and will shrink as the larger bubble expands. Thus, paradoxically, the smaller bubble will blow up the larger one, collapsing in the process.

This is quite unexpected and different from a similar experiment that involves blowing up two balloons.



863 Since the lines radiating from John's shot are the source for the others that branch off from them, John was first.

864 The shortest route along the cracks is 13 units long.



865 The end of the strip under the paper will not move. In fact, if you strike the wood hard enough, it may snap, but the newspaper won't budge.

The weight of the atmosphere presses on the newspaper and resists being squeezed up suddenly. This holds the stick firmly to the table.

The pressure of air is 1 kilogram on every square centimeter. The force of air pressure on the newspaper—about 2.25 metric tons over its entire surface—is strong enough to hold the newspaper and stick firmly in place for the split second it takes you to break the stick.

866 When you push the plungers together, you remove most of the air between their cups. The air on the outside presses in on the plungers and forces them together.

867 The pressure of the air in the balloon increases as you blow into it—but so does the counterpressure of the air enclosed in the bottle.

The air around the balloon inside the bottle takes up a certain amount of space and has nowhere to escape. As you try to inflate the balloon, the balloon compresses the air inside the bottle until the inside air pressure becomes so great that you cannot inflate the balloon any further.

868 Bernoulli's principle shows that the train carries low-pressure air around it, and atmospheric pressure may force you toward the train.

869 The balls will actually move toward each other. The air moving between the balls has a lower pressure than the surrounding air, which pushes the two balls together.

This is a simple demonstration of the Bernoulli's principle, which links air speed and air pressure. This is also the basis of airplane flight.

870 It takes longer to drop than to fly up.

The ball has to work against air resistance on its way up and so continuously loses energy. Thus, the total energy of a ball at a point on its way up is greater than its energy at the same height on its way down. Since the potential energy (its energy due to its height) is the same at both instances, the difference in energy must be due to a reduced kinetic energy. That means the falling ball is moving more slowly and will take more time to cover the same distance.

871 The wings of an airplane are designed so that air will rush across their upper surface faster than it rushes past the lower surface. For this reason the top surface of the wings is made longer than the bottom.

As described in Bernoulli's principle, that extra speed lowers the pressure above the wings, producing a net force from below called lift. That force keeps the airplane in the air as it moves forward. When an airplane is in midflight, the combined weight of the plane, fuel, passengers and cargo exerts a heavy pull downward. However, that total weight is overcome by the lift, allowing the airplane to remain airborne.

872 The lightweight Ping-Pong ball will rise very quickly in still water.

But when the water is agitated, the buoyancy of the ball is drastically reduced. The movement of the liquid produces higher pressures that make the displacement of the water by the ball more difficult.

873 Your thumb prevents the surrounding air from entering one end of the tube. The open end of the tube allows air to enter and press down on the water on that side. The weight of the air pressing down on the water prevents the level from returning to its initial balanced position.

This is a simple proof that air has weight.

874 According to Archimedes's principle, an object floats because it displaces an amount of water equal to the weight of the object. So to float when the ring was placed on it, the duck must displace a volume of water that equals the weight of the ring.

Since the metal ring is denser than water, the volume of the displaced water is greater than the volume of the ring. When the ring falls in the water and sinks, it displaces only its own volume of water.

The water level, then, drops when the ring slips off the duck and into the tub.

875 Rapidly moving air has low pressure, and a column of upward-rushing air can actually imprison a lightweight object like a Ping-Pong ball. As soon as the ball wobbles a bit to one side, the greater pressure outside of the airstream forces the ball back to the middle.

876 The stream of air creates a low-pressure area, drawing the flames together.

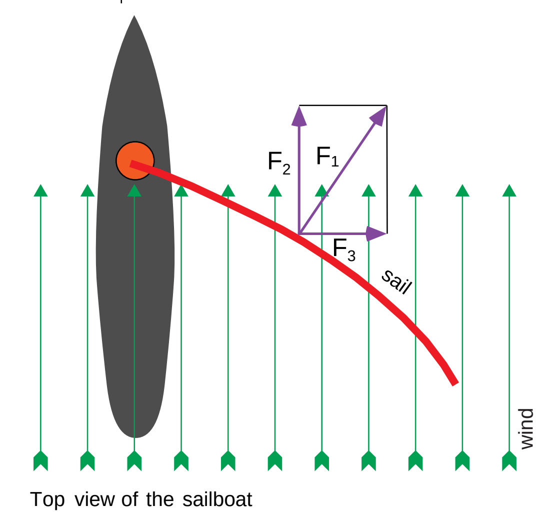
877 You can attain a speed of nearly 40 kilometers per hour. If the forces of water friction on the boat were 0, you could attain the speed of the wind, but no higher.

If the boat were traveling as fast as the wind, there would be no impact of air against the sail. The sail would sag, as on a windless day, because there would be no wind relative to the sail.

878 This configuration will decrease the speed of the boat for two reasons. First, the impact of the wind against the sail is lessened because the sail catches less wind at such angles. Second, the direction of the wind impact force is not in the direction of the boat's motion, as shown in the parallelogram of forces.

Whenever any fluid—gas or liquid—interacts with a smooth surface, the force of interaction is perpendicular to the surface. Not only is this magnitude of force smaller than what it would be if the wind were hitting the sail face on, but only a fraction of the force is directed along the direction of the boat's motion. That's the component that drives the boat forward. The other component simply tips the boat.

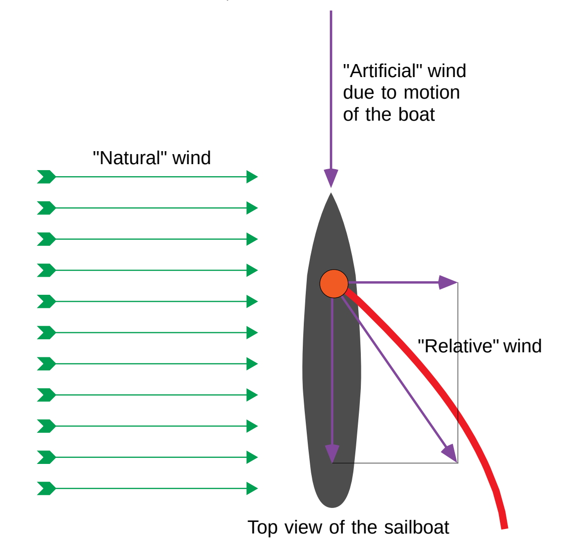
As the sail is pulled farther in, this force vector decreases until it reaches 0, when the sail is pulled in so that it is parallel to the keel.



Top view of the sailboat

879 You can go faster. The force vector is greater because the sail doesn't catch up with the wind speed, so it will not eventually sag.

Even when the boat is traveling as fast as the wind, there is still an impact of wind against the sail, so it can sail even faster than the wind. The boat will reach its maximum speed when the relative wind—the vector made by the natural wind and the "artificial wind" caused by the boat's motion through the air—is directed parallel to the sail.

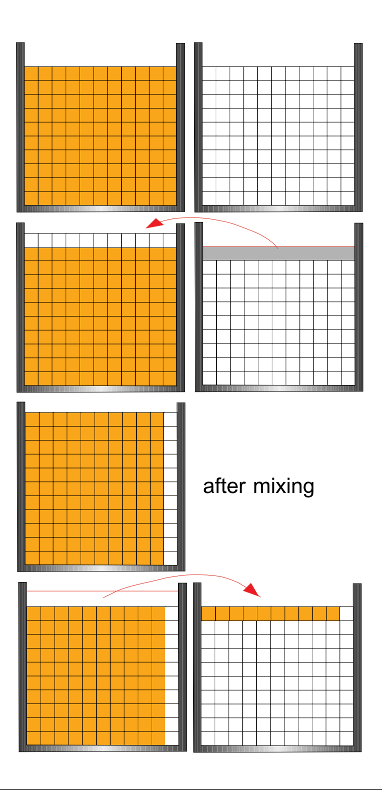


Top view of the sailboat

880 Surprisingly, only boat 4 will move in a forward direction, even though it is sailing into the wind.

The force vector has a small component that will propel the boat. In fact, the faster the boat travels, the greater the force of the wind's impact. As counterintuitive as it may seem, a sailboat's maximum speed comes at an angle upwind. The boat cannot sail directly into the wind, so to reach a straight upwind destination, it has to zigzag back and forth. That strategy is called tacking.

881 As tricky as the problem sounds, there is exactly the same amount of milk in the tea as there is tea in the milk. As you can see in the diagram above, the total volume in each glass is unchanged by the transfer; the net volume transferred from glass A to glass B exactly cancels that which went from glass B to glass A.



882 When you stick your finger in the water, your finger takes the place of some of the water, and so the water level goes up.

Your finger not only takes the place of some of the water but also stands in for the weight of that water. The glass weighs more, by the weight of that displaced water. The weight of the object displacing the water is not a factor; it could be a balloon or a lead cylinder.

883 The ship will float as long as there is enough water to surround it completely. The amount of water does not matter. The ship's hull cannot tell whether it is surrounded by an ocean or by just a thin layer of water. The water pressure on the hull is the same in both cases.

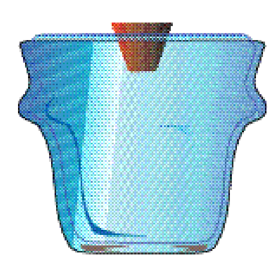
To float, the ship must displace its weight in water. The displacement refers to the water that would fill the ship's hull if the inside of the ship's hull were filled to the waterline.

This principle is exploited at the Mount Palomar Observatory, where the 550-metric-ton telescope actually floats on a thin cushion of oil.

884 The small bottle will drop.

The pressure exerted on a confined liquid is transmitted in all directions. When you squeeze the big bottle, you increase the pressure on the water. The air bubble in the small bottle is compressed and gets smaller. As more water rises into the small bottle, it sinks to a depth where water pressure is greater. When you loosen your grip on the large bottle, the pressure is released and the small bottle rises to its original position.

885 Fill the glass with water until it forms a convex lip above the rim. Then place the cork in the glass. The cork will seek the highest point, which is now in the middle, and stay there.



886 Large raindrops fall faster.

A falling drop is subject to two opposing forces—gravity and air resistance. Air resistance is proportional to the drop's cross section, and it increases with velocity. At first, the slowing effect of air resistance is very small, and the drop keeps falling faster because of the constant force of gravity. As the speed increases, so does air resistance—until the speed is so great that the force of air resistance equally opposes the force of gravity. From that point the drop starts falling at a uniform speed, the so-called terminal velocity.

The force of gravity grows in proportion to the drop's volume, which is the cube of the radius. On the other hand, air resistance builds up at the cross-section area of the drop, which is the square of the radius. As the drop's radius increases, the force of gravity increases faster than the opposing force of air resistance. The drop can reach a greater terminal velocity before the air resistance catches up with it.

887 The water level will stay exactly as before.

The weight of the water displaced by the iceberg exactly equals the weight of the iceberg. When the iceberg melts, it turns back into water and fills the volume of water it displaced.

The volume of the iceberg above the water must exactly equal the increased volume of the water that froze and expanded to become ice.

888 When the bottle is inverted, the paper will bulge a bit. That bulge causes a change in the volume of the air inside the bottle. According to Boyle's law, any change of volume is accompanied by a change in pressure. What is really surprising is that such a small change in volume as that caused by the bulge in the card is sufficient to drop the pressure enough to prevent water from pouring out.

It should be noted that the necessary change in volume is easier to achieve when the bottle is nearly full.

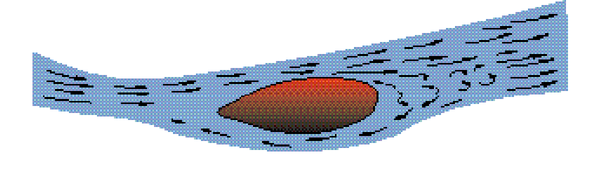
889 The speed of the flow depends on how far below the surface the outlet is located. The depth is the same for both outlets, so water will leave both holes at the same speed.

890 The 6-centimeter drain has a cross section that is three times that of the total for the three smaller drains, so it will drain three times as fast.

891 The current will flow backward.

The flow actually speeds up through the narrow passage but slows down where the channel widens. Where does that excess speed go? The water sheds the speed by flowing uphill. The water flows back around behind the rock from a section of lower elevation to one of slightly higher elevation.

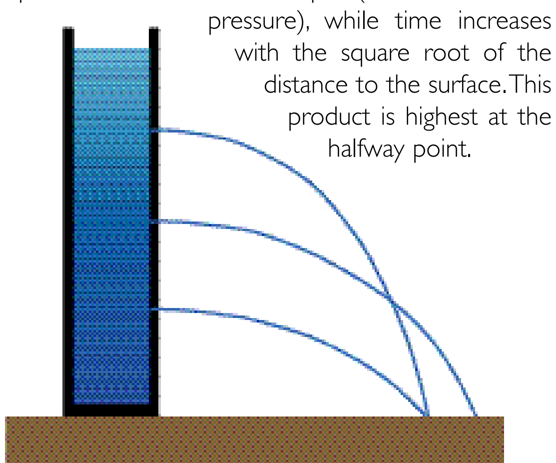
Such backwaters create dangerous turbulence behind boulders in fast rivers.



892 The last time I tried this, I was able to add fifty-two pennies to a supposedly full cup of water before it overflowed.

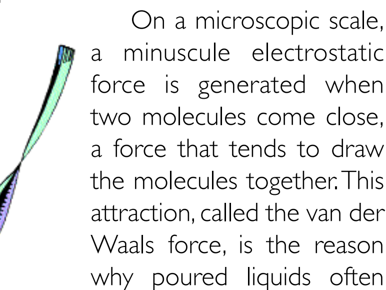
Water has a high surface tension. It behaves as though it had a flexible skin on its surface; that skin pulls inward and resists breaking. Not only can a glass of water develop a great bulge before it flows over the edge of the container, but the surface tension can support the weight of light objects. If you place a clean razor blade flat against the surface of a glass of water, the blade can actually "float"—not because of buoyancy but because of the support of surface tension.

893 The distance the water will squirt depends on the exit speed of the water out of the hole multiplied by the time it takes the water to reach the table. The middle hole has the greatest range because speed increases with the square root of the water depth (because of water pressure), while time increases with the square root of the distance to the surface. This product is highest at the halfway point.



894 The stream will follow the curve of the spoon; that's called the Coanda effect.

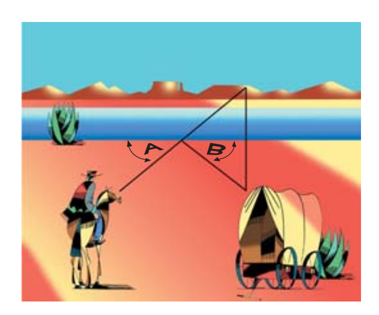
On a microscopic scale, a minuscule electrostatic force is generated when two molecules come close, a force that tends to draw the molecules together. This attraction, called the van der Waals force, is the reason why poured liquids often dribble down the side of a glass rather than exit cleanly over the side.



895 If the flow of water is continuous, the volume of water that is discharged is constant along the entire stream. The same volume of water per second must pass through any given cross section of the stream, including the top and the bottom. But as the velocity of the falling water increases (because of the acceleration due to gravity) the cross section of the stream becomes thinner.

896 The air pressure at the moving end of the tube is lower than the pressure at the end being held. That pressure difference causes the air to flow through the tube, and the air vibrates as it passes over the corrugated walls of the tube.

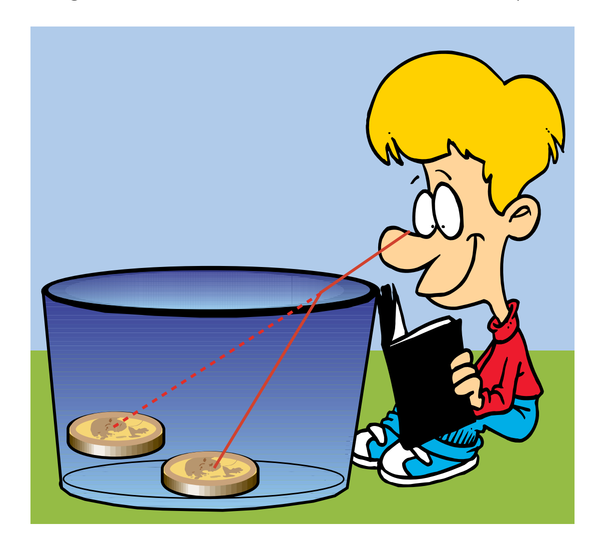
897 The path is shortest when angles A and B are equal, as shown below. (This is the same as the reflection of light off a mirror.) In fact, if the cowboy imagined that the wagon was on the other side of the riverbank but the same distance from it, he could ride toward that point to reach the proper spot to water his horse.



898 As the container fills with water, the coin will come into view.

Light travels at different speeds through different substances. It travels more slowly through water or glass than it does through air. When light passes across the border between two different "speed zones," it changes direction. This change in direction is called refraction; it makes light rays look like they "bend" at the point where two substances meet.

When the light from the coin reaches the surface of the water, it is bent back toward your eyes. But since your brain does not sense what is happening, you perceive the light as coming from a place that is higher and farther back than the coin actually is.



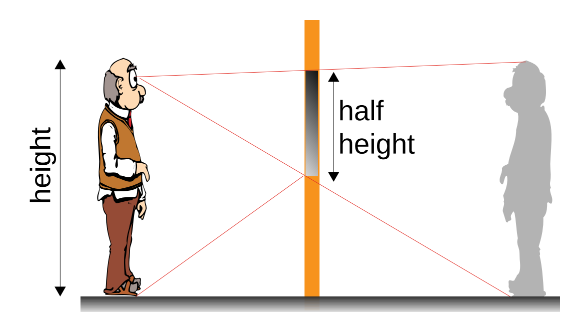
899 The magnification will actually decrease. The amount by which a lens can bend rays of light depends on both the curvature of the glass and the difference in the speed of light between air and glass. The difference in speed from water to glass is less than that between air and glass, so the lens will not bend the light as powerfully and therefore will not magnify the image as much.

900 Because the sun is so large, the shadow will be smaller, but the difference in size is imperceptible. But if the sun is at an angle to the shadow surface, such as an hour or less before sunset, the shadow can be much larger.

Light rays from a distant object may appear parallel, but this is not necessarily the case. If the light source is larger than the object, the shadow (on a flat surface perpendicular to the light source) will be smaller. If the light source is smaller than the object, then the shadow will be larger. The difference in size, however, is scarcely perceptible if the distance between the two objects is great.

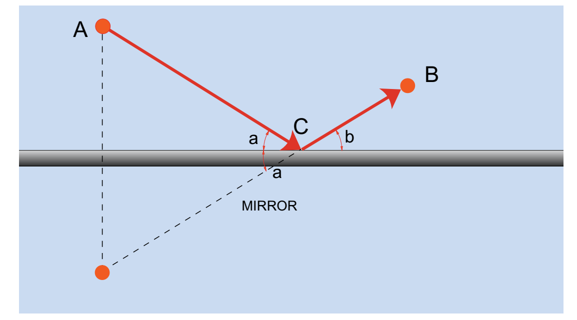
901 The angle will remain 15 degrees. Some measurements do not change when dimensions are magnified.

902 It doesn't matter how far you are from the mirror, as long as it is hung at the correct height—with the lower edge at half the height of the eyes of the person looking in the mirror.

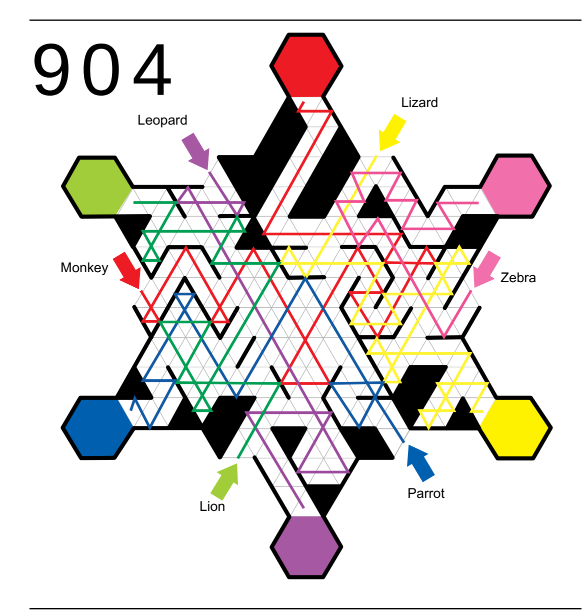


903 The ancient Greek geometer Euclid also studied optics. Euclid found that light travels through space along straight lines, and he laid down the fundamental laws of reflection:

- The plane of incidence of the ray coincides with its plane of reflection.
- The angle of incidence of the ray equals its angle of reflection (in the diagram, angle a = angle b).
- Light always travels via the shortest path.

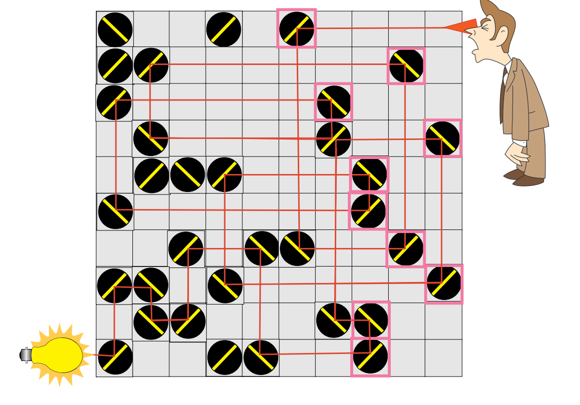


904



905 In the water container—because at 20° F, water is frozen solid.

906 One way to route the light rays is shown; ten mirrors have been rotated.



907 Scientists and historians have long dismissed the story as an impossible feat. But over the centuries a few enthusiasts have tried to prove otherwise. Rather than using one giant mirror, these people say, Archimedes created the effect of a large mirror by using a great number of small reflectors that were aligned in the proper way. The highly polished shields of the Syracuse army may have done the job, they say.

But even if Archimedes lined up his men and had them focus the sun's rays on the Roman ships, was it physically possible for the ships to catch fire?

In 1747 the French naturalist Georges-Louis Leclerc de Buffon conducted an experiment using 168 ordinary rectangular flat mirrors. Aligning them in just the right way, he was able to ignite a piece of wood at a distance of about 100 meters. The port of Syracuse was not nearly that large; the Roman ships were probably less than 20 meters from land.

A Greek engineer conducted a similar experiment in 1973, employing 70 mirrors to focus sunlight on a rowboat some 80 meters from shore. Within a few seconds after the mirrors were properly aligned, the boat burst into flames. Those mirrors were slightly concave, but it is likely that Archimedes could have built such mirrors.

908 Three meters.
The image of the flower in the hand mirror is as far behind that mirror as the flower is in front of it: .5 meter. That puts the image of the flower .5 + .5 + 2, or 3 meters, in front of the large mirror, so that is the distance behind the large mirror that its reflected image forms.

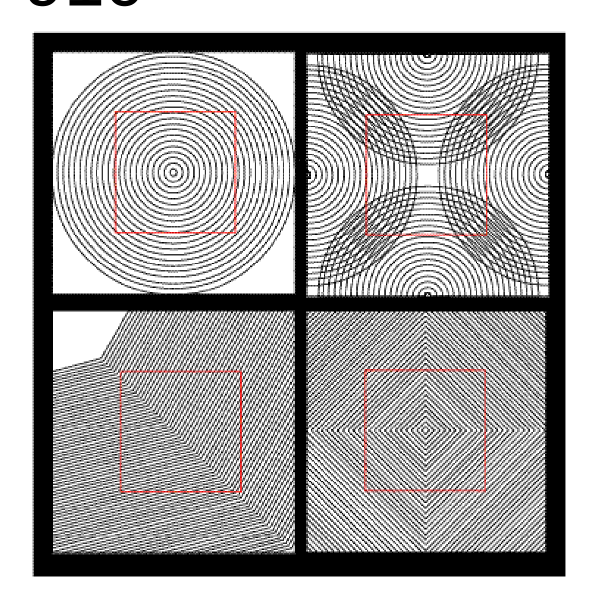
CHAPTER 13 SOLUTIONS

909 Your scorecard should look like the chart below. The visual shortcut requires turning the page upside down. That makes the missing cubes appear solid.

SCORE BOX

Missing Cubes	1	2	3	4	5
Cubes colored on three sides	1	1	1	1	1
Cubes colored on two sides	6	3	6	6	10
Cubes colored on one side	12	3	12	12	19
Cubes not colored	7	0	1	0	6
TOTALS	26	7	20	19	36

910 1, concave; 2, convex; 3, skewed; 4, bent



911 Dots, number 9; arrows, number 7; semi-circles, number 5.
The Illusion Wheel was inspired by one of the simplest and most striking optical illusions, the so-called Müller-Lyer illusion and its variants.

912 To make the butterfly disappear, close your right eye and stare at the red dot with your left eye. From a certain distance, the circle containing the butterfly should disappear, and the line should appear to be continuous. The disappearance of the butterfly is sudden and striking.

This illusion is due to a phenomenon called the blind spot. Researchers have shown that one eye cannot cover the entire visual field. There are no visual receptors over an area of about 1.5 millimeters in diameter at the place where the optic nerve enters the retina.

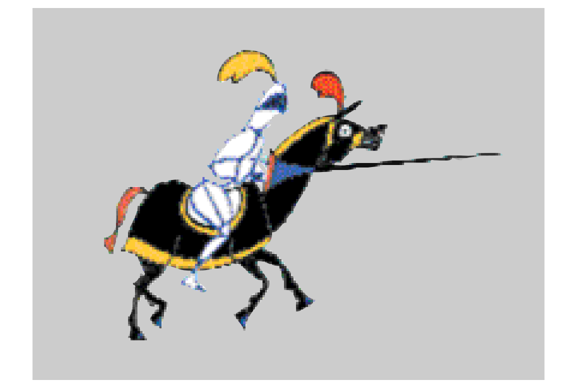
When the incomplete signal from the eye reaches the brain, the brain uses simple rules to calculate what the blind spot of the retina ought to be seeing. In this case the brain extrapolates between the two black lines and deduces that it is one straight line and fills the gap. Although the brain behaves in this way to enable us to make sense of the world, sometimes this property can be exploited to make nonsense, such as illusions.

913 Stare at the red bird for a minute and then look at the center of the birdcage. You will see an illusory afterimage—a green bird—in the cage.

There are three types of color receptors in the eye—one each for red, green and blue. The red of the bird in the picture causes the red receptors to adapt, temporarily decreasing their sensibility to red. Since the figure does not reflect much green or blue light, receptors for those colors become considerably more sensitive. When you shift your gaze to the gray area, the effect of adaptation makes your green and blue receptors overly sensitive—and the red receptors dulled—and therefore you see the gray area temporarily as green.

In short, afterimages are a signal that our visual receptors have become fatigued from seeing too much of the same color.

914 Stare at the black knight for a while, then look at the gray area at right. You'll see the afterimage reversal—a white knight on a black horse.



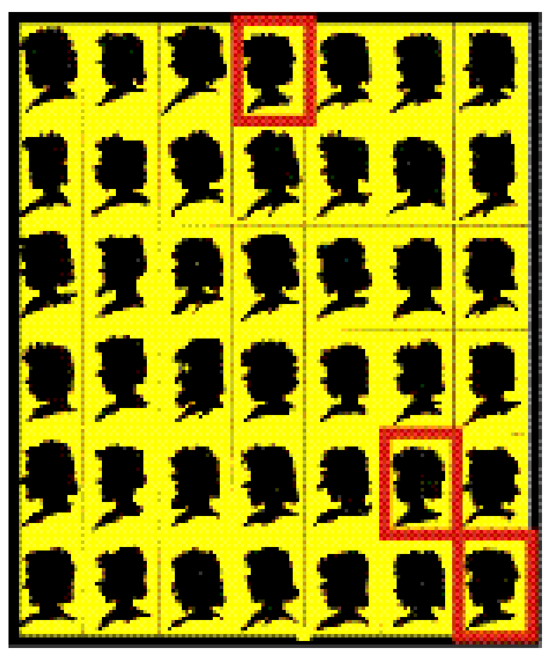
915 It's always the one you are looking at.
Look steadily at the pattern, and you'll see a positive afterimage effect that results in an illusion of small gray spots at the intersections. But if you try to look directly at a spot, the new visual information from the center of your field of vision erases the afterimage effect—and the spot.

916 The blue lid fits the red coffin, and the red lid fits the blue coffin.

917 When you look at the page at a very slanted angle along the direction of the two lines, a third or even fourth line appears as a strong illusion. Such optical effects and illusions appear when two lines or a group of lines intersect at very small angles.

918 You have supervision, just like Superman, since you can close the gap and the bridge simply by looking at it. All you have to do is look squint-eyed at the picture from a distance.

919

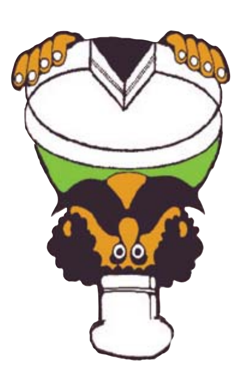


920 A palace guard. If you can't make it out, stand about 1 meter from the picture and squint.

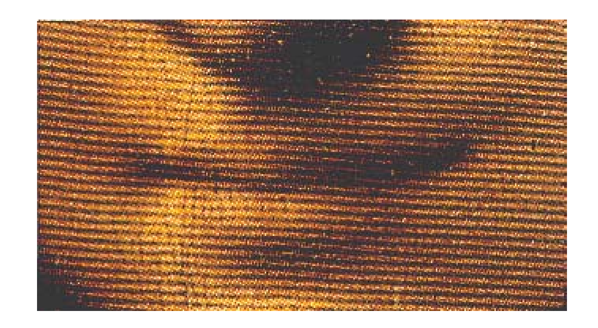
There are more than 120 million photoreceptors that split up the images projected on the retina into point-sized messages—not unlike newspaper pictures printed in halftone dots, computer monitors broken into pixels, and pointillistic paintings.

921 Position number 5 is in direct line with the tank.

922 The missing slice of cake can be found by turning the picture upside-down.



923 When seen with squinting eyes from three or four feet away, the pattern resolves into the famous smile of the Mona Lisa.



924 The pairs are 1-8, 4-10 and 7-5. The set of three is 2-3-9. The odd one out is 6.

925 The letters in the words at the bottom all have horizontal symmetry. For such an image, a reflection is the same as a 180-degree rotation. The rotation is more easily perceived because the image still spells out recognizable English words.

926 The fly can be in one of three positions:

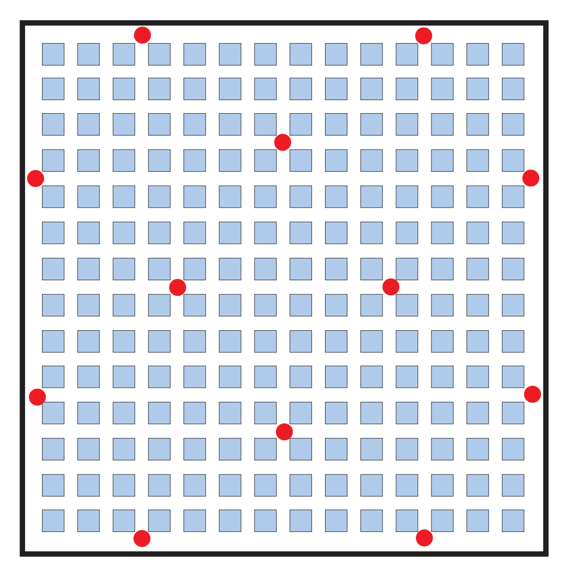
1. On the outside of the box, on the checkered vertical side facing you.
2. On the outside of the box, on the bottom.
3. On the inside, on the checkered floor.

927 Just turn the picture upside-down.

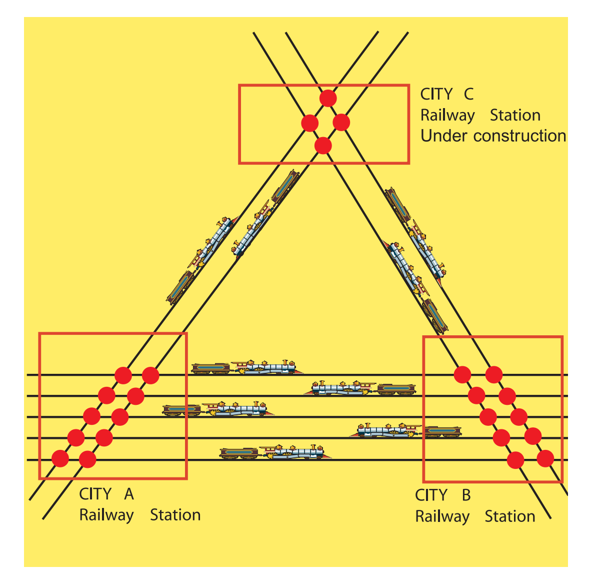
CHAPTER 14 SOLUTIONS

928 Building 1—Blueprint 11 (top view)
Building 2—Blueprint 9 (top)
Building 3—Blueprint 13 (top)
Building 4—Blueprint 5 (top)
Building 5—Blueprint 7 (top)
Building 6—Blueprint 16 (front view)
Building 7—Blueprint 8 (front)
Building 8—Blueprint 15 (front)

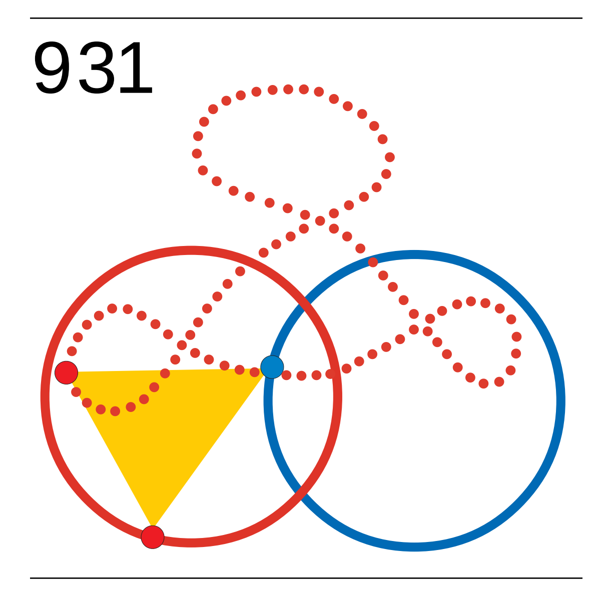
929 The design shown below requires only twelve outlets.



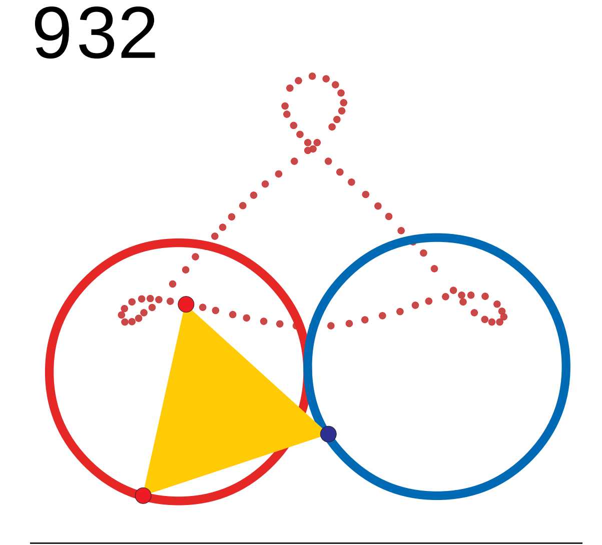
930 Through trial and error, you can determine that the three groups of nine tracks can be arranged 3, 3 and 3 (for 27 intersections), 2, 3 and 4 (for 26 intersections) or the minimal solution, shown, of 2, 2 and 5 (for 24 intersections).



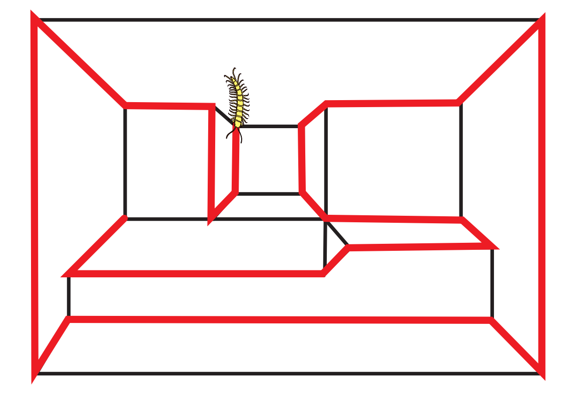
931



932



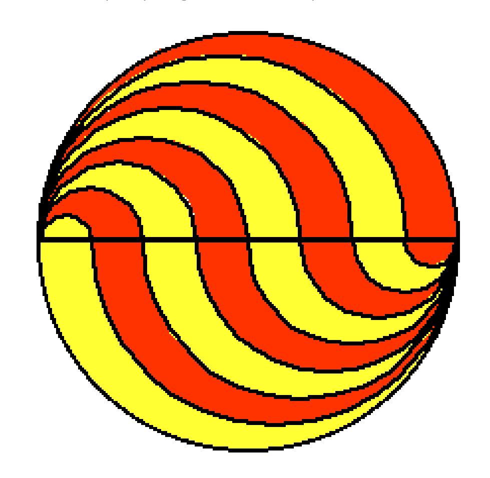
933 It is problematic to solve this sort of problem by looking at the three-dimensional figure; some corners and edges will always be hidden. Instead, one can create a topologically equivalent two-dimensional diagram, such as this one, on which to work out the solution.



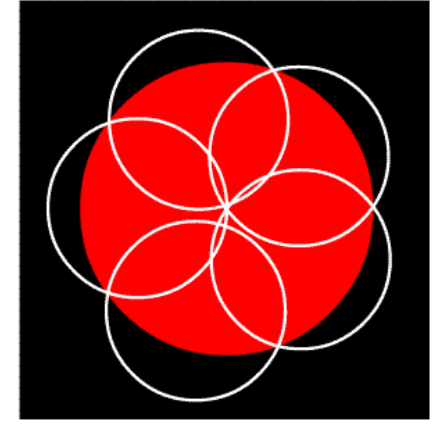
934

A circle can be divided into any number of regions of equal area using a compass and a ruler. Simply divide the diameter into the number of equal divisions required and from those points draw semicircles, as shown.

Ancient Chinese mathematicians knew of this method; the yin-yang is an example.

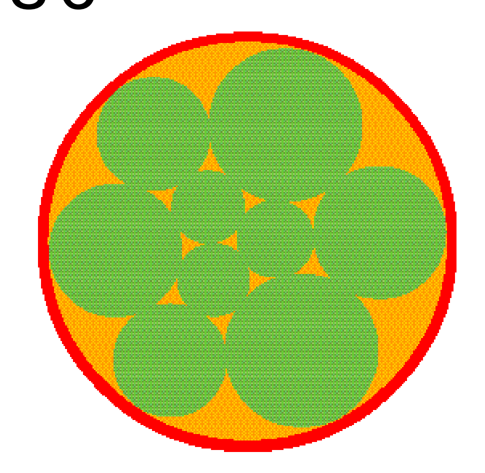


935



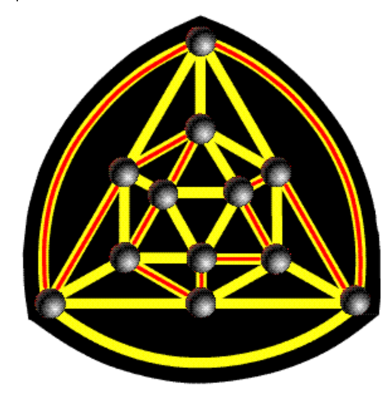
936

This is the best solution found so far.



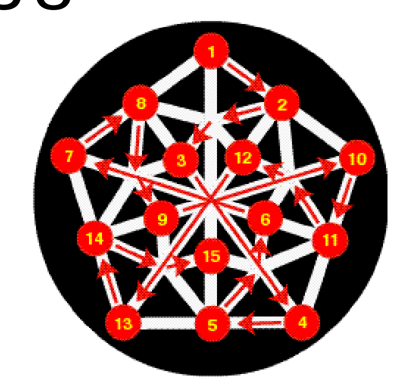
937

One possible answer is shown below. If the puzzle had required you to traverse each line once and only once, it would have been impossible!



938

One of many solutions.



939

The results are independent of the way the smaller shapes intrude on the larger. After all, the overlap is removed from both the red and blue areas. Therefore, one easy method for comparing the red and blue areas is to find the difference between the sum of the areas of the smaller shapes and the area of the largest shape.

Circles ($r^2\pi$): Red and blue areas are equal.

Squares (a^2): Blue area is larger.

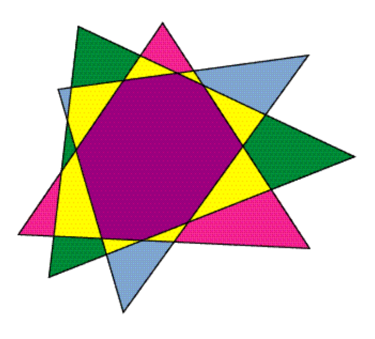
Triangles ($\frac{a^2}{4}\sqrt{3}$): Sum of the red areas is larger.

940

To take into account the worst possible scenario (five red, five yellow, five green and one blue), you must grab sixteen wires.

941

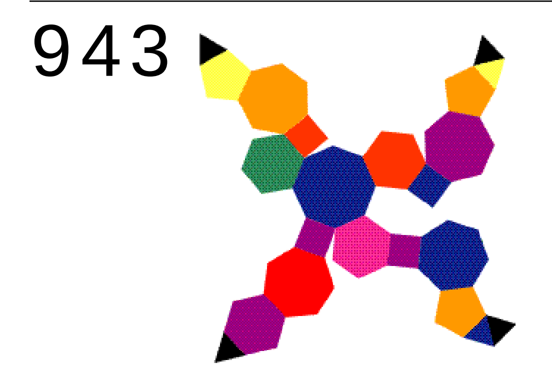
These triangles can overlap to form as many as nineteen regions.



942

For whichever of the seven horses comes in first, there are six different horses that can come in second; for each of the forty-two different combinations of first- and second-place horses, there are five different horses that can come in third. That means there are 7 x 6 x 5, or 210, different combinations of horses.

943



944

The number of nonrepeating three-letter combinations are

26 x 25 x 24, or 15,600

That means his chances are .0064 percent.

945

The red region is two-thirds the area of the original triangle.

946

There must be at least two such boys.

947

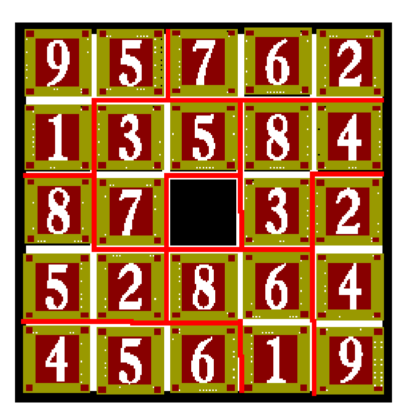
The answer can be found by simple multiplication: 26 x 10 x 10 x 10 x 26 x 26 x 26, or 456,976,000.

948

There are fifteen unique pairs of dogs. If the dogs are named, say, A, B, C, D, E and F, the possible pairs are: AB, AC, AD, AE, AF, BC, BD, BE, BF, CD, CE, CF, DE, DF and EF.

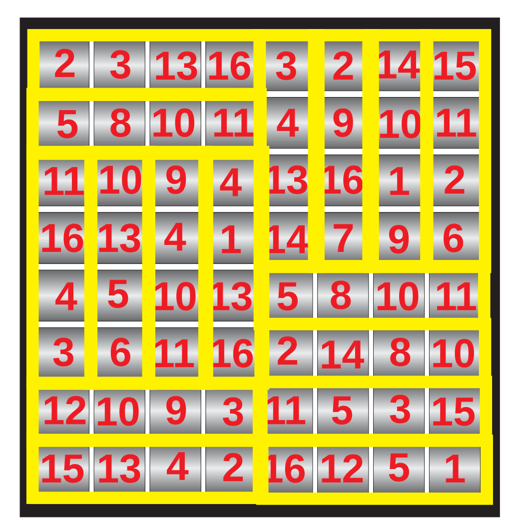
949

The eight groups represent the eight possible ways to create different triplets of the numbers 1 through 9 that add up to 15.



950

These sixteen combinations of four numbers are part of a larger set of eighty-six possible combinations of numbers from 1 through 16 that total 34.



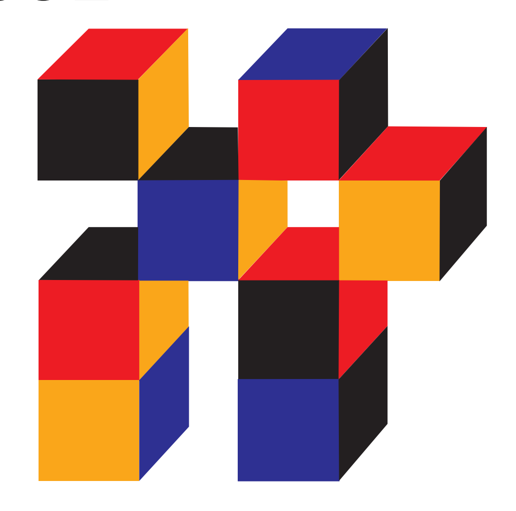
951

Four colors are needed, as shown.

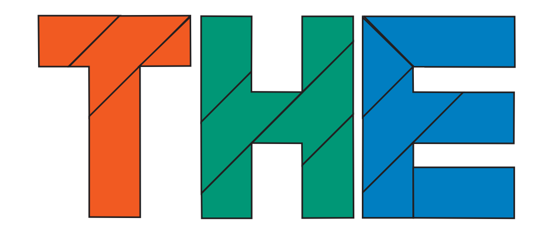


952

Eight.

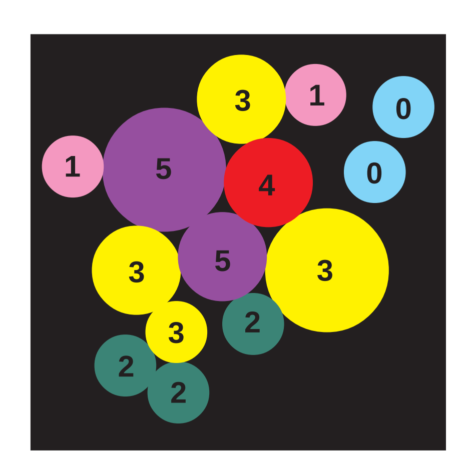


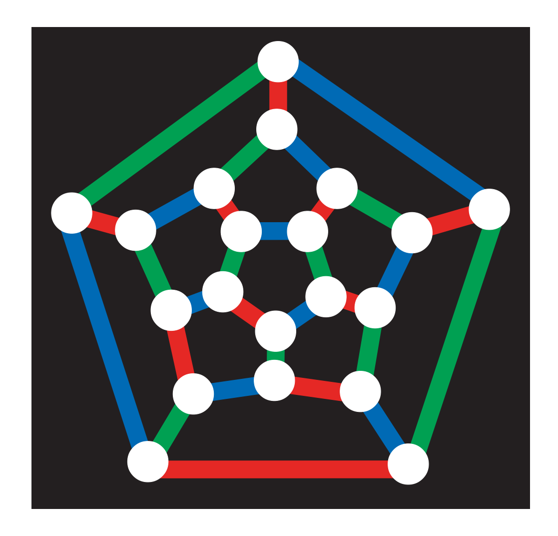
953



954

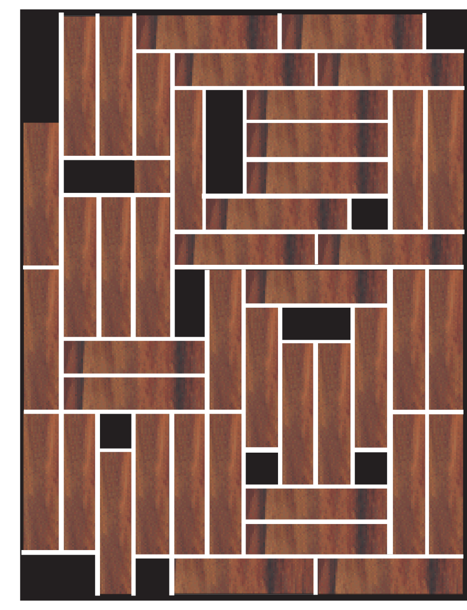
The color of each circle is determined by the number of other circles it touches.





955

Only three colors are necessary, as shown.



956

957

Each spot on the robot's electronic display can either show 1, 2, 3 or be blank. That means it can show three different one-digit numbers:

1, 2, 3

nine different two-digit numbers:

11, 12, 13, 21, 22, 23, 31, 32, 33,

and twenty-seven different three-digit numbers:

111, 112, 113, 121, 122, 123, 131, 132, 133,
211, 212, 213, 221, 222, 223, 231, 232, 233,
311, 312, 313, 321, 322, 323, 331, 332, 333

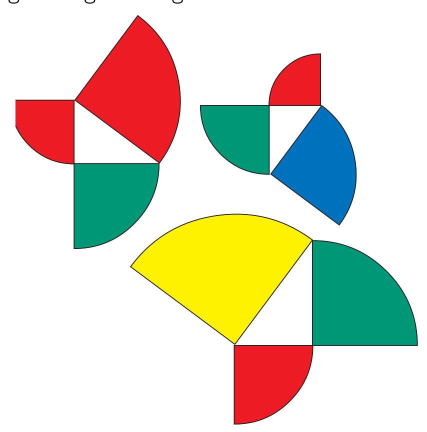
for a total of thirty-nine numbers.

You can solve this easily with this formula:

$$3 + 3^2 + 3^3 = 39$$

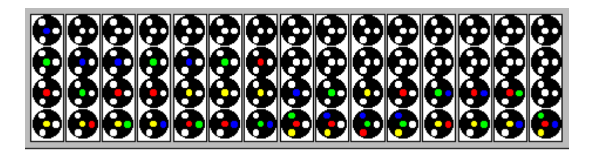
958

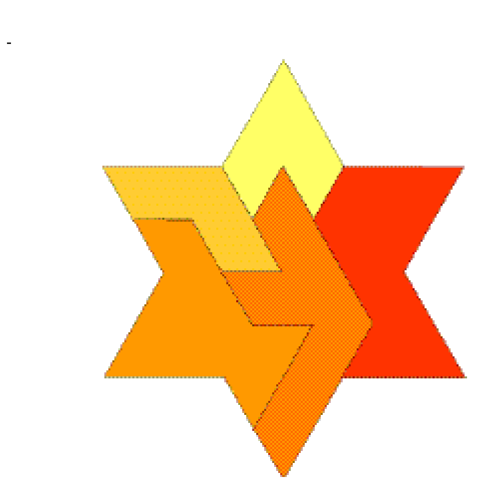
The Pythagorean theorem ensures that the areas are exactly equal. Two radii of a pair of touching quarter circles are at right angles to each other, and the radius of the matching quarter circle stretches across the hypotenuse, completing the right triangle.



959

As shown, there are fifteen different ways to distribute four pieces of fruit over four plates.



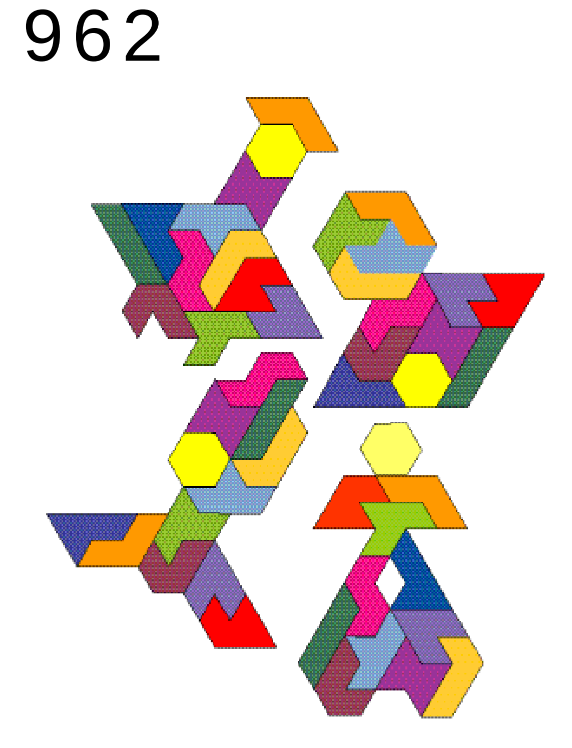


961

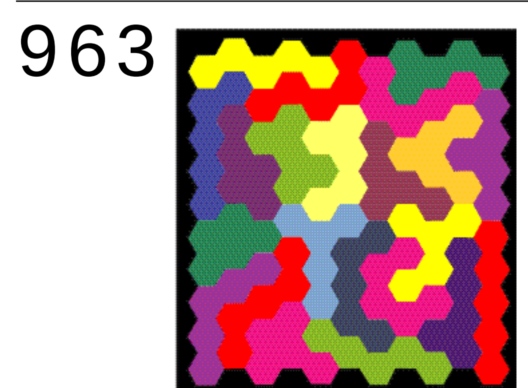
This sequence of numbers details the number of new pairs of rabbits produced each month, starting with the first new pair born in January. The total number of pairs is 376.

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1	2	1	3	5	8	13	21	34	55	89	144

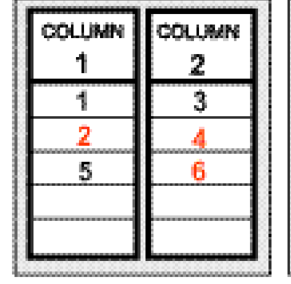
962



963



964



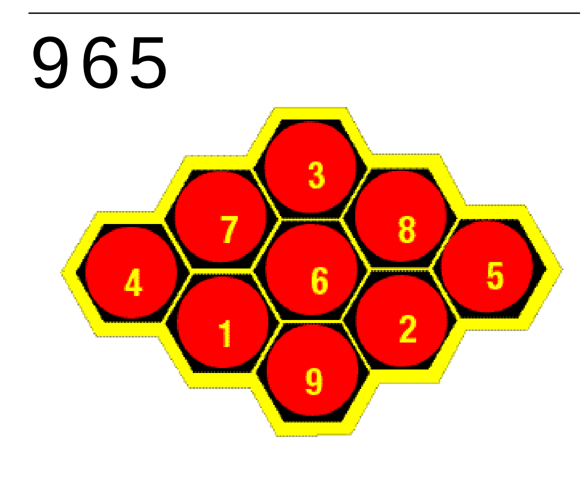
COLUMN 1	COLUMN 2
1	3
2	4
6	5

As you can see in the first diagram, it doesn't matter where player 1 places the 5 because player 2 wins when placing the 6.

COLUMN 1	COLUMN 2
1	3
2	5
4	6
8	7

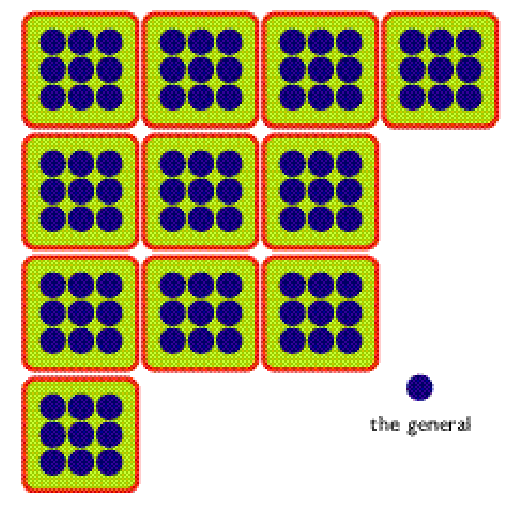
In the second diagram you can see it is always impossible to place the 9.

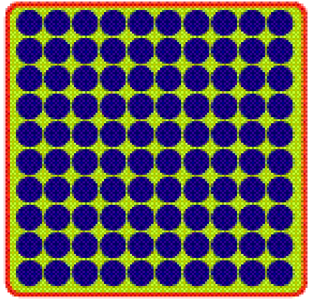
965



966

The total number of soldiers plus the general must be a square number. The smallest square that is also equal to 1 plus a multiple of eleven is 100, which is $9 \times 11 + 1$.





967

The general answer to this problem, called the hailstone problem because of the way the numbers cycle in much the same way as hailstones growing in a thundercloud, is not known. But none of the numbers up to 26 survive for long. Beginning with 7, you get:

7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5,
16, 8, 4, 2, 1, 4 and so on

The number 27 takes an interesting journey, making it up to 9,232 at step 77 before crashing. It reaches the 1-4-2-1-4-2 loop in step 111. Every number up to a trillion has been tested, and every one eventually collapses into the rut.

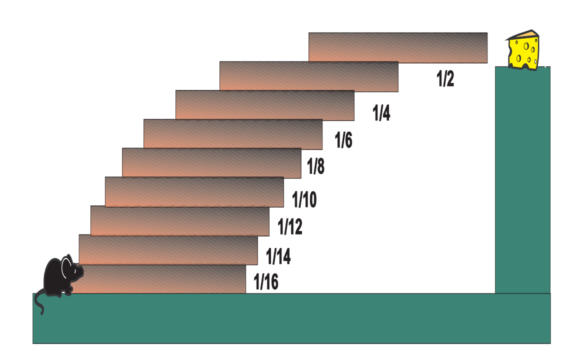
968

The area of the one gold square of the first generation is simply $\frac{1}{9}$ the area of the original blue. The area of the eight gold squares of the second generation is $\frac{1}{9}$ the area of the smaller blue squares, which themselves are $\frac{1}{9}$ the area of the original. The third generation finds sixty-four gold squares, each of which is $(\frac{1}{9})^3$ the area of the original blue square. The pattern emerges:

$$1 \times \tfrac{1}{9} + 8 \times (\tfrac{1}{9})^2 + 8^2 \times (\tfrac{1}{9})^3 + 8^3 \times (\tfrac{1}{9})^4 + \ldots$$

If you carry out the calculation to the twenty-fifth generation, you will find that gold covers an area equal to almost 95 percent of the original blue square. It's clear that the area of the gold will come increasingly close to 100 percent of the original square, but it will never reach total coverage.

969 Counting from the top, the *n*th plank can have an overhang relative to the plank immediately below it equal to $\frac{1}{2n}$ meters. This leads to the sequence $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \frac{1}{12}, \frac{1}{14}, \frac{1}{16}$; the corresponding overhangs are .500, .250, .167, .125, .100, .83, .71 and .62 meters. The total overhang is then 1.358 meters—just shy of the cheese.



970 I got the inspiration for this puzzle during a lecture by American mathematician and logician Raymond Smullyan. As he explained, the answer is simple: the young man simply asks, "Are you married?"

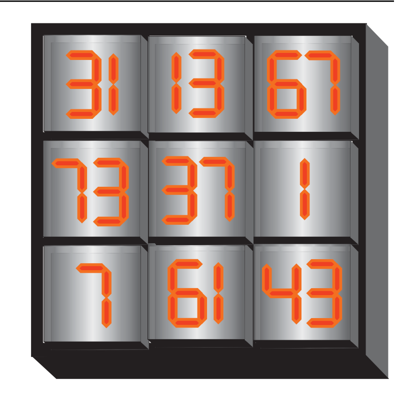
Regardless of who answers his question, he knows that a "yes" means that Amelia is married and a "no" means Leila is married. Virtuous Amelia will tell him the truth—"yes" if she is, "no" if Leila is—no matter what, and wicked Leila will say "no" if she is married and "yes" if she is single and Amelia is married.

971 You simply move each guest into the room with the number that is twice that of the room he or she is in now. The person in room 1 goes to room 2, the person in room 2 goes to room 4, the person in room 3 goes to room 6 and so on. All the odd-numbered rooms will be vacated, and since there are an infinite number of odd numbers, all your new guests can be accommodated.

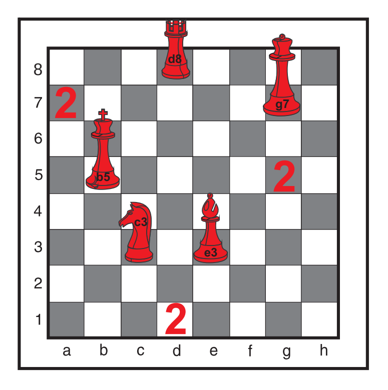
972 You should ask him, "Which way to your hometown?"

If he is from Truth City, he'll point to it; if he is from Lies City, he will also point to Truth City.

973



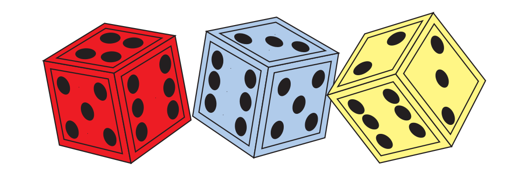
974



975 You should ask, "Am I in Las Wages?" and "Am I in Las Wages?"

Two yeses will come from a truth teller; two nos from a liar. And a yes and a no will mean that the person alternated between truth and lie.

976 The largest sum you can see on any given die is 15, that is, the sum of 4, 5 and 6. Therefore, the only possible combinations of three different numbers that total 40 are 15 + 14 + 11 and 15 + 13 + 12. But a sum of 13 is impossible to see on the three faces of a real die. (Try it if you doubt this.) That leaves the only answer as 15, 14 and 11, as shown.



977 This is a classic game, here in a form suggested by Peter Gabor. There are six *F*s. The *F*s in *of* are easy to overlook.

978 Although the coin has an equal chance of landing on heads after every throw, the player who tosses first has a decided advantage, no matter how long the game lasts. The probability that the first player will win is the sum of the probabilities that occur at every turn:

$$\frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^7 + \ldots$$

This is a series with an infinite number of terms that approaches two-thirds in value. Therefore, the player who tosses first has a chance of winning that is almost twice that of the second player. If you are surprised at this result, play a number of games and keep track of who comes out on top.

979 Simply multiply together the chances that each ball thrown will land in an empty box:

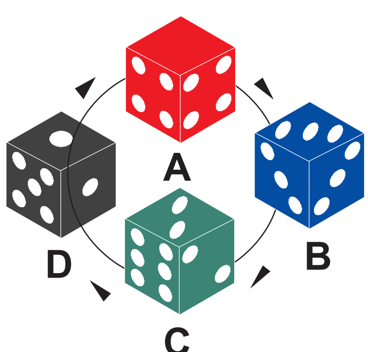
$$\frac{4}{4} \times \frac{3}{4} \times \frac{2}{4} \times \frac{1}{4} = \frac{6}{64} = 0.09$$

That means there is roughly one chance in ten that each of the four boxes will contain a single ball.

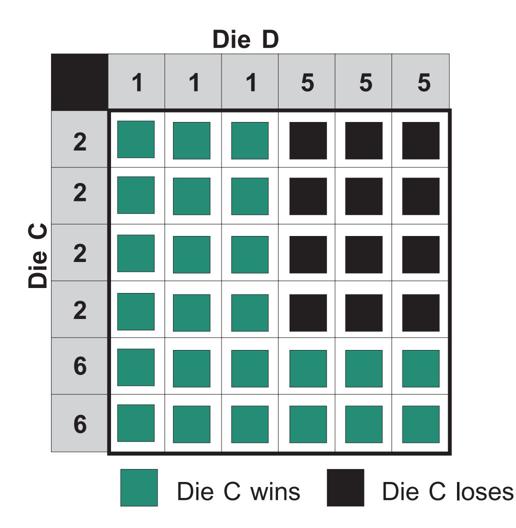
The general formula for this problem is $n!/n^n$.

980 The first spinner is always the best to choose. Against the third spinner, the first spinner will win 51 percent of the time because its 3 will beat the third spinner's 1. The second spinner has a higher average spin (3.33), but the 3 on the first spinner beats the second spinner's 2, which comes up 56 percent of the time.

981

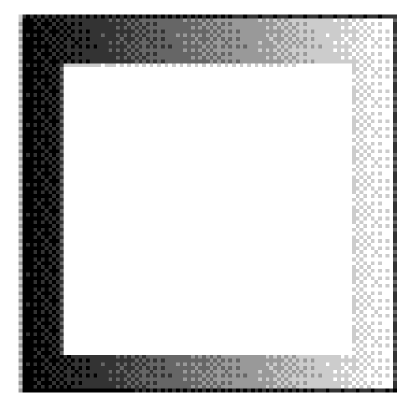


When two six-sided dice are thrown against each other, there are thirty-six possible outcomes. The table below shows the results of die C versus die D: C wins twenty-four times, D wins just twelve. Similar results can be found with D versus A, A versus B, and B versus C. No matter what die your opponent selects, you can pick the die to its immediate left (or D if your opponent chooses A) and win two out of every three times.



982 The result is two bands—one with a right-hand twist, one with a left-hand twist.

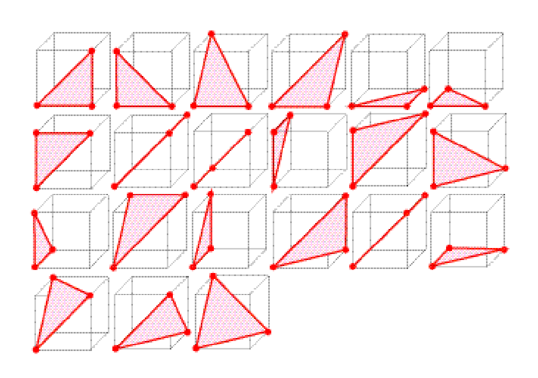
983 You will get an ordinary square "ring"— two sides, two edges and no twists.



984 The structure is made up of two separate pieces and could be pulled apart.



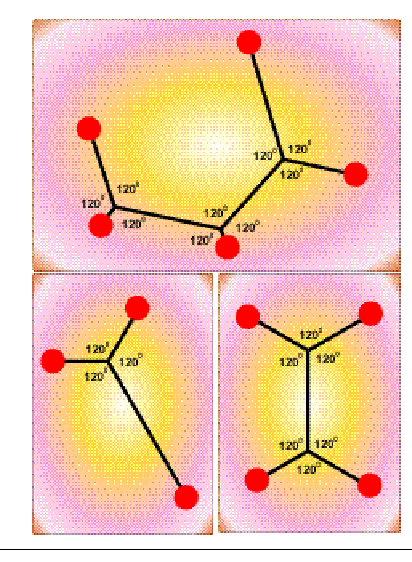
985



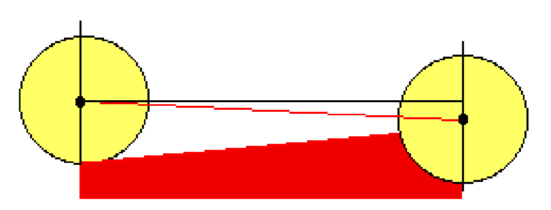
987 The diagram shows all the possible orientations, starting with point 1. As you can see, a right triangle is formed eighteen out of twenty-one times, which means the probability is 6/7.

988 The minimal path is a tree—a graph with no closed loops—on which lines are joined together at angles of 120 degrees to one another. For large numbers of points, it is difficult to predict the minimal path. Interestingly, though, a three-dimensional model immersed in a soapy solution will give the solution for even the most complex configurations in an instant.

The five-town solution was provided by Nick Baxter.

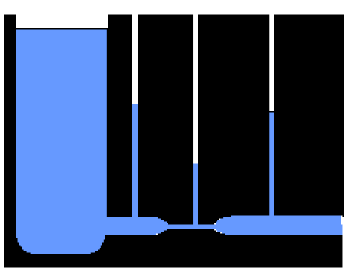


989 As the double cone seemingly rolls "upward," the increasing width of the tracks actually lowers the center of gravity of the cone. In spite of what we think we see, the double cone is actually rolling downhill.



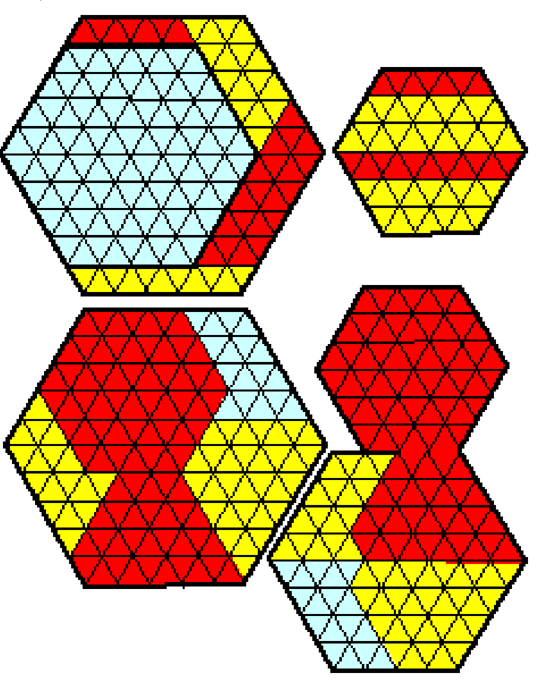
990 If the objects to be weighed are on one pan and the weights on the other, you must have weights of 1, 2, 4, 8, 16 and 32 grams. But if the weights can be on either pan, then a smaller set of masses may be employed: 1, 3, 9 and 27 grams. Claude-Gaspar Bachet first worked out this solution in 1623.

991 The water levels will be as shown here. Where the water flows the fastest, the water pressure will be the lowest and push the water up with the least force. As you can see, the water will flow the fastest in the narrowest part of the pipe.

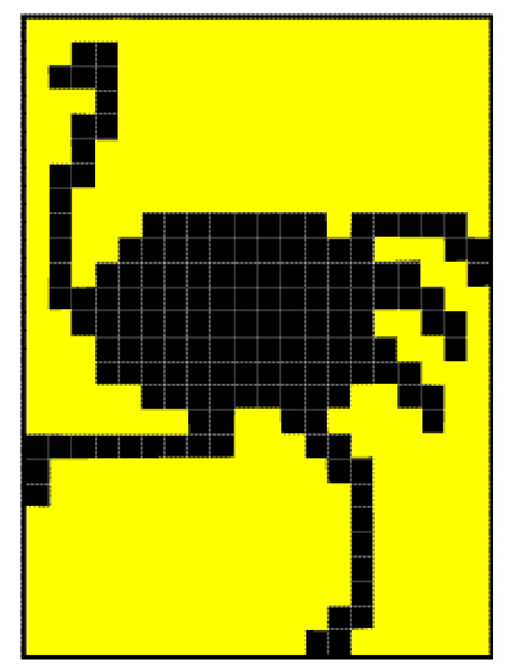


992 In fact, the Pythagorean theorem is valid not only for hexagons and squares but for any set of geometrically similar figures.

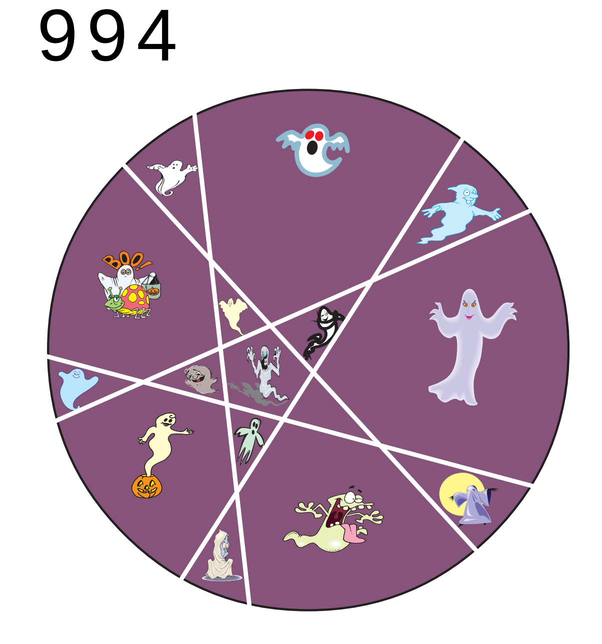
Schmerl found a five-piece solution to his problem (shown below, left and right) and American mathematician Greg Frederickson found a "sneaky" four-piece solution. Both are shown.



993



994



995

There are 21 possible pairs among seven birds. You can use such a list to systematically work out a foraging schedule:

Day 1: 1, 2, 3, involving the pairs 1-2, 1-3 and 2-3

Day 2: 1, 4, 5, involving the pairs 1-4, 1-5 and 4-5

Day 3: 1, 6, 7, involving the pairs 1-6, 1-7 and 6-7

Day 4: 2, 4, 6, involving the pairs 2-4, 2-6 and 4-6

Day 5: 2, 5, 7, involving the pairs 2-5, 2-7 and 5-7

Day 6: 3, 4, 7, involving the pairs 3-4, 3-7 and 4-7

Day 7: 3, 5, 6, involving the pairs 3-5, 3-6 and 5-6

996

In the linked tubes, the water level will be the same. Pressure is independent of the volume or shape of the tube and depends only on the height of the liquid. This is called the hydrostatic paradox.

997

The eleventh square will have sides of 32 units. For every two steps in the progression, the length of the sides doubles.

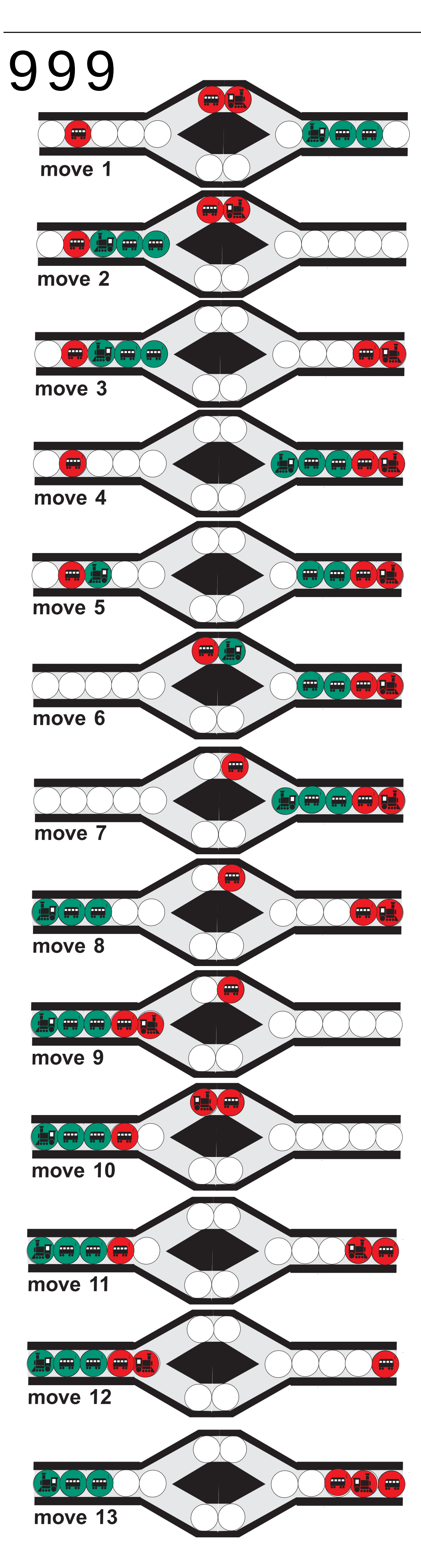
998

There is less than a 2 percent chance:

$$\frac{6}{6} \times \frac{5}{6} \times \frac{4}{6} \times \frac{3}{6} \times \frac{2}{6} \times \frac{1}{6} = 0.015,$$

or 1.5 percent

999



1000

There are exactly eight possibilities for the product of three ages to be 36:

Son 1	Son 2	Son 3	Product	Sum
1	1	36	36	38
1	2	18	36	21
1	3	12	36	16
1	4	9	36	14
1	6	6	36	13
2	2	9	36	13
2	3	6	36	11
3	3	4	36	10

Since Ivan could not solve the problem when he knew the sum of the three numbers—the date of the encounter—that meant the sum must have been 13, for which there are two possibilities. The added information about the youngest son means that one of the possibilities—a nine-year-old and two two-year-olds—can be ruled out, since there is no one youngest son in that case.

That left Ivan with one solution: 1, 6 and 6.

REFERENCES

Ball, W. W.; Rouse, and H.S.M. Coxeter. *Mathematical Recreations & Essays.* New York: Dover Publications, 1987.

Barbeau, Edward J.; Murray S. Klamkin; and William O. Moser. *Five Hundred Mathematical Challenges.* Washington, D.C.: The Mathematical Association of America, 1995.

Barr, Stephen. *Experiments in Topology.* New York: Dover Publications, 1989.

———. *Mathematical Brain Benders: Second Miscellany of Puzzles.* New York: Dover Publications, 1982.

Berlekamp, Elwyn, and Tom Rodgers. *The Mathemagician and Pied Puzzles: A Collection in Tribute to Martin Gardner.* Natick, Mass.: A. K. Peters, 1999.

Berlekamp, Elwyn R.; John H. Conway; and Richard K. Guy. *Winning Ways for Your Mathematical Plays.* Natick, Mass.: A. K. Peters, 2001.

Bodycombe, David J. *The Mammoth Book of Brainstorming Puzzles.* London: Constable Robinson, 1996.

———. *The Mammoth Puzzle Carnival.* New York: Carroll and Graf, 1997.

Brecher, Erwin. *Surprising Science Puzzles.* New York: Sterling Publishing, 1996.

Burger, Edward B., and Michael Starbird. *The Heart of Mathematics: An Invitation to Effective Thinking.* New York: Springer-Verlag, 2000.

Case, Adam. *Who Tells the Truth?: A Collection of Logical Puzzles to Make You Think.* Suffolk, UK: Tarquin Publications, 1991.

Comap. *For All Practical Purposes: Introduction to Contemporary Mathematics.* New York: W. H. Freeman and Company, 1988.

Conway, John H., and Richard K. Guy. *The Book of Numbers.* New York: Copernicus Books, 1997.

Cundy, H. M., and A. P. Rollett. *Mathematical Models.* Suffolk, UK: Tarquin Publications, 1997.

Devlin, Keith. *Mathematics: The Science of Patterns: The Search for Order in Life, Mind, and the Universe.* Scientific American Paperback Library. New York: W. H. Freeman and Company, 1997.

Dewdney, A. K. *The Armchair Universe: An Exploration of Computer Worlds.* New York: W. H. Freeman and Company, 1988.

Dudeney, Henry Ernest. *Amusements in Mathematics.* New York: Dover Publications, 1958.

Epstein, Lewis Carroll. *Thinking Physics: Is Gedanken Physics; Practical Lessons in Critical Thinking.* San Francisco: Insight Press, 1985.

Fomin, Dmitri; Sergey Genkin; and Ilia Itenberg. *Mathematical Circles (Russia Experience).* Providence, R.I.: American Mathematical Society, 1996.

Frederickson, Greg N. *Dissections: Plane & Fancy.* Cambridge, UK: Cambridge University Press, 1997.

Gale, David. *Tracking the Automatic Ant and Other Mathematical Explorations.* New York: Copernicus Books, 1998.

Gamow, George. *One Two Three . . . Infinity: Facts and Speculations of Science.* New York: Dover Publications, 1988.

Gardiner, A. *Mathematical Puzzling.* New York: Dover Publications, 1999.

Gardiner, Tony. *More Mathematical Challenges: Problems from the UK Junior Math Olympiad 1989–95.* Cambridge, UK: Cambridge University Press, 1997.

Gardner, Martin. *Aha! Gotcha: Paradoxes to Puzzle and Delight.* New York: W. H. Freeman and Company, 1982.

———. *Aha! Insight.* New York: W. H. Freeman and Company, 1978.

———. *Entertaining Mathematical Puzzles.* New York: Dover Publications, 1986.

———. *Fractal Music, Hypercards and More: Mathematical Recreations from* Scientific American Magazine. New York: W. H. Freeman and Company, 1991.

———. *Knotted Doughnuts and Other Mathematical Entertainments.* New York: W. H. Freeman and Company, 1986.

———. *The Last Recreations: Hydras, Eggs, and Other Mathematical Mystifications.* New York: Copernicus Books, 1997.

———. *Mathematical Carnival.* New York: Penguin Books, 1965.

———. *Mathematical Circus: More Puzzles, Games, Paradoxes, and Other Mathematical Entertainments.* Washington, D.C.: Mathematical Association of America, 1992.

———. *Mathematical Magic Show.* Washington, D.C.: Mathematical Association of America, 1988.

———. *Mathematical Puzzles of Sam Loyd.* New York: Dover Publications, 1959.

———. *More Mathematical Puzzles and Diversions.* New York: Penguin Books, 1961.

———. *More Mathematical Puzzles of Sam Loyd.* New York: Dover Publications, 1959.

———. *The New Ambidextrous Universe: Symmetry and Asymmetry, from Mirror Reflections to Superstrings.* Rev. ed. New York: W. H. Freeman and Company, 1991.

———. *Penrose Tiles to Trapdoor Ciphers: And the Return of Dr. Matrix.* Washington, D.C.: Mathematical Association of America, 1997.

———. *Perplexing Puzzles and Tantalizing Teasers.* New York: Dover Publications, 1988.

———. *Riddles of the Sphinx: And Other Mathematical Puzzle Tales.* Washington, D.C.: Mathematical Association of America, 1988.

———. *Second Scientific American Book of Mathematical Puzzles and Diversions.* Chicago: University of Chicago Press, 1987.

———. *Time Travel and Other Mathematical Bewilderments.* New York: W. H. Freeman and Company, 1987.

———. *The Unexpected Hanging: And Other Mathematical Diversions.* Chicago: University of Chicago Press, 1991.

———. *Wheels, Life and Other Mathematical Amusements.* New York: W. H. Freeman and Company, 1983.

Gay, David. *Geometry by Discovery.* New York: John Wiley & Sons, 1998.

Golomb, Solomon W. *Polyominoes: Puzzles, Patterns, Problems, and Packings.* Princeton, N.J.: Princeton University Press, 1996.

Gruenbaum, Branko, and G. C. Shephard. *Tilings and Patterns.* New York: W. H. Freeman and Company, 1986.

Gullberg, Jan. *Mathematics: From the Birth of Numbers*. New York: W. W. Norton & Company, 1997.

Higgins, Peter M. *Mathematics for the Curious.* London: Oxford University Press, 1998.

Hoffman, Paul. *Archimedes' Revenge*. New York: Ballantine Books, 1997.

———. *The Man Who Loved Only Numbers: The Story of Paul Erdös and the Search for Mathematical Truth*. New York: Little, Brown and Company, 1999.

Ishida, Non, and James Dalgety. *The Sunday Telegraph Book of Nonograms*. London: Pan Books, 1993.

Konhauser, Joseph D. E.; Dan Velleman; and Stan Wagon. *Which Way Did the Bicycle Go?: And Other Intriguing Mathematical Mysteries*. Washington, D.C.: Mathematical Association of America, 1996.

Kordemsky, Boris A. *The Moscow Puzzles: 359 Mathematical Recreations*. New York: Dover Publications, 1992.

Krause, Eugene F. *Taxicab Geometry*. New York: Dover Publications, 1986.

Lines, Malcolm E. *Think of a Number*. Bristol, UK: Institute of Physics Publishing, 1990.

Madachy, Joseph S. *Madachy's Mathematical Recreations*. New York: Dover Publications, 1979.

Nelsen, Roger B. *Proofs Without Words: Exercises in Visual Thinking*. Classroom Resource Materials, No. 1. Washington, D.C.: The Mathematical Association of America, 1993.

———. *Proofs Without Words II: More Exercises in Visual Thinking*. Washington, D.C.: The Mathematical Association of America, 2000.

Pappas, Theoni. *More Joy of Mathematics: Exploring Mathematics All Around You*. San Carlos, Calif.: Wide World Publishing/Tetra, 1991.

Pentagram. *The Puzzlegram Diary*. London: Ebury Press Stationery, 1994.

Peterson, Ivars. *Islands of Truth: A Mathematical Mystery Cruise*. New York: W. H. Freeman and Company, 1991.

———. *The Mathematical Tourist: New and Updated Snapshots of Modern Mathematics*. New York: W.H. Freeman and Company, 1998.

Pickover, Clifford A. *The Loom of God: Mathematical Tapestries at the Edge of Time*. New York: Perseus Books, 1997.

Salem, Lionel; Frederic Testard; Coralie Salem; and James D. Wuest. *The Most Beautiful Mathematical Formulas.* New York: John Wiley & Sons, 1997.

Schechter, Bruce. *My Brain Is Open: The Mathematical Journeys of Paul Erdös*. Oxford, UK: Oxford University Press, 1998.

Schuh, Fred. *The Master Book of Mathematical Recreations*. New York: Dover Publications, 1969.

Smith, David E. *A History of Mathematics, Volume 1*. New York: Dover Publications, 1978 (reprint).

———. *A History of Mathematics, Volume 2*. New York: Dover Publications, 1972 (reprint).

Smullyan, Raymond. *To Mock a Mockingbird*. Oxford, UK: Oxford University Press, 2000.

Stein, Sherman K. *Strength in Numbers: Discovering the Joy and Power of Mathematics in Everyday Life*. New York: John Wiley & Sons, 1996.

Steinhaus, Hugo. *Mathematical Snapshots*. New York: Dover Publications, 1999.

Stewart, Ian. *Another Fine Math You've Got Me Into . . .* New York: W. H. Freeman and Company, 1992.

———. *From Here to Infinity*. London: Oxford University Press, 1996.

———. *Game, Set and Math*. New York: Penguin Books, 1991.

———. *The Magical Maze: Seeing the World through Mathematical Eyes.* New York: John Wiley & Sons, 1999.

Trigg, Charles W. *Mathematical Quickies: 270 Stimulating Problems with Solutions*. New York: Dover Publications, 1985.

Tuller, Dave, and Michael Rios. *Mensa Math & Logic Puzzles*. New York: Sterling Publications, 2000.

van Delft, Pieter, and Jack Botermans. *Creative Puzzles of the World*. Emeryville, Calif.: Key Curriculum Press, 1995.

Walker, Jearl. *The Flying Circus of Physics*. New York: John Wiley & Sons, 1975.

Wells, David. *Can You Solve These? Series No. 2*. Jersey City, N.J.: Parkwest Publications, 1985.

———. *Can You Solve These? Series No. 3*. Jersey City, N.J.: Parkwest Publications, 1986.

———. *The Guinness Book of Brain Teasers*. London: Guinness Publishing, 1993.

———. *Hidden Connections, Double Meanings*. Cambridge, UK: Cambridge University Press, 1988.

———. *The Penguin Book of Curious and Interesting Geometry.* New York: Penguin Books, 1992.

———. *The Penguin Book of Curious and Interesting Math*. New York: Penguin Books, 1997.

———. *The Penguin Book of Curious and Interesting Puzzles*. New York: Penguin Books, 1993.

———. *You Are a Mathematician*. New York: Penguin Books, 1995.

Wells, David, and Robert Eastaway. *The Guinness Book of Mind Benders*. London: Guinness Publishing, 1995.

DIFFICULTY INDEX

Each puzzle in the book has been assigned a level of difficulty from 1 to 10. Level One puzzles are appropriate for the beginner, Level Ten for the puzzler looking for a challenge.

When you've done a puzzle, mark your accomplishment by putting a check next to the puzzle's name. Boxes have been provided for this purpose, for you and two other *PlayThinks* users.

LEVEL ONE

LEVEL TWO

LEVEL THREE

LEVEL FOUR

LEVEL FIVE

LEVEL SIX

LEVEL SEVEN

LEVEL EIGHT

LEVEL NINE

LEVEL TEN

"AN OPUS... MIXING MATH WITH WONDER." —*Washington Post Book World*

Can you cross the IMPOSSIBLE DOMINO BRIDGE?
Break through the QUEENS' STANDOFF?
Wield the SICKLE OF ARCHIMEDES?
Or figure out how to avoid the booby prizes in GAME SHOW?

A compulsive, exuberant cornucopia of puzzles, *The Big Book of Brain Games* is like salted peanuts for the mind. Here are mental games, visual challenges, logic posers, riddles and illusions.

Comprised of both original puzzles and mind-boggling adaptations of classics, this book, written by a man *Wired* magazine called "a living inspiration for the rest of us," celebrates that unique place where pure play and problem solving coexist.

998 THE TOSS OF THE DIE

Start solving, and right away you'll feel smart, intuitive, curious, successful and at one with the beauty of mathematics.

681 COIN TOSSING

IVAN MOSCOVICH is an internationally known and acclaimed inventor, puzzler and artist. He is the author of many books besides this one, which was originally published as *1000 PlayThinks*. They include *The Think Tank*, the *MindGames* series of mathematical puzzle books for younger readers, and more.

816 EGG OF COLUMBUS

WORKMAN PUBLISHING · NEW YORK
ISBN-10: 0-7611-3466-2
ISBN-13: 978-0-7611-3466-4
$22.95 U.S. / $29.95 CAN. www.workman.com PRINTED IN CHINA



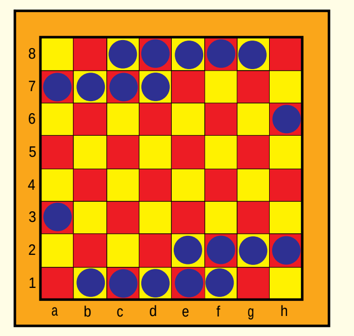
Find the Perfect Puzzle

WARM-UPS

249 TUBE ILLUSION
709 MÖBIUS STRIP
835 BOMBS AWAY
913 BIRD IN THE CAGE

CHALLENGING

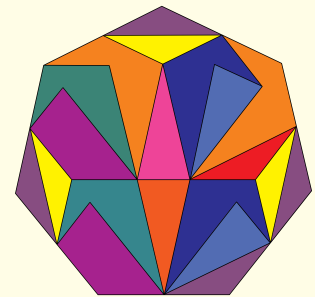
270 KISSING SPHERES
368 PERMUTINO
445 SEPARATING MONKEYS
758 JUMPING DISKS



80 KNIGHTS ATTACK
181 HAMILTONIAN CIRCUIT

PURE GENIUS

172 CRANKSHAFT
242 APOLLONIUS'S PROBLEM



42 HEPTAGON MAGIC
165 MATCH POINT
714 M-PIRE COLORING GAME

Bookland EAN
9 780761 134664
52295>